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Analysis of Impact of Shaft Speed and External Load on the Radial Ball Bearing Lubrication Regimes

Appropriate lubrication of the rolling bearing is needed to lower the friction between the surfaces in mutual contact and their wear. A lubricant should completely separate the rolling elements from the raceways. The values that affect the efficiency and regimes of lubrication are analyzed in the paper, after which it is introduced a lubrication regime coefficient. This coefficient makes it possible to choose in a simple and fast manner an optimum combination of rolling bearing and lubricant based on the known shaft speed, external load, and rolling bearing operating temperature. For certain bearings with radial contact and certain lubricants, the lubrication regime dependence on the shaft speed and contact load is shown.

Keywords: *lubrication regime coefficient, lubrication, rolling bearing, angular speed, external load.*

1. INTRODUCTION

The friction in the contact of rolling bearing surfaces causes their wear and damage. It also generates heat, and by it, the thermal dilatations, which can in a negative way affect the rolling bearing geometrical properties and lubricant features.

In their monograph [1], Harris and Kotzalas gave a comprehensive analysis of the above-mentioned phenomena. They consider the rolling bearing geometric and kinematic features, calculation of contact stress and deformation, load distribution within the rolling bearing, mechanisms of heat generated in bearing, friction mechanisms, operating surfaces lubrication, etc.

Load distribution between rolling elements is one of the important factors influencing the magnitude of the contact load and over it the level of friction between the rolling bearing operating surfaces. The first attempt to describe the distribution of external radial load was made by Stribeck [1]. His model enables the calculation of the maximum ball load in a very simple way, as a function of external load, several balls, and Stribeck's number. However, the model is rough, especially when used on bearings with internal radial clearance, because it doesn't take into account the clearance value, but only the fact that clearance exists.

On the other side, Tomović in his studies [2-5] came up with the external load distribution model for the radial rolling bearings. He developed a mathematical model of load distribution and analyzed the influence of certain factors on the number of rolling elements taking part in the load transfer from one ring to another. He concluded that the number of active rolling elements and load distribution are affected by the contact stiffness, the total number of rolling elements, internal radial clearance, and external load size.

The influence of the load distribution on some rolling bearing properties, such as service life, load capacity, and radial stiffness has been studied and introduced in the papers of Mitrović [6,7]. In [8] he analyzed the dependence of equivalent-von Mises stresses, material strength, and radial stiffness on the bearing operating temperature, while in [9] he analyzed the interaction between high-speed radial rolling bearing operating ability and its tribological and structural parameters. He concluded that bearing limiting operating speed depends on a very large number of factors, such as load intensity, type and amount of lubricant, heat removal, internal geometry, radial clearance, energy losses, etc.

Another, very important, value that affects the intensity of friction in a significant way is the coefficient of friction between the contact surfaces. It highly depends on the quality of lubrication, i.e. the lubricant properties, operating temperature, and the height of surface micro-asperities in the first place.

Due to its high importance, the lubrication of rolling bearing was investigated by a lot of researchers [10-16]. Hamrock and Dowson in their papers [14-16] came up with significant results. Their research activities resulted in the relation between the lubricant film thickness and parameters of speed, load, material, and geometry. Based on the expression for the minimum film thickness and the expression for contact elements elastic deformation, it is possible to determine the film thickness at any point of the contact area. Adding

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additives to lubricant influences its properties and lubricating abilities. In [17], the authors analyzed the mechanism in which additives impact the lubricant viscosity and proposed the rheological model of a thin oil lubricating film.

Besides the primary function of friction reduction between the contacting surfaces, lubricant protects them from corrosion and reduces the noise level caused during rolling bearing operation. Technical greases and oils are used for lubrication and in special cases, solid lubricants, depending on the load, rotational frequency, and operating temperature.

Jones et al. [18] have dealt with the experimental determination of the individual lubricant properties. They analyzed the property's dependence on the operating pressure and temperature. Due to significant changes in the lubricant viscosity during rolling bearing operation, Harris [1], based on their empirical results, developed the expressions that establish the viscosity dependence on the temperature and pressure values. A brief description of the properties of certain types of lubricants with the indicated areas of their application is given in [19].

Surfaces roughness has a very great impact on the quality of lubrication and the lubrication film thickness necessary to ensure the complete separation of surfaces, to completely avoid the metal contact. Wang et al. [20] were evaluating the impact of lubricated surfaces roughness with the initial point contact on the sliding friction magnitude. On the other hand, Assoudi et al. [21] analyzed the interaction between surface roughness of the wall and the flow of fluid over the wall, in the terms of streamlines and pressure and velocity components.

Generally, proper lubrication can significantly affect the extension of rolling bearing service life and improve its performance. If the lubricant film between contact surfaces fully enables their separation, as is the case with elastohydrodynamic lubrication (EHL), the friction will be substantially less, which will cause the bearing service life extension and greater energy efficiency. In contrast, if the balls come into direct contact with raceways, friction and wear of contact surfaces would be much higher, causing the enhancement of the heat generation intensity. Therefore, bearing lubrication is performed, and a very thin layer (film) of lubricant enters the contact zone and becomes an intermediary in transferring the load. This prevents direct metal surfaces contact and extends the bearing life.

It is very important to choose the right combination of bearing and lubricant to meet anticipated operating conditions and operate in such a lubrication mode that allows a longer bearing service life. In the present case, the entire bearing lubrication is analyzed through the lubrication of individual balls.

The effects of applied lubricant, operating conditions, and bearing construction parameters on the different lubrication regimes which can occur in rolling bearings are analyzed in this paper. Based on this analysis, the paper introduces the so-called lubrication regime coefficient. It shows the dependence of the regime of lubrication on the rolling bearing geometry, operating conditions, lubricant properties. The main purpose of the coefficient is to facilitate the choice of the proper combination of the rolling bearing and lubricant to achieve the most favorable regime of lubrication.

The paper also presents the impact analysis of lubricant type, shaft speed, contact load/external load, operating temperature, and bearing micro and macro-geometry on different regimes occurrence for specific radial rolling bearings, with the introduced assumption of negligibility of mutual slipping contact surfaces.

2. ROLLING BEARING LUBRICATION REGIMES

The surfaces of balls and raceways can never have a perfect geometric shape, due to the occurrence of micro-asperities. In some cases, asperities of balls and raceways may penetrate the thin lubricant layer coming into contact with each other.

