Analysis of Dynamic Stress Intensity Factor of Finite Piezoelectric Composite Plate Under a Dynamic Load

Piezoelectric materials are widely utilized in electronic device applications such as sensors. This paper investigates the dynamic response of a central crack in a finite piezoelectric layered composite plate under impact load. This novel project studies the crack effect on the finite geometries of a piezoelectric layered composite plate. A finite element method for transient dynamic crack analysis in two-dimensional, homogeneous, anisotropic and linear elastic composites was used for this investigation. Initially, the simulation method was verified and calibrated using a simple case study of a piezoelectric material (BaTiO$_3$) with a pre-crack. The results were in good agreement with previous studies in this field. Later, the verified model was used to simulate the piezoelectric layered composite plate. The results showed that the maximum dynamic stress intensity factor depends on the thickness and material properties of the piezoelectric layer and these are crucial parameters in the design and evaluation of a piezoelectric.

Keywords: Fracture mechanics, Finite element method, Piezoelectric composite plate, Dynamic stress intensity factor, Dynamic load.

1. INTRODUCTION

In the development of electromechanical systems and devices, smart structures with integrated functional ceramics are gaining growing interest. The electromechanical coupling behavior of piezo ceramics makes this class of materials useful in electronic device applications, particularly for transducers, sensors, etc. [1].

Piezoelectric ceramics, however, are very brittle and susceptible to fracture during service. Therefore, it is important to understand the fracture behavior of piezoelectric materials. In [2–4] anti-shear of cracked piezoelectric ceramics strip and in [5,6] cracked piezoelectric composites were investigated.

In the past years, extensive studies have presented valuable analytical solutions for the dynamic fracture problem in piezoelectric materials. For example, Shindo et al [7] dealt with the dynamic problem of an infinite orthotropic piezoelectric ceramic containing a Griffith crack under in-plane impact loading. Their results showed that for the piezoelectric materials, the dynamic stress intensity factor and the dynamic energy release rate exceed the corresponding static value by about 27-31% and 70-88%, respectively. Chen et al. [8] studied the transient response of a piezoelectric strip with a vertical crack under electromechanical impact load. Their numerical computation of the resulting coupled dynamic equations revealed that the electric field retards or promotes the propagation of the crack in a piezoelectric strip, at different stages of the impact loading process, depending upon the crack length and the electric load. Wang and Noda [9] investigated the electro-elastic interaction and fracture behavior of a piezoelectric layer bonded to a dissimilar elastic layer. A novel time-domain BEM for transient dynamic crack analysis of linear piezoelectric solids was implemented by Garcia et al. [10,11]. The obtained results were more stable and less sensitive to the selected time-steps than the classical one.

Some differences were observed between experiments and analytical solutions in piezoelectric composite plates and one reason for the differences was the existence of defects such as crack which affect the response of piezoelectric sensors.

While many piezoelectric devices are constructed in laminated form, only a few studies have been made for the transient response of the piezoelectric layered composite plates. Ueda used an analytical method to study the dynamic behaviour of an infinite piezoelectric layered composite plate with a central crack under normal impact loading [12]. However, the piezoelectric sensors have finite dimensions, where no work has been done for the finite geometries.

The aim of this study is to investigate the effect of crack on the response of the piezoelectric layered composite plates with finite dimensions. The results would be useful to have an optimum sensor design and to consider the crack effects on the piezoelectric behaviour. In this work, the dynamic behaviour of a piezoelectric layered composite plate with a central crack is examined under normal impact loading. The crack is oriented normally to the interfaces and ends at the interface between the orthotropic piezoelectric plate and the elastic layer. Finite element modelling is
utilized to find dynamic stress intensity factor near the crack tip. The simulation method is checked and calibrated with a simple case study [13]. Later the numerical results for the dynamic stress intensity factor are obtained for several piezoelectric layered composite plates and are reported in the following sections.

2. MATERIALS AND METHODS

There are few studies in the simulation of the transient response of the piezoelectric layered composite plates. Therefore, at first, the simulation method is checked and calibrated using simple case studies, i.e. an anisotropic piezoelectric material (BaTiO$_3$) with a pre-crack. This case study is available in the previous study and it is possible to compare our results with the previously obtained results [13].

