Spherical Perspective in Design Education

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The spherical perspective has not yet been widely introduced into design education. Its ability to serve as a meta-class model of vanishing point perspective systems, giving a teacher the opportunity to present approximations of the straight linear perspective models (with one, two or three vanishing points) all in one system, is presented in this article. The mathematical basis for a spherical grid as a curvilinear approximation to one-eyed human vision and a didactic approach for its integration into design oriented perspective freehand drawing are also discussed.

Keywords: *Spherical perspective; curvilinear perspective; vanishing point perspective; design education; augmented reality; immersion.*

1. SPHERICAL PERSPECTIVE AS AN APPROXI–MATION TO HUMAN VISION

With a simple example Bruno Ernst [1] provided an explanation for a work of M.C. Escher who is known for having played an important role in introducing the use of curvilinear (in this case cylindrical) perspective into modern art. Ernst did this by asking the reader to picture him/herself lying underneath two parallel infinite telegraph wires and then establishing two basic rules for perspective spatial perception:

1. Distant objects are perceived smaller than objects of the same size located closer to the viewer;

2. Lines or edges that are continuous and diffe–rentiable in reality are perceived preserving these qualities if they are in eye-focus and unobstructed by other objects.

One consequence is, e.g. that, these rules are to be strictly observed, buildings viewed by people standing on street-level can only be displayed correctly on a flat picture plane in spherical perspective (Fig. 5, left-hand object).

1.1 Mathematics

To construct a spherical grid, an orthogonal grid based on horizontal and vertical straight lines in a two-dimen–sional plane is projected onto a sphere. The resulting grid on the sphere's surface is then rolled out cylindrically in the horizontal and vertical direction onto a plane parallel to the original grid, following its orthogonal logic. To achieve this, each projection from straight line onto the sphere, resulting in a half circle on the sphere's surface, is again projected onto the sphere's equator plane as an ellipse. The distance p from the sphere's center to the points defining the contour of each ellipse is calculated; finally the *arccos* of this distance represents the *y*-value of the correponding points of cylindrically rolled out

spherical projection plane.

$$\sqrt{\frac{b^2}{1-\left(\frac{e}{a}\right)^2 \cdot \cos^2 \varphi}} \quad (1)$$

$$e = \sqrt{1 - \cos^2 \alpha} \quad (2)$$

$$b = \cos \alpha \wedge a = 1 \quad (3)$$

$$p = \sqrt{\frac{\cos^2 \alpha}{1 - \left(1 - \cos^2 \alpha\right) \cdot \cos^2 \alpha}} \quad (4)$$

$$\arccos p = \arccos \sqrt{\frac{\cos^2 \alpha}{1 - \sin^2 \alpha \cdot \cos^2 \varphi}} \quad (5)$$

1.2 A Macro to Display Sinoid Curves

Thus it becomes feasible to construct a curvilinear grid, e.g. in AutoCAD, in this case using the CAD-software's VBA (Visual Basic for Applications) macro progra–mming API. VBA for AutoCAD in the used version only allows for calculating *arctan* (atn) - to avoid division by zero. Therefore *arccos* is a reference to an additional function not listed here. Also 0° and 90° are not calculated by the macro program.

```
Private Sub
Linsenraster_CommandButton_1_Click()

Dim pt1(0 To 2) As Double
Dim pt2(0 To 2) As Double
Dim dblX1 As Double
Dim dblX2 As Double
Dim intZaehl1 As Integer
Dim intZaehl2 As Integer
Dim pi As Double
pi = 3.14159265358979

dblX1 = pi / 2 - 0 / 360 * 2 * pi
dblX2 = pi / 2 - 1 / 360 * 2 * pi

For intZaehl1 = 1 To 89
For intZaehl2 = 1 To 89
```

$$pt1(0) = (pi / 2) / 90 * (intZaehl2 - 1)$$

$$pt1(1) = arccos(Sqr((Cos((pi / 2) / 90 * (intZaehl1)))$$
$$\text{^} 2 / (1 - ((Sin((pi / 2) / 90 * (intZaehl1))) \text{^} 2) *$$
$$Cos(dblX1) \text{^} 2)))$$

$$pt2(0) = (pi / 2) / 90 * (intZaehl2)$$
$$pt2(1) = arccos(Sqr((Cos((pi / 2) / 90 * intZaehl1)) \text{^}$$
$$2 / (1 - ((Sin((pi / 2) / 90 * intZaehl1)) \text{^} 2) *$$
$$Cos(dblX2) \text{^} 2)))$$

