Directional Stability of an Agricultural Tractor

The discrepancy between the plow width and the tractor width leads to the asymmetry of plowing units. The geometry of the plowshare surface of the moldboard plow contributes to the generation of lateral forces on the working tool. All this leads to the imbalance of the tool and the deviation of the tractor from straight-line movement during plowing. To maintain straight-line movement, the driver has to adjust the machine every 5–10 meters, which is highly tiresome. To study the causes of lateral slips of the plowing unit, we constructed a mathematical model, which consists of the equations of controlled movement and equations of the tractor’s uncontrolled shear under the action of external forces from the plow. The description of the force interaction of the drive with the ground is based on the mathematical theory of friction, taking into account anisotropy and elastic properties in contact. Based on the passive shear model, we constructed a hodograph diagram of the maximum tractor shear force from the side of the working tool. We found that the shear force reaches its maximum friction value only in the case of a translational shear, when its line of action passes through the tractor’s center of gravity. In all other cases, the shift (slip) of the tractor is caused by a lower force. We formulated the features and assumptions of the model as applied to caterpillar and wheeled tractors. As a result, we found that, regardless of the direction of the lateral displacement of the plow’s traction resistance, the tractor is slipped towards the plowed field. The result of the numerical experiment showed that the main reason for the slip of the wheeled plowing unit is the difference in soils along the sides of the tractor but not the deviation of the plow traction resistance.

Keywords: directional stability; mathematical model; lateral slip; moldboard plow; ground contact; plowing unit; mathematical theory of friction.

1. INTRODUCTION

The initial step to a good harvest is high-quality soil treatment [1]. Today, modern agriculture includes various pre-seeding treatment technologies. However, the main method is still traditional plowing with a moldboard plow, which has a positive effect on the further growth and development of plants [2]. In terms of energy intensity, plowing takes 30–35% of the entire energy consumption in field cultivation [3]. The arable layer inversion technology does not only cuts and embeds weeds to a depth inaccessible for germination but also ensures mixing of soil layers and protects it from infectious agents [4].

The discrepancy between the plow width and the tractor width leads to the asymmetry of plowing units. When the plowing unit is in operation, a turning moment is created from the side of the working tool, which deflects the tractor from straight-line movement [5]. To maintain straight-line movement, the driver has to constantly adjust the machine. In wheeled tractors, the impact on the steering wheel is 15-20 times in a 100-meter travel section [6]. In caterpillar tractors, the impact on the friction control lever is every 4–6 m of the unit travel [7]. This leads to the operator’s increased fatigue and a decrease in the productivity of up to 10-15% [8]. The main reason for the slip of the plowing unit from the straight-line direction is the impact of the plow on the tractor [9–10]. The composite surface of the moldboard plow leads to a deviation of the normal resistance force $P$ by a certain angle $\beta=15^\circ...25^\circ$ relative to the longitudinal axis of the tractor [11]. To compensate for the lateral component $P_l$ of the plow resistance force, a landside plate is installed in the horizontal plane. The length of the plate is limited by the technological process and the structural dimensions of the plow. The friction of the landside plate consumes up to 17% of the total traction resistance of the tractor, which makes us look for other ways of balancing the plow in the horizontal plane [12].

Studies of the directional stability of a tractor unit during plowing began long time ago [13–15]; however, this issue remains relevant today [16–18]. The solution to the problem of the directional stability of a plowing unit is often limited by the balance of only one working tool (plow) [12, 19]. At the same time, studying the movement of the entire plowing unit in general will allow us to understand better the causes of the side slip of the machine. We will construct a mathematical model
of the unit’s slip to assess the influence of each factor on the deviation from straight-line movement.

**The purpose of the research** is to construct a mathematical model of the plowing unit movement allowing us to assess the influence of various factors on the directional stability of a tractor with an asymmetric external load.

2. **MATHEMATICAL SLIP MODEL**

The movement of the plowing unit under the action of eccentric resistance forces from the plow side is a combination of controlled straight-line movement and a passive uncontrolled shift [20].

