A Computational Methodology for Synthesis of Epicyclic Gear Transmission system configurations with Multiple Planetary Gear Trains

An Epicyclic gear transmission system (EGT) is sought when more than a one-speed ratio is required from a gearbox. This necessitates the usage of a multiple planetary gear train (PGT). In short, an EGT is an assemblage of PGTs, compounded together in specific ways to yield multiple speed ratios. The word 'compounding' here means how the PGTs are interconnected along with the constraining elements like brakes and permanent connections as well as friction devices (clutches) to produce a variety of speed ratios. In other words, some elements of a PGT are connected to some elements of other PGTs.

The gear trains (or EGTs) used in early automatic transmission systems possessed two d.o.f. When one of the d.o.f is suppressed, usually by applying a band brake to prevent the rotation of some members of the gear train or by engagement of the clutch, the gear train has one d.o.f, thereby resulting in a single ratio between the input and output shafts. For example, in the case of the Wilson gearbox, which is a two d.o.f system possessing six-speed ratios (5 forward + 1 reverse), it consists of six band brakes, which need to be applied one at a time. A variety of speed ratios are achieved by using brakes or clutches for different members of EGT.

1. INTRODUCTION

An Epicyclic gear transmission system (EGT) is sought when more than a one-speed ratio is required from a gearbox. This necessitates the usage of a multiple planetary gear train (PGT). In short, an EGT is an assemblage of PGTs, compounded together in specific ways to yield multiple speed ratios. The word 'compounding' here means how the PGTs are interconnected along with the constraining elements like brakes and permanent connections as well as friction devices (clutches) to produce a variety of speed ratios. In other words, some elements of a PGT are connected to some elements of other PGTs.

A Computational Methodology for Synthesis of Epicyclic Gear Transmission system configurations with Multiple Planetary Gear Trains

Epicyclic Gear Transmission Systems (EGTs) find a wide variety of applications in automatic transmission systems due to their flexibility to deliver a variety of speed ratios. The design of EGT involves the enumeration of all possible layouts, typically called structural synthesis. Thus, the structural synthesis of an EGT consists of defining a topology for EGT with coupled planetary gear trains (PGTs) along with constraining elements such as connections, brakes, and clutches while delivering a definite transmission ratio between the input and output shafts. Given the number of PGTs, permanent connection, clutches, and brakes, the methodology systematically introduces these elements of PGTs to deliver the given number of distinct speed ratios. Determining the number of distinct speed ratios for a given combination of brakes and clutches has been expressed as a transformation of a graph representation of the EGT layout. A computer program has been developed for the structural synthesis of EGTs and validated against the 3 epicyclic, 6-speed EGT systems. The viability aspects of the synthesized structure are carried out. Also, an isomorphism test is addressed in the synthesis methodology to detect unique EGT structures at every synthesis stage.

Keywords: Epicyclic Gear Transmission system, Planetary Gear Train, Structural synthesis, Isomorphism problem.
The basic element of an EGT is a simple planetary gear train (PGT). It consists of a sun gear (S), annulus (A) and planet carrier (C), and several planet gears (P). It consists of 3 shafts and exhibits 2 degrees of freedom (d.o.f). A schematic of a simple PGT is shown in Figs.1a&1b. Figure. 1 (a) Simple PGT; (b) Schematic of a PGT; (c) Graph of a PGT; (d) Circle representation of a PGT;

1.1 Representation of an EGT

The basic element of an EGT is a simple planetary gear train (PGT). It consists of a sun gear (S), annulus (A) and planet carrier (C), and several planet gears (P). It consists of 3 shafts and exhibits 2 degrees of freedom (d.o.f). A schematic of a simple PGT is shown in Figs.1a&1b.

Fig.1 shows different representations of a simple PGT. Each of the PGT elements is permanently connected to two other elements in it. Fig.1a shows the simplified diagram of PGT; Fig.1b the schematic of PGT; Fig.1c shows the graph of a PGT wherein each shaft of PGT maps to a vertex; Fig.1d, a circle notation of a PGT useful to illustrate connectivity of multiple PGTs.