Lubricants task is to detach the operating contact surfaces, wherein the full or partial separation may occur, depending on the lubricant film thickness and asperities height. The contact surfaces separation degree and the bearing lubrication efficiency are described with the parameter λ:

$$\lambda = \frac{h_{\min}}{\sqrt{{R_{qr}}^2 + {R_{qb}}^2}}, \tag{1}$$

wherein R_{qr} and R_{qb} [mm] represent the roughness mean square of raceways and rolling elements, and $h_{\min}$ [mm] minimum film thickness. Depending on the λ parameter value, there are three forms of lubrication (Figure 1):

- complete or elastohydrodynamic lubrication (EHL), for λ>3 (film allows complete separation of contact surfaces, and the load is fully transmitted through the lubricant),
- boundary lubrication, for λ<1 (film thickness is not large enough to achieve complete separation of the contact surfaces, and the load is transferred only through asperities peaks, while lubricant fills the empty space and does not participate in load transfer) and
- mixed lubrication, λ=1÷3 (the combination of the first two forms of lubrication, so both peaks and lubricant take a part in load transfer) [1,13,14,24].

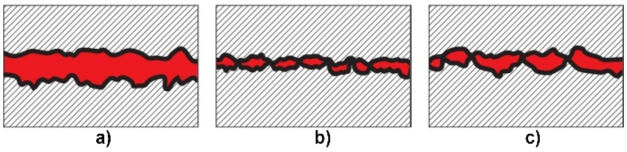


Figure 1. Lubrication: a) EHL, b) boundary c) mixed [22]

EHL is the best in terms of friction, heat generation, and bearing service life. Mixed lubrication is unfavorable because the film thickness is not large enough to prevent peaks penetration and the establishment of partially metal contact. Even worse is the case of boundary lubrication, where the load is only transmitted over the asperities.

Which of the above-mentioned three cases of lubrication will occur, primarily depends on the film thickness and surface roughness parameters, according to equation (1).

3. MINIMUM OIL FILM THICKNESS

Lubricant thickness is not constant in the zone of contact of rolling elements and raceways. The thickness at any contact area point is expressed as a function of the minimum film thickness, as shown in Figure 2 [12].

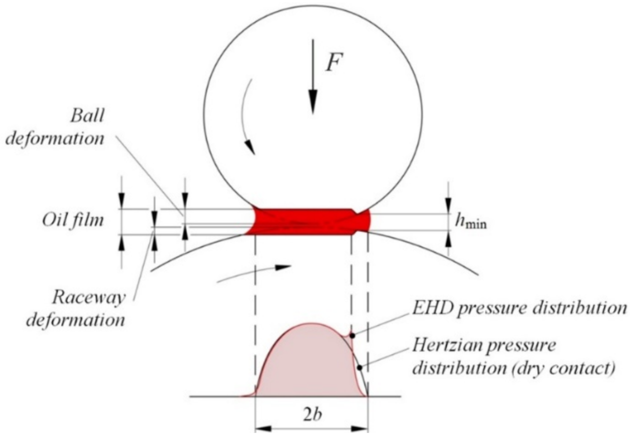


Figure 2. The pressure distribution in the contact zone during EHL [12]

The minimum oil film thickness for the initial contact in point is defined by Hamrock and Dowson [15] according to the following expression:

$$h_{\min} = 3.63\underline{\mathrm{U}}^{0.68}\underline{\mathrm{G}}^{0.49}\underline{\mathrm{W}}^{-0.073}\left(1-\exp\left(-0.68\kappa\right)\right)R_x\left[\mathrm{m}\right] \quad (2)$$

wherein the dimensionless parameters of speed $\underline{\mathrm{U}}$, material $\underline{\mathrm{G}}$, and load $\underline{\mathrm{W}}$ are given by the expressions:

$$\underline{\mathrm{U}} = \frac{\eta_0 U}{E' R_x}, \quad (3)$$

$$\underline{\mathrm{G}} = \alpha_0 E', \quad (4)$$

$$\underline{\mathrm{W}} = \frac{F}{E' R_x^{\,2}}, \quad (5)$$

wherein: κ is an elliptical eccentricity parameter (the ratio of the major and minor axis of the contact ellipse), η_0 [Pa·s] lubricant viscosity at the atmospheric pressure, U [m/s] entrainment velocity (velocity with which fluid is swept into the rolling element-raceway contact), E' [Pa] reduced modulus of elasticity, R_x [m] reduced curvature radius in the direction of the contact ellipse main axis (x), α_0 [Pa^{-1}] atmospheric pressure-viscosity coefficient (the lubricant characteristics which is determined experimentally), while F [N] is a contact load of particular rolling element, which depends on the external load Q distribution, and can be determined according to the load distribution models presented in [2,7-9].

There is a fluid viscosity at atmospheric pressure η_0 in the expression (3), not viscosity at contact pressure η, but also atmospheric pressure-viscosity coefficient α_0 in the expression (4), not contact pressure-viscosity α, as the oil film thickness is the function of the fluid characteristic at the entrance of the contact zone [1], and lubricant at the entrance is not exposed to the contact pressure. The above-mentioned characteristics for the particular oil lubricants are presented in [18].

The reduced radius of curvature in the x-axis direction is given by [1]:

$$R_x = \frac{1}{\rho_{\mathrm{I}x} + \rho_{\mathrm{II}x}}, \quad (6)$$

wherein $\rho_{\mathrm{I}x}$ and $\rho_{\mathrm{II}x}$ [1/m] are the curvature radii of the rolling elements and raceways surfaces in the direction of the x-axis. They are equal to the reciprocal values of rolling elements and raceways radii and have negative values for concave curved surfaces, i.e. positive values for convex curved surfaces.

The reduced modulus of elasticity E' is determined from the following expression [1]:

$$E' = \frac{E}{1-\xi^2}, \quad (7)$$

where E is the modulus of elasticity and ξ Poisson's ratio.

Oil entrainment velocity U is the mean value of ring velocity and ball velocity in contact point:

$$U = \frac{U_{\mathrm{ir}} + U_{\mathrm{b}}}{2}, \quad (8)$$

wherein U_{ir} and U_{b} are velocities of the inner ring and ball, respectively, in the point of their contact. They can be determined based on equations derived in Appendix A.

If we introduce an assumption of negligibility of mutual slip between contact surfaces, balls velocities are equal to rings velocities in the contact points, and they equal the velocity U. As the ball circumferential velocity in its contact with the inner raceway is equal to its circumferential velocity in its contact with the outer raceway, entrainment velocity will have the same intensity for ball contact with both inner and outer raceway, so, according to the expressions in Appendix A, we have [23]:

$$U = U_{\mathrm{b}} = \frac{D_{\mathrm{b}}}{2}\omega_{\mathrm{b}} = \frac{D_{\mathrm{b}}}{2}\frac{D_{\mathrm{m}}^{\,2} - D_{\mathrm{b}}^{\,2}}{2D_{\mathrm{m}}D_{\mathrm{b}}}\omega$$

$$U = \frac{D_{\mathrm{m}}^{\,2} - D_{\mathrm{b}}^{\,2}}{4D_{\mathrm{m}}}\omega. \quad (9)$$

4. MEAN-SQUARE ROUGHNESS OF ROLLING ELEMENTS AND RACEWAYS

According to the international standard ISO 1302 [24], surface roughness is defined as the set of micro-geometrical peaks that form the actual surface area relief.

