Multicriteria Optimal Selection of a Hydraulic Cylinder for Drive Mechanisms

Rosen P. Mitrev
Associate Professor
Technical University of Sofia
Faculty of Mechanical Engineering
Bulgaria

Todor S. Todorov
Full Professor
Technical University of Sofia
Faculty of Industrial Technology
Bulgaria

The paper reports the optimization synthesis of a hydraulically actuated drive mechanism. A mathematical model of the mechanism using vector closure equations is developed. Based on the functional purpose of the mechanism, a set of geometric and force/moment requirements are defined which must be met by a proper selection of a standardized hydraulic cylinder and its points of attachment. A multiobjective design optimization task is defined with three objective functions whose minimum is searched – the mass of the hydraulic cylinder, the squared total deviation of the developed by the hydraulic cylinder moments from the predefined values of the external moments and the force in the hydraulic cylinder. The defined multiobjective optimization task is considered as a mixed variable nonlinear constrained optimization problem containing 5 continuous and 2 discrete variables and the multistage Monte Carlo method is used for its solution. Using different weighting schemes several Pareto-optimal compromise solutions are obtained.

Keywords: *optimization, drive mechanism, continuous and discrete variables, Monte Carlo method.*

1. INTRODUCTION

In modern society, along with the introduction of new technologies, there is an increasing trend to improve the characteristics of the existing products and devices. Especially of paramount importance is the optimization of the drive mechanisms used in the energy-intensive industrial equipment such as hydraulically driven manipulators - hydraulic excavators, truck-mounted cranes, mining and material handling machines etc. The optimization synthesis [1-3] is a key technique for designing mechanisms satisfying predefined geometric, force, mass, power etc. requirements. Using a suitable optimization method the dimensions of the mechanism links are determined to satisfy certain requirements, expressed by one or more objective functions, interval, and functional constraints. The optimization synthesis is applied successfully many times for obtaining competitive machines and mechanisms. The authors of [4] presented an optimal dimensional synthesis of the hydraulic excavator working mechanism performed using multiobjective optimization and achieved an improvement of the digging performance. The paper [5] presents a method for the selection of optimal transmission mechanism parameters of a hydraulic forest crane by the solution of an optimization task and thus the machine performance is enhanced. A systematic method for optimal component selection of electrohydraulic servo systems including hydraulic cylinders is presented in [6]. The paper [7] presents mathematical modelling and optimum synthesis of oscillating slide actuators used in mechatronics devices. A complete methodology for optimal synthesis of drive mechanisms of hydraulic excavators taking into account the digging processes is developed in [8]. The authors of [9] developed a mathematical model of an excavator considering the kinematics of the links drive mechanisms.

A problem that has not been given enough attention in the existing research is the selection of a hydraulic cylinder, whose parameters largely determines the performance characteristics of the drive mechanism. Usually, this design task is related to the selection from a given finite discrete set, and most often it is a selection of a standardized in diameter and stroke hydraulic cylinders from a manufacturer's catalogue [10]. The optimization problems in which some of the design variables accept only discrete values are very common in engineering. Over the last few decades, a large number of studies have been devoted to this problem [11,12]. The methods for solving optimization problems containing simultaneously continuous and discrete/integer variables are classified in [13] and depend mainly on the type of the objective function and design variables. Since in most cases it is difficult to achieve the optimal solution within one iteration, it is necessary to specify the solution in several consecutive steps, accompanied by decision-making and changing the parameters of the task. For this reason, the process of finding the optimal solution is iterative and has a certain degree of interactivity, i.e. the designer can influence the process of obtaining the final solution [14]. In all cases, finding the best feasible design is not guaranteed.

With a great application for solving mechanism dimensional synthesis the problems are the evolutionary algorithms [15], which have been implemented in dedicated software products [16]. Less commonly used, but with great potential in solving optimization problems

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Correspondence to: Dr Rosen Mitrev, Associate Professor, Technical University of Sofia, Faculty of Mechanical Engineering, Bulgaria
E-mail: rosenm@tu-sofia.bg
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with mixed variables is the Monte Carlo method based on the multiple evaluations of the objective function value for randomly generated sets of values of the design variables [17]. The effectiveness of the Monte Carlo method is considerably increased when it is applied in several consecutive stages with successive narrowing of the search region based on the previous stage results [18,19]. The Parameter Space Investigation method has the same conceptual simplicity, but it is not based on the generation of uniformly distributed random numbers but the use of quasi-random LP_τ sequences [20,21]. Both methods are very suitable for optimization of functions, in which numerical problems arise related to the presence of discrete variables, inability to differentiate functions etc. or when, in addition to the optimal solution, suboptimal solutions are also accepted for practical use.

The paper aims to develop a method for an optimal selection of a standardized hydraulic cylinder for a hydraulically actuated drive mechanism considering the multicriteria character of the problem. For the achievement of this goal, a kinematic model of the mechanism is developed and an optimization dimensional synthesis is performed using the multistage Monte Carlo method.

2. KINEMATIC ANALYSIS OF THE HYDRAULICALLY ACTUATED MECHANISM

The kinematics and force equations are derived for the case of a hydraulically actuated boom [22] with kinematic length L lifting a payload with weight G_p, see Fig. 1.