During structural synthesis, since the three shafts of a PGT are considered indistinguishable from one another, and since all three are interconnected, a PGT can be represented as a complete graph of three vertices, Fig. 1(c), and hence the vertices 1,2,3 represents 'S', 'A' and 'R' which are identical in the synthesis point of view and thus, the edges connected through vertices 1-2, 2-3 and 1-3 represents the PGT with the elements connected.
It is also evident from the figure that brake $B_1$ is connected to the shaft of the sun gear on PGT-1, brake $B_2$ is connected to the permanent connection between $A_1$ and $R_2$, and brake $B_3$ is connected to the shaft between $R_3$ and $S_4$. Also, there is a clutch (C) between the input shaft (i/p) and $S_1$. Similarly, the output shaft (o/p) is located on the permanent shaft connecting $A_3$ and $A_4$.

1.2 Kinematics of an EGT

In a simple planetary gear train, the rotational speeds of sun gear ($n_s$), annulus or ring gear ($n_a$) and planet carrier ($n_c$) respectively are related by,

$$n_s - R n_a = (1 - R) n_c \tag{1}$$

where 'R' is the basic gear ratio, which is the ratio between the number of teeth on the annulus or ring gear and the number of teeth on sun gear.

In the above equation, the signs of $n_a$, $n_s$, and $n_c$ are alike for rotations in the same sense. The equation also shows that the speeds of two shafts of PGT must be known to determine the third's speed. If one of the shafts is locked, corresponding to one of the variables in the equation being set to zero, the other two shafts are then connected by a definite gear ratio. Note that a PGT with one locked shaft is indistinguishable from an ordinary gear train. If, on the other hand, two of the three shafts rotate at identical speeds, the third shaft also rotates at the same speed as the other two.

An EGT system comprised of $N$ PGTs can be mathematically described by a system of $N$ equations in the $3N$ variables as

$$n_{sk} - R n_{Ak} = (1 - R) n_{ck} \tag{2}$$

where $n_{sk}$, $n_{Ak}$, and $n_{ck}$ are the speeds of the sun, annulus and planet carrier respectively of the $k^{th}$ PGT.

Since certain elements of these PGTs are either permanently connected or intermeshed, mathematically, it can be expressed as,

$$n_x + R_{xy} n_y = 0 (x \neq y) \tag{3}$$

where $x$ and $y$ separately stand for any one of $C_k$, $A_k$ or $S_k$ ($k = 1$ or 2 or $\ldots$ $N$), and $R_{xy}$ is the multiplication factor.

Also, it must be noted that temporary connections are caused by the engagement of clutches, which introduces additional equations of type 2.

Finally, when certain shafts are held stationary by braking, the third set of equations results as

$$n_{sk} = 0 \tag{4}$$

where $x = C$ or $A$ or $S$, and $k = 1$ or 2 or $\ldots$ $N$.

The complete set of equations (2), (3), and (4) determines the kinematics of the epicyclic gear transmission system.

The systematics of an EGT can be explained as follows. Suppose an epicyclic gearbox consists of $N$ PGTs with $L$ permanent connections. Let the gearbox activate $c$ number of clutches and $b$ number of brakes to provide a positive transmission ratio. As mentioned in the last section, there shall be $N$ equations of type (2) that relate to the speeds of $3N$ shafts. Also, there shall be $(L + c)$ a number of equations of type 3 and $b$ of equations of type 4. If $M$ denotes the mobility of the gearbox, then the $M$ number of shafts’ speeds must be specified so that the system attains 1 d.o.f. But the speeds of $M$ input shafts, as well as the three sets of equations, viz., (2), (3), and (4), uniquely determine the speeds of all the shafts in the gearbox. This leads to the Molian-Salamoun equation as follows.

$$N + L + c + b = 3N - M \quad \text{or} \quad L + c + b = 2N - M \tag{5}$$

2. METHODOLOGY

The flowchart depicting the methodology followed for the synthesis of EGT is shown in Fig. 3. The procedure starts with the presumption of the number of PGTs present in the EGT under synthesis.
After that, the number of control elements is decided, i.e., the value of \((c + b)\), based on which the minimum value permanent connections are determined from the Molian-Salamoun equation. Deducing the permanent shafts from the total number of available shafts gives the prospective free shafts available to serve as port elements. The output of the synthesis methodology is the speed ratios possible from the synthesized EGT layouts.

