Theoretical Research and Analysis of Work Process of a Road Roller With Local Deformation of Roll in Compaction Zone

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The paper presents a new simple mathematical model of a fundamental and applied elasticity theory problem, considering the stress-strain state of road-roller working members with elliptic surfaces and the road-building material interacting with them. The derived mathematical relations and the developed calculation theory prove the possibility of adjusting (optimizing) the parameters of contact interaction of flexible working members in road-rollers with elliptic surfaces, with the compacted layer by means of variating the design-constructive dimensions of semi-axles of the ellipse-shaped surface of the roll under service conditions, thus adjusting force action on the compacted road-building material.
The paper deals with an operation peculiarity of flexible working members with elliptic surfaces, offers a selection of materials and determines geometric parameters of flexible working members with elliptic surfaces.
Based on the analysis of stress-strain state of the roll, two cases are considered suitable for constructive realization of the examined mathematical model.

Keywords: *roll, road roller, distortion (deformation), compaction.*

1. INTRODUCTION

The development of a network of hard motor roads is a topical issue. Rates of motor transport and freight traffic development require intensification of motor roads construction process and increase service life. The increasing scope of motor roads construction and modernization involves, in addition to qualitatively new construction methods, high production machines and complexes. Implementation of these measures based on effective road-building machinery and equipment ensures a significant economy of material, energy and labor resources in construction and public road system. This calls for application of new effective road machinery, including smooth rollers. One important element of the motor road construction process is compaction of the road concrete mix. Compaction is also an important constituent of construction of buildings and structures. Reliability, quality and life time of a completed project depend on how the compacting works are planned and implemented.

For this reason, major attention is paid to the arrangement of a strong and stable road bed which is the foundation of a structure. Compaction process is most important in road building, as the attained compacting factor holistically determines strength and tolerance of the whole structure to the influence of natural climatic and operational factors. Today, unfavorable influence of natural climatic factors gets more and more intensive. Due to an increase in weight and driving speed of transport vehicles, the dynamic loads on road structure constantly increase. Accordingly, requirements to compaction toughen, which in its turn stimulates further theoretical research in this field and, based on their results, perfection of design of road-rollers as one of the most common compacting means.

By their nature, soils are rather varied, and therefore their physical and mechanical properties are different. During construction, soils are compacted in different conditions – on large areas, in fills, slopes, trenches, ditches etc. All this necessitates different requirements to soil-compacting machines. These requirements are often contradictory, and this is why compaction of soils in road building cannot be performed by any one or even two types of existing machines. High density of material is achieved by correct selection of compaction methods, parameters of machines and compaction modes used.

Pressures of compacted medium on contact surface of a machine should never exceed the ultimate strength of the medium. They must gradually increase from passage to passage or from stroke to stroke. Under high pressures, a plastic flow (pressing-out) of the material from under the working members appears on the surfaces of working members contacting with the material. During rolling operations, this causes wave formation. Under insufficient pressures, required density is not achieved, which negatively effects the strength, bearing capacity and life of the road surface. This predetermines the selection of necessary “dosing” of compacting impact, and therefore the method of road surface compaction by rollers usually implies using

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several nominal sizes of rollers. Using conventional rolls does not allow to design a universal road roller ensuring optimal parameters of road-building materials compaction, even during several passages along a trace. Designing a roller and a roll with a changeable curve would make it possible to adjust its pressure on the compacted material, therefore it is a topical issue. Such a roller would be able to replace several rollers with conventional rolls, which is undoubtedly important with the current equipment costs.

The task to improve road construction quality can be solved under the condition that road rollers used in the construction process will ensure high quality at maximum efficiency. Improving the efficiency of, and intensifying the compaction process are based on the development of theory and calculation methods applied to compacting machines and to rolls of road rollers as conventional multipurpose machines in particular.

This determined the character and structure of theoretical research. Based on the results of the work conducted, the paper describes substantiation of the road roller structure with a locally distorted roll and the method of calculating its parameters.

2. THEORETICAL RESEARCH OF THE INTER–ACTION PROCESS OF A LOCALLY-DISTORTED ROLL WITH COMPACTED MATERIAL IN THE COMPACTION ZONE

Issues of stress-strain state of working members in road rollers and of compacted medium belong to a constantly researched field of road-building industry. With regard to resolving these issues, the paper introduces a new simple mathematical model of a fundamental and applied problem of elasticity theory, previously not described in literature.