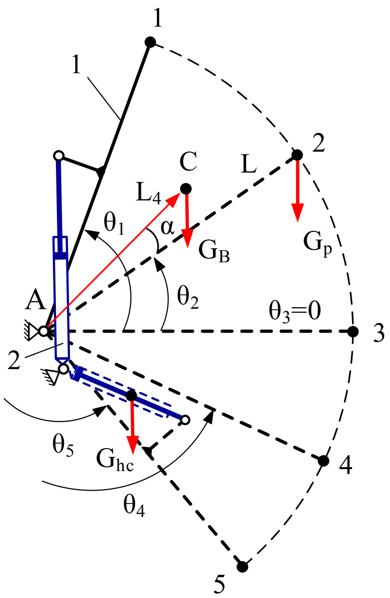


Figure 1. Graphical layout of the boom specific positions

Another two loads applied to the boom are: 1) The weight of the boom G_B applied to its gravity center whose position is determined by the vector with length L_4 and angle α; 2) The weight of the hydraulic cylinder G_{hc}, applied to its gravity center. In the current paper, for simplicity, it is considered that for every position gravity center of the hydraulic cylinder is situated in the middle of its current length. Another important assumption is that the movements of the mechanism are relatively slow and the inertia forces can be neglected.

Figure 1 shows five specific positions of the boom, determined by the corresponding angles θ_i (i=1,…,5) measured counterclockwise from the horizontal axis. The angles θ_1 and θ_5 correspond to the maximum and minimum angles of inclination of the boom respectively, and the angle θ_3 corresponds to the horizontal position of the boom. The angles θ_2 and θ_4 are defined as averages between the horizontal and the maximum and the horizontal and minimum angles of the boom inclination, respectively.

Figure 2 depicts the schematic layout of the mechanism comprised of a hydraulic cylinder 1 and a boom 2.

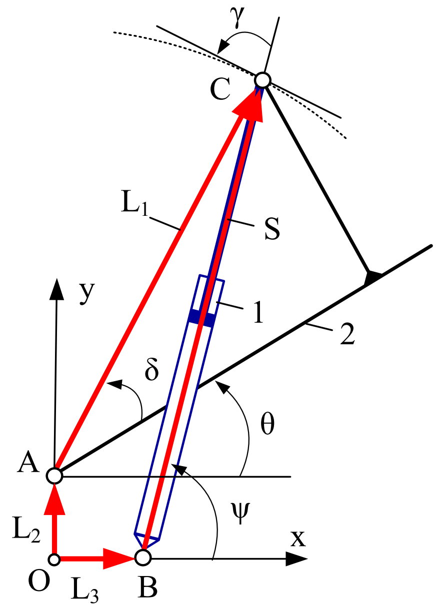


Figure 2. Schematic layout and vector closure diagram of the inverted slider-crank mechanism

By θ is denoted the current angle of rotation of the boom measured between the line representing the kinematical length of the boom and the horizontal axis. The mechanism is classified as an inverted slider-crank mechanism with input the changeable length of the hydraulic cylinder and output – the angle of rotation of the boom θ.

The kinematic analysis of the mechanism is conducted using the vector closure diagram shown in Figure 2:

$$\mathbf{L_2} + \mathbf{L_1} = \mathbf{L_3} + \mathbf{S} \tag{1}$$

whose projections onto the fixed $\{xOy\}$coordinate system are:

$$\begin{aligned} L_1 \cos(\delta + \theta) &= L_3 + S\cos\psi \\ L_2 + L_1 \sin(\delta + \theta) &= S \sin\psi \end{aligned} \tag{2}$$

After certain transformations and simplification, the equations (2) are presented as a single equation:

$$K_1 \cos\Delta + K_2 \sin\Delta + K_3 = 0 \tag{3}$$

where the following notations are used: $K_1 = -2L_1L_3$, $K_2 = 2L_1L_2$, $K_3 = L_1^2 + L_2^2 + L_3^2 - S^2$, $\Delta = \delta + \theta$.

The solution of this equation gives the following relation for the hydraulic cylinder length S and the angle of the boom rotation θ:

$$S(\theta)=\sqrt{L_1^2+L_2^2+L_3^2+2L_1\left(L_2\sin\Delta-L_3\cos\Delta\right)} \quad (4)$$

Also, the angle of inclination ψ of the hydraulic cylinder is calculated as:

$$\psi=\operatorname{atan}_2\left(\frac{L_1\cos\Delta-L_3}{S},\frac{L_1\sin\Delta+L_2}{S}\right) \quad (5)$$

The hydraulic cylinder force F_{hc} is transformed into an equivalent torque M_{hc}, applied in the joint A. Using the principle of virtual work, the mapping $T_{hc}(F_{hc})$ is obtained:

$$T_{hc}(\theta)=F_{hc}k(\theta) \quad (6)$$

where $k(\theta)$ is the influence coefficient calculated as:

$$k(\theta)=-L_1\sin(\delta+\theta-\psi) \quad (7)$$

3. DEFINITION OF FUNCTIONAL REQUIREMENTS FOR THE HYDRAULICALLY ACTUATED MECHANISM

Based on the functional purpose of the considered hydraulically actuated mechanism, the coordinates of the points of attachment C and B and the standardized hydraulic cylinder parameters should be determined in such a way so that the predefined geometric and force /moment requirements are met.

3.1 Geometric requirements

The following two types of geometric conditions must be satisfied:

1) The stroke of the hydraulic cylinder should be determined in such a way so that the predefined values of the maximum θ_1 and minimum θ_5 angles of the link 2 rotation are achieved (see Fig.1). The angle θ_1 is achieved for the maximum extended hydraulic cylinder with length L_{hc}^{max}:

$$S(\theta_1)=L_{hc}^{max}=L_T+2h \quad (8)$$

where h denotes the stroke of the hydraulic cylinder and by L_T is denoted the specific distance, computed as a difference between the length L_{hc}^{min} of the fully retracted cylinder and the stroke. Similarly, the angle θ_5 is achieved at the maximum retraction of the hydraulic cylinder:

$$S(\theta_5)=L_{hc}^{min}=L_T+h \quad (9)$$

In (8) and (9), $S(\theta_1)$ and $S(\theta_5)$ denote the maximum and the minimum lengths of the hydraulic cylinder, computed by (4).

