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Double Diffusion in Square Porous Cavity Subjected to Conjugate Heat Transfer

Double diffusion inside a square porous cavity being subjected to conjugate heat transfer arising because of the square solid block at the center is studied under homogeneous properties. The double diffusive equations along with the conjugate effect due to square block are subjected to solution through finite element method (FEM). The right vertical surface of cavity remains at cool temperature (T_c) and concentration (C_c), whereas the hot temperature (T_h) and higher concentration (C_h) is maintained at the left surface of the cavity. The investigated parameters considered are the Rayleigh number (Ra), Thermal conductivity ratio (K_r), Lewis number (Le), Buoyancy ratio (N) and the height of solid block (B). The outcomes are presented in terms of contours of concentration, isotherms and streamlines, along with local Sherwood number (Sh) and Nusselt number (Nu). The fluid is found to rush around the top left corner of solid. The mass transfer has sharp rise around the top left corner of the solid block.

Keywords: Double Diffusion; Porous media; Solid Square Block; Finite Element Method

1. INTRODUCTION

The double diffusion has drawn the attention of several researchers due to its immense importance and vital applications in the field of science and engineering. Prominent authors [1-6] have discussed thoroughly the behaviour of fluid flow and heat transfer encompassing various relevant issues pertaining to porous media. Some of the studies like heat transfer, mass transfer, viscous dissipation, magneto hydrodynamic etc. are studied extensively. The literature reveals that various heat transfer characteristics of porous cavities have been investigated and reported by many researchers. For instance, the heat transfer has been investigated in porous cavities and porous annulus by applying hot temperature at left vertical wall and maintaining cold temperature at the right wall [7-13]. This included the investigation of heat transfer along with difficult boundary conditions or various other phenomenon such as viscous irregular cavity [7], viscous dissipation [8-10], Soret and Dufour effect [11], heat and mass transfer [12], the effect of heat generating strip placed inside the porous cavity [13], wavy tube heat exchanger [14], flow of nanofluid inside the porous media [15-17] etc.

The addition of any solid or subtraction of some section of porous domain from square cavity can affect the heat

transfer quite substantially. For instance, presence of a hollow section at the center of cavity can lead to multiple variations in the heat transfer along the hot surface [18-19]. Similarly, an addition of any solid substance in the porous domain is known to produce different heat transfer characteristics depending upon the solid and its location in the domain. Arasteh et al. [20] while studying the heat transfer of metal foam reported that the metal foam addition to the channel increases heat transfer and pressure drop. Nazri et al. [21] analysed the mixed convection in porous cavity. They found that the volume fraction increment of nanoparticles and reduction in Richardson number improved the temperature distribution.

The process of heat transfer across the porous medium via a solid substance that emanates complex boundary conditions between solid and fluid is coined as conjugate heat transfer. This concept did not draw much attention of the researchers due to its complex phenomenon when any solid obstruction arises in the porous medium that substantially changes the flow behaviour leading to change in heat and mass transfer characteristics. Lamnato et al. [22] reported that the drying process of porous media produced an extreme mass transfer at the surface of porous sliced wood with the aid of upstream flow divider. It is reported that [23] the wall thickness and ratio of wall to fluid conductivity, helps in enhancing the heat transfer characteristics. In an identical work Tao [24], found that heat capacity ratio of the fluids affects the conjugate heat transfer in finned tube. Saeid [25] investigated the effect of natural convection due to presence of solid vertical wall with emphasis on conduction in the solid wall. Amiri et al [27] studied the steady state heat transfer with conjugate

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effect in porous cavity. Heat transfer analyses of solid on inner radius of a vertical annulus was analyzed by Ahmed et al [28] which was further extended to analyze the heat and mass transfer [29]. Alhashash et al [30] found that the radiation intensity enhances the fluid circulation and so does the Nusselt number. Al-Farhany et al [31], have shown in their work that the Darcy number is inversely related with average Nusselt number. Badruddin et al [32] analyzed the role of varying conductivity ratio as well as the relative thickness of two solid surfaces in an annulus. They reported that the conductivity ratio as well as the relative thickness of participating solids have impacts on heat transfer characteristics. Recent investigations have shown that the size and location of a rectangular solid block inside the porous domain would have significant influence on the double diffusion [33]. Ghalambaz et al. [34] investigated the conjugate heat transfer using local thermal non-equilibrium with a porous enclosure with hybrid nano-particles. Ghalambaz et al. [35] also studied the conjugate heat transfer in a cavity using thermal non-equilibrium model. The mixed convection conjugate heat transfer due to nanofluid in inclined flat plate about a porous regime was analysed by Khademi et al [36].