The measure of surface roughness is described by the profile arithmetic mean deviation R_{a} which represents the arithmetic mean value of the distance of all effective profile points from the centerline, on the reference length (Figure 3):

$$R_{\mathrm{a}} = \frac{1}{l}\int_0^l \left|y\left(x\right)\right| \mathrm{d}x, \quad (10)$$

where l is the reference length [24].

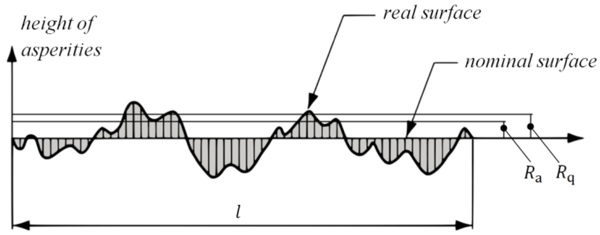


Figure 3. Profile lines

Surface classification is performed according to the profile arithmetic mean deviation. In addition to the profile arithmetic mean deviation (arithmetic mean roughness) R_a, there is a surface mean square roughness (geometric mean roughness) R_q. Assuming Gaussian distribution of asperities heights, there is [1,13]:

$$R_q = 1.25 R_a \, . \tag{11}$$

The roughness depends on many factors, but primarily on the process and quality of the surface finish. More accurate determination is possible only by direct measurement. However, based on the experience in rolling bearings production, roughness intervals for balls and raceways are given in some literature sources, which are presented in Table 1.

Table 1. Surface roughness

	R_a [μm] [14]	R_a [μm] [25]
Raceways	0.050-0.30	0.100-0.30
Rolling elements	0.025-0.20	0.025-0.12
Higher values of roughness correspond to the bigger rolling bearings		

5. REGIME LUBRICATION COEFFICIENT

Previous considerations have shown that the minimum film thickness depends on the lubricant properties, bearing geometry, contact load, and rotating shaft speed. In addition to the aforementioned factors, based on the expression (1) it seems that the parameter λ, i.e. lubrication regime, is influenced by the contact surfaces' roughness. Thus, for each rolling bearing and appropriate lubricant, it is possible to establish a lubrication regime dependent on the rotational speed and contact load. By substituting (3) into (2) we get:

$$h_{\min} = 3.63 \underline{\mathrm{U}}^{0.68} \underline{\mathrm{G}}^{0.49} \underline{\mathrm{W}}^{-0.073} \left(1-\exp\left(-0.68\kappa\right)\right) R_x$$

$$\underline{\mathrm{U}}^{0.68} = 0.275 h_{\min} \underline{\mathrm{G}}^{-0.49} \underline{\mathrm{W}}^{0.073} \left(1-\exp\left(-0.68\kappa\right)\right)^{-1} R_x^{-1}$$

$$\underline{\mathrm{U}} = \left[\frac{0.275 h_{\min} \underline{\mathrm{W}}^{0.073}}{\underline{\mathrm{G}}^{0.49} \left(1-\exp\left(-0.68\kappa\right)\right) R_x} \right]^{1/0.68}$$

$$U = \frac{E' R_x}{\eta_0} \left[\frac{0.275 h_{\min} \underline{\mathrm{W}}^{0.073}}{\underline{\mathrm{G}}^{0.49} \left(1-\exp\left(-0.68\kappa\right)\right) R_x} \right]^{1/0.68} .$$

Substitution of expression (9) in the previous expression gives:

$$\omega = \frac{4D_m}{D_m^2 - D_b^2} \frac{E'}{\eta_0} \left[\frac{0.275 h_{\min} \underline{\mathrm{W}}^{0.073}}{\underline{\mathrm{G}}^{0.49} \left(1-\exp\left(-0.68\kappa\right)\right) R_x^{0.32}} \right]^{1/0.68} \tag{12}$$

Substitution of expressions (1), (4) and (5) in expression (12) gives:

$$\omega = \frac{4D_m}{D_m^2 - D_b^2} \frac{E'}{\eta_0} \left[\frac{0.275\lambda \sqrt{R_{qr}^2 + R_{qb}^2} \left(\frac{Q}{E' R_x^2} \right)^{0.073}}{\left(\alpha_0 E'\right)^{0.49} \left(1-\exp\left(-0.68\kappa\right)\right) R_x^{0.32}} \right]^{1/0.68}$$

$$\omega = \frac{4}{\eta_0} \frac{D_m F^{0.107}}{D_m^2 - D_b^2} \left[\frac{0.275\lambda E'^{0.117} \sqrt{R_{qr}^2 + R_{qb}^2}}{\alpha_0^{0.49} \left(1-\exp\left(-0.68\kappa\right)\right) R_x^{0.466}} \right]^{1/0.68}$$

$$\omega = C_m\left(\lambda\right) F^{0.107} , \tag{13}$$

where: D_m, D_b, R_{qr}, R_{qb} and R_x are in [m], η_0 in [Pa·s], E' in [Pa], α_0 in [Pa^{-1}], ω in [s^{-1}], and F in [N]. Coefficient $C_m(\lambda)$ [s^{-1}N$^{-0.107}$]:

$$C_m = \frac{4D_m}{D_m^2 - D_b^2} \frac{1}{\eta_0} \left[\frac{0.275\lambda E'^{0.117} \sqrt{R_{qr}^2 + R_{qb}^2}}{\alpha_0^{0.49} \left(1-\exp\left(-0.68\kappa\right)\right) R_x^{0.466}} \right]^{1.47} \tag{14}$$

is a regime lubrication coefficient, i.e. $C_m(\lambda=1)$ is the coefficient of transition from boundary to mixed lubrication regime, and $C_m(\lambda=3)$ is the coefficient of transition from mixed to EHL regime.

The lubrication regime coefficient $C_m(\lambda)$ depends on the lubricant properties and rolling bearing micro and macro geometry. Machine designer usually has in advance the information about rolling bearing operating conditions, that is the shaft speed, contact load, and operating temperature. If so, the lubrication regime coefficient is what makes the choice of the optimal combination of rolling bearing and lubricant very easy. If the machine operates at room temperature and in normal operating conditions, the temperature of its rolling bearings is within the interval from 40°C to 90°C, depending on the machine type [26].

Table 2 and Table 3 provide $C_m(\lambda)$ coefficient values for particular oil lubricants and eight radial ball bearings with operating at 38°C to 99°C (almost boundary temperatures of the above-mentioned range), according to the lubricant characteristics given below, in Table 4. Tables 2 and 3 could be extended to a larger number of combinations of rolling bearings and lubricants.