**The controlled movement equations**, taking into account small lateral deviations and low motion speeds, can be written in the form of ordinary curvilinear integrals [21]:

\[
\begin{aligned}
x_c &= \int_0^T V \cos \left( \int_0^t \frac{V}{\rho} d\tau \right) dt \\
y_c &= \int_0^T V \sin \left( \int_0^t \frac{V}{\rho} d\tau \right) dt
\end{aligned}
\]

(1)

where \(x_c, y_c\) are the coordinates of the unit’s center of mass in a fixed reference, \(t=0\ldots T\) is the movement time; \(V\) is the speed, \(\rho\) is the radius of the trajectory curvature.

**The uncontrolled shift equations** are the equations of the power balance of the entire plowing unit (Fig. 1).

\[
\begin{aligned}
F_{x1} + F_{x2} - P_f - P \cos \beta &= 0 \\
F_{y1} + F_{y2} - (P \sin \beta - R_\theta) &= 0 \\
\left( y_k + y_l - 0.5 B \right) \cos \phi - x_k \left( P \sin \beta - R_\theta \right) + y_1 P_{f1} + y_2 P_{f2} - \sum M_i &= 0,
\end{aligned}
\]

(2)

where \(P\) is the resulting plow resistance force; \(\beta\) is the tilt angle of the resistance force to the longitudinal axis of the tractor; \(x_k, y_l\) are the coordinates of point \(K\) of applying the resulting resistance force of the plow relative to the tractor’s center \(C\) of mass; \(R_\theta\) is the resistance force of the sideslip plate; \(B\) is the tractor wheel track; \(P_f\) are the forces of resistance to the movement of the \(i\) caterpillar band (side); \(y_i\) are the coordinates of the instantaneous center \(O\) of sliding of the contact area of the \(i\) caterpillar band (side); \(F_{xi}, F_{yi}, M_i\) are the total force factors in the contact of each caterpillar band (side) with the ground [22].

The force factors in the drive-ground contact are essentially friction forces. A shift relative to the ground is observed when the friction forces reach their limiting value. There is no relative slip in cases of all lower values of the forces in the contact. The mathematical theory of friction [23] allows us to determine the maximum (limiting) values of the force factors in the contact of the drive with the ground, when the external force \(P\) causes a rotational shift relative to some instantaneous sliding center \(O\). Then, the force factors \(F_{xi}, F_{yi}, M_i\) in the \(i\) contact are the functions of the unknown coordinates \(x_i, y_i\) of the instantaneous sliding center \(O\) in the coordinate system linked with the geometric contact center [24]:

\[
\begin{aligned}
F_{xi} &= q - \frac{\mu_i (y_i - \gamma)}{\sqrt{(y_i - \gamma)^2 + (x_i - \gamma)^2}} d\phi d\theta, \\
F_{yi} &= -q \frac{\mu_i (x_i - \gamma)}{\sqrt{(y_i - \gamma)^2 + (x_i - \gamma)^2}} d\phi d\theta, \\
M_i &= q \mu_i \sqrt{(y_i - \gamma)^2 + (x_i - \gamma)^2} d\phi d\theta
\end{aligned}
\]

(3)

where \(\mu_i\) is the coefficient of friction in the \(i\) contact with the ground; \(x_i, y_i\) are the coordinates of the instantaneous sliding center of the \(i\) ground contact area in the coordinate system linked with its geometric center; \(\gamma, \eta\) are the present coordinates of the contact points.

The introduction of a variable friction coefficient \(\mu_i\) under the integral of the force factors \(F_{xi, F_{yi}, M_i}\) allows us to take into account the elastic properties at each contact point [25] and establish the relationship between the force factors and the radius \(\rho\) of the trajectory curvature:

\[
\begin{aligned}
x_i &= \max \left( \frac{(y_i - \gamma)}{(\rho 0.5 B + y_i)} \right), \\
y_i &= \max \left( \frac{(x_i - \gamma)}{(\rho 0.5 B + y_i)} \right),
\end{aligned}
\]

(4)

where \(\mu_{max} = \mu_{max}\) is the maximum coefficient of friction in the longitudinal and transverse directions; \(\theta\) is the hyperbolic tangent function; \(\lambda\) is the empirical coefficient characterizing the elastic properties of the ground; \(\rho\) is the radius of the trajectory curvature.