$$ThisDrawing.ModelSpace.AddLine\ pt2,\ pt1$$

$$dblX1 = pi / 2 + ((intZaehl2 - 1) / 360 * 2 * pi)$$
$$dblX2 = pi / 2 + ((intZaehl2) / 360 * 2 * pi)$$

$$\text{Next intZaehl2}$$
$$\text{Next intZaehl1}$$

$$\text{End Sub}$$

Putting this macro to use the following partial (π/2 * π/2) representation of horizontal lines in a spherical grid can be produced - only one third of the calculated curves are shown here (Fig. 1). This calculated quadrant can be mirrored and assembled to form a full (π * π) spherical grid (Fig. 2).
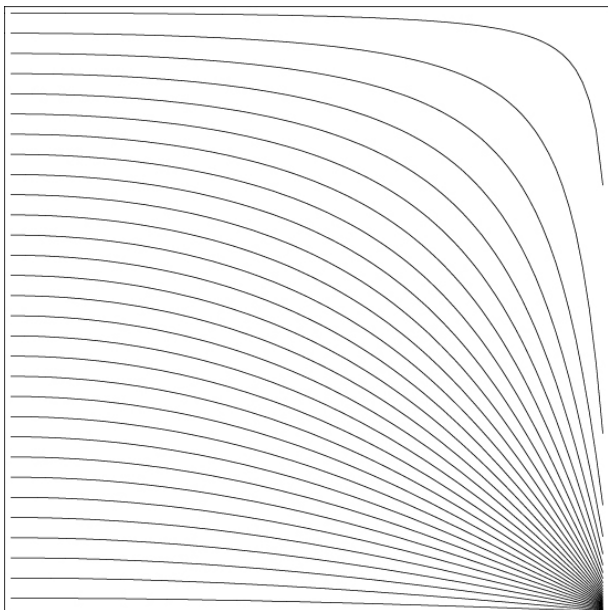

Figure 1. Top right quadrant of the spherically projected and rolled out horizontal lines (Kulcke 2018)

1.3 Changes in the Grid according to View Point

In contrast to straight-line vanishing point perspective systems the curvature of lines is changed according to eye-focus (see e.g. [2]). This explains why different spherical systems can be found in literature.

Systems showing straight lines going through a central vanishing point (Fig. 2) show actually a very special case; when exactly this vanishing point is in eye-focus. And, geometrically speaking, it never is, since it is defined only by the lines pointing to it (for an artistic discussion on this issue see [3]). As soon as the eye

chooses one of the lines rather than the point itself, the system should change accordingly (Fig. 3 and Fig. 4).
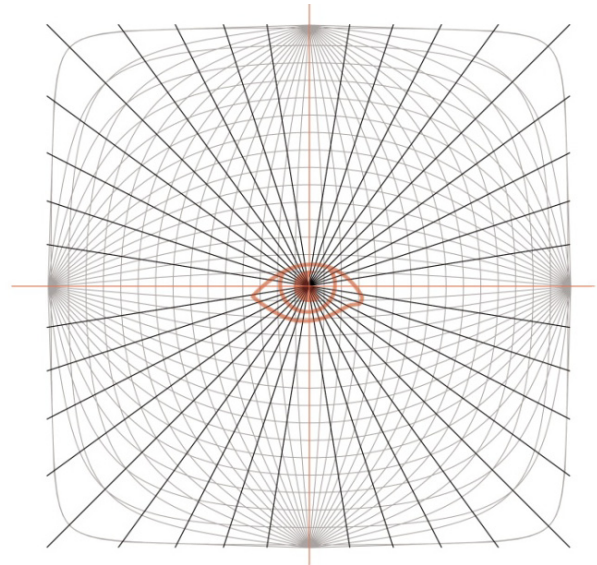

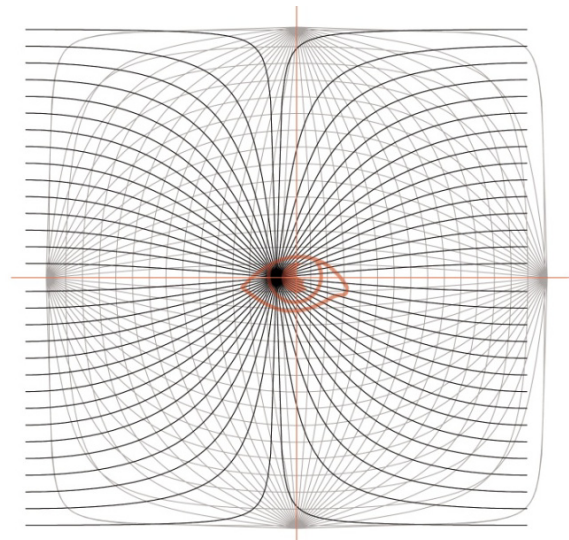**Figure 2. Curvilinear system with exact center focus (Kulcke 2018)**