The scientific and technical literature [1, 2] considers a general case of flat contact distortion, when the contact of compressed bodies happens along a straight line perpendicular to plane XOУ (Fig. 1*a*), and functions $f_1(x)$ and $f_2(x)$ that determine configuration of cylindrical surfaces have continuous first and second derivatives in the area of point x = y = 0. Directing the axis *Ox* at a common tangent to curves $f_1(x)$ and $f_2(x)$ that limit the elastic bodies, we obtain:

$$f_1'(0) = f_2'(0) = 0 \tag{1}$$

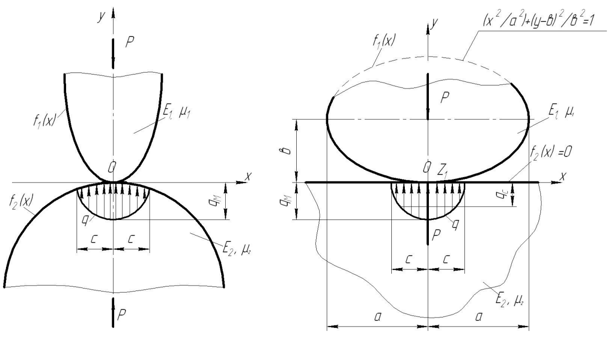

Figure 1. General case of flat contact distortion of a road roller roll

The sum of the second derivatives $f_1''(0) + f_2''(0)$ is considered here as different from zero and, assuming that elastic displacements are small, we approximate as follows [1, 3]:

$$f_1'(x) + f_2'(x) = \left[f_1''(0) + f_2''(0) \right] \cdot \frac{x^2}{2} \tag{2}$$

Regarding distributed contact forces q = q(x), we introduce a supposition that their resulting force, perpendicular to axis Ox, is directed to point O of the beginning of the interacting surfaces contact, that is, to the origin of coordinates. As initial clearance between the contacting bodies, according to (2), is symmetric with respect to axis *Oy*, pressure q on cylindrical surfaces is also an axially symmetric elliptic function by argument x (Hertz-Staierman function), which, in compliance with [1], looks as follows:

$$q = q(x) = \frac{2 \cdot P}{\pi \cdot c^2} \cdot \sqrt{c^2 - x^2} = \frac{q_м}{c} \cdot \sqrt{c^2 - x^2} = \frac{4 \cdot q_c}{\pi \cdot c} \cdot \sqrt{c^2 - x^2} \tag{3}$$

$$-c \le x \le c$$

where q_m, q_c are, respectively, the maximum and average values of functional dependence q (x) (Fig. 1*a*), determined by the following formula:

$$q_м = \frac{2 \cdot P}{\pi \cdot c} = \frac{4 \cdot q_c}{\pi} \tag{4}$$

where c is half-width of the contact area [1].

$$c = \sqrt{\frac{2 \cdot P \cdot (y_1 + y_2)}{f_1''(0) + f_2''(0)}} \tag{5}$$

Where y_1, y_2 are physical-mechanical constants of interacting materials depending on their elasticity modules E_1 and E_2, and Poisson ratios μ1, μ2:

$$y_1 = \frac{P}{\pi \cdot E_1} \cdot (1 - \mu_1^2)\,,\; y_2 = \frac{2}{\pi \cdot E_2} \cdot (1 - \mu_2^2) \tag{6}$$

Force P is related to reactive pressure q by the following integral relation:

$$P = \int_{-c}^{c} q(x)dx = 2 \cdot \int_{0}^{c} q(x)dx \tag{7}$$

In the context of the current applied mechanical-mathematical problem, we modify formulae of I. F. Staierman (3) – (6) [1], where an immobile steel roll modelled by an absolutely stiff and smooth cylindrical punch with an elliptic profile $f_1(x)$ ($y_1 = 0$, $E_1 >> E_2$ or $E_1 = \infty$) applies static pressure on an elastic deformable semi-plane $f_2(x) = 0 \Rightarrow f_2''(0) = 0$ that is a layer of soil or road carpet, compacted to cessation of residual displacements, with the average Poisson ratio μ2 = 0.25 (0.2…0.3) and modulus of deformation $E_2 = E_к$ [4] (Fig. 1*b*). It ought to be noted that the parameter μ2 comparatively insignificantly influences the stress-strain state of road pavement [4].