![Figure 1. A diagram of the forces acting on the plowing unit](image-url)
Characteristics of the caterpillar unit movement. The length of the caterpillar band-to-ground contact $L$ is much larger than its width $b$. This allows us to replace the double integral in equations (4) with a single one with an error of no more than 2% [26]. In this case, the average normal pressure in the contact is $q = G/2Lb$.

A caterpillar plow tractor generally moves along the unplowed part of a field at a certain distance from the edge of the furrow (provided that the furrow wall is not shedding) (Fig. 2a), which allows us to accept identical ground conditions under both caterpillar bands.

However, due to grousers, the interaction of the caterpillar band with the ground acquires anisotropic properties, which can be expressed by introducing different friction coefficients $\mu_i$ in the longitudinal and transverse $\mu_i$ directions [26]. Because of the transverse arrangement of grousers, there is almost no elastic deformation of ground in this direction, since the ground cut begins almost at once $\mu_i = \mu_{sv}$. 

As a result, the force factors $F_{yi}$, $F_{yi}$, $M_i$ are as follows:

$$
F_{yi} = \frac{G}{2Lb} \int_{-L/2}^{L/2} \mu_{sv} N_i \left[ \frac{(x_i - y)}{\sqrt{y^2 + (x-y)^2}} \right] \, dx,
$$

$$
F_{yi} = \frac{G}{2Lb} \int_{-L/2}^{L/2} \mu_{sv} N_i \left[ \frac{(x_i - y)}{\sqrt{y^2 + (x-y)^2}} \right] \, dx,
$$

$$
M_i = \frac{G}{2Lb} \int_{-L/2}^{L/2} \mu_{sv} N_i \left[ \frac{(x_i - y)}{\sqrt{y^2 + (x-y)^2}} \right] \, dx.
$$

Characteristics of the wheeled unit movement. The technology of plowing with a wheeled plowing unit implies the movement of the wheels of one side along the bottom of the furrow (Fig. 2b). This results in a constant lateral tilt of the machine. The consequence is a different normal load on the sides determined by [12]:

$$
q_i = \frac{G}{2ab} \left( \frac{\cos \alpha}{2} \pm \frac{h \sin \alpha}{B} \right)
$$

where the “+” sign is used for the lower side moving in the furrow, the “-” sign is used for the upper side moving along the virgin soil; $\alpha$ is the tilt angle of the tractor, $h$ is the height of the tractor’s center of mass; $ab$ is the length and width of the wheel track.

The lateral tilt of a wheeled tractor leads to a redistribution of the weight load between the sides (6). According to calculations, the tilt angle is 5–6°, which corresponds to the 13–15% difference in the normal load. A change in the normal load on the wheels leads to a change in the size of the contact patch (Fig. 3), increasing the track length $a$, while its width $b$ remains unchanged [27]. Besides, the movement on a loose ground contributes to an additional increase in the contact patch of one of the tractor sides [28].

The symmetrical arrangement of the tire tread pattern allows us to accept identical friction coefficients in the longitudinal and transverse directions $\mu_{sv} = \mu_{svy}$. The elastic properties in the contact are provided in the longitudinal direction due to the crushing of the ground, and in the transverse direction due to the deformation of the tire. Besides, the wheels of different sides move on the ground with different properties. Taking into account the aforesaid, the force factors $F_{yi}$, $F_{yi}$, $M_i$ (3) will be as follows:

$$
F_{yi} = \frac{G}{2ab} \int_{-L/2}^{L/2} \mu_{sv} N_i \left[ \frac{(x_i - y)}{\sqrt{y^2 + (x-y)^2}} \right] \, dx
$$

$$
M_i = \frac{G}{2ab} \int_{-L/2}^{L/2} \mu_{sv} N_i \left[ \frac{(x_i - y)}{\sqrt{y^2 + (x-y)^2}} \right] \, dx.
$$

Figure 2. Movement of (a) a caterpillar and (b) wheeled tractor during plowing
where $\mu_{ad}$ is the maximum friction coefficient in the $i$ side-to-ground contact; $f_i$ is the coefficient of the $i$ side rolling resistance.