The current work emphasizes the analysis of the heat and mass transfer in presence of a solid block at the centre of porous cavity. It was noted that the solid block at the cavity centre would produce unique heat transfer behaviour [37]. However, the literature needs to be updated concerning the double diffusive flow in a cavity containing a solid block. The present work is an effort to fulfil this missing information.

2. MATHEMATICAL METHOD

A square cavity as depicted in figure 1 with a square solid placed at the center was considered. The equations governing heat and fluid flow for this particular problem are given as [38]:

Figure 1. Physical domain and co-ordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{gK}{\nu} \left(\beta_T \frac{\partial T}{\partial x} + \beta_c \frac{\partial C}{\partial x} \right) \quad (2)$$

$$u \frac{\partial T_p}{\partial x} + v \frac{\partial T_p}{\partial y} = \alpha \left(\frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2} \right) \quad (3)$$

$$\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} = 0 \quad (4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) \quad (5)$$

Subjected to the boundary conditions as shown in table 1:

Table 1: Boundary conditions

Coordinate Position	Boundary Condition
$x = 0$	$T = T_h, C = C_h$ $u = 0, v = 0$
$x = H$	$T = T_c, C = C_c$ $u = 0, v = 0$
$y = 0$ and $y = H$	$\frac{\partial T}{\partial y} = 0, \frac{\partial C}{\partial y} = 0$ $u = 0, v = 0$
$\frac{H-B}{2} \leq x \leq \frac{H+B}{2}, y = \frac{H-B}{2}$	$u = 0, v = 0$
$\frac{H-B}{2} \leq x \leq \frac{H+B}{2}, y = \frac{H+B}{2}$	$u = 0, v = 0$
$\frac{H-B}{2} \leq y \leq \frac{H+B}{2}, x = \frac{H-B}{2}$	$u = 0, v = 0$
$\frac{H-B}{2} \leq y \leq \frac{H+B}{2}, x = \frac{H+B}{2}$	$u = 0, v = 0$
$x = x_{sp}$	$k_s \frac{\partial T_s}{\partial x} = k_p \frac{\partial T_p}{\partial x}$ $T_s = T_p, u = 0, v = 0$
$y = y_{sp}$	$k_s \frac{\partial T_s}{\partial y} = k_p \frac{\partial T_p}{\partial y}$ $T_s = T_p, u = 0, v = 0$

Using stream function ψ as:

$$u = \frac{\partial \psi}{\partial y}; v = -\frac{\partial \psi}{\partial x} \quad (6)$$

The above equations are dimensionless as:

$$\bar{x} = \frac{x}{H}, \bar{y} = \frac{y}{H}, \bar{\psi} = \frac{\psi}{\alpha}, \bar{T} = \frac{(T - T_c)}{(T_h - T_c)}, \bar{C} = \frac{(C - C_\infty)}{(C_w - C_\infty)},$$

$$Ra = \frac{g\beta \Delta T K H}{\nu \alpha}, Le = \frac{\alpha}{D}, N = \left(\frac{\beta_c \Delta C}{\beta_T \Delta T} \right)$$

Substituting equations (6) into equation (2-4) yields:

$$\frac{\partial^2 \bar{\psi}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} = -Ra \frac{\partial \bar{T}}{\partial x} \quad (7)$$

$$\left[\frac{\partial \bar{\psi}}{\partial y} \frac{\partial \bar{T}}{\partial x} - \frac{\partial \bar{\psi}}{\partial x} \frac{\partial \bar{T}}{\partial y} \right] = \left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right) \quad (8)$$

$$\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} = 0 \quad (9)$$

$$\left[\frac{\partial \bar{\psi}}{\partial y} \frac{\partial \bar{C}}{\partial x} - \frac{\partial \bar{\psi}}{\partial x} \frac{\partial \bar{C}}{\partial y} \right] = \frac{1}{Le} \left(\frac{\partial^2 \bar{C}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \right) \quad (10)$$