Figure 3. Change of curvature due to line focus, horizontal direction (Kulcke 2018)
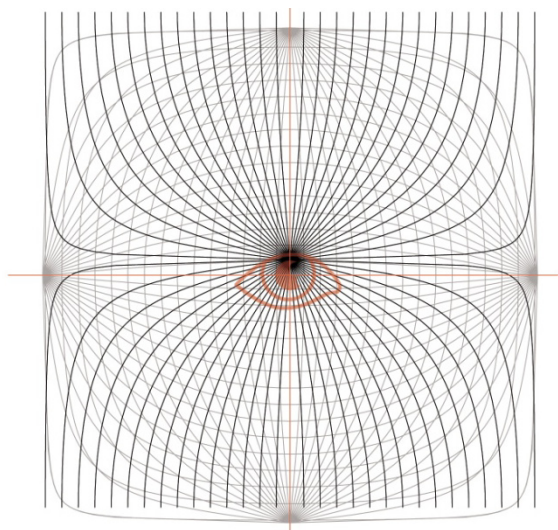

Figure 4. Change of curvature due to line focus, vertical direction (Kulcke 2018)

To raise awareness with the students considering this phenomenon and to also stay within the explanatory logic of Ernst's example, the spherical grid introduced to the students (Fig. 5) uses sinoid curves representing the third dimension going toward the systems center. Also the curves are simplified to sinus curves so they are easier to draw in freehand.
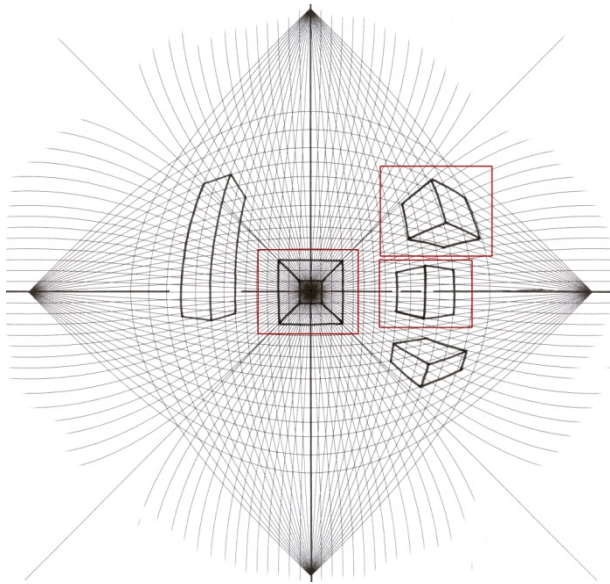

Figure 5. Spherical perspective (using simplified sinoid curves) as an explanatory model (Kulcke 2007)

2. SPHERICAL PERSPECTIVE AS AN APPROXI– MATION TO HUMAN VISION

An interest for using curvilinear perspective or distor–tions of orthogonal architectural concepts, e.g. hinting at a wide-angle (fish-eye) distortion, is seen frequently in contemporary architecture, as in building structures like Norman Fosters "Chesa Futura" [4]. These designs are not merely free form but can be perceived as a reference to the use of distortion via a curvilinear perspective grid. One of the reasons for the current use of such concepts might be that architects and builders have only recently significantly increased their ability to handle them in design and construction processes by using CAD-Software (like Rhino/Grasshopper) in combination with CAM (computer aided manufacturing).

However, the spherical perspective (as the curvilinear perspective par excellance) has not been widely introduced into design education; literature on applied perspective construction is often confined to straight linear perspective (e.g. [5], [6]). And if spherical perspective is mentioned, then it is usually presented as an "outside the box" model (e.g. [7], [8], [9]) rather than as what it should be viewed as: A meta-class model of perspective representation, giving a teacher the opportunity to present approximations of straight linear perspective models (with one, two or three vanishing points) all in one system (Fig. 5).