In order to re-express and adapt the fundamental correspondences (3) – (6), we cite the necessary analytical relations (Fig. 2) [3, 5]:

- functions of the lower part of cylindrical surface of the roll $f_1(x)$ and its second derivative $f_1''(x)$ from equation of an ellipse (see Fig. 1*b*):

$$f_1(x) = y(x) = -b\cdot(\sqrt{1-\frac{x^2}{a^2}}-1), 0 \le y \le b \quad (8)$$

$$f_1''(x) = \frac{d^2y}{dx^2} = \frac{b}{a^2}\cdot(1-\frac{x^2}{a^2})^{\frac{-3}{2}}, -a \le x \le a \quad (9)$$

- value of $f_1''(0)$ at x = 0:

$$f_1''(0) = \frac{b}{a^2} \quad (10)$$

- curvature radii R = R(x) and R(0) of the elliptical directrix of the cylinder (8), considering expressions (9) – (10):

$$R = R(x) = \frac{\left[1+(y')^2\right]^{\frac{3}{2}}}{y''} = \left[1-\frac{x^2}{a^2}\cdot(1-\frac{b^2}{a^2})\right]^{\frac{3}{2}}\cdot\frac{a^2}{b} \quad (11)$$

$$R(0) = \frac{1}{f_1''(0)} = \frac{a^2}{b} \quad (12)$$

- formula connecting specific linear force P (N/m) with the width B of the roll and with the vertical load G_B, applied to its centre:

$$P = \frac{G_в}{B} \quad (13)$$

- depth h of the roller dipping into the compacted material layer (height of segment КОК', Fig. 2), which is found from ellipse equation (Fig. 1*b*), where x = ± c and y = h:

$$\frac{c^2}{a^2} + \frac{(h-b)^2}{b^2} = 1 \quad (14)$$

or, at h - b < 0:

$$h = b\cdot\left(1-\sqrt{1-\frac{c^2}{a^2}}\right) \quad (15)$$

where c is the value of semichord of the curve КОК' (Fig. 2).

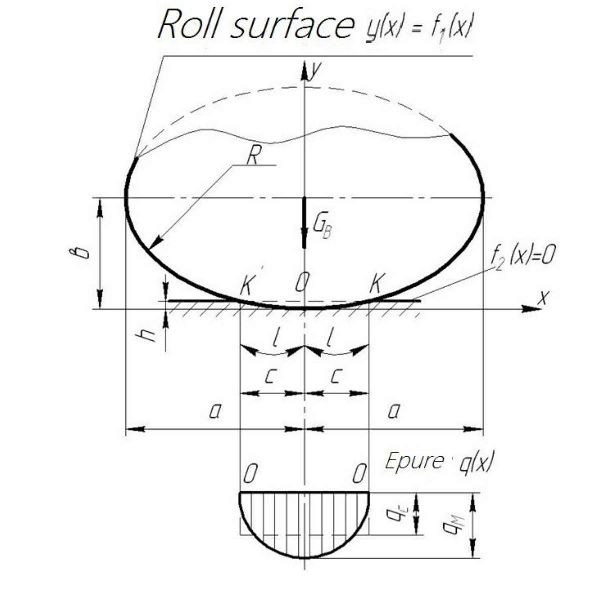

Figure 2. Diagram of locally-distorted roll contact with compacted material in the compaction zone

Using formulae (8) – (15), we render expressions (3) – (6) and (15) concrete, after the above substitutions $E_2 = E_к$, $f_2(x) = 0$, $f_2''(0) = 0$, $y_1 = 0$, $\mu_2 = 0.25$:

$$q = q(x) = \frac{2\cdot G_в}{\pi\cdot B\cdot c^2}\cdot\sqrt{c^2-x^2} = \frac{g_M}{c}\cdot\sqrt{c^2-x^2} = \frac{4g_O}{\pi\cdot c}\cdot\sqrt{c^2-x^2} \quad (16)$$

$$q_M = \frac{2\cdot G_в}{\pi\cdot B\cdot c}; \qquad q_C = \frac{G_в}{2\cdot B\cdot c} \quad (17)$$

$$q_M = \frac{4}{\pi}\cdot q_C \quad (18)$$

$$Y_2 = \frac{2}{\pi\cdot E_K}\cdot(1-\mu_2^2) = \frac{1.875}{\pi\cdot E_K} \quad (19)$$

$$c = \sqrt{\frac{2\cdot G_B\cdot Y_2}{B\cdot f_1''(0)}} = a\cdot\sqrt{\frac{3.75\cdot G_B}{\pi\cdot E_K\cdot b\cdot B}} \quad (20)$$

$$h = b\cdot\left(1-\sqrt{1-\frac{3.75\cdot G_B}{\pi\cdot E_K\cdot b\cdot B}}\right) \quad (21)$$