Because contemporary technological methods allow the achievement of high quality bearing contact surfaces, the following arithmetic means roughness values have been adopted in the results analysis process, according to [10]:

$$\left.\begin{aligned} R_{ar} &= 0.14 \\ R_{ab} &= 0.05 \end{aligned}\right\} \overset{(11)}{\Rightarrow} \left.\begin{aligned} R_{qr} &= 0.1750 \\ R_{qb} &= 0.0625 \end{aligned}\right\} \Rightarrow \sqrt{R_{qr}^2 + R_{qb}^2} = 0.1858 \ \mu\mathrm{m}$$

For known rolling bearing external load and shaft speed it is possible, from the equation (13), to determine the C_m coefficient value necessary to achieve the desired lubrication regime:

$$C_m = \frac{\omega}{F^{0.107}} . \tag{15}$$

Table 2. Regime lubrication coefficient $C_m(\lambda)$ values for four oil lubricants and eight radial ball bearings, with contact surfaces roughness of R_{ar}=0.14 μm and R_{ab}=0.05 μm and operating temperature of 38°C

Advanced Ester									
Rolling bearing		6004	6005	6204	6206	6304	6305	6306	6312
o	$C_m(\lambda=1)$	178.61	154.60	148.87	93.10	117.50	84.87	68.73	24.55
	$C_m(\lambda=3)$	898.57	777.81	748.97	468.36	591.13	426.98	345.78	123.50
i	$C_m(\lambda=1)$	235.65	189.99	194.93	120.35	166.70	121.17	92.97	33.78
	$C_m(\lambda=3)$	1185.52	955.82	980.69	605.46	838.64	609.58	467.74	169.97
Polyalkyl Aromatic									
Rolling bearing		6004	6005	6204	6206	6304	6305	6306	6312
o	$C_m(\lambda=1)$	143.86	124.53	119.91	74.98	94.64	68.36	55.36	19.77
	$C_m(\lambda=3)$	723.75	626.48	603.26	377.24	476.12	343.91	278.51	99.47
i	$C_m(\lambda=1)$	189.80	153.02	157.01	96.93	134.27	97.59	74.88	27.21
	$C_m(\lambda=3)$	954.87	769.86	789.89	487.66	675.48	490.98	376.74	136.90
Synthetic Paraffinic Oil (Lot 4)									
Rolling bearing		6004	6005	6204	6206	6304	6305	6306	6312
o	$C_m(\lambda=1)$	7.68	6.65	6.40	4.00	5.05	3.65	2.96	1.06
	$C_m(\lambda=3)$	38.64	33.45	32.21	20.14	25.42	18.36	14.87	5.31
i	$C_m(\lambda=1)$	10.13	8.17	8.38	5.17	7.17	5.21	4.00	1.45
	$C_m(\lambda=3)$	50.98	41.10	42.17	26.03	36.06	26.21	20.11	7.31
Super-Refined Naphthenic Mineral Oil									
Rolling bearing		6004	6005	6204	6206	6304	6305	6306	6312
o	$C_m(\lambda=1)$	39.89	34.53	33.25	20.79	26.24	18.95	15.35	5.48
	$C_m(\lambda=3)$	200.68	173.71	167.27	104.60	132.02	95.36	77.23	27.58
i	$C_m(\lambda=1)$	52.63	42.43	43.54	26.88	37.23	27.06	20.67	7.55
	$C_m(\lambda=3)$	264.77	213.47	219.03	135.22	187.30	136.14	104.46	37.96
o – contact of the ball and outer raceway; i – contact of the ball and inner raceway									

Table 3. Regime lubrication coefficient $C_m(\lambda)$ values for four oil lubricants and eight radial ball bearings, with contact surfaces roughness of R_{ar}=0.14 μm and R_{ab}=0.05 μm and operating temperature of 99°C

Advanced Ester									
Rolling bearing		6004	6005	6204	6206	6304	6305	6306	6312
o	$C_m(\lambda=1)$	1019.92	882.85	850.12	531.62	670.97	484.64	392.48	140.18
	$C_m(\lambda=3)$	5131.17	4441.58	4276.92	2674.54	3375.59	2438.21	1974.54	705.22
i	$C_m(\lambda=1)$	1345.63	1084.90	1113.14	687.23	951.90	691.91	530.91	192.92
	$C_m(\lambda=3)$	6769.77	5458.08	5600.13	3457.40	4788.97	3480.95	2670.97	970.57
Polyalkyl Aromatic									
Rolling bearing		6004	6005	6204	6206	6304	6305	6306	6312
o	$C_m(\lambda=1)$	1004.75	869.72	837.48	523.71	660.99	477.43	386.64	138.09
	$C_m(\lambda=3)$	5054.85	4375.52	4213.31	2634.76	3325.38	2401.94	1945.17	694.73
i	$C_m(\lambda=1)$	1325.61	1068.77	1096.58	677.01	937.75	681.62	523.01	190.05
	$C_m(\lambda=3)$	6669.07	5376.90	5516.83	3405.97	4717.74	3429.17	2631.25	956.14
Synthetic Paraffinic Oil (Lot 4)									
Rolling bearing		6004	6005	6204	6206	6304	6305	6306	6312
o	$C_m(\lambda=1)$	91.87	79.52	76.57	47.88	60.44	43.65	35.35	12.63
	$C_m(\lambda=3)$	462.18	400.07	385.24	240.91	304.05	219.62	177.85	63.52
i	$C_m(\lambda=1)$	121.21	97.72	100.26	61.90	85.74	62.32	47.82	17.38
	$C_m(\lambda=3)$	609.78	491.63	504.42	311.42	431.36	313.54	240.58	87.42
Super-Refined Naphthenic Mineral Oil									
Rolling bearing		6004	6005	6204	6206	6304	6305	6306	6312
o	$C_m(\lambda=1)$	580.83	502.77	484.14	302.75	382.11	276.00	223.51	79.83
	$C_m(\lambda=3)$	2922.14	2529.43	2435.66	1523.12	1922.36	1388.53	1124.48	401.62
i	$C_m(\lambda=1)$	766.32	617.84	633.92	391.37	542.10	394.03	302.35	109.87
	$C_m(\lambda=3)$	3855.30	3108.31	3189.21	1968.95	2727.26	1982.36	1521.09	552.73
o – contact of the ball and outer raceway; i – contact of the ball and inner raceway									

Table 4. Oil lubricants characteristics [18]

	η_0 [10^{-3} Pa·s]		α_0 [10^{-8} Pa^{-1}]	
	38°C	99°C	38°C	99°C
Advanced Ester	25.3	4.75	1.52	1.38
Polyalkyl Aromatic	25.3	4.08	2.03	1.74
Synthetic Paraffinic Oil	375	32.6	2.84	2.69
Super-Refined Miner. Oil	68.1	6.86	3.08	1.81

The goal is to obtain EHL regime. This requires lower values of coefficient $C_m(\lambda=3)$, obtained by the (14), than those obtained by the (15). Also, higher values of the contact load give fewer values of the necessary coefficient C_m, that is, the contact zone is the critical zone for achieving this goal. Besides that, (15) indicates that angular speed ω has a much higher impact on the lubrication regime than contact load F. Only bearing-oil combinations, from Tables 2 and 3, with C_m lower than the value obtained by (15) would be selected.

