Sonja Gorjanc

Assistant Professor
University of Zagreb
Faculty of Civil Engineering
Croatia

Ema Jurkin

Associate Professor
University of Zagreb
Faculty of Mining, Geology and Petroleum
Engineering
Croatia

Web Textbook for Descriptive Geometry

In this paper we present educational material which is a basis for a web textbook designed for the lectures on geometric subjects. The textbook is available online for free. It differs from previous textbooks in the application of information technologies in presenting classic contents and introducing 3D modelling in descriptive geometry teaching.

Keywords: geometry education, descriptive geometry, web textbook

1. INTRODUCTION

In this paper we present educational material which is a basis for a web textbook designed for the lectures on geometric subjects, [1]. The authors of the textbook are geometry teachers from the University of Zagreb, three of them, Sonja Gorjanc, Helena Koncul and Iva Kodrnja, coming from the Faculty of Civil Engineering, and one, Ema Jurkin, from the Faculty of Mining, Geology and Petroleum Engineering. The contents of the textbook cover the topics that are taught within the courses *Descriptive Geometry* and *Perspective* at the Faculty of Civil Engineering, and courses *Descriptive Geometry* and *Descriptive Geometry with Computer Graphics* at the Faculty of Mining, Geology and Petroleum Engineering, however, some of its content is also applicable to other faculties in Croatia.

The textbook is available online for free and licensed under CC BY-NC-SA 4.0 licence. One can include the full textbook, or only the parts of the textbook, into various e-learning platforms. Materials can be used free under the same conditions, i.e. must not be used commercially.

The importance of clear and careful spatial reaso–ning and its graphic representation in the education of future engineers makes descriptive geometry a tradit–ional mathematical course at technical faculties, parti–cularly at those related to civil engineering. A series of high-quality descriptive geometry textbooks were published in Croatian during last fifty years, and this textbook presents their sequel. However, it really differs from previous textbooks in the application of infor–mation technologies in presenting classic contents and introducing 3D modelling in descriptive geometry teaching. Therefore, a different approach to some course units has been required. Due to specific characteristics of descriptive geometry as a discipline of mathematics, the textbook is graphically rich and it contains 38 interactive files, 493 original drawings and figures, 83 original animations, 82 original presentations and 93 video clips (up to five minutes), a total of more than six hours of audio and video material.

It took quite a long time to prepare this extensive teaching material, which required interdisciplinary knowledge and skills, in parallel with lesson prepa–ration. The starting point of the textbook was repository of educational materials created within the project *Introducing 3D Modeling into Geometry Education at Technical Colleges* that was supported by Fund for Development of the University of Zagreb, [2]. During the year 2012 twelve teachers worked on the project, and four faculties were involved. The head of the pro–ject was Sonja Gorjanc. The main goals of the project were strengthening the professional and scientific colla–boration among teachers, developing teaching metho–dology for 3D computer modeling to enhance geometry courses, and harmonization of educational material sta–ndards and their implementation in the e-learning sys–tems of the technical faculties that were involved. The focus of the project was creating a basic repository of material related to common teaching topics and those customized to profiles of each faculty, [3]. The expe–rience of teachers using created educational materials were described in [4].

During the last few years we enriched the existing repository with new material and increased the number of topics it used to cover, giving it a form of textbook. We tried to connect the benefits of classical and modern approaches to teaching geometry guided by the thoughts presented in [5], as well as in [6] and [7].

2. CONTENTS OF TEXTBOOK

Table of contents is the following:
1. Plane Geometry
   1.1. Extended Euclidean Plane
   1.2. Second Order Curves - Conic Sections
   1.3. Transformations of Plane
2. Space Geometry – Method of Projections
   2.1. Introduction to Space Geometry
   2.2. Monge's Method of Projections
   2.3. Axonometry
   2.4. Elevated Projection
   2.5. Perspective
3. Space Geometry – Surfaces and Space Curves
   3.1. Surfaces
   3.2. Intersections of Surfaces with Planes
   3.3. Intersections of Surfaces
   3.4. Undevelopable Ruled Surfaces
4. Space Geometry - 3D Modeling with program *Rhinoceros*
   4.1. Introduction to *Rhinoceros*

4.2. Solved Examples and Tasks

The first chapter deals with plane geometry in more detail than a descriptive geometry courses. The chapter contains more than 30 interactive figures designed by *GeoGebra*, and by allowing the user to move certain elements one can study the plane constructions and their properties in detail. The links in the textbook enable the user to connect to certain contents when dealing with other contents in the textbook.