2.1 Characteristics of the sample plate

Characteristics of the case study that is used for validation of the simulation method are illustrated in Fig. 1. This case study is a piezoelectric (BaTiO$_3$) plate with a central through thickness crack of length $2a=4.8$ mm in a rectangular plate with the dimensions $2b_1·2b_2=(20·40)$mm$^2$. Material properties and geometry of this case study are observable from table 2. As depicted in Fig. 2, mechanical load $\sigma_0(t)$ is applied at the opposite ends of the plate, following a ramp function with the time constant $\tau=1.e-6$ s and $\sigma_0=5.0$ MPa. Due to symmetries with respect to the plate in Fig. 2, is discretized by a two-dimensional finite element mesh, applying symmetry boundary conditions $u_2=0$ and on the ligament and $u_1=0$ along the vertical symmetry line. A generalized state of plain strain ($u_{i,3}=0$) is assumed in the model.

Two-dimensional FE models are used for simulation of this simulation. A sample view of the model with and without mesh elements is illustrated in fig. 2.

The results obtained from simulation of the piezoelectric (BaTiO$_3$) plate and a previous study by M. Enderlein [13], are indicated in fig. 3. Good agreement can be observed between our results and Enderlein’s work.

![Figure 3. Comparison of the piezoelectric (BaTiO$_3$) plate and Enderlein’s work](image)

Percentage of errors of our results compared to Enderlein’s work is listed in table 1. Where $K_{max}$ is the maximum dynamic stress intensity factor and $t_{max}$ is the required time to reach this stress intensity factor. The errors are small and acceptable.

Table 1. Percentage of errors of our results and Enderlein’s work for the steel plate

<table>
<thead>
<tr>
<th></th>
<th>Obtained results</th>
<th>Enderlein’s work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{max}$ (MPa)</td>
<td>9.5</td>
<td>8.85</td>
</tr>
<tr>
<td>$t_{max}$ (µs)</td>
<td>4.027</td>
<td>3.444</td>
</tr>
</tbody>
</table>

After verification of the model using the simple case study, it is possible to apply it for the investigation of a finite piezoelectric layered composite plate. In the following, effect of the thickness and material properties of the piezoelectric layer on dynamic stress intensity factor and stress distribution are investigated.

2.2 Characteristics of the piezoelectric layered composite plate

The layered composite plate is made of a cracked piezoelectric layer of width $2h_1$ bonded between two elastic layers of width $h_2 - h_1$ as shown in Fig 4. The ends of the crack length $2c$ ($c\leq h_1$) are situated at equal distances away from the interfaces which are directed normal to the crack plane. The system of rectangular Cartesian coordinates (x,y,z) is introduced in the composite in such a way that the crack is located centrally along the x-axis, and the z-axis is parallel to the interfaces. The piezoelectric layer is poled in the z-direction exhibiting transversely isotropic behaviour. A plane strain condition perpendicular to the y-axis is assumed [12].

Also, use of superposition reduces the problem to one of perturbation in which the crack surface tractions are the only nonzero external loads [14]. Because of the assumed symmetry in geometry and loading, it is sufficient to consider the problem for $0\leq x\leq h_2$ (h/2) and $0\leq z \leq h_1$ only, therefore, we simulate the problem in $0\leq x \leq h_2$. The piezoelectric composite plate is subjected to a tensile impact loading of the form $\bar{\sigma}_0(t)$, where $\bar{\sigma}_0(t)$ is 400 MPa and $\tau$ is 2 µs. The size of the cracked plate is defined by $h=20$ mm [15].
Table 2. Properties of materials [2]

<table>
<thead>
<tr>
<th></th>
<th>Elastic stiffnesses (*10^10 N/m²)</th>
<th>Piezoelectric coefficients (C/m²)</th>
<th>Dielectric constants (*10^10 C/Vm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>c₁₁</td>
<td>c₁₂</td>
<td>c₁₃</td>
</tr>
<tr>
<td>PZT-4</td>
<td>13.9</td>
<td>11.3</td>
<td>2.56</td>
</tr>
<tr>
<td>PZT-5H</td>
<td>12.6</td>
<td>11.7</td>
<td>2.3</td>
</tr>
<tr>
<td>Al</td>
<td>8.84</td>
<td>8.84</td>
<td>2.7</td>
</tr>
<tr>
<td>BaTiO₃</td>
<td>15</td>
<td>14.6</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Figure 4. Geometry of the crack in the piezoelectric layered composite plate

The elastic, piezoelectric and dielectric properties are listed in Table 2. The shear modulus and Poisson's ratio of aluminium are μ = c44 and ν = 0.28, respectively.

3. NUMERICAL RESULTS AND DISCUSSION

3.1 The thickness effect of the piezoelectric

The thickness effect of piezoelectric layer on stress intensity factor are considered for three cases: 1) Al/PZT-4/Al, 2) Al/PZT-5H/Al and 3) Al/BaTiO₃/Al laminates with h₁/c = 2. The dynamic stress intensity factor of the piezoelectric composite plates versus time obtained from the simulation is shown in Figure 6.