2) The transmission angle γ is defined as (see Figure 2):

$$\gamma(\theta)=\frac{\pi}{2}+\delta+\theta-\psi \quad (10)$$

must not exceed a predefined value $[\gamma]$. This requirement must be satisfied for the larger by the absolute value angle $\theta_{max}=max(|\theta_1|,|\theta_5|)$:

$$\gamma(\theta_{\max})\le[\gamma] \quad (11)$$

3.2 Force/moment requirements

The defined geometric constraints are accompanied by the following force/moment constraints:

1) Moment constraints ensure the achievement of predefined moment values for the predefined angles of rotation. The moment $T_{hc}(\theta)$, developed by the hydraulic cylinder must overcome the total moment $M_{ext}(\theta)$ of the external forces about point A (see Fig.1):

$$T_{hc}(\theta)\ge M_{ext}(\theta) \quad (12)$$

For the considered hydraulically actuated boom the total external moment $M_{ext}(\theta)$ is calculated as a sum of the moments developed by the weights of the payload, boom and hydraulic cylinder (see Fig.1):

- Moment of the payload

$$M_p(\theta)=G_pL\cos\theta \quad (13)$$

- Moment of the boom

$$M_B(\theta)=G_BL_4\cos(\alpha+\theta) \quad (14)$$

where G_B is the weight of the boom, L_4 and α are the length and the angle of rotation of the vector defining the position of the center of gravity of the boom, respectively;

- Moment of the hydraulic cylinder

$$M_{hc}(\theta)=z\frac{m_{hc}g}{2}L_1\cos\Delta \quad (15)$$

where m_{hc} is the mass of the hydraulic cylinder; z is the number of the connected in parallel hydraulic cylinders (typical values are z=1 or z=2); g is the gravity acceleration.

The force F_{hc} developed by the hydraulic cylinder is calculated as:

$$F_{hc}=zp\frac{\pi.D^2}{4}\eta \quad (16)$$

where p is the pressure in the hydraulic cylinder chamber; D is the hydraulic cylinder piston diameter; η is the hydraulic cylinder efficiency which takes into account the friction forces between the piston and body of the hydraulic cylinder, the pressure in the rod chamber, the friction moments in the hydraulic cylinder joints etc.

2) Force constraint imposed by the hydraulic cylinder buckling. The admissible force in the hydraulic cylinder is constrained by the buckling condition:

$$F_{hc}\le z[F_{buck}] \quad (17)$$

where $[F_{buck}]$ is the admissible force in the hydraulic cylinder due to the buckling constraint [10]:

$$[F_{buck}]=\begin{cases}\dfrac{\pi^2EI}{\nu\left(L_{hc}^{max}\right)^2}, if\ \lambda>\lambda_g\\[2ex] \dfrac{\pi d^2(335-0.62\lambda)}{4\nu}, if\ \lambda\le\lambda_g\end{cases} \quad (18)$$

where $L_{hc}^{\max}$ (in mm) is the free buckling length of the hydraulic cylinder; λ is the slenderness ratio:

$$\lambda = \frac{4L_{hc}^{\max}}{d} \tag{19}$$

λ_g is the critical value of the slenderness ratio:

$$\lambda_g = \pi\sqrt{\frac{E}{0.8R_e}} \tag{20}$$

Also, the following values of the constants are used: E=2.1×10^5 MPa is the modulus of elasticity for steel; I=0.0491d^4 is the moment of inertia of circular cross-sectional area, (in mm^4); ν=3.5 is the safety factor; d is the diameter of the piston rod (in mm); R_e=400 MPa is the yield strength of the piston rod material.

4. DEFINITION OF A DESIGN OPTIMIZATION PROBLEM

There is a great variety of technical aspects concerning the design of the considered mechanism. The solution of the task of hydraulic cylinder selection and its points of attachment is ambiguous due to the possibility to select more than one standardized hydraulic cylinders, satisfying with different degree of approximation the predefined requirements. Typically, the requirements for the diameter and stroke of the hydraulic cylinder to be simultaneously minimal are contradictory because moving the axis of the hydraulic cylinder away from the axis of rotation of the boom reduces its diameter but increases its stroke and vice versa. A multiobjective optimization task should be defined here, the solution of which will minimize both the stroke and the diameter of the hydraulic cylinder at the same time and will satisfy the defined geometric and force/moment requirements. The common characteristic combining the mentioned parameters is the working volume of the hydraulic cylinder whose minimal value is searched:

$$V_{hc} = zh\frac{\pi.D^2}{4} \tag{21}$$

4.1 Determination of design variables

As design variables of the considered mechanism determining its output geometric and force/moment characteristics the following variables are considered:

$$\mathbf{x} = \begin{bmatrix} L_1 & L_2 & L_3 & \delta & p & D & h \end{bmatrix}^T \tag{22}$$

where L_1, L_2, L_3 and δ are geometric variables (see Figure 2) determining the coordinates of the attachment points of the hydraulic cylinder; p is the pressure in the hydraulic cylinder piston chamber; D and h are the piston diameter and the stroke of the hydraulic cylinder, correspondingly.

The variables L_1, L_2, L_3, δ and p are considered as continuous in a predefined interval of values. The values of D and h are of discrete type and are determined by the standardized manufacturer's size range. Table 1 shows the numerical values of the following parameters of the range of standardized double-acting hydraulic cylinders: diameter D of the piston; diameter d of the piston rod; specific length L_t; basic mass m_b of the cylinder body for stroke h=0; mass m_h which must be added to the basic mass m_b for every 1 m stroke.