The second chapter deals with space geometry and methods of projection, i.e. basic classic contents of descriptive geometry. Modern technology is used for visualization. The chapter mostly deals with Monge's method, with solved exercises presented as a slideshow with video clips of spatial representations.

A large number of figures and animations are supported by *Mathematica*, while slideshows are created using *Adobe Illustrator*.

The third chapter presents space curves and surfaces in great detail. The chapter contains approximately 70 animations created by *Mathematica* that visualize the generation of surfaces (rotational, translation, ruled sur– faces, helicoids) and space curves (intersection curves of surfaces, helixes).

In the final chapter the contents are presented thro– ugh the application of computer technologies. For teac– hing 3D modeling we use *Rhinoceros*. It was chosen for the following reasons: a good experience of the colle– agues from the University of Innsbruck, a relatively low cost of the educational lab licence and the free trial version available for students' downloads. Therefore, the textbook also contains 88 five-minute video clips helping students to learn basics of *Rhinoceros* 3D modeling.

They were produced by the screencasting program *Jing*. By using video clips, students are introduced to the basics of Rhinoceros on their own. Every time when we want to teach them something about Rhinoceros, we use a geometry example. So, at the same time students master the computer program and learn geometrical contents chosen by geometry teachers.

3. EXAMPLES

In this section we present the textbook material by sho– wing the examples of using aforementioned programs.
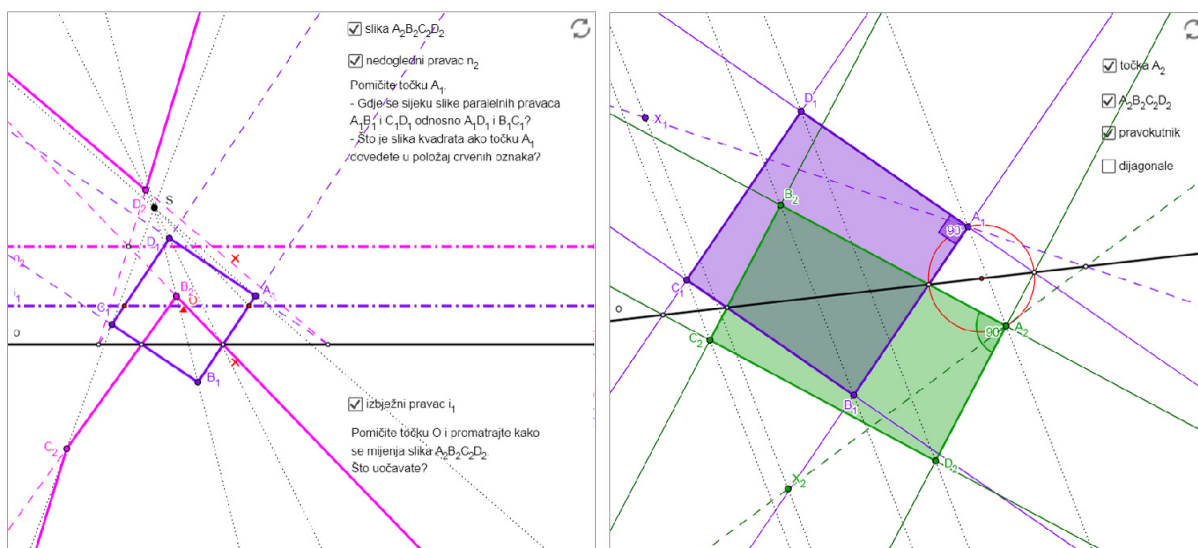