Equation (13) gives the relation between contact load F of the specific ball, and angular speed ω, through parameter $C_m(\lambda)$ which is a function of rolling bearing geometry and lubricating oil properties. In terms of practical usage of equation (13), it would be more favorable to get the relation between rolling bearing external radial load Q and the number of revolutions n [rpm]. The simplest way to perform it would be the usage of Stribeck's number to relate maximal contact load F and external load Q, since higher contact load is more unfavorable. The well-known relation between F_{max} and Q, obtained by Stribeck, is:

$$F_{max} = \frac{StQ}{z}, \qquad (16)$$

where: F_{max} [N] is the maximal contact load, z is the number of balls, and St is the Stribeck's number, which is equal to 4.37 for ball rolling bearing with zero clearance, and 5 if the clearance is positive.

Substitution of the previous expression and relation $\omega = \pi n/30$ in expression (13) gives:

$$n = \frac{30}{\pi} C_m \left(\frac{StQ}{z} \right)^{0.107}. \qquad (17)$$

6. NUMERICAL ANALYSIS

A plot that shows a lubrication regime dependence of angular speed ω and contact load F for rolling bearing 6206 and Advanced Ester oil is shown in Figure 4.

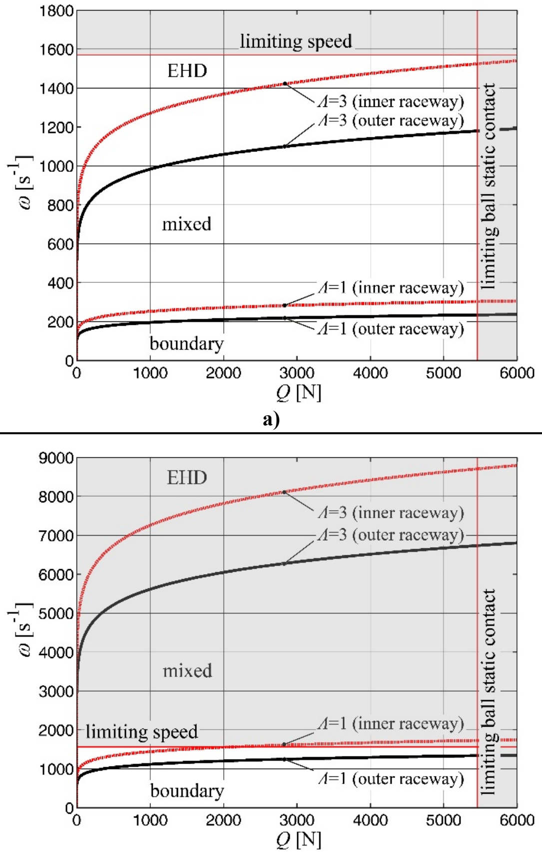


Figure 4. A plot that shows a lubrication regime dependence of angular speed ω and contact load F for rolling bearing 6206 and lubricant Advanced Ester, with surfaces roughness of R_{ar}=0.14 μm and R_{ab}=0.05 μm and operating temperature of a) 38°C, b) 99°C

Figure 4 is drawn based on expression (13). It also indicates a limiting speed value prescribed by the manufacturer [27] and maximum allowable contact load calculated based on permissible static load given in [27] and external load distribution mathematical model presented in [2-5].

A plot that shows a lubrication regime dependence on the number of revolutions n and external radial load Q for rolling bearing 6206 and lubricant Advanced Ester is drawn based on expression (17) and shown in Figure 5.

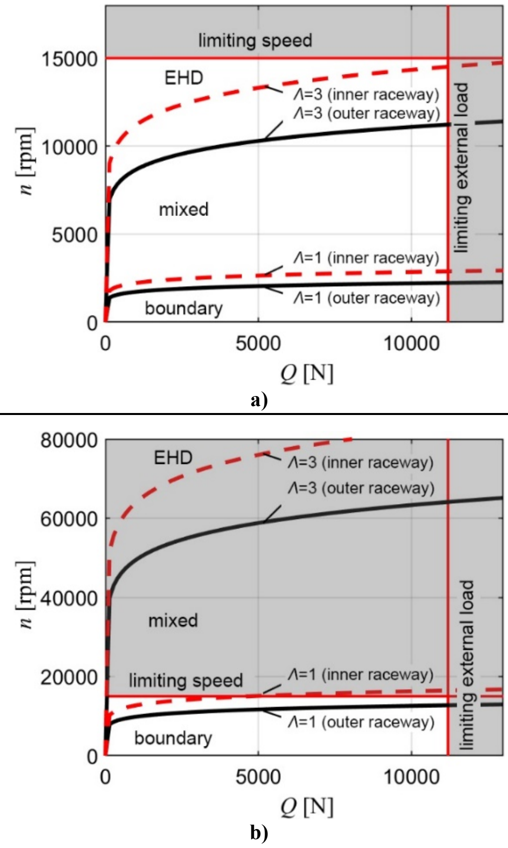
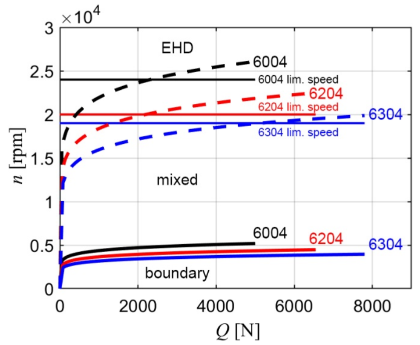


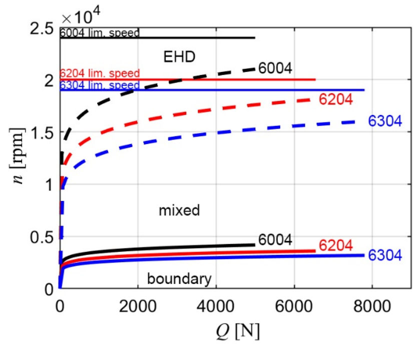
Figure 5. A plot that shows a lubrication regime dependence on the number of revolutions n and external radial load Q for rolling bearing 6206 and lubricant Advanced Ester, with contact surfaces roughness of R_{ar}=0.14 μm and R_{ab}=0.05 μm and operating temperature of a) 38°C, b) 99°C

It is noticeable, from the previous figures and Tables 2 and 3, that the EHL will occur earlier in ball-outer raceway contact than in ball-inner raceway contact. The reason for this is the fact that the inner raceway surface curvature radius in the direction of the x-axis is lower than the outer raceway surface curvature radius. Therefore, it is sufficient to consider only a contact of balls and inner raceway to perceive the conditions under which the entire bearing will operate with the desired lubrication regime.