Using the data from Table 1 the overall mass of a hydraulic cylinder with stroke h is calculated as:

$$m_{hc} = m_b + m_h h \tag{23}$$

where the stroke h is measured in meters.

4.2 Definition of a multi-objective optimization problem

Here, the optimization problem is considered as multi-objective [23] consisting of three objective functions.

The first objective function is defined from the consideration that the hydraulically actuated mechanism is a part of a mobile machine and minimum mass of the selected hydraulic cylinder is required:

$$f_1(\mathbf{x}) = zm_{hc} \rightarrow \min \tag{24}$$

The mass of the hydraulic cylinder is proportional to its working volume (21) so the minimum of the mass will correspond to the minimum of the working volume. An additional important consideration is that the minimization of the mass improves the mechanical system dynamics and increases the tip-over stability of the machine [24].

Since it is necessary to achieve a certain degree of proximity of the moment $T_{hc}(\theta)$ developed by the hydraulic cylinder to the predefined external moments $M_{ext}(\theta_i)$ for five angles according to (12) and Figure 1, as a second objective function is accepted the squared total deviation of the developed by the hydraulic cylinder moments from the predefined values of the external moments:

$$f_2(\mathbf{x}) = \sum_{i=1}^{5}\left(T_{hc}(\theta_i) - M_{ext}(\theta_i)\right)^2 \rightarrow \min \tag{25}$$

As a third objective function defined by the mechanism links and supporting structure strength requirements was adopted the minimum of the force in the hydraulic cylinder:

Table 1. The size range of standardized double-acting hydraulic cylinders [10]

№	1	2	3	4	5	6	7	8	9	10	11	12	13	14
D, mm	40	50	63	80	100	125	140	160	180	200	220	250	280	320
d, mm	28	36	45	56	70	90	100	110	125	140	160	180	200	220
L_t, mm	482	505	570	620	695	790	860	930	1025	1090	1260	1300	1465	1630
m_b, kg	7	10	16	26	44	80	112	169	239	309	452	582	753	1125
m_h, kg	10	15	26	36	57	92	119	139	168	215	309	369	488	604

$$f_3(\mathbf{x}) = F_{hc} \to \min \tag{26}$$

Taking into account the defined geometric requirements (8)÷(11), force/moment requirements (12)÷(20) and the defined objective functions (24)÷(26) the following optimization task is defined:

Minimize the set of objective functions

$$f(\mathbf{x}) = \left[f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})\right] \tag{27}$$

subject to the following set of:

- Functional constraints

$$\begin{aligned}
&g_1(\mathbf{x}) \equiv \left|S(\theta_1) - L_{hc}^{\max}\right| - \varepsilon \le 0\\
&g_2(\mathbf{x}) \equiv \left|S(\theta_5) - L_{hc}^{\min}\right| - \varepsilon \le 0\\
&g_3(\mathbf{x}) \equiv \gamma(\theta_{\max}) - [\gamma] \le 0\\
&g_{4,\dots,8}(\mathbf{x}) \equiv M_{ext}(\theta_i) - T_{hc}(\theta_i) \le 0,\ i = 1,\dots,5\\
&g_9(\mathbf{x}) \equiv F_{hc} - z\left[F_{buck}\right] \le 0
\end{aligned} \tag{28}$$

- Interval constraints:

$$\begin{aligned}
&L_1^{\min} \le L_1 \le L_1^{\max},\ L_2^{\min} \le L_2 \le L_2^{\max}\\
&L_3^{\min} \le L_3 \le L_3^{\max},\ \delta_2^{\min} \le \delta \le \delta_2^{\max}\\
&p^{\min} \le p \le p^{\max}
\end{aligned} \tag{29}$$

- Discrete variables h and D.

For the definition of the constraint $g_1(\mathbf{x})$ corresponding to the maximum angle of rotation θ_1 is accepted that when hydraulic cylinders with standardized discrete strokes are used it is not always possible to satisfy (8) exactly. Therefore an allowable deviation ε between the theoretical length $S(\theta_1)$ and the length $L_{hc}^{\max} = L_t + 2h$ of the standardized hydraulic cylinder is defined. As a consequence of the presence of an allowable deviation, the required value of the angle θ_1 will not be achieved, but for small ε the error will be acceptable. The same considerations are used in defining the constraint $g_2(\mathbf{x})$ for the minimum angle of rotation θ_5. Besides, the following values of the constants are used: $\theta_1 = 70^0$, $\theta_2 = 35^0$, $\theta_3 = 0^0$, $\theta_4 = 335^0$, $\theta_5 = 310^0$, $[\gamma] = 85^0$, $\varepsilon = 0.001\,\mathrm{m}$, $G = 50\,\mathrm{kN}$, $L = 10\,\mathrm{m}$, $\eta = 0.95$, $G_B = 10\,\mathrm{kN}$, L_4=4.2 m, α=10^0.

5. NUMERICAL EXPERIMENTS AND DISCUSSION

Many researchers applied a variety of optimization methods to improve the performance of systems with a different degree of complexity and belonging to different domains [25-28] using appropriate analytical and numerical tools [29]. The defined multiobjective optimization task (27) is considered as a mixed variable nonlinear constrained optimization problem containing 5 continuous and 2 discrete variables. For its solution is used the well-known Monte Carlo method based on uniform random sampling with a large number of trial points [18]. The steps of the method are as follows:

a) Specification of the number of trial points n;

b) For i=1 to n a trial point with coordinates $\mathbf{x}^{(i)}$ is generated:

$$\mathbf{x}^{(i)} = \left(L_1^{(i)}, L_2^{(i)}, L_3^{(i)}, \delta^{(i)}, p^{(i)}, D^{(i)}, h^{(i)}\right) \tag{30}$$

The values $C^{(i)}$ of the continuous variables for the trial point i are computed as:

$$C^{(i)} = C_L + \left(C_U - C_L\right)u^{(i)} \tag{31}$$

where C_L and C_U denote the lower and upper bounds of the corresponding variable; $u^{(i)}$ denotes values of uniformly distributed random numbers between 0 and 1 different for each design variable to ensure their mutual independence.