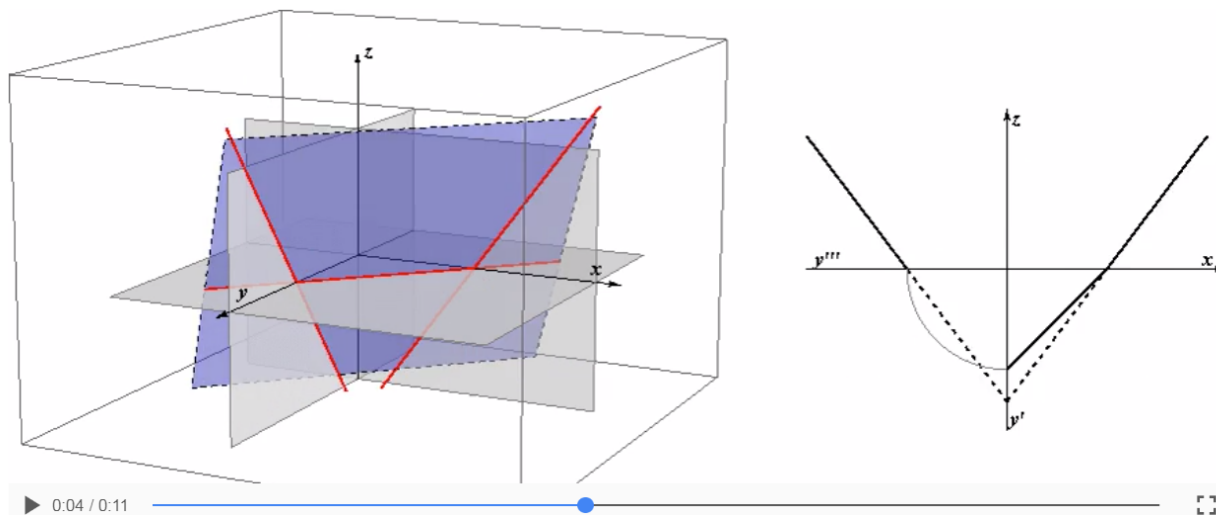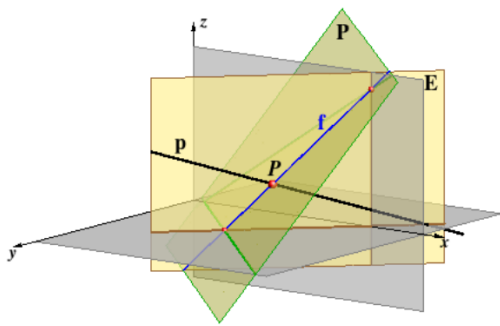**Figure 1. Interactive files with the images of squares in perspective collineation and affinity**
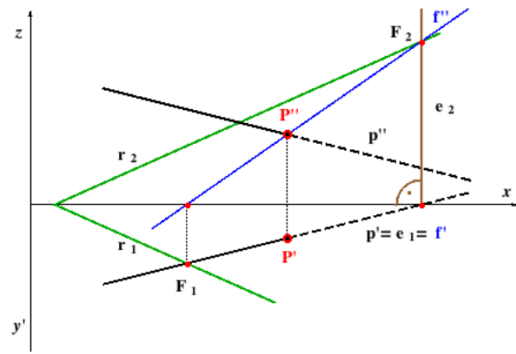

Figure 2. Animation - the representation of the concept of traces of plane in the Monge's method

• Pravcem *p* postavimo 1. projicirajuću ravninu E.
• Uočimo 1. i 2. probodište presječnice $f = P \cap E$.
• Uočimo presječnicu $f = P \cap E$.
• Uočimo točku u kojoj je sijeku f i p.
• Ta je točka traženo probodište $P = p \cap P$.

• Konstruiramo tragove e_1 i e_2 ravnine E.
• Konstruiramo 1. i 2. probodište presječnice $f = P \cap E$.
• Konstruiramo projekcije presječnice $f = P \cap E$.
• Uočimo točku $P'' = f'' \cap p''$.
• $P' \in p'$ i P'' su tlocrt i nacrt traženog probodišta $P = p \cap P$.
• **Vidljivost pravca *p* u odnosu na ravninu P.**