The corresponding dimensionless Boundary Conditions are given in table 2:

Table 2: Dimensionless Boundary Conditions

Coordinate Position	Boundary Condition
$\bar{x} = 0$	$\bar{\psi} = 0, \bar{T} = 1, \bar{C} = 1$
$\bar{x} = 1$	$\bar{\psi} = 0, \bar{T} = 0, \bar{C} = 0$
$\bar{y} = 0$ and $\bar{y} = 1$	$\bar{\psi} = 0, \frac{\partial \bar{T}}{\partial \bar{y}} = 0, \frac{\partial \bar{C}}{\partial \bar{y}} = 0$
$\frac{1-\bar{B}}{2} \leq \bar{x} \leq \frac{1+\bar{B}}{2}, \bar{y} = \frac{1-\bar{B}}{2}$	$\bar{\psi} = 0$
$\frac{1-\bar{B}}{2} \leq \bar{x} \leq \frac{1+\bar{B}}{2}, \bar{y} = \frac{1+\bar{B}}{2}$	$\bar{\psi} = 0$
$\frac{1-\bar{B}}{2} \leq \bar{y} \leq \frac{1+\bar{B}}{2}, \bar{x} = \frac{1-\bar{B}}{2}$	$\bar{\psi} = 0$
$\frac{1-\bar{B}}{2} \leq \bar{y} \leq \frac{1+\bar{B}}{2}, \bar{x} = \frac{1+\bar{B}}{2}$	$\bar{\psi} = 0$
$\bar{x} = x_{sp}$	$\bar{\psi} = 0, Kr \frac{\partial \bar{T}_s}{\partial \bar{x}} = \frac{\partial \bar{T}_p}{\partial \bar{x}}$
$\bar{y} = y_{sp}$	$\bar{\psi} = 0, Kr \frac{\partial \bar{T}_s}{\partial \bar{y}} = \frac{\partial \bar{T}_p}{\partial \bar{y}}$

The Nusselt number (Nu) is calculated as:

$$Nu = - \left(\frac{\partial \bar{T}}{\partial \bar{x}} \right)_{\bar{x}=0} \quad (11)$$

The Sherwood number (Sh) is given by:

$$Sh = \left(- \frac{\partial \bar{C}}{\partial \bar{x}} \right)_{\bar{x}=0} \quad (12)$$

The above-mentioned 4 partial differential equations (7-10) with boundary conditions given in Table 2 are simplified using Finite Element Method (FEM), following the procedure as given in [39-42]. The domain of the porous cavity is discretized using three noded triangular elements. This results in a simpler form of three algebraic equations for each of the elements. We achieved the solution by utilizing the algorithm as suggested by Badruddin et al [43]. The domain is meshed by utilizing the triangular elements. The solution is obtained by an iterative method with a convergence criterion of 10^{-6} for temperature as well as concentration and 10^{-7} for stream function. The mesh independency is ensured to have good accuracy of results. Table 3 shows the Average Nusselt number obtained at different number of elements. It is clear from table 1 that the solution does not vary much when the number of elements has increased from 2048 to 3200. However, the time taken to solve for the case of 3200 elements is 131.42% higher than that of the case with 2048 elements. Thus, the current work utilized 2048 triangular elements for simulation of problem under consideration. Table. 4 shows the results that are compared with the data in literature by reducing the size

of the block to zero. It is obvious from table 4 that the current method has good accuracy.

Table 3: Mesh Characteristics

No of Elements	Avg.Nu	Time, s
800	2.7202	30.0
2048	2.8702	140.7
3200	2.9012	324.7

Table 4. Comparison with previously published data for the case N=0.

Author	Ra=10	Ra=100
Present	1.084	3.076
Ref. [43]	1.070	3.09
Ref. [44]	1.065	2.801
Ref. [45]	1.079	3.16
Ref. [46]	1.119	3.05