From the physical-technical point of view, correctness of formulae (16) – (21) based on classic relations (3) – (6) [1] results primarily from axial symmetry calculation models (Fig. 1) and initial contact of interacting bodies along axis $z \perp xOy$, passing through point x = y = 0 (Fig. 1*b*). Thereat, the model of flat deformed state underlying the correspondences (16) – (21) is adequate in the context of the given structural (non-classic) problem [7] for contact areas of the roll cylindrical surface with the compacted material, when adhering to specification (see (12) and [1])

$$c \ll R(0) = \frac{a^2}{b} \quad (22)$$

when congruence (2) is fulfilled, confirming the possibility of crude approximation of function (8) in compliance with (10) and [3], i. e.: $-c$

$$f_1(x) = b\cdot\left(1-\sqrt{1-\frac{x^2}{a^2}}\right) \approx b\cdot\left[1-\left(1-\sqrt{1-\frac{x^2}{2\cdot a^2}}\right)\right] = \frac{b}{a^2}\cdot\frac{x^2}{2} \quad (23)$$

$$f_1(x) + f_2(x) = \left[f_1''(0) + f_2''(0)\right]\cdot\frac{x^2}{2} = \frac{b}{a^2}\cdot\frac{x^2}{2} \quad (24)$$

where (Figures 1*b* and 2).

Practical implementation of analytical corres–pondences (16) – (21) calls for introduction of an additional prerequisite on invariable (of longitudinal non-deformability) length S of elliptical cylinder generatrix in the process of its transformation into a circle with radius R = const. In this connection and with the purpose of comparability of successive calculated values at different axes *a*, *b* of the ellipse, we introduce a method of calculating S and linear dimension 2*l* of *КОК'* contact arc (Fig. 2).

For mathematical formulation of the given procedure, it is more convenient to write down the canonical equation of the same ellipse (Fig. 1*b* and 2):

$$\frac{x_э^2}{a^2} + \frac{y_э^2}{b^2} = 1 \quad (25)$$

In coordinate system $x_э O_э y_э$ and in parametric form [3, 5]:

$$x_Э = a\cdot\sin\varphi \;\; y_Э = b\cdot\cos\varphi \tag{26}$$

Geometrical meaning of the parameter ϕ is clear from Fig. 3, where ANA' is a semicircle of radius a and point N taken on the same vertical line with point M of the ellipse, on the same side of axis AA'. In the current problem, angle ϕ has two numeric values:
(1) for calculation of the fourth part of parameter S, where in (26) $x_Э = a$ and then

$$\phi = \phi_S = \frac{\pi}{2} = 90^\circ \tag{27}$$

(2) for determination of the elliptic half-arc length l at $x_Э = c$ (formula (20), (26) and Fig. 3)

$$\varphi = \varphi_1 = \arcsin\frac{c}{a} = \arcsin\sqrt{\frac{3.75\cdot G_B}{\pi\cdot E_K\cdot b\cdot B}} \tag{28}$$

Differential dS of arc S (Fig. 3) is as follows:

$$dS = a\cdot\sqrt{1-\xi^2\sin^2\phi}\;d\phi \qquad]\backslash, \tag{29}$$

where is the eccentricity of the ellipse [3, 5]

$$\xi = \frac{\sqrt{a^2-b^2}}{a} \tag{30}$$

with the bigger semi-axis $a \ge b \quad (0 \le \xi \le 1)$.

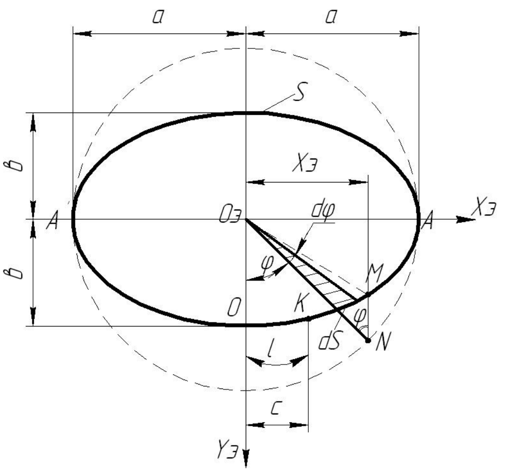

Figure 3. Determination of elliptic roll half-arc length

For the circle (a = b), being a subcase of an ellipse, ξ= 0.

Taking into consideration (29), we present unknown dimensions l and S by elliptic integrals $E(\phi_i,\xi)$, $E(\frac{\pi}{2},\xi) = E(\xi)$ of the second type in Legendre form [5, 6] (complete and incomplete, respectively), which, as is known [6], cannot be expressed via elementary functions and are not taken in their final shape, while reference tables [3, 8] are compiled for their calculation.

$$l = a\cdot\int_0^{\phi_i}\sqrt{1-\xi^2\cdot\sin^2\phi}\;d\phi = a\cdot E(\phi_i,\xi) \tag{31}$$

$$S = 4\cdot a\cdot\int_0^{\frac{\pi}{2}}\sqrt{1-\xi^2\cdot\sin^2\phi}\;d\phi = 4\cdot a\cdot E(\xi) \tag{32}$$

In generally accepted (standard) designations $E(\phi,\xi)$ and $E(\xi)$, eccentricity ξ is called an elliptic module, while ϕ is called an amplitude [9].