2.1 The Didactic Approach

Thus it becomes easier to explain to students in which cases to use a certain straight linear model and also to show the limits of each of these models. The author has used spherical perspective as an explanatory system to teach students the basics of freehand-drawn perspective within the subject "Prozesse der Innenraumgestaltung" ("the interior design process") at the Hamburg University of Technology since 2007 and in the course "Methoden der Darstellung" ("presentation methods") since 2011. The free use of the spherical grid (Fig. 7) and the benefits as well as the limits of the straight linear perspective models are told in several sets of lecturing, practical exercises and corrections, leading to an understanding of the basic principles of perspective and their application in design processes.

2.2 Freehand Drawing in the CAD Era

The author also intends to raise the awareness with the students that a general understanding of CAD doesn't have to be limited to 2D and 3D technical drawing and modeling on the computer. The refinement of sketches e.g. with image enhancing software may also serve as a feedback regarding the status of their designs and often lead to the creation of alternative concepts and sketches. Therefore, in contrast to the craft of drafting with a ruler and similar devices, freehand drawing hasn't been replaced to the same extent by digital tools and techniques; it has, on the contrary, even gained importance since image enhancing software can easily enrich a quick sketch with color and contrast.

2.3 Course Structure

The course "Methoden der Darstellung" contains freehand drawing instruction on free sketching, parallel projections, vanishing point perspective systems as well as layout and design exercises. Due to the articles's subject only the part on vanishing point perspectives will be explained in further detail.
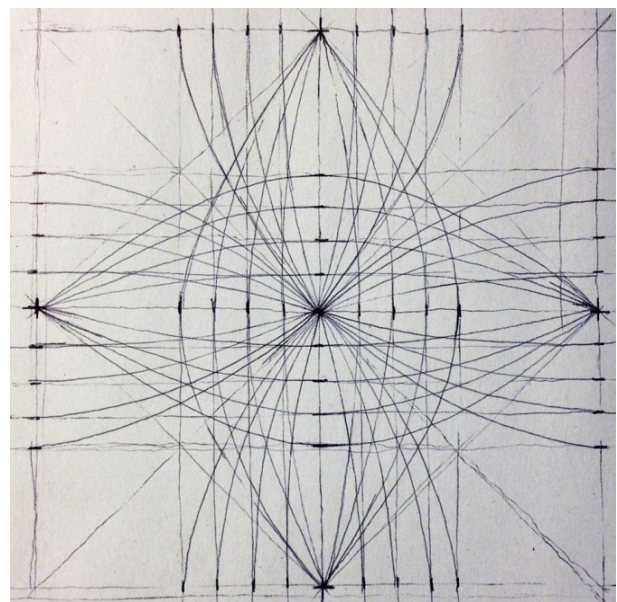

Figure 6. Spherical perspective in freehand drawing, grid construction (Kulcke 2011)

In four sessions curvilinear and straight line vanishing point perspectives are explained to the participants. The first session introduces Ernst's example and thus a

spherical grid is successively discussed in its representation of all three dimensions. In this order the lessons on vanishing point perspective systems start with the seemingly most complicated, but actually with the least abstract system of the four, since it is closest to actual human vision. After the introduction the grid is constructed in a freehand drawing exercise and simple objects are drawn into the system (Fig. 6 and Fig. 7).
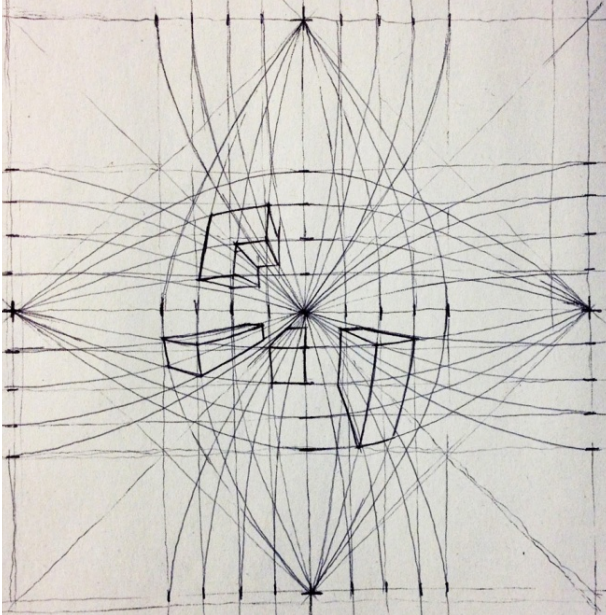