The values of the discrete variables at step i are randomly selected from vectors containing their discrete values. The position $f^{(i)}$ of the randomly selected discrete value in the vector with length q is calculated as:

$$f^{(i)} = round\left[1 + (q-1)u^{(i)}\right] \tag{32}$$

where *round* denotes the rounding to the nearest integer;

c) If for the current trial point the constraints are satisfied then the values of the three objective functions are calculated and together with the coordinates of the corresponding trial points $\mathbf{x}^{(i)}$ are stored:

$$f\left(\mathbf{x}^{(i)}\right) = \left[f_1\left(\mathbf{x}^{(i)}\right), f_2\left(\mathbf{x}^{(i)}\right), f_3\left(\mathbf{x}^{(i)}\right)\right] \tag{33}$$

The set of the stored values $f(\mathbf{x}^{(i)})$ satisfying the constraints represents a discrete approximation of the feasible space.

d) The optimal solution of the defined multiobjective optimization task consists not of a single point but of a set of points representing compromise solutions among the defined three objectives. This set is called Pareto front [23] and includes nondominated (efficient) solutions for which none of the three objectives can be improved without causing deterioration of the value of at least one of the other objectives. That's why after determination of the discrete feasible space a Pareto filter is applied to determine the points belonging to the Pareto front. Then by certain criteria, an optimal point from the obtained Pareto front is selected.

Typically, to discover the region in which the global optimum is located, one repeats the steps from a) to d) several times with decreased bounds of the design variables. The change of the bounds at each subsequent stage is based on the conducted analysis of the obtained Pareto front from the previous stage.

The choice of a single compromise point from the obtained Pareto front is based on the minimum value of the calculated squared distance d^2 between the normalized Utopia point and every point belonging to the Pareto front (Salukvadze optimal solution [30]):

$$d^2 = \sum_{l=1}^{3} w_l \left(f_l^n\right)^2 \tag{34}$$

where w_l is the weighting coefficient serving as a quantitative expression of the importance of the l-th objective function. The weighting coefficients are positive, satisfying

$$\sum_{l=1}^{3} w_l = 1,\ 0 \le w_l \le 1 \tag{35}$$

Also, in (34) f_l^n denotes the value of the l-th normalized objective function:

$$f_l^n = \frac{f_l - f_l^{\min}}{f_l^{\max} - f_l^{\min}} \tag{36}$$

where $f_l^{\min}$ and $f_l^{\max}$ are the minimum and maximum values of the l-th objective function.

The multistage Monte Carlo method combined with the Pareto front determination is applied for the solution of the defined optimization task (27). The used data, obtained results and the analysis for every stage are as follows:

Stage I. The number of the trial points used at this stage is 10×10^9. The bounds of the continuous design variables are determined from the design limitations of the real-world structure and are shown in Table 2 where by C_L and C_U are denoted the lower and upper bounds of the variables respectively.

Table 2. Design variables bounds for Stage I

	L_1, m	L_2, m	L_3, m	δ, deg	p, MPa
C_L	3	0.4	0.4	14	8
C_U	4.1	1.5	1.5	21	25

Table 3. Pareto optimal solutions - Stage I

№	f_1, kg	$f_2\times10^9$, (Nm)^2	$f_3\times10^5$ N	L_1, m	L_2, m	L_3, m	D, m	h, m	δ, rad	p, MPa
1	1082.8	8.02	5.69	3.680	0.949	0.452	0.18	1.8	0.278	11.19
2	1015.6	11.54	6.49	3.378	0.826	0.443	0.18	1.6	0.343	12.74
3	1049.2	16.69	6.26	3.551	0.839	0.523	0.18	1.7	0.359	12.31
4	1116.4	19.45	5.60	3.799	1.003	0.499	0.18	1.9	0.341	11.00
5	1183.6	17.64	4.99	4.075	1.140	0.504	0.18	2.1	0.316	9.80
6	1150	27.89	5.55	3.979	1.066	0.478	0.18	2.0	0.246	10.90
7	1150	34.53	5.42	3.909	1.109	0.448	0.18	2.0	0.316	10.65
8	982	47.64	7.70	3.270	0.767	0.406	0.18	1.5	0.256	15.14
9	948.4	48.54	8.26	3.105	0.707	0.402	0.18	1.4	0.323	16.22
10	1435	5.31	5.33	3.881	0.990	0.509	0.20	1.9	0.328	8.49

Table 4. Design variables bounds for Stage II

	L_1, m	L_2, m	L_3, m	δ, deg	p, MPa
C_L	3.1	0.7	0.4	14	8
C_U	4.1	1.2	0.53	21	16

Table 5. Pareto optimal solutions - Stage II

№	f_1, kg	$f_2\times10^9$, (Nm)^2	$f_3\times10^5$ N	L_1, m	L_2, m	L_3, m	D, m	h, m	δ, rad	p, MPa
1	1015.6	3.41	6.12	3.391	0.802	0.474	0.18	1.6	0.360	12.04
2	982	3.25	6.51	3.243	0.769	0.414	0.18	1.5	0.321	12.80
3	1049.2	3.24	5.77	3.534	0.877	0.467	0.18	1.7	0.322	11.33
4	1082.8	3.88	5.50	3.689	0.959	0.426	0.18	1.8	0.237	10.80
5	1116.4	3.27	5.18	3.835	0.999	0.479	0.18	1.9	0.268	10.17
6	948.4	9.23	7.31	3.103	0.707	0.403	0.18	1.4	0.328	14.36
7	1150	3.801	4.96	3.982	1.041	0.528	0.18	2	0.290	9.74
8	1183.6	9.96	4.89	4.096	1.131	0.503	0.18	2.1	0.289	9.61