2.1 Determination of \(L\) and \((c + b)\)

A simple PGT unit is a two d.o.f system. The mobility of the gearbox must be reduced to unity if it provides a positive transmission ratio. This necessitates an imposition of constraints on each d.o.f of the system till the mobility of the entire system becomes unity. Let an EGT system consist of \(N\) PGTs with \(L\) permanent connections between its shafts. Let \(c\) and \(b\) denote the number of clutches and brakes that need to be activated to constrain the gearbox. As mentioned in the kinematics of EGT, when the EGT is providing a positive transmission ratio (also called the 'operating mode'), the number of constraints \((L + c + b)\) must be equal to \((2N - 1)\). For a given \(N\), \((L + c + b)\) is a constant. This implies that for a fixed speed ratio. A decrease in the value of \(L\) is accompanied by an increase in the \((c + b)\) value. Larger values for \((c + b)\) are undesirable for a gearbox because they call for activation of \((c + b)\) number of control devices. In this work, the quantity \((c + b)\) is assigned a value of either 2 or 3, and thus the number of permanent connections \(L\) will correspondingly be \((2N - 3)\) or \((2N - 4)\), respectively.

2.2 Restrictions of permanent connections

The speed relationship between the three shafts of a PGT unit (equation 1) disallows arbitrary connections amongst the shafts of uncoupled epicycles. The following are the restrictions on having permanent connections between the shafts of PGT units.

1. Two elements of a single PGT must not be permanently connected.
2. One element of a PGT unit must not be permanently connected to two elements of another PGT unit.
3. Two PGT units should not have more than two permanent connections between them.

Violation of (1) and (2) causes a PGT unit to lose its capability to produce differential motion.

These situations are illustrated in Figs. 4a to 4c. In Fig. 4b, PGT-2 becomes ineffective. Violation of (3), as shown in Fig. 4c, renders the six shafts of the two PGTs to behave like a single shaft.

![Figure 4. Redundant permanent connections](image)

2.3 Determination of the number of clutches and brakes in EGT

Let an EGT system consist of \(C\) number of clutches and \(B\) number of brakes. Let \(c\) denote the number of engaged clutches, and \(b\) represent the number of applied brakes necessary to provide a positive transmission ratio. Such a system yields \(C + B\) number of speed ratios. It is desirable that the quantity \((c + b)\) takes a lower value so that fewer speed-controlling devices need to be activated to constrain the entire system. In the present work, the quantity \((c + b)\) has been assumed to be either 2 or 3. Thus, if a greater number of speed ratios are desired from an EGT system, one has the option to maximize \((C + B)\). All the \(\binom{C + B}{c + b}\) number of speed ratios obtained from the gearbox ought to be distinct to be an additional requirement on the derived EGTs, then, \(C \leq (c + b)\). Otherwise, there will be \(\binom{C}{c + b}\) number of distinct combinations of clutches for which the speed ratio is unity. It should be noted that when all the controllable types of constraints are clutches, the gear train simply reduces to one of direct connection. Thus, the only way to obtain a maximum number of distinct speed ratios is through proper selection of \(B\).

Determination of \(B\) necessitates the definition of port elements. Port elements of an EGT system are those shafts that become an input shaft, an output shaft, or a brake. Thus, port elements represent shafts directly connected to the exterior via the input source, output member, and the frame of EGT (through brakes). A more significant number of port elements in a gearbox automatically increases \(B\) since only two-port elements become the input and the output shafts. However, not all the shafts qualify as port elements.

Following are the conditions that govern the introduction of port elements.

1. A PGT unit can have the utmost 2 ports.
2. Violation of the condition leads to the following situations.
   i. The input shaft is held stationary. This situation arises when two of the three-port elements are brakes while the third is the input shaft. (Fig. 5a).
   ii. The output shaft is held stationary. This situation arises when two of the three-port elements are brakes while the third is the output shaft. (Fig. 5b).
   iii. A single PGT alone can take part in the speed transmission process. This situation arises when the input, output, and brake are the three-port elements. (Fig. 5c).
   iv. The epicyclic unit becomes an over-constrained structure with unit redundancy. This occurs when all the three-port elements are brakes. The release of one of the three brakes still leads to a PGT unit whose elements are fused into a single stationary element. (Fig. 5d).
ways of introducing the EGT system, the next step is to find possible ways of inter-shaft clutches, respectively.

permanent connections, there shall be 2 brakes and 2 following assumptions.

limits the enumeration of EGT layouts with the solution space is immense. Thus, the present study distinct speed ratios capable of being produced that the of viability. This shall be dealt with briefly subsequently accordingly increase in the number of brakes in the EGT system. This is because the number of clutches in the system has already been assumed to be either 2 or 3. Having excessive brakes in an EGT may not permit the insertion of the requisite number of clutches since the feasible locations for the introduction of the clutch is subject to the following conditions.