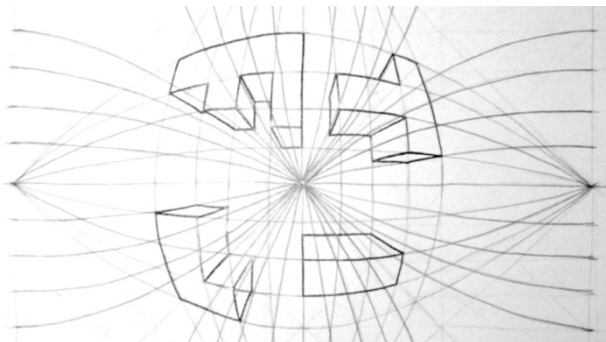**Figure 7. Spherical perspective in freehand drawing, filling in objects (Kulcke 2011)**
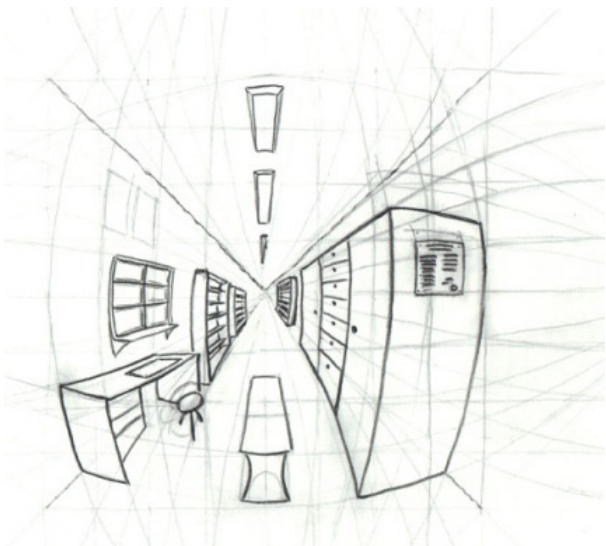

Figure 8. Student work (Grocholl 2016)

Figure 9. Student work (Volkmann 2013)

The freehand drawing approach exempts the stu–dents from worrying too much about actual dimensions, even if they draw more specific objects (Fig. 9). This allows them to focus on questions of edge constellations and directions.

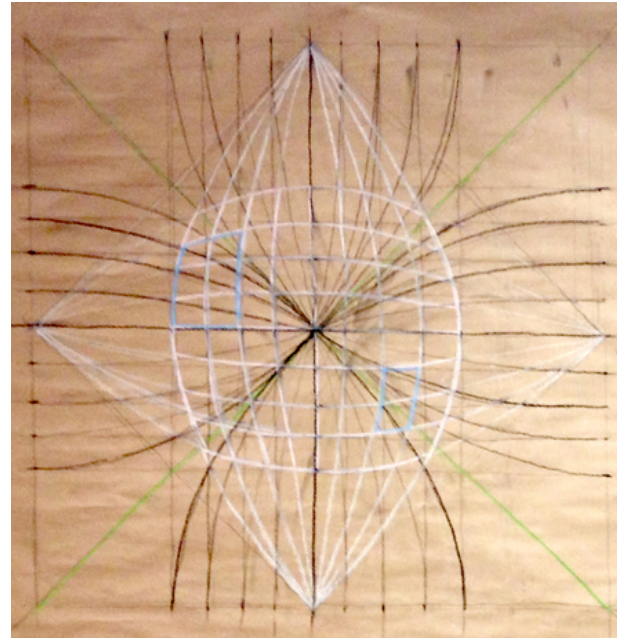## 2.4 Explaining Straight Line Perspective as a Fol-low-Up

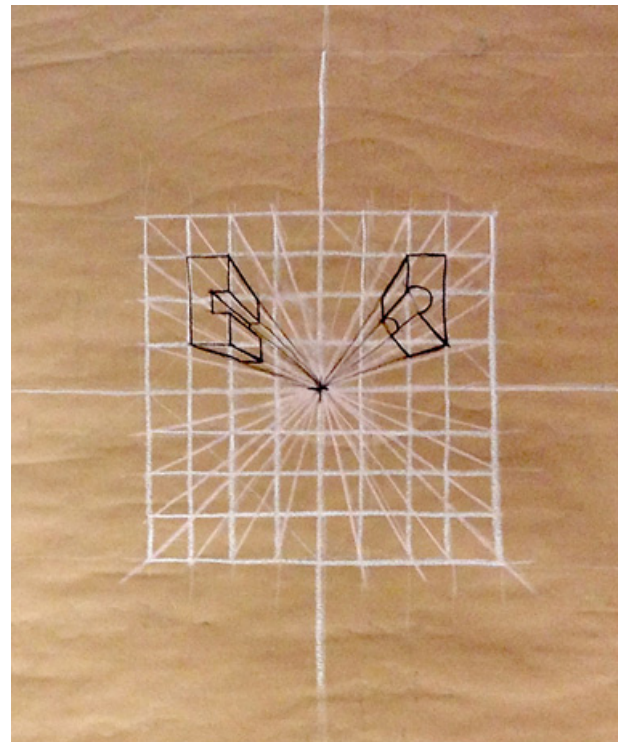**Figure 10. From curvilinear to straight line perspective 1 (Kulcke 2014)**