Table 6. Design variables bounds for Stage III

	L_1, m	L_2, m	L_3, m	δ, deg	p, MPa
C_L	3.1	0.75	0.4	14	9
C_U	4.1	1.13	0.53	21	14

Table 7. Pareto optimal solutions - Stage III

№	$\boldsymbol{f_1}$, kg	$\boldsymbol{f_2}\times10^9$, (Nm)^2	$\boldsymbol{f_3}\times10^5$ N	$\boldsymbol{L_1}$, m	$\boldsymbol{L_2}$, m	$\boldsymbol{L_3}$, m	$\boldsymbol{D}$, m	$\boldsymbol{h}$, m	$\boldsymbol{\delta}$, rad	$\boldsymbol{p}$, MPa
1	1049.2	3.22	5.76	3.538	0.892	0.435	0.18	1.7	0.280	11.32
2	1015.6	3.21	6.13	3.386	0.839	0.414	0.18	1.6	0.295	12.05
3	1082.8	3.47	5.47	3.689	0.949	0.447	0.18	1.8	0.259	10.75
4	1082.8	3.46	5.48	3.688	0.949	0.446	0.18	1.8	0.258	10.75
5	982	3.03	6.50	3.239	0.776	0.406	0.18	1.5	0.316	12.77
6	1116.4	3.47	5.18	3.831	0.983	0.517	0.18	1.9	0.312	10.19
7	1116.4	3.46	5.19	3.834	0.975	0.527	0.18	1.9	0.320	10.20
8	1150	3.51	4.93	3.985	1.048	0.514	0.18	2.0	0.272	9.69
9	1183.6	7.78	4.84	4.099	1.125	0.513	0.18	2.1	0.294	9.51

Table 8. Compromise solutions for different weighting schemes

№	w_1	w_2	w_3	f_1, kg	$f_2\times10^9$, (Nm)^2	$f_3\times10^5$, N	L_1, m	L_2, m	L_3, m	D, m	h, m	δ, rad	p, MPa
1	0.(3)	0.(3)	0.(3)	1082.8	3.47	5.47	3.689	0.949	0.447	0.18	1.8	0.259	10.75
2	0.7	0.15	0.15	1015.6	3.21	6.13	3.386	0.839	0.414	0.18	1.6	0.295	12.05
3	0.15	0.7	0.15	1049.2	3.22	5.76	3.538	0.892	0.435	0.18	1.7	0.280	11.32
4	0.15	0.15	0.7	1116.4	3.47	5.18	3.831	0.983	0.517	0.18	1.9	0.312	10.19
5	1	0	0	982	3.03	6.50	3.239	0.776	0.406	0.18	1.5	0.316	12.77
6	0	1	0	982	3.03	6.50	3.239	0.776	0.406	0.18	1.5	0.316	12.77
7	0	0	1	1183.6	7.78	4.84	4.099	1.125	0.513	0.18	2.1	0.294	9.51

The vectors containing the discrete values for the standardized diameters and strokes of the hydraulic cylinders are taken from Table 1:

$$D \in \begin{Bmatrix} 0.1, 0.125, 0.14, 0.16, 0.18, \\ 0.20, 0.22, 0.25, 0.28, 0.32 \end{Bmatrix} m$$

and

$$h \in \begin{Bmatrix} 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, \\ 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7 \end{Bmatrix} m .$$

By the conducted numerical experiment at this stage, 488 feasible points are found. In Figure 3a) they are shown in the 3D objective functions space together with the points belonging to the Pareto front. The Pareto front determined for the first stage contains 10 points for which objective functions values and its corresponding set of design variables are shown in Table 3. It must be noted that the ratio of the feasible points and the number of trial points is equal to 4.8×10^{-8}, which points out that the problem is difficult to solve due to the imposed very restrictive constraints;

Stage II. The analysis of the values of the objective functions and design variables for the obtained Pareto front at Stage I (see Table 3) makes it possible to narrow the intervals of change of the variables. The accepted bounds of the continuous design variables for Stage II are shown in Table 4.

The sets of the values of the discrete variables are reduced to $h \in \{1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1\}\, m$ and $D \in \{0.18, 0.20\}\, m$.

At this stage, a numerical experiment with 5×10^9 trial points is conducted. The number of the feasible points is 10582 whose number is much greater than the points obtained at Stage I. The ratio of the feasible points and the number of trial points is equal to 2.12×10^{-6}, which means that the procedure of determination of feasible points is more effective than during Stage I. The obtained feasible space and Pareto front points are shown in Figure 3b), and the numerical values for the Pareto front are shown in Table 5. As one can see, the value of the piston diameter D is equal to 0.18 m for all points in the Pareto front and thus this value can be considered optimal;

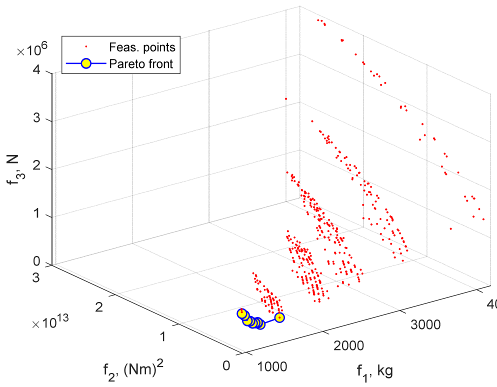
Stage III. Based on the results for Stage II, Table 6 shows the accepted bounds of the continuous design variables.