3. RESULTS AND DISCUSSION

Figures 2–4 displays the isotherms, concentration lines and streamlines for different parameters. Figure 2 shows the consequences of changing the solid block height at mid of cavity for $N=0.2$, $Le=5$, $Ra=100$, $Kr=2$. This figure is obtained for 2 different sizes of the block that are obtained by changing the non-dimensional height $\bar{B} = 0.125$ (left column of figure 2) and $\bar{B} = 0.5$ (right column of figure 2). The increased solid due to increased $\bar{B}$ occupies larger area of the cavity that leads to decrease in the fluid velocity as indicated by streamlines. This in turn increases the conduction effect in the whole domain reflected by straightened isotherms. The isotherms penetrated into solid because of thermal conductivity of solid. If there is no solid, the isotherms spread across the empty space as observed in other studies [19]. The mass transfer is significantly affected by the increased size of solid as represented by iso-concentration lines of figure 2. As the solid size increases (owing to increased $\bar{B}$), the larger area of cavity is influenced by concentration. This happens because of the obstruction to mass transfer posed by solid block. The impact of changing buoyancy ratio from $N=0.1$ to $N=1$ is depicted in figure 3, having the constants as $\bar{B} = 0.375$, $Le=5$, $Ra=50$, $Kr=2$. As expected [33], the effect of N is negligible on isotherms as compared to the mass transfer. The increased buoyancy ratio generates better mass transfer, which in turn allows the concentration lines to shift towards the lower side of the hot surface. This is consistent with available literature covering the topic of heat and mass transfer. The magnitude of the streamlines rises with the increase in N and surrounds the square block.

Figure 4 draws the conclusion of Lewis number (Le) at $\bar{B} = 0.375$, $Kr=2$, $Ra=100$, $N=0.2$. As observed in case of heat and mass transfer [12], Lewis number influences the mass transfer substantially. This is obvious from present case (figure 4), that shows highly distorted iso-concentration lines as compared to isotherms, when Lewis number (Le) is increased from 1 to 25. The iso-concentration substantially comes closer to the hot surface that creates a large concentration gradient, which in turn should increase the mass transfer in the domain.

Figures 5-10 shows the influence of distinct parameters on local Nusselt number (Nu) and

Sherwood number (Sh) with respect to different parameters. The Nu decreases along the height of cavity, which is in line with that of cavity having no solid [7]. The Nu is found to decrease with increase in the solid size in the porous medium, which acts as a barricade. This is because the fluid movement gets restricted due to increase in solid size that reduces the heat carrying capability thus reducing the Nu at hot surface of the domain. The Nu is found to be higher for higher value of buoyancy ratio (N) until almost half of the hot surface as shown in figure 7. The Lewis number is found to have lesser impact on heat transfer as shown in figure 9, which follows the trend as in other studies [29]. The local Sh too decreases with the rise in cavity height but not for smaller dimensions of solid. However, there is peculiar behavior of mass transfer for $\bar{B} = 0.375$ where a sharp rise and fall of Sherwood number is observed at cavity height of about 0.7. This happens due to the reason that the larger size of solid makes concentration distribution split into two halves at around cavity height 0.65, as observed in figures 2-4.

The presence of larger sized solid block forces the fluid to confine within a narrow vertical channel thus reducing the mass carrying capacity from the hot surface. However, the fluid rushes towards the cold surface when it gets the opportunity at the end of solid block, which leads to sharp rise in mass transfer as reflected in terms of increased Sherwood number. This is further vindicated by the streamline flow pattern as shown in figures 2-4. This behaviour is consistently seen for Sherwood number with respect to all other parameters such as buoyancy ratio (figure 8), Lewis number (figure 10), Thus it can be conveniently said that the mass transfer has a unique behaviour at higher size of solid that shoots up the mass transfer at a particular position of cavity height. Such behaviour is not reported in literature for solid being placed at other positions. It can be said that the Nu increases with increase in buoyancy ratio (N) (figure 7), However, Nu diminishes as Lewis number (Le) increases (figure 9). This is consistent with the findings of other researchers [36-37].

4. CONCLUSION

The problem of double diffusion in a porous square cavity with a solid square block is analysed numerically. The main concluding points of this work are:

The heat and mass transfer in porous media deteriorated with increasing the solid height. This significantly depends on the Buoyancy ratio, the local Nusselt number increases for higher values of Buoyancy ratio (N), along the height of the cavity, but decreases at a certain point of the cavity.

The Sherwood number shows a sharp increase at $\bar{B} = 0.375$. In all cases, the most common effect of Buoyancy ratio was that the Nusselt number becomes the increasing function of N except the Lewis number (Le) and height of the solid block ($\bar{B}$). The concentration distribution got divided into two distinct regimes when solid height is increased to 0.375.