Figure 3. Slideshow – the representation of the construction of a point of intersection of a line and a plane by a projecting plane
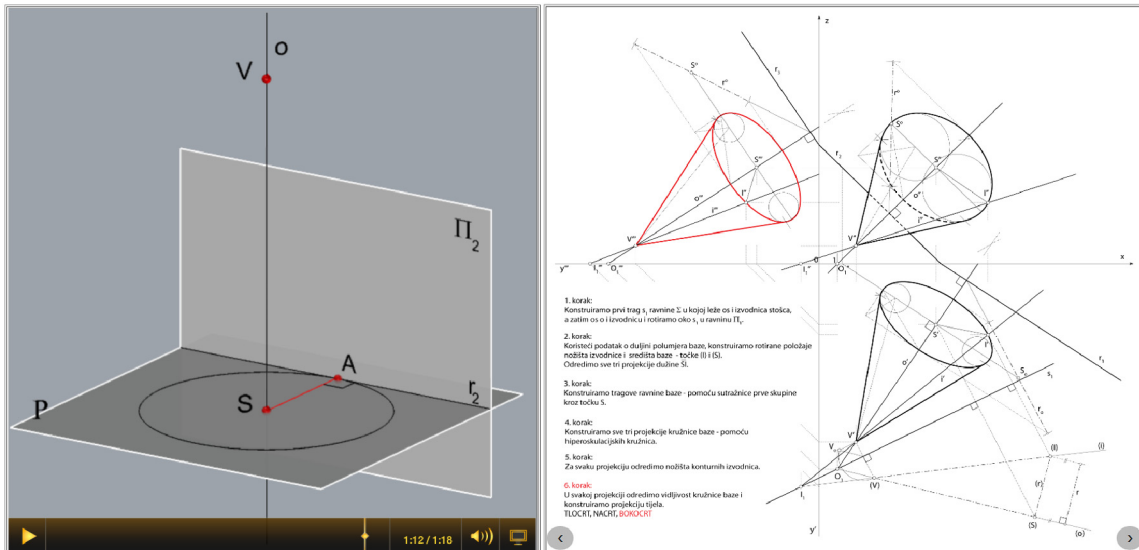

Figure 4. A video clip with a spatial representation and a slideshow with a constructive representation of the exercise
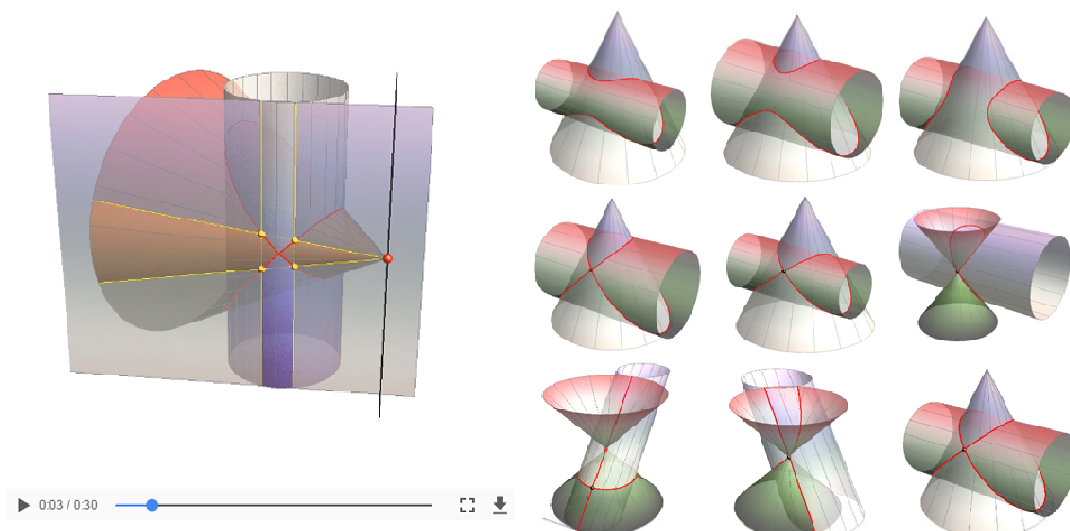