The sets of the values of the discrete variables are reduced to $h \in \{1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1\}\, m$ and $D \in \{0.18\}\, m$.

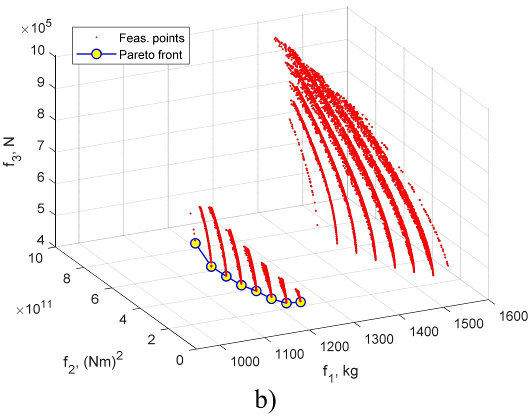
From the conducted numerical experiment with 5×10^9 trial points, 4557 feasible points are obtained. The feasible points and the Pareto front are shown in Figure 3c) and the numerical values of the points belonging to the Pareto front are shown in Table 7.

As one can see from Tables 3, 5 and 7, narrowing the limits of the variables during the different stages leads to an improvement in the values of the solutions belonging to the Pareto front. The objective functions have different sensitivity - the value of criterion f_2 has the biggest improvement, the value of the objective function f_3 is less improved, and the value of f_1 has not changed.

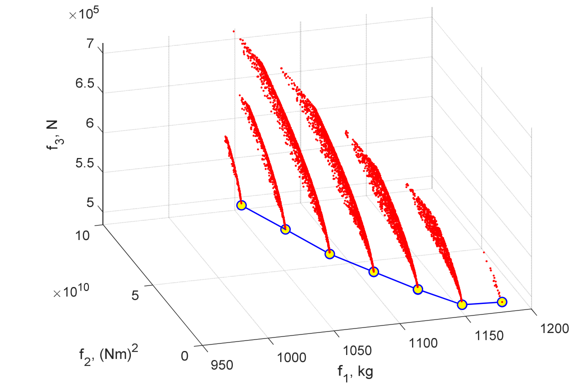
Table 8 shows the optimal solutions calculated according to (34) for seven different combinations of the weighting factors. As one can see the optimal solution strongly depends on the chosen values of the weighting coefficients. In the first combination of weights, it is assumed that all objective functions are of equal importance, i.e. $w_1=w_2=w_3=0.(3)$. In the combinations №2, №3 and №4, a higher priority is given to each of the three objective functions through assigning higher values of the corresponding weighting factor. The combinations №5, №6 and №7 present absolute minimums of the three objective functions.



a)



b)



c)

Figure 3. The feasible points and Pareto front for: a) Stage I; b) Stage II; c) Stage III

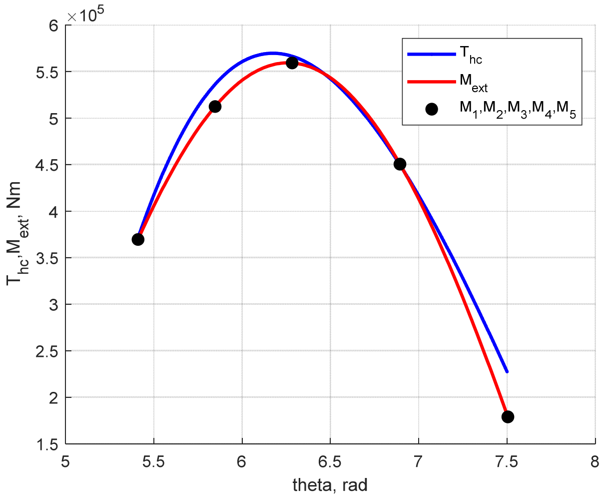


Figure 4. Moments $T_{hc}(\theta)$ and $M_{ext}(\theta)$ for solution №5 from Table 8

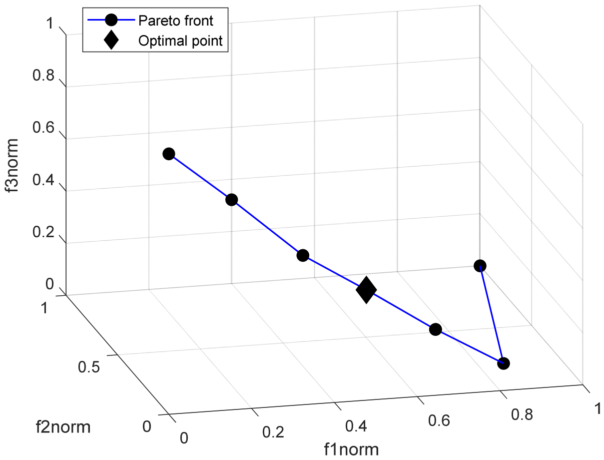


Figure 5. The normalized Pareto front and the optimal point for solution №5 from Table 8

Although at the present stage the obtained solutions can be considered acceptable, the values of the design variables for every single solution can serve as initial points for further refinement. To do this, the obtained values of the discrete variables are fixed and an additional searching in a small region, including the obtained values of the continuous variables is performed, i.e. an additional refinement of the attachment points of the selected hydraulic cylinder is performed. The values thus obtained for solution №5 from Table 8 are as follows: f_1=982 kg, f_2=2.84×10^9 (Nm)^2, f_3=6.48×10^5 N, D=0.18 m, h=1.5 m, L_1=3.241 m, L_2=0.764 m, L_3=0.425 m, δ=0.339 m, p=12.73 MPa. As one can see, there is a more noticeable improvement for the value of the objective function f_2.