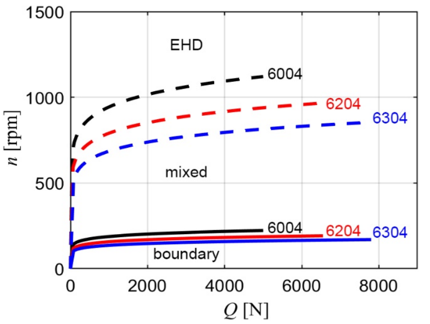
For the contact of balls and inner raceways, a lubrication regime dependence of the number of revolutions n and external radial load Q for particular rolling bearings and particular oil lubricants are shown in Figure 6 for the operating temperature of 38°C and in Figure 7 for the temperature of 99°C. Limiting speed values are also indicated, and curves are cut at the points of limiting external static loads, given in [27].



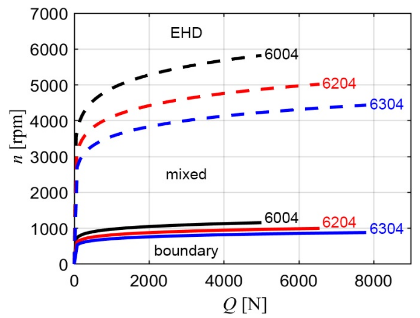
a)



b)



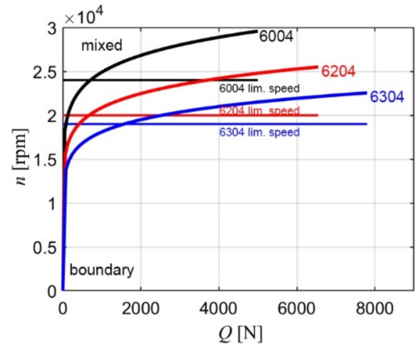
c)



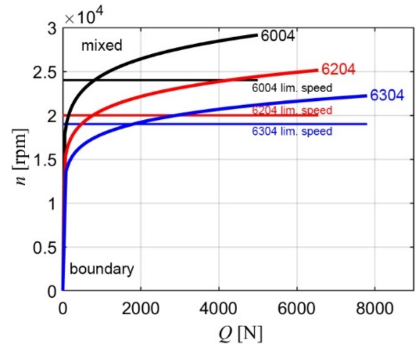
d)

Figure 6. A plot that shows a lubrication regime dependence of the number of revolutions *n* and external radial load *Q* for particular rolling bearings and balls-inner raceway contact, with contact surfaces roughness of R_{ar}=0.14 μm and R_{ab}=0.05 μm, operating temperature of 38°C and oil lubricant: a) Advanced Ester, b) Polyalkyl Aromatic, c) Synthetic Paraffinic Oil (Lot 4), d) Super-Refined Naphthenic Mineral Oil

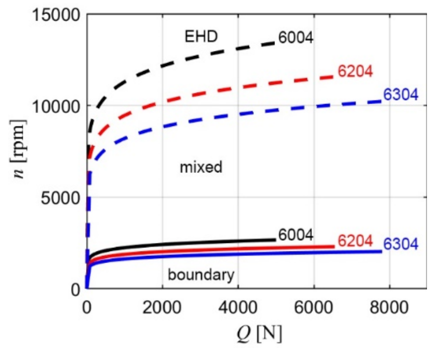
For the bearing contact surfaces roughness interval, which is shown in Table 1, it is possible to define an interval for $C_m(\lambda)$ coefficient required to achieve the EHL regime for particular rolling bearing and oil lubricant. In this situation, it can be formed a plot in which the curve of transition from mixed lubrication to EHL (for the ball-inner raceway contact), shown in Figure 4.a), takes the form of surface (area), whose boundaries are determined with the surfaces roughness boundary values from Table 1. Such a graph for rolling bearing 6206 and particular oil lubricants are shown in Figure 8 for the operating temperature of 38°C, and in Figure 9 for the operating temperature of 99°C.



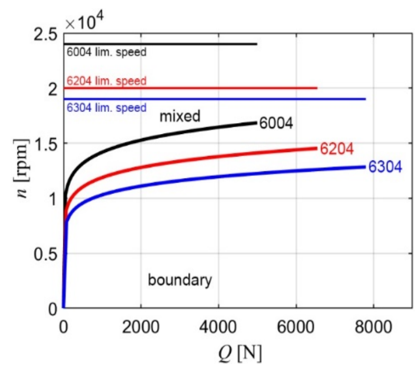
a)



b)



c)



d)

Figure 7. A plot that shows a lubrication regime dependence of the number of revolutions *n* and external radial load *Q* for particular rolling bearings and balls-inner raceway contact, with contact surfaces roughness of R_{ar}=0.14 μm and R_{ab}=0.05 μm, operating temperature of 99°C and oil lubricant: a) Advanced Ester, b) Polyalkyl Aromatic, c) Synthetic Paraffinic Oil (Lot 4), d) Super-Refined Naphthenic Mineral Oil

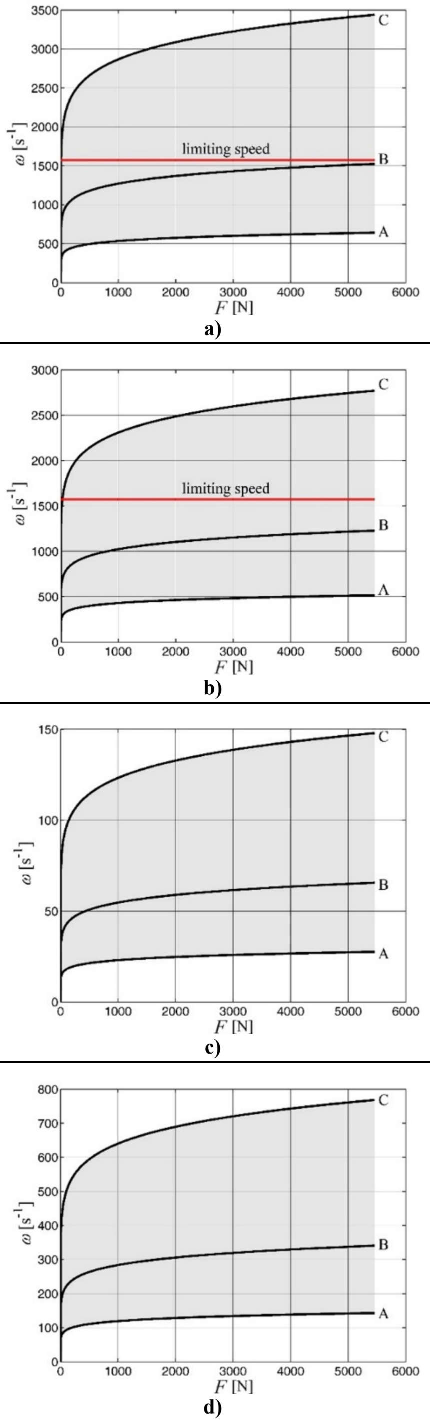


Figure 8. A plot that shows a lubrication regime dependence of angular speed *ω* and contact load *F* for rolling bearing 6206 and balls-inner raceway contact, with an operating temperature of 38°C and oil lubricant: a) Advanced Ester, b) Polyalkyl Aromatic, c) Synthetic Paraffinic Oil, d) Super-Refined Naphthenic Mineral Oil

Curve A (Figures 8 and 9) corresponds to the contact surfaces roughness of $R_{qr}^2+R_{qb}^2 = (0.103\ \mu m)^2$, curve B to the contact surfaces roughness of 0.1858 μm (used in all previous analyses), and curve C to the contact surface roughness of 0.323 μm.