In figures 5 and 6, stress intensity factor versus time, for infinite piezoelectric composite plates, are illustrated and are compared with the results of this paper. From the figures, it can be seen that at the start the infinite piezoelectric composite plate and finite composite plate, i.e. the results of this paper, has similar increasing trend. For both cases, after reaching a maximum value, their trends become descending, but, for the infinite plate, the stress intensity factor never takes zero value and after small decrease it becomes constant.

Fig. 5. Dynamic stress intensity factor of piezoelectric composite plates versus time for h₁/c=2

Fig. 6: Normalized stress intensity factor versus time for infinite piezoelectric composite plates with different thicknesses [12]

In the third section, the effect of h₂/h₁ on the stress intensity factor of the broken laminate is considered. It can be seen that the values of K₁ max and t max increase with decreasing h₂/h₁.

In figures 5 and 6, stress intensity factor versus time, for infinite piezoelectric composite plates, are illustrated and are compared with the results of this paper. From the figures it can be seen that at the start for the infinite piezoelectric composite plate and finite composite plate, there is a similar increasing trend and for both cases, after reaching a maximum value, their trends decrease. But, in the infinite plate, the stress intensity factor never takes zero value and, after a small decrease, it becomes constant.
3.2 The material effect of the piezoelectric

The effect of piezoelectric properties on the dynamic stress intensity factor is investigated for three cases with $h_2/h_1=2$ and $h_1/c=2$ and the results are shown in Fig. 7.

Fig. 7. The effect of the piezoelectric material properties on the dynamic stress intensity factor for $h_1/c=2$ & $h_2/h_1=2$

It can be seen from the figure that the values of $K_{I_p}^{\max}$ and $t_{\text{max}}$ are changing using different material properties in the piezoelectric layer. Note that the magnitudes of the dynamic stress intensity factor of the case studies decrease suddenly by increasing $h_2/h_1$.

The sudden release of the uniaxial stress at the ends results in arrival of stress waves to the crack surface after a finite period of time. This period depends on the geometry of the problem and the material properties.

4. CONCLUSIONS

In this paper, the dynamic response of a piezoelectric layered composite plate having a central crack was investigated. The finite element method was used to simulate the dynamic response of infinite piezoelectric composite plates. The dynamic stress intensity factor versus time were evaluated and were compared with the finite piezoelectric composite results. The results show that dynamic stress intensity factor depends on the thickness and material properties of the piezoelectric layer. The results revealed that at the start of the loading, the infinite piezoelectric composite plate and the finite composite plate has similar increasing trend and, for both cases, after reaching a maximum value, their trends become decreasing. But for the infinite plate, the stress intensity factor never takes zero value and after a small decrease it becomes constant. Broadly speaking, the results indicated that the maximum dynamic stress intensity factor depends on the thickness and material properties of the piezoelectric layer under dynamic load. Therefore, these factors should be considered carefully in design and evaluation of the piezoelectric.

REFERENCES


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АНАЛИЗА ФАКТОРА ИНТЕЗИТЕТА ДИНАМИЧКОГ НАПОНА КОНАЧНЕ ПИЕЗО-ЕЛЕКТРИЧНЕ КОМПОЗИТНЕ ПЛОЧЕ ПОД УТИЦАЈЕМ ДИНАМИЧКОГ ОПТЕРЕЋЕЊА

С. Фотухи, С. Мохamed Реза Халили

Пиезоелектричне материјале се широко користе у применама електронских уређаја као што су сензори. Овај рад истражује динамички одговор централне пукотине у коначном пиезоелектричном слоју композитне плоче под утицајем оптерећења. Овај рад проучава утицај пукотине на коначну геометрију пиезоелектричних слојева композитне плоче. Метода коначних елемената за транзитну динамичку аналиzu пукотина у двумерном, хомогеном, анизотропном и линеарно еластичном композиту је коришћена за ово истраживање. У почетку, метод симулација је проверен и калибрисан коришћењем једноставаног случаја пиезоелектричног материјала (BaTiO3) са иницијалном прслином. Резултати су били у доброј сагласности са претходним студијама у овој области. Касније, потврђени модел је коришћен за симулацију пиезоелектричне слојеве композитне плоче. Резултати су показали да максимални динамички фактор интензитета напона зависи од дебелине и материјалних својстава пиезоелектричног слоја и то су кључни параметри у дизајну и евалуације пиезоелектричних материјала.