**Coupling equations.** Uncontrolled movement (without the driver’s control action) is characterized by a fixed relative movement of the drive supports (wheels or caterpillar bands) corresponding to a single sliding center (point $O$) for the contacts of both sides [8,29], which allows us to write the missing coupling equation:

$$y_1 = y_2 + B \quad (8)$$

where 1,2 is the caterpillar band (side) index.

A quasi-static slip model includes the system of equations (1–8), where the unknown radius $\rho$ of the trajectory curvature is unambiguously determined [23] from the equilibrium equations (2), taking into account formulas (3–7). Thus, each value of the external plow resistance force $P$ corresponds to its own radius $\rho$ of the trajectory curvature and the instantaneous sliding center $O$.

We will conduct a numerical experiment for a detailed analysis of the impact of various factors on the movement trajectory of a plowing machine-tractor unit.

### 3. RESULTS OF THE NUMERICAL EXPERIMENT

The plow resistance force is characterized by the module $P$, direction, and point of application.

The determination of the plow resistance force module $P$ is a very complicated task, depending on many factors: the type of the ground being treated, the depth of treatment, the speed of movement, the weight, shape and width of the plow (number of bodies), etc. [30]. The transverse component $P_x$ is generally compensated by the response of the landside plate $R_{Ox}$. Its increase leads to an increase in the length of the landside plate and, consequently, an increase in the coverage and tractive effort. The correct choice of the landside plate size allows us to achieve the equality [12]. $P \sin \beta = R_{Ox}$. Then, the uncontrolled shear is preconditioned only by the lateral displacement $y_1$ of the longitudinal component $P_x$ of the plow resistance force.

The longitudinal component $P_x$ is perceived by the traction force $\Sigma F_{ii}$ of the tractor characterized by friction. Consequently, there is no side slip (cross slip) when the force $P_x$ is less than the limiting value according to the equilibrium conditions. The limiting (maximum) value of the shear force $P$ corresponds to a certain direction (line of action) [29]. F.A. Opeiko proved the uniqueness of the connection between the ultimate strength and the line of action.

The direction of the shear force is determined by the shape of the ploughshare surface and is taken into account through the tilt angle $\beta$ of the resultant plow resistance force $P$ to the longitudinal axis of the tractor [12]. For a comprehensive study of the dependence of the limiting force $P$ on the direction, on the basis of the equilibrium equations (2), we constructed a hodograph diagram of the shear force using the example of a caterpillar tractor (Fig. 4). It allows us to determine the limiting values of the shear force $P_{max}$ and its components $P_x$ and $P_y$ for each line of action. The accounting of the anisotropy reduces the values of the maximum shear force and rotates the hodograph diagram towards a lower friction coefficient by an angle $\delta$ which depends on the ratio of the friction coefficients in the longitudinal $\mu_{max}$ and transverse $\mu_{my}$ directions.

**Figure 4. Shear force hodograph diagram**

The analysis showed that at a passive shear, the maximum value of the shear force $P_{max} = G(u - f)$ is possible only in the case of a straight-line shear, when the line of the force action passes through the machine’s center $C$ of the mass. This private case characterizes translational sliding. The movement trajectory, taking into account the slip, is a straight line (line 1 in Fig. 5) and is provided by a displacement of the hitch point relative to the longitudinal axis of the tractor [31–32]. In all other cases, there is an instantaneous rotational shear, and the shear force is less than its maximum value $P < P_{max}$. The presence of the instantaneous rotational shear leads to the fact that if the driver takes no control action, the value of the lateral deviation $y_c$, quadratically increases with an increase in the distance covered $x_c$ (lines 2-4 in Fig. 5).

**Figure 5. Movement trajectories of the plowing unit: 1 - translational slip, 2 - slip caused by the displaced traction resistance of the plow, 3 - slip caused by different ground contact conditions; 4 - total slip**
An increase in the lateral displacement $y_L$ (the arm of the force action) leads to an increase in the rotating effect and, consequently, to an increase in the side slip of the tractor. The calculations have shown that regardless of the direction of the lateral shift $y_L$, the deflection of the tractor was always observed in one direction - towards the plowed field.