These two figures show that the area limited by curves A and C is very wide. This is because the entire range of surface roughness from Table 1 is taken. However, the choice of roughness class depends on the bearing dimensions, so if we know the roughness interval for concrete bearing this area will be significantly narrowed.

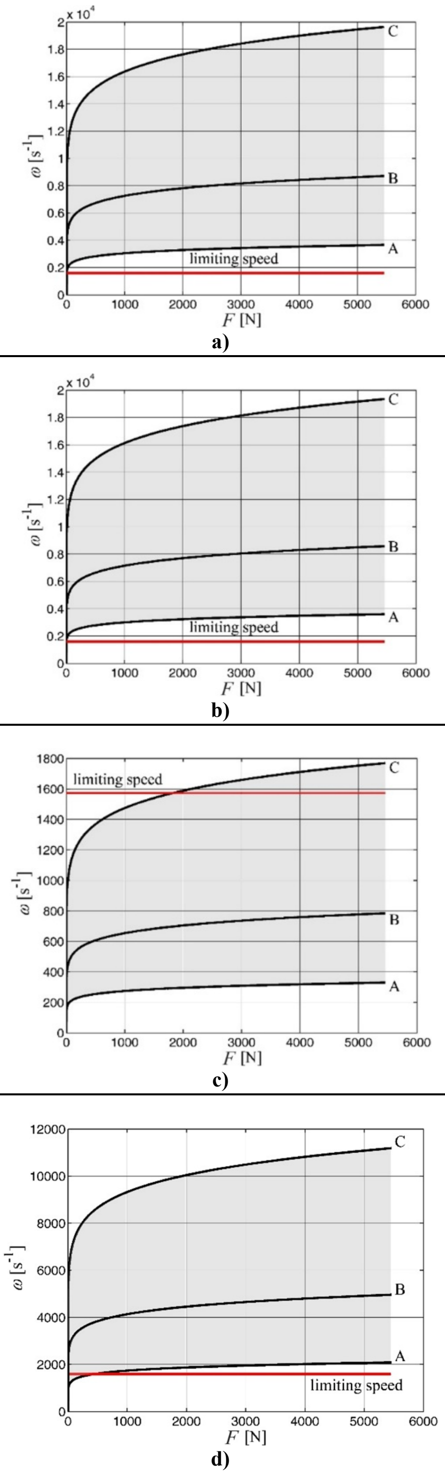


Figure 9. A plot that shows a lubrication regime dependence of angular speed *ω* and contact load *F* for rolling bearing 6206 and balls-inner raceway contact, with an operating temperature of 99°C and oil lubricant: a) Advanced Ester, b) Polyalkyl Aromatic, c) Synthetic Paraffinic Oil, d) Super-Refined Naphthenic Mineral Oil

7. DISCUSSION OF RESULTS

The results shown in the previous chapter pertain to the individual lubrication regimes between the balls and raceways. However, in practical application, if only on a single point of balls-raceways contact does not come to the complete separation of operating surfaces, it can be considered that the rolling bearing operates in the zone of mixed lubrication. Thus, it is a justified assumption that the lubrication regime of complete bearing can be observed about the lubrication regime of each rolling element. Therefore, the results obtained for the individual rolling element can be considered valid for the entire rolling bearing. In this way, we can consider that the overall rolling bearing is lubricated in EHL mode if the ball with the highest contact load is lubricated in EHL mode.

From Figure 4-7, it can be noticed that higher contact loads and lower rotational speeds have a negative influence on the rolling bearing lubrication regime. As EHL mode is the most suitable in the terms of friction and service life, it is essential to perform a correct selection of lubricant according to the rolling bearing operating conditions, and in that sense, expressions (13) and (17) and the previous analysis facilitate the selection of the most suitable lubricants, because they help to easily predicted the achieved lubrication mode. For example, if bearing 6206 operates at a temperature of about 38°C and at a very high speed (close to limiting value), it can be seen, from Figure 4.a), that it is convenient to use Advanced Ester oil for lubrication. However, if bearing 6206 operates at lower rpm, then complete separation of the contact surfaces cannot be achieved, and metal contact between balls and rings will occur. A disadvantage of Advanced Ester oil application for the lubrication of bearing 6206 is particularly evident if the bearing operates at higher temperatures (of about 99°C), because oil viscosity becomes significantly decreased, and $C_m(\lambda)$ value (Table 3) significantly increased. Figure 4.b) shows that it is not possible to achieve a combination of angular speed and contact load at which bearing will operate under the EHL regime. Even a mixed lubrication area is very narrowed. It can be seen that the mixed lubrication can be achieved for angular speed close to the limiting speed and with the contact loads not exceeding 1500 N.

From Table 4, it can be noticed that the coefficient of atmospheric pressure-viscosity α_0 does not significantly change with the change of temperature, while the viscosity η_0 changes can be extremely large. This affects the possibility of realizing the desired lubrication regime, which is noticeable if make a comparison of the diagrams shown in Figure 6 and Figure 7. Figure 6 indicates that lubricants Synthetic Paraffinic Oil (Lot 4) and Super Refined Naphthenic Mineral Oil are suitable for the considered rolling bearings (6004, 6204, 6304) lubrication in the given temperature range, because complete separation of the metal contact surfaces, with the appropriate angular speed, can be achieved even at a temperature of 99°C, which is not always the case with oils Advanced Ester and Polyalkyl Aromatic.

Figures 8 and 9 show that roughness has a very important role in achieving the desired lubrication regime. Looking at Figure 8, it can be noticed that EHL mode of the bearing 6206, at a temperature of 38°C, can be achieved for any surface roughness from the range defined by Table 1, if lubrication is performed by Synthetic Paraffinic Oil (Lot 4) or Super Refined Naphthenic Mineral Oil, wherein the speed required for its achievement is significantly lower than limiting. At a temperature of 99°C, Synthetic Paraffinic Oil (Lot 4) can be used for lubrication of bearing 6206 in the EHL mode for different contact surfaces' roughness values, but only if shaft speed is much higher compared to the previously considered case (up to 10 times higher).