For different values of ξ (in general case $0 \le \xi \le 1$) function $E(\phi,\xi)$ changes from $E(\phi,\xi) = \phi$ at $\xi = 0$ (for a circle, Fig. 3) to $E(\phi,\xi)$, if ξ =1. When ϕ = 0, we have $E(0,\xi) = 0$, while at $\phi = \phi_S = \frac{\pi}{2}$ we obtain a complete elliptic integral $E(\xi)$.

The proposed and fully described theoretical model can be used for rolls with a circular outline and outer radius R, after replacing the corresponding letter symbols:

$$c \Rightarrow c_0,\; h \Rightarrow h_0,\; q_M \Rightarrow q_{MO},\; q_c \Rightarrow q_{co},$$
$$a = в = R,\; \xi = 0,\; \phi_i \Rightarrow \phi_{io},\; S = S_{O},\; l = l_O$$

As a result, expressions (15), (17), (18), (20) – (22), (28), and (32) are converted as follows:

$$q_{CO} = \frac{G_в}{2B\cdot c_O}; \quad q_{MO} = \frac{4}{\pi}\cdot q_{CO} \tag{33}$$

$$c_O = \sqrt{\frac{3.75\cdot R\cdot G_B}{\pi\cdot E_K\cdot B}},\;\; c_O << R \tag{34}$$

$$h_O = R\cdot\left(1-\sqrt{1-\frac{G_O^2}{R^2}}\right) = R\cdot\left(1-\sqrt{1-\frac{3.75\cdot G_B}{\pi\cdot E_K\cdot R\cdot B}}\right) \tag{35}$$

$$\phi_{iO} = \arcsin\sqrt{\frac{3.75\cdot G_B}{\pi\cdot E_K\cdot R\cdot B}} = \arcsin\frac{c_O}{R} \tag{36}$$

$$S_O = 2\pi R, \quad l_O = R\cdot\phi_O \tag{37}$$

The derived formulae and the developed calculation theory confirm the assumption by A. F. Zubkov [9], that the length of arc "c" is equal to the length of its contracting chord l, and prove that it is possible to adjust (rationalize) the parameters of contact interaction of a roller with compacted material by varying the design-structural dimensions of semi-axes of the ellipse-shaped surface of the flexible shell of a road roller roll in service conditions, thus increasing the quality of compacted road carpet.

In order to determine correspondences between the roller parameters and characteristics of the road surface compacted layer, let us consider the known model of compacting efforts distribution during the roller roll movement on the low-density base shown in Fig. 4 [7]. Contact pressure buildup continues until the resistance to friction and coupling between solid phase particles, and to pressing-out (with filtration) of air through the layer pores is overcome.

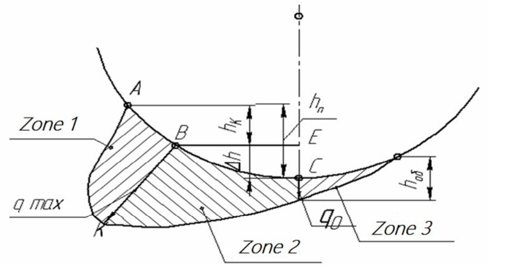

Figure 4. Distribution of compacting zones, pressures and residue during movement of a locally-deformed roll on a low-density base

Zone 1 predetermines the compaction process connected with pressing out the air, the latter being actively pressed out in zone 2. While in the area of zone 1, along with vertical deformations, shear deformation can happen as well, in zone 2, the volume of material under the roll is mostly subject to vertical deformations. In zone 2, a creeping process related to pressing out the air is observed. Zone 3 is related to the unloading process and pressure drop.

Alignment of the maximum pressure line BK with the vertical axis of the roll means cessation of the process of compacting with the roll in the given conditions. The character of contact pressures distribution reflects the degree to which the roll is used in the compaction process.

In order to maintain connection between the roller parameters and compacted material properties, it is not sufficient to maintain connection between pressure and density. It is also necessary to maintain it between energy (power) consumption or pressure pulse and density increment – decrease in the amount and volume of air.

Let us look at the operation scheme of a locally deformed roll during its interaction with a compacted road-building material (Fig. 5) [3]. While the metal roll passes, indentation resistance forces appear, which causes deformation of the compacted material. The resulting contact zone of the locally-deformed roll and compacted material has a certain value of contact area Fk and average value of contact stresses σк. At depth Z of compacted material, in any point, the stress state occurs, and its value depends on values Fk and σк and on the thickness of the compacted layer.