Figure 5. Animation – the principle of construction of an intersection curve of a cone and a cylinder and the examples of their intersection curves
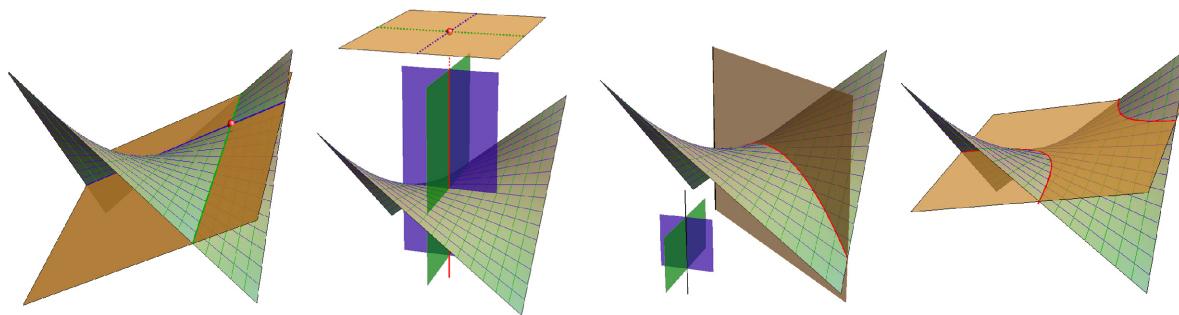
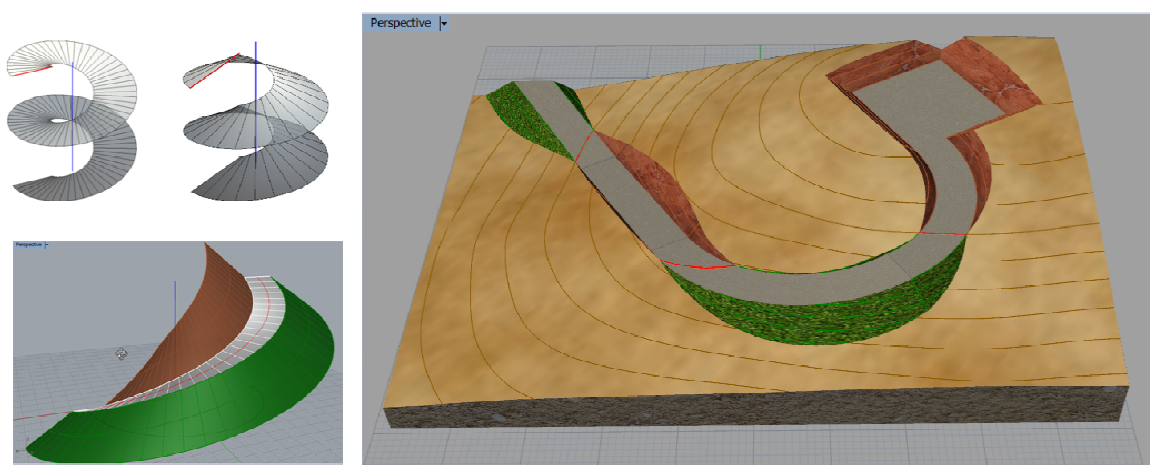**Figure 6. Intersections of a hyperbolic paraboloid and a plane**

Figure 7. The application of helicoid

Interactive files with the images of squares in pers–pective collineation and affinity are shown in Fig 1. The visualisation of the Monge's method is shown in Fig. 2 and Fig. 3, while the spatial representation and a slide–show with a constructive representation of the corres–ponding exercise in Fig 4. Fig. 5 demonstrates the prin–ciple and an example of an intersection curve const–ruction of a cone and a cylinder while Fig. 6 exhibits the intersections of a hyperbolic paraboloid and a plane. Finally, the application of helicoid is shown in Fig. 7.

4. CONCLUSION

This paper discloses the nowel educational material for delivering the lectures in descriptive geometry and per–spective on the Faculty of Civil Engineering and the Faculty of Mining, Geology and Petroleum Engineering, University of Zagreb. This approach provides the set of highly efficient educational activities and exercises by which students - the future engineers can develop clear and careful spatial reasoning and its graphic repre–sentation.

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WEB УЏБЕНИК ЗА НАЦРТНУ ГЕОМЕТРИЈУ

С. Горјанц, Е. Јуркин

У овом раду приказујемо наставни материјал који је саставни дио WEB уџбеника израђеног за предавања из геометријских колегија. Уџбеник је слободно доступан на интернету. Од претходних се разликује по примјени информацијске технологије у презен–тирању класичних садржаја и по увођењу 3Д моделирања у подучаване дескриптивне геометрије.