Figure 2. Contours of a) Isotherms b) Iso-concentration c) Streamlines for $\bar{B}$ at $N = 0.2$, $Le = 5$, $Ra = 100$, $Kr = 2$ (I) $\bar{B} = 0.125$ (II) $\bar{B} = 0.5$

Figure 3. Contours of a) Isotherms b) Iso-concentration c) Streamlines for Buoyancy Ratio at $\bar{B} = 0.375$, $Le = 5$, $Ra = 50$, $Kr = 2$ (I) $N = 0.1$ (II) $N = 1$

Figure 4. Contours of a) Isotherms b) Iso-concentration c) Streamlines for Lewis Number at $\bar{B} = 0.375, Kr = 2, Ra = 100, N = 0.2$ (I) $Le = 1$ (II) $Le = 25$

Figure 5. Nu variation due to change in Block width ($\bar{B}$) at $Kr = 2, Le = 5, N = 0.2, Ra = 100$

Figure 6. Sh variation due to change in Block width ($\bar{B}$) at $Kr = 2, Le = 5, N = 0.2, Ra = 100$

Figure 7. Nu variation due to change in Buoyancy ratio (N) at $\bar{B} = 0.375, Kr = 2, Le = 5, Ra = 50$

Figure 8. Sh variation due to change in Buoyancy ratio (N) at $\bar{B} = 0.375, Kr = 2, Le = 5, Ra = 50$

Figure 9. Nu variation due to change in Lewis number (Le) at $\bar{B} = 0.375, Kr = 2, N = 0.2, Ra = 100$

Figure 10. Sh variation due to change in Lewis number (Le) at $\bar{B} = 0.375, Kr = 2, N = 0.2, Ra = 100$

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NOMENCLATURE

$C, \bar{C}$	Species concentration
D	Mass diffusivity (m^2/s)
$B, \bar{B}$	Dimensional height/width (m) of solid and dimensionless Ratio of solid height/width to cavity height/width
g	Acceleration due to gravity (m/s^2)
H	Height or length of cavity (m)
k	Thermal conductivity ($W/m^{\circ}C$)
k_p, k_s	Porous and Solid thermal conductivity respectively ($W/m^{\circ}C$)
K	Permeability of porous medium (m^2)
K_r	Conductivity ratio
Le	Lewis number
Nu	Nusselt number
N	Buoyancy ratio

R_a	Modified Raleigh number
Sh	Sherwood number
$T, \bar{T}$	Temperature
u, v	Velocity components in x and y direction respectively (m/s)
x, y	Cartesian co-ordinates
$\bar{x}, \bar{y}$	Dimensionless co-ordinates

Greek

α	Thermal diffusivity (m^2/s)
ρ	Density (kg/m^3)
ν	Coefficient of kinematic viscosity (m^2/s)
β_T	Thermal coefficient ($1/m$)
β_C	Concentric Coefficient ($1/m$)
ψ	Stream function
$\bar{\psi}$	Dimensionless stream function

Superscripts

$_h$	Hot
$_c$	Cold
$_p$	Porous
$_s$	Solid

ДВОСТРУКА ДИФУЗИЈА У КВАДРАТНОЈ ПОРОЗНОЈ ШУПЉИНИ ИЗЛОЖЕНОЈ КОЊУГОВАНОМ ПРЕНОСУ ТОПЛОТЕ

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Истражује се, у условима хомогености, двострука дифузија унутар квадратне порозне шупљине изложене коњугованом преносу топлоте који настаје због квадратног чврстог тела лоцираног у центру шупљине. Једначине двоструке дифузије и коњуговани ефекат преноса топлоте због квадратног тела се решавају применом ФЕМ методе. Десна вертикална површина шупљине остаје на нижој температури и концентрацији, док се висока температура и већа концентрација одржавају на левој страни површине шупљине. Разматрају се следећи параметри: Рајлијев број, однос топлотне проводљивости, Луисов број, однос потиска и висина чврстог тела. Резултати су приказани помоћу контура концентрације, изотерми и струјница заједно са Шервудовим бројем и Нуселтовим бројем. Утврђено је да налет флуида настаје око врха левог горњег угла чврстог тела. Пренос масе нагло расте око врха левог горњег угла чврстог тела.