Different movement conditions of the tractor sides have the maximum impact on the directional stability of the plowing unit. The calculations have shown that due to the difference in the coefficients of friction $\mu_i$ and of rolling resistance on the sides $f_r$, the tractor loses its directional stability even when there is no impact of the external load from the plow. In this case, the lateral deviation (line 4 in Fig. 5) exceeds the slip value from the plow resistance force (line 2 in Fig. 5).

4. CONCLUSIONS

The mathematical slip model consists of the equations of controlled straight-line movement and the equations of uncontrolled passive shear under the influence of external forces, which represent the power balance of the plowing unit. The interaction of the tractor drive with the ground is presented on the basis of the mathematical theory of friction, taking into account the anisotropy and elastic properties in the contact.

Each line of action of the shear force has a corresponding limiting value. When the force is less than this value, there is no sides hear and slip of the tractor from straight-line movement. The shear force reaches its maximum friction value when its line of action passes through the machine’s center of mass. In this case, there is a translational shear, and the slip trajectory is a straight line. When the line of action of the shear force does not pass through the tractor’s center of mass, the static balance is violated at its lower value, and an instantaneous rotational shear occurs. In this case, the value of the lateral deviation from the straight-line direction quadratically increases depending on the distance covered. The interaction anisotropy additionally reduces the value of the limiting shear force.

The movement of one side of the tractor along the bottom of the furrow creates different conditions in the wheel-to-ground contact. The normal load, track size, friction, and rolling resistance coefficients differ on the sides, which leads to the loss of the machine’s directional stability (even if there is no resistance on the plow).

In the future, the proposed mathematical model can be used as the basis for the development of an automatic (unmanned) tractor control system.

REFERENCES


**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Greek symbols</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>is tractor lateral tilt angle</td>
</tr>
<tr>
<td>$\beta$</td>
<td>the tilt angle of the resistance force to the longitudinal axis of the tractor</td>
</tr>
<tr>
<td>$\rho$</td>
<td>the radius of trajectory curvature[m]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>is the empirical coefficient characterizing the elastic properties of the ground</td>
</tr>
<tr>
<td>$\mu$</td>
<td>the coefficient of friction in the contact with wheeled machines.</td>
</tr>
</tbody>
</table>

$\text{NOMENCLATURE}$

$a,b$ wheel contact length and width

$B$ the tractor wheel track

$L$ the longitudinal tractor base

$H$ the height of the tractor’s center of mass

$F_x, F_y, M$ the force factors in the contact with the ground

$P$ the resulting plow resistance force

$P_l$ the forces of resistance to movement

$R_o$ the resistance force of the landside plate

$q$ the normal contact pressure

$V$ the tractor speed

$x_0, y_0$ the coordinates of the unit’s center of mass

$x_c, y_c$ the coordinates of point of applying the result of resistance force $P$ of the plow relative to the tractor’s center of mass

$x_c, y_c$ the coordinates of the instantaneous sliding center
Stability of the Direction of Cultivator Wheel Position

I. Trojanovska, A. Zakov, O. Grebennyikova, C. Bojnak, E. Timofeyev


The ground

\( \mu_m \)

the maximum coefficient of friction in the contact with the ground

the coefficient of friction in the longitudinal and transverse directions

\( \gamma, \eta \)

the present coordinates of the contact points

The maximum coefficient of friction in the contact with the ground for the longitudinal and transverse directions are \( \mu_x, \mu_y \), respectively. The present coordinates \( \gamma, \eta \) of the contact points are used to describe the stability of the cultivator's direction.

The paper presents a mathematical model that consists of equations for controlled motion and equations for non-controlled swerving under the effect of external forces acting on the plow. The interaction of forces between the drive and the ground is described based on the theory of friction, with account taken of anisotropy and elastic properties of the contact. On the basis of the model of passive swerving, we have constructed an hodogram of the maximum force of swerving at the tractor. We have established that the force of swerving reaches its maximum value only in the case of translatory swerving when the attack line passes through the center of the tractor. In all other cases, the force is of lower intensity and is the cause of the tractor slipping. The characteristics and assumptions of the model are formulated for the tractor and the cultivator. It is shown that regardless of the lateral movement of the plow in the case of resistance on the wheel, the numerical result of the experiment shows that the main cause of the tractor's swerving is the difference in the earth's surface along both sides of the tractor and the lack of movement of the plow.