8. CONCLUSION

To prolong service life and improve the operational performance of rolling bearing, it is important to choose a proper lubricant that can establish the EHL regime. In that sense, this paper introduces a lubrication regime coefficient $C_m(\lambda)$, which depends on the lubricant properties and rolling bearing geometry, by following the research of Hamrock and Dowson [14-16]. It enables the constructor to make a very fast and easy selection of proper bearing-lubricant combinations. Knowing the operating conditions (number of revolutions n and external radial load Q) and expression (17), the constructor just has to calculate the maximum value of regime lubrication coefficient C_m that will ensure the proper lubrication of rolling bearing, and then choose the appropriate combination of rolling bearing and lubricant from the tables with many combinations, like Table 2 and Table 3 that will ensure the EHL regime.

The analysis in this study showed that rolling bearing achieves favorable lubrication mode in ball-outer ring contact before then in ball-inner ring contact. Also, angular speed has a much higher influence on the regime of lubrication than contact load.

APPENDIX: KINEMATIC ANALYSIS OF ROLLING BEARING

For kinematic analysis of rolling bearing, a presumption of slip negligibility between contact surfaces will be introduced. To determine the individual rolling bearing elements rotating speeds, the entire system relative motion about the rolling element (ball) center will be observed, in such a manner that all elements of the system get a negative cage rotating speed (orbital speed of ball) ω_m, forming a kinematic equivalent model [1]. Before system transformation, the ball rotated around its axis by ω_b speed and around the cage center by the cage speed ω_m (Figure 10.a). After transformation, the ball rotates only about its axis, while its center has a zero speed (Figure 10.b), and the primarily stationary outer ring rotates around the rolling bearing axis with rotating speed ω_m.

Ignoring the slip between the balls and raceways, balls velocities are equal to raceways velocities in their mutual points of contact. Thus, the point “I” velocity on the ball equals the point “I” velocity on the inner ring, and the point “O” velocity on the ball equals the point “O” velocity on the outer ring (Figure 10.b):

$$v_I = \frac{D_b}{2}\omega_b = \frac{D_m - D_b}{2}\left(\omega - \omega_m\right), \tag{18}$$

$$v_O = \frac{D_b}{2}\omega_b = \frac{D_m + D_b}{2}\omega_m, \tag{19}$$

where: v_I is ball circumferential velocity in contact with the inner raceway, v_O is ball circumferential velocity in contact with the outer raceway, D_b is ball diameter, D_m is bearing pitch diameter, ω bearing inner ring absolute rotating speed (shaft speed), ω_b speed of ball about its axis, and ω_m cage rotating speed (orbital speed of ball).

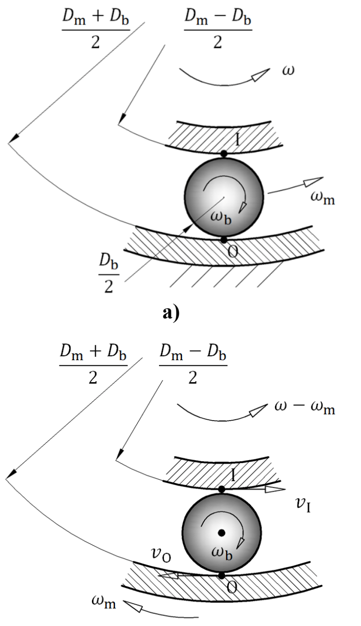


Figure 10. Balls and raceways speed: a) before system transformation, b) after system transformation

As the speeds v_I and v_O are equal by the intensity, based on the previous two expressions, it follows:

$$\frac{D_m - D_b}{2}\left(\omega - \omega_m\right) = \frac{D_m + D_b}{2}\omega_m$$

$$\omega_m = \frac{D_m - D_b}{2D_m}\omega . \tag{20}$$

By substituting expression (20) to (19), it is possible to obtain a speed of ball about its own axis ω_b:

$$\frac{D_b}{2}\omega_b = \frac{D_m + D_b}{2}\frac{D_m - D_b}{2D_m}\omega$$

$$\omega_b = \frac{{D_m}^2 - {D_b}^2}{2D_m D_b}\omega . \tag{21}$$

In some analyzes, it is important to keep in mind that ω_b speed has an opposite direction compared to the ω speed (Figure 10.a). Then it is necessary to change the sign in the expression (21).

ACKNOWLEDGMENT

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NOMENCLATURE

$C_m(\lambda)$	regime lubrication coefficient
D	diameter
E	modulus of elasticity
E'	reduced modulus of elasticity
F	contact load of particular rolling element
$\underline{G}$	dimensionless parameter of the material
h_{min}	minimum film thickness
n	number of revolutions
Q	rolling bearing external radial load
R_a	arithmetic mean roughness
R_q	geometric mean roughness
R_{qb}	roughness mean square value of rolling elements
R_{qr}	roughness mean square value of raceways
R_x	reduced curvature radius in the direction of the contact ellipse main axis (x)
St	Stribeck's number
$\underline{U}$	dimensionless parameter of speed
U	lubricant entrainment velocity
v	ball circumferential velocity
$\underline{W}$	dimensionless parameter of load
z	number of balls
α_0	atmospheric pressure-viscosity coefficient
η_0	lubricant viscosity at the atmospheric pressure
κ	ratio of the major and minor axis of the contact ellipse
λ	parameter describing the rolling bearing lubrication efficiency
ξ	Poisson's ratio
ρ_{Ix}	curvature radii of the rolling elements in the direction of the x-axis
ρ_{IIx}	curvature radii of the raceways surfaces in the direction of the x-axis
ω	rolling bearing angular speed

Indexes

i / I	contact of the ball and inner raceway
o / O	contact of the ball and outer raceway
b	ball
m	cage

АНАЛИЗА УТИЦАЈА БРЗИНЕ ВРАТИЛА И СПОЉАШЊЕГ ОПТЕРЕЋЕЊА НА РЕЖИМЕ ПОДМАЗИВАЊА РАДИЈАЛНИХ КУГЛИЧНИХ ЛЕЖАЈЕВА

Р. Грујичић, М. Тица, Б. Стојановић, Л. Ивановић, Р. Митровић, Р. Томовић

Потребно је одговарајуће подмазивање котрљајућег лежаја како би се смањило трење између површина у међусобном контакту и њихово хабање. Мазиво би требало да у потпуности одвоји котрљајуће еле–менте од стаза. У раду су анализиране вредности које утичу на ефикасност и режиме подмазивања, након чега се уводи коефицијент режима подма–зивања. Овај коефицијент омогућава да се на једно–ставан и брз начин одабере оптимална комбинација котрљајућег лежаја и мазива на основу познате

брзине осовине, спољашњег оптерећења и радне температуре котрљајућег лежаја. За поједине лежа–јеве са радијалним контактом и одређена мазива приказана је зависност режима подмазивања од брзине осовине и контактног оптерећења.