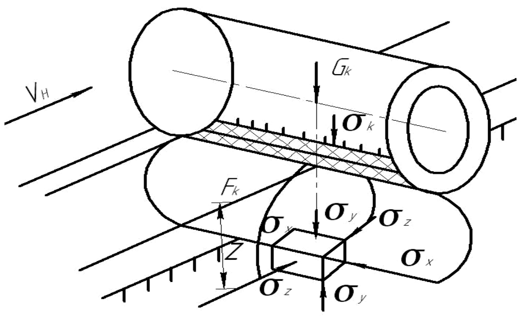

Figure 5. Stress state of an elementary volume of compacted material

As a result of the stress state, deformation of the compacted material occurs, accompanied, first of all, by displacement of the material particles into a new position, and then – by modification of internal relations.

Due to elastic-viscous-plastic relations in the compacted material, deformation does not proceed immediately, but requires a certain period of time. The period the material remains stressed is determined by the traveling speed of the locally deformed roll, ceteris paribus. Therefore, the compaction effect depends on the stress state that occurs in every elementary volume of material and on its duration. The stress state value and time of duration are determined, apparently, by the value of the average contact pressure and the contact area, character of stress distribution on the contact area and the traveling speed.

The main contact parameters of the locally deformed roll are as follows: Fk – contact area; σк – contact pressures.

Average pressure on the contact surface of the locally deformed roll is determined from the expression:

$$\sigma = Q / F_k \qquad \sigma = Q / F_k \tag{38}$$

where Q is load on the roll, H;
F_k is the contact area, m^2.

Therefore,

$$\sigma_к = f(F_к) \tag{39}$$

In order to determine F_k, let us look at Fig. 6, from where we find:

$$F_к = B \cdot R \cdot \alpha \tag{40}$$

where B is the width of the roll, m;
R is the radius of the roll, m;
α is the angle of contact of the roll with the compacted medium.

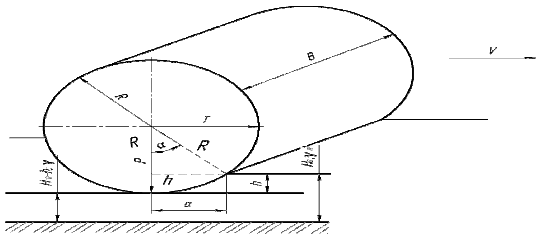

Figure 6. Main parameters of contact of a locally deformed roll with a compacted medium

Accepting ground deformation depth h, from Fig. 6 we find:

$$\cos\alpha = \frac{R-h}{h} \tag{41}$$

from where:

$$\alpha = \arccos\frac{R-h}{h}\,; \tag{42}$$

Having inserted expression (21) into equation (42), we obtain:

$$\alpha = \arccos\frac{R - b\cdot\left(1-\sqrt{1-\dfrac{3.75\cdot G_B}{\pi\cdot E_K\cdot b\cdot B}}\right)}{b\cdot\left(1-\sqrt{1-\dfrac{3.75\cdot G_B}{\pi\cdot E_K\cdot b\cdot B}}\right)} \tag{43}$$

Having inserted the received expression (43) into equation (40), we obtain:

$$F_k = B\cdot R\cdot\arccos\frac{R - b\cdot\left(1-\sqrt{1-\dfrac{3.75\cdot G_B}{\pi\cdot E_K\cdot b\cdot B}}\right)}{b\cdot\left(1-\sqrt{1-\dfrac{3.75\cdot G_B}{\pi\cdot E_K\cdot b\cdot B}}\right)} \tag{44}$$

Having inserted expression (44) into formula (38), we receive the law of contact pressures σ, MPa for the smooth metal roll with local deformation in compaction zone:

$$\sigma_k = G_B \cdot [B \cdot R \cdot \arccos \frac{R - b \cdot \left(1 - \sqrt{1 - \frac{3.75 \cdot G_B}{\pi \cdot E_K \cdot b \cdot B}}\right)}{b \cdot \left(1 - \sqrt{1 - \frac{3.75 \cdot G_B}{\pi \cdot E_K \cdot b \cdot B}}\right)}]^{-1} \quad (45)$$

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3. ESTIMATE CALCULATIONS OF FLEXIBLE ROLL

Figure 7 represents a flexible roll and shows its main dimensions. Tolerance limit of ordinary steels varies between ±200 and ±400 MPa. In special steels, e. g. composed of C = 0.3%, Mn = 0.56%, Ni = 4.3%, Cr = 1.4%, it can reach ±500 … ±700 MPa.