1. There should not be a clutch between the two ports.
2. There should not be a clutch between the elements of an epicyclic unit in which two of its elements are identified as ports.
3. There should not be a clutch between the elements of two epicyclic units, which have two-port elements and two permanent connections between them.
4. There should not be more than one clutch between the elements of two epicyclic units having two permanent connections but with less than 2 port elements.

Another factor that significantly points against having more port elements in the system is the question of viability. This shall be dealt with briefly subsequently in a separate section. It is to be noted that the interconnections between various elements of PGT units with retaining the coaxial nature of the entire layout tend to become more difficult as the number of port elements in the EGTs increases. Thus, it appears that whether an EGT system with a given number of port elements is viable at all is unpredictable. On the contrary, EGT layouts with N PGTs possess all possible combinations of Ls, Cs, and Bs without considering the number of distinct speed ratios capable of being produced that the solution space is immense. Thus, the present study limits the enumeration of EGT layouts with the following assumptions.

1. The degree of freedom of the derived EGT system is either 3 or 4, which calls for the engagement of two friction devices for a 3 DOF system and three friction devices for a 4 DOF system to exhibit unit mobility.
2. There exist two inter-shaft clutches in a 3 DOF EGTs and three inter-shaft clutches for 4 DOF EGTs.

Thus, in the present case of 3 PGTs with 3 permanent connections, there shall be 2 brakes and 2 inter-shaft clutches, respectively.

Having decided upon the number of brakes in the EGT system, the next step is to find possible ways of disposing of them in the system where there are permanent connections between the shafts. Theoretically, every shaft of the system can become a port element. Therefore, it is necessary to assign the status of the port element to the available (3N – L) shafts in all possible ways without violating the following rules.

1. A PGT unit cannot have three port elements.
2. There can be a maximum of two-port elements between two PGT units having two permanent connections.

Also, the isomorphism test based on rooted tree structures (RTS) proposed by the authors [19,20] is implemented to avoid duplicate structures. RTS method was found to be robust as it can address isomorphism while synthesizing various structures reported by the same authors [21-24].

2.4 Number of port elements and their assignments in the EGT system

A low value of L implies a corresponding increase in the number of brakes in the EGT system. Therefore, among the available P port elements, there are \( \binom{P}{2} \) ways of assigning input/output elements.

2.5 Derivation of disposition of ‘C’ clutches

In an EGT system, a clutch may be incorporated between the elements of a simple PGT unit having utmost one port element. However, the speed relationship of the entire system is unchanged irrespective of which two of the three shafts are coupled by means of a clutch. Therefore, from among the three possibilities, only one must be chosen for the purpose. Given the graph of a system with permanent connections in which the port elements are identified, it is required to derive distinct ways of introducing the C number of clutches into the system. If there are C ways, the following situation arises depending upon the relative magnitude of C and 2.

i. When \( C < 2 \), the system cannot accommodate a requisite number of clutches, and hence the layout is rejected as it cannot yield the desired number of speed ratios.

ii. When \( C = 2 \), the system permits the inclusion of just one set of C clutches.

iii. When \( C > 2 \), there are \( \binom{2}{2} \) ways of introducing C clutches.

Thus, the need to detect isomorphism arises again here, for which the RTS-based isomorphism test is adopted.

2.6 Selection of distinct Input/Output port elements

Having identified the shafts qualifying for port elements earlier, two of them become input and output ports. Also, the layout structure does not change by merely reversing the locations of input and output members. Therefore, among the available P port elements, there are \( \binom{P}{2} \) ways of assignment of input/output elements; hence, an RTS-based isomorphism test is carried out to identify distinct configurations.