Fig. 6 depicts a scaled geometrical layout of the mechanism for solutions №1 and №5 of Table 8. It demonstrates that, by giving different weights to different objective functions, the mechanism dimensions can be adjusted. It can be seen that sol. №5 has smaller dimensions than sol. №1, and the hydraulic cylinder is lighter, but on the other hand the force in the hydraulic cylinder is greater in solution №5.

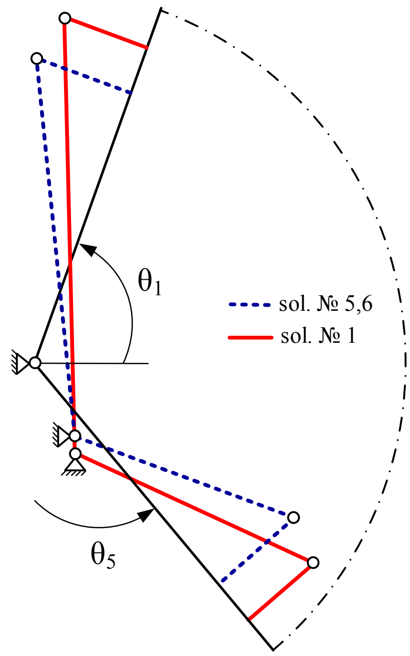


Figure 6. Scaled geometric layout of the attachment points for solutions № 1 and 5 from Table 8

The solutions №5 and №6 of Table 8 deserves special attention since they simultaneously have a minimum of the objective functions f_1 and f_2 equal to 982 kg and 3.03×10^9 (Nm)^2 respectively, while the function f_3 has a maximum of 6.5×10^5 N. From a practical point of view, this solution does not differ much from the solution №1 from Table 8 with equal importance of all objective functions, and it is also acceptable for implementation in a real-world structure. Figure 4 shows the graphs of the moments $T_{hc}(\theta)$ and $M_{ext}(\theta)$ for solution №5, while Figure 5 depicts the normalized Pareto front and the optimal point for the same case. As one can see, the constraints $g_5(\mathbf{x})$÷$g_9(\mathbf{x})$ are completely satisfied and the difference between the two curves is acceptable.

The verification of the developed optimization model is conducted using the ε-constraint method [22]. To preserve the multiobjective character of the optimization problem is used as an objective function f_2→min and the objective functions f_1 and f_3 are used as additional constraints and are added to the set of constraints (28), i.e. f_1-1000≤0 and f_3-7×10^5≤0. Using a nonlinear programming solver based on genetic algorithms, the single objective optimization problem is solved and the following optimal values of the design variables are obtained: L_1=3.244 m, L_2=0.761 m, L_3=0.427 m, δ=0.337 rad, p=13.27 MPa, D=0.18 m,

h=1.5 m. The obtained values for the objectives are f_1=982 kg, f_2=2.72×10^9 (Nm)^2 and f_3=6.42×10^5 N. As one can see, the differences in the objective functions compared to the refined solution №5 from Table 8 are negligible.

6. CONCLUSION

In the designer's practice, the process of selecting a hydraulic cylinder for hydraulically actuated mechanisms is inherently iterative. The designer of such mechanisms repeatedly uses analytical and graphical techniques to select a suitable standardized hydraulic cylinder and checks whether the resulting solution meets predefined requirements. To some extent, the design process is also intuitive and based on the experience of the designer. Using trial and error method it is challenging to achieve an optimal selection that cannot be improved or an improvement can be achieved with a large expenditure of time and effort.

The developed method for optimization synthesis of hydraulically driven mechanisms allows multiobjective optimal selection of a standardized hydraulic cylinder satisfying preset geometric and force requirements. By applying the multi-stage Monte Carlo method the Pareto front is obtained. It contains several compromise solutions that are Pareto-optimal. The choice of a certain solution from the Pareto set is made based on Salukvadze's criterion by setting proper weighting factors for each of the objective functions. Although the Monte Carlo method is computationally inefficient, with reasonable effort optimal and suboptimal solutions can be obtained that can be implemented in real-world machines. A great advantage of this approach is its interactivity - in the process of optimization, the designer can adjust the intervals of change of the design variables and constraints, as well as not to comply with their type - continuous or discrete.

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ВИШЕКРИТЕРИЈУМСКИ ОПТИМАЛНИ ИЗБОР ХИДРАУЛИЧКОГ ЦИЛИНДРА ЗА ПОГОНСКЕ МЕХАНИЗМЕ

Р.П. Митрев, Т.С. Тодоров

Рад приказује синтетизовани метод оптимизације примењен код хидраулички активираног погонског механизма. Развијен је математички модел механизма помоћу једначине затварања вектора. На основу функционалности механизма дефинисан је скуп захтева, геометријских и сила/момент, који се морају задовољити адекватним избором стандардизованог хидрауличког цилиндра и тачака везивања. Задатак вишециљне оптимизације дизајна је дефинисан помоћу три објективне функције код којих се тражи минимум – маса хидрауличког цилиндра, квадрат укупног одступања момента цилиндра из унапред одређених вредности спољашњих момената и сила цилиндра. Задатак оптимизације се разматра као проблем оптимизације мешовитих променљивих и нелинеарних ограничених вредности и садржи 5 непрекидних и 2 дискретне променљиве, док се Монте Карло метод користи за решавање проблема. Коришћењем различитих шема пондерисања добијено је неколико Парето оптималних решења.