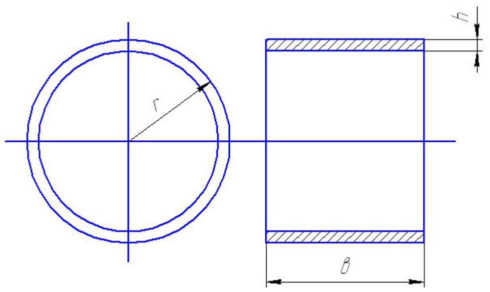

Figure 7. Symbolic notations of roll dimensions

Based on these stress values, let us determine the necessary roll thickness that would allow to change the curvature radius by 10-15 times without exceeding the specified stresses.

Let us find the sheet thickness that can be rolled from square to circular with a radius of 1 m.

As is known, when a beam is bent with a moment (pure bending), curvature radius depends on the moment $\frac{M}{E \cdot J} = \frac{1}{R}$. Having inserted the expressions for stress and section inertia moment, we obtain:

$$M = \sigma \cdot \frac{в \cdot h^2}{6}; \quad J = \frac{в \cdot h^3}{12}; \quad \frac{1}{R} = \frac{[\sigma] \cdot \frac{в \cdot h^2}{6}}{E \cdot \frac{в \cdot h^3}{12}} = \frac{2 \cdot [\sigma]}{E \cdot h}; \quad h = \frac{2 \cdot [\sigma] \cdot R}{E} \quad (46)$$

For steel with [σ] = 500 MPa:

$$h = \frac{2 \cdot [\sigma] \cdot R}{E} = \frac{2 \cdot 500}{2 \cdot 10^6} \cdot 1000 = 5\ mm$$

If steel [σ] = 700 MPa:

$$h = \frac{2 \cdot [\sigma] \cdot R}{E} = \frac{2 \cdot 700}{2 \cdot 10^6} \cdot 1000 = 7\ mm$$

Let us find to what radius these thicknesses allow bending the roll from the first curvature meter.

In this case, curvature change is determined by the formula:

$$\frac{1}{R} \pm \frac{1}{r} = -\frac{M}{E \cdot J} = -\frac{2 \cdot [\sigma]}{E \cdot h} \quad (47)$$

(1) h = 5 mm

$$\frac{2 \cdot [\sigma]}{E \cdot h} = \frac{2 \cdot 5000}{2 \cdot 10^6 \cdot 0.5} = \frac{1}{100}$$

$$\frac{1}{R} + \frac{2 \cdot [\sigma]}{E \cdot h} = \frac{1}{r}; \quad \frac{1}{r} = \frac{1}{100} + \frac{1}{100} = \frac{2}{100}; \quad r = 0.5 m$$

(2) h = 7 mm

$$\frac{2 \cdot [\sigma]}{E \cdot h} = \frac{2 \cdot 7000}{2 \cdot 10^6 \cdot 0.7} = \frac{1}{100}; \ r = 0.5 m$$

Let us examine what forces cause, at h = 7 mm, a change of curvature from r = 1 m to r = 0.5 m. (Fig. 8).

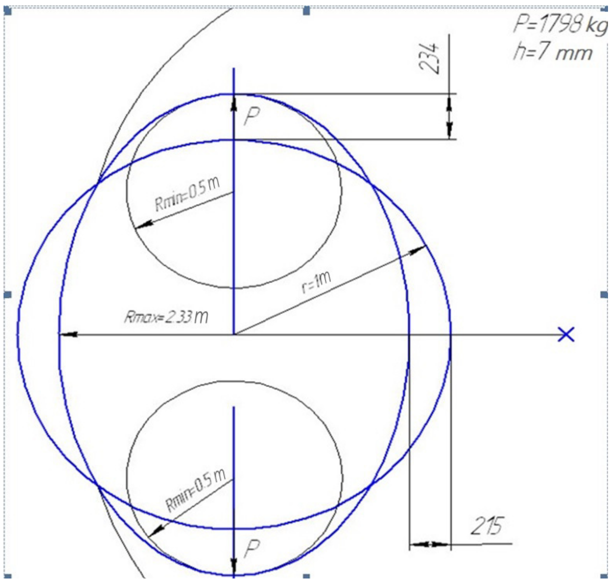

Figure 8. Roll deformation caused by different forces

As already noted, stress in the roll is equal to [σ] = 700 MPa

$$M = [\sigma] \frac{B \cdot h^2}{6} = 700 \frac{1000 \cdot 0,007^2}{6} = 5717\ H \cdot м \quad (48)$$

From here, $P = \frac{5717}{0.318 \cdot 1} = 17977 H$.