Figure 11. From curvilinear to straight line perspective 2 (Kulcke 2014)

The results are promising, almost every student manages to comply with the basic rules of perspective drawing within the one and a half hours of the first

session. In the following lessons straight linear pers–pective systems, with one, two or three vanishing points, are successively introduced and explained as simplifications in reference to the spherical grid.

In the age of the geometrical CAD System that creates images in vanishing point perspectives on the basis of 3D models, it is important to adapt and refine didactic strategies in the academic realm when teaching students about the essence of vanishing point perspec–tive representations. Using spherical perspective as an explanatory model may serve as a foundation regarding such approaches.

3. THE SPHERICAL PERSPECTIVE IN THE CON–TEXT OF AUGMENTED REALITY

While it still remains to be proven, that the described didactic approach is superior to others in regards to un–derstanding straight line vanishing point perspective, there are further reasons to teach students about cur–vilinear systems.

An awareness and deeper knowledge of spherical perspective could prove to be crucial for advanced aug–mented reality (AR) applications and the design and pla–nning processes using them. Since the human eye perceives straight edges next to an edge in focus, due to the curvature of its lens and other geometrical factors, in spherical distortion (e.g. [2]), the simulated objects that complete an urban situation, for example as simulated image parts on an optical see-through display seen within the already existing surroundings, will be more con–vincing if the simulated part is itself distorted accor–dingly.

Virtual reality (VR) software usually uses a static representation of curvature, which is still a crutch, beca–use eye movement constantly changes the focal point and therefore the vanishing point of the system and thus the relationship between object and edge curvature as well as constellation (see also [2] and [10]).

VR glasses and the accompanying applications keep the vanishing point at the center of the calculated image (as if the eye would stare all the time in a right angle on the image), thereby potentially weakening immersive illu–sion as soon as eye movement without head movement occurs.

4. CONCLUSION AND OUTLOOK

The application of curvilinear grids to willingly distort an object or a building should not be reduced to simply displaying a prerogative of the designer or to create a superficial effect. Consciously integrating the category of spherical distortion into the design process might actually lead to better designs, especially in but not limited to the urban design context. Thus designers and architects may reconnect modern practise to design techniques already known and used, e.g. for the shaping of columns in ancient Greek architecture.

In architectural and urban design the inclusion of public and/or customer opinion at an early state into the planning process becomes an increasingly important factor, especially if participatory planning processes are put into practice. The use of VR and AR technologies, as described under 3, on building sites, in learning envi–

ronments and excavation sites (combining the excavation with 3D models of reconstructed building elements) that are open to the public is also a tempting possibility, if not already in use. In the light of these developments aug–mented reality and its immersive perfection, through a better understanding of and awareness for spherical perspective, and its implementation into 3D-simulation CAD-software, also taking stereo vision (see [11]) and last but not least eye movement into consideration, are bound to become more important. To integrate especially the latter into modern CAD and visualization systems a deeper and widespread understanding of spherical pers–pective and its dynamic grid changes, due to eye move–ment are a critical factor for its designers and developers.

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СФЕРНА ПЕРСПЕКТИВА У ОКВИРУ ДИЗАЈНЕРСКОГ ОБРАЗОВАЊА

М. Кулчке

Сферна перспектива још увек није у већој мери уведена у дизајнерско образовање. Могућност њене примене као модела мета-класе система перспективе нестајуће тачке, која даје предавачу прилику да представи апроксимацију модела перспективе праве линије (применом једне, две или три нестајућих тачака) у оквиру истог система је приказана у овом раду. Такође је разматрана и математичка основа

сферне мреже са криволинијском апроксимацијом једнооког људског вида и дидактичким приступом њеној интеграцији у концепт дизајна слободном руком у оријентисаној перспективи.