2.7 Detection of degeneracy in EGTs

An EGT layout generates multiple speed ratios because the engagement of a requisite number of friction devices causes some of the PGT units to be actively involved in the speed-changing process. Some of these elements play the definite role of input, output, and brake ele-
ments, while others merely act as a connection between the active PGT units. But there exist some instances wherein one or more PGT units are prevented from actively participating in the speed-changing process, which is listed below.

i. The braking of two elements of a PGT unit causes the braking of the third element; such a PGT unit becomes inactive.

ii. When the engagement of speed controlling devices gives rise to three permanent connections between two PGT units, all six elements belonging to the pair of PGTs have identical speeds. Such a pair of PGT units becomes inactive.

iii. If two shafts of a PGT unit rotate at the same speed by the engagement of a clutch between them, then the third shaft also rotates at the same speed.

2.8 Viability aspects of EGTs

Planarity check is done in the synthesized layouts to check the viability of EGT layouts synthesized using PYTHON programming. The viability of an EGT is defined by Lloyd [7], which states that an epicyclic gearbox is viable if all internal connections can be made coaxially. All EGT layouts synthesized according to the preceding steps need not generally comply with this requirement. However, using additional geared drives, one can make all the interconnections that the system requires. But this can only be done at the expense of its structural simplicity and the benefits of epicyclic gearings, such as its compactness and hence lesser weight. Also, it is proved that an epicyclic mechanism is viable if a two-dimensional portrayal when lines are drawn representing the internal and external connections, the connecting lines do not cross each other, and only pass-through representations of gear sets if they are connected to the planet carriers.

3. RESULTS

EGT layouts possessing up to 4 PGTs and 4 permanent connections are synthesized in the present work. Planarity and Viability checks are done, and the final complete collections are as tabulated in Table 1.

With the input as 3 PGT units and 2 permanent connections, the number viable of EGT layouts synthesized is 10. With 4 PGTs and 5 permanent connections, 246 layouts are found to be viable, consisting of three brakes and two clutches. Whereas, with 4 PGT units and 4 permanent connections, 8960 viable EGT layouts were synthesized, which consisted of two brakes and three clutches.

Table 1 EGT synthesis results

<table>
<thead>
<tr>
<th>No. of PGTs (N)</th>
<th>No. of permanent connections (L)</th>
<th>Total no. of brakes (B)</th>
<th>Total no. of clutches (C)</th>
<th>No. of Viable EGT Layouts</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>8960</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>246</td>
</tr>
</tbody>
</table>

4. CONCLUSION

Based on the aforementioned concepts, a Python program has been developed to perform the structural synthesis of EGTs. Input data to the program consists of the number of PGT units, permanent connections, brakes, and clutches required to be present in the EGT system. The procedure is automated up to the stage wherein the number of distinct speed ratios obtainable from the synthesized layouts is determined. The EGT layouts thus synthesized using the methodology are found to be viable and non-planar. Thus, a complete collection of several viable EGT layouts consisting of up to 4 PGTs and 5 permanent connections with 3 brakes and 2 clutches are presented in this work.

REFERENCES


[34] Maja Cavic et al. " Graphical synthesis of 6-bar dwell linkage mechanism"FME Transactions (2019) 47, 226-233


Епициклични системи преноса зупчаника (ЕГТ) налазе широк спектар примена у системима аутоматског мењача због своје флексибилности да испоруче различите односе брзина. Дизајн ЕГТ-а укључује набрајање свих могућих распореда, који се обично називају структурна синтеза. Дакле, структурна синтеза ЕГТ-а се састоји од дефинисања топологије за ЕГТ са спојеним планетарним зупчанима (ПГТ) заједно са ограничавајућим елемен-тима као што су везе, кочнице и квачила уз испоруку одређеног преносног односа између улазних и излазних вратила. С обзиром на број ПГТ-ова, трајну везу, квачила и кочнице, методологија систематски уводи ове елементе ПГТ-а како би се испоручио број различитих односа брзина. Одређивање броја различитих односа брзина за дату комбинацију кочница и квачила је изражено као трансформација графичког приkaza ЕГТ распореда. Развијен је компјутерски програм за структурну синтезу ЕГТ-а и валидиран у односу на 3 епициклична, 6-брзинска ЕГТ система. Спроводени су аспекти одрживости синтетизоване структуре. Такође, тест изоморфизма је обрађен у методологији синтезе да би се откриле јединствене ЕГТ структуре у свакој фази синтезе.