Herewith, deformations occur along the vertical diameter:

$$\delta_в = 0.149 \cdot \frac{P \cdot r^3}{E \cdot J} = 0.149 \cdot \frac{17977 \cdot 1000^3}{2 \cdot 10^6 \cdot 2.858 \cdot 10^4} = 469 mm \quad (49)$$

$$J = \frac{B \cdot h^3}{12} = \frac{1000 \cdot 0.007^3}{12} = 2.858 \cdot 10^4 mm^4 \quad (50)$$

Deformations along the horizontal diameter:

$$\delta_r = 0.137 \cdot \frac{P \cdot r^3}{E \cdot J} = 0.137 \cdot \frac{17977 \cdot 1000^3}{2 \cdot 10^6 \cdot 2.858 \cdot 10^4} = 431 mm \quad (51)$$

Now, let us find the roll curvature for these two directions:

Along the vertical diameter:

$$\frac{1}{R} = \frac{1}{r} + \frac{0.318 \cdot P \cdot r}{E \cdot J} = \frac{1}{100} + \frac{0.318 \cdot 17977 \cdot 1000}{2 \cdot 10^6 \cdot 2.858 \cdot 10^4} = \frac{2}{1000}$$

$$R = 0.5 m$$

As expected, the smallest roll diameter is equal to R_{min} = 0.5 m.

Along the horizontal diameter:

$$\frac{1}{R}=\frac{1}{r}+\frac{0.182\cdot P\cdot r}{E\cdot J}=\frac{1}{1000}+\frac{0.182\cdot 17944\cdot 1000}{2\cdot 10^{6}\cdot 2.858\cdot 10^{4}}=\frac{1}{1000}-\frac{1}{1754}$$

$$R_{\max}=2.33\,m\ .$$

The smallest curvature radius is related to the largest as 1:4.6.

It would be possible to increase this ratio by increasing R_{max} and here is the strength reserve.

However, this would require a different system of forces.

Supporting the flexible wheel with numerous rolls could increase R_{max}, the value of R_{min} remaining unchanged, however this would lead to an increase in longitudinal forces and additional roll load.

In the case considered, the highest stresses are located in the minimum diameter zone and equal:

$$\sigma_P=\frac{P}{2\cdot F}=\frac{17977}{2\cdot 1000\cdot 7}=1.28\,MPa \tag{52}$$

Thus, normal stresses do not reach considerable values.

4. CONCLUSIONS

The derived mathematical relations and the developed calculation theory prove the possibility of adjustment (optimization) of parameters of contact interaction of a road roller with the compacted layer by varying, in service conditions, the design-constructive dimensions of semi-axles of the ellipse-shaped surface of the roller, thus adjusting force action on the compacted road-building material and improving the created quality of the road or dirt-road surface.

The analysis of stressed and deformed roll state shows that two cases are suitable for constructive implementation. The first case is that of the roll stretched by two forces. The second one implies that the roll with local deformation allows to change, at comparatively small efforts and in wide ranges, the roll curvature radius in the rolling zone.

Based on the results of the estimated calculations for the locally deformed roll, it is possible to make the following conclusions:

(1) It is possible to design a flexible roll with walls 7 mm or even 100 mm thick, and to ensure the necessary (1:10) ratio of the profile curvature radii change, as well as the required strength.

(2) The most rational structure, in terms of flexibility and strength, is that with a deformable flexible roll.

In the roll with free deformation, the main force factors are the bending moments. Longitudinal and transverse forces are small. This allows to design a roll with maximum thickness h and acceptable values of thrust forces and curvature radii.

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ТЕОРИЈСКО ИСТРАЖИВАЊЕ И АНАЛИЗА РАДНОГ ПРОЦЕСА ДРУМСКОГ ВАЉКА СА ЛОКАЛНОМ ДЕФОРМАЦИЈОМ ВАЉКА У ЗОНИ КОМПАКТНОСТИ

С. Речицки

Рад приказује нов једноставан математички модел проблема фундаменталне и примењене теорије ела–стичности. Проблем се бави стањем напона и деформације радних делова друмског ваљка са елиптичним површинама и њиховом интеракцијом са материјалом за изградњу путева. Изведене мате–матичке релације и развијена теорија израчунавања доказују да постоји могућност прилагођавања (оптимизације) параметара интерак–ције контакта флексибилних радних делова код друмских ваљака са елиптичним површинама, са компактним слојем варирањем пројектованих конструкционих димен–зија полуосовина елиптичне површине ваљка у радним условима, чиме се прилагођава дејство силе на компактан материјал за изградњу пута.

Рад се бави специфичношћу рада покретних радних делова са елиптичним површинама, нуди избор мате–ријала и одређује геометријске параметре покретних радних делова са елиптичним површинама. На основу анализе стања напона и деформације ваљка, сматрамо да постоје два случаја погодна за конструкциону реализацију испитаног математичког модела.