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Structural Optimisation of Planetary Gearbox Components

The high-speed reduction in a relatively small space, coupled with a torque load capacity larger than that of any other standard transmission, positions planetary gear systems as one of the most vital components in gearing applications today. Structural analysis of a gear train is conducted using CATIA software, employing strain full tensor distribution analysis through Finite Element Method (FEM). The analysed natural frequencies and vibration modes of each component provide essential information for fine-tuning resonances away from the assembly's operating speeds. Based on these results, component dimensions are optimized using CATIA's optimizer module to achieve a structurally compact gearbox design. The minimization of gearbox mass serves as an objective function, with gear dimensions as variable parameters constrained by ultimate bending tooth root stress, safety factors, and critical frequencies expressed as inequalities. Structural optimization results are presented in tables, comparing initial natural frequencies with those obtained from the optimized solution. The final optimized design for each gear train component is also presented and discussed in the paper's concluding section. In conclusion, the paper outlines its primary objectives, summarizing key findings and proposing new ideas for further research to enhance and optimize planetary gear transmissions in practical applications.

Keywords: *planetary transmission; machine design; structural optimization; CATIA*

1. INTRODUCTION

Planetary gearboxes are widely used in various mecha–nical applications such as industrial, automotive, and aerospace machinery due to their compact size, high power density, and excellent torque transmission capa–bilities [1-4]. However, despite their numerous advan–tages, planetary gearboxes often face challenges related to their structural integrity and efficiency. Optimization plays a prominent role in structural analysis and design. The importance of minimum weight design of structures is recognized in most industries because the weight of the system affects its performance or because of the reduced energy loss saving conventional energy sour–ces. Optimization in the design process is not only a matter of weight but could be used to optimize any type of data. In real engineering problems, it is also common to minimize an objective function describing data such as the total volume, the lifetime or the cost of a struc–ture. On another hand, optimization can use objective functions not only to be minimized, but sometimes to target, or maybe maximize value for a defined desirable solution.

Structural optimization techniques have emerged as an effective means to enhance the performance and reliability of planetary gearboxes, offering opportunities to address the limitations associated with conventional designs. Structural optimization aims to find the most efficient and cost-effective design for a given structure to meet specific performance criteria while considering various constraints [5]. The goal is to minimize or maximize certain objective functions, such as reducing mass/weight, minimizing deflection, or maximizing stiffness, while ensuring the structure remains safe, and stable, with desired load capacity and operational ability, as well as cost-effective. It involves mathe–matical and computational methods to optimize the st–ructure's size, shape, and material distribution i.e. topology [6]. The process typically begins with defining the objectives, which could be minimizing weight, maximizing stiffness, or reducing cost. Next, the design variables, such as the dimensions and properties of the components, should be identified. Constraints are also established, which may include stress limits, displa–cement limitations, or manufacturing constraints.

The process of structural optimization involves mathematical modelling, computational analysis, and iterative algorithms. Engineers use mathematical opti–mization techniques to search for the best combination of design variables, such as material properties, geometries, and support conditions, that will lead to the desired performance. The optimization process may employ various algorithms, such as gradient-based methods, evolutionary algorithms, or genetic algorithms [7], depending on the complexity of the problem and the available resources. Finite element method (FEM) analysis and other numerical methods are used to eva–luate the structural response and behaviour under

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different loading conditions. So, structural optimization enables engineers to create lighter, stronger, and more efficient structures, leading to cost savings, improved sustainability, and enhanced overall performance.

The analysis of the existing available literature on the structural optimization of planetary gear systems highlights the growing interest in the structural optimization of planetary gearboxes to meet the demanding requirements of modern mechanical systems. Various approaches, such as gear tooth profile modifications, material selection, and multi-objective optimization, have been explored to improve the gear system's performance, reliability, and efficiency. The studies explore various techniques and demonstrate the importance of tailoring the design of planetary gearboxes to specific applications.

The paper [8] provides an extensive analysis of planetary gears, considering influencing factors that impact their functionality. The author employs a systematic approach to design and optimization, including theoretical modelling, numerical simulations, and practical experiments. Three basic planetary gear transmission concepts, using different gear ratios are presented in the paper [9]. The study emphasizes reducing volume and weight, applying the lowest feasible number of gear teeth, and maximizing the number of planets to achieve high power density. Additionally, special low-loss involute gears are designed to reduce power losses due to sliding in the loaded gear meshes. The review paper [10] focuses on the optimization of epicyclic gear train design to reduce gear load failure. A method that combines structural optimization with tooth contact analysis to optimize the housing structure of ring gears in planetary gearboxes is proposed by the study [11]. This paper presents a genetic algorithm-based structural optimization, leading to variable rim thicknesses and improved stiffness in critical areas. A novel multi-objective optimization approach for planetary gearbox design using a discrete version of the Non-Dominated Sorting Genetic Algorithm (NSGA-II) is presented in [12]. The objectives are to minimize the gearbox weight and total power loss. Several design variables, including the number of teeth in gears, module, face width, input shaft, and planet pin diameter, are considered. The study takes into account regular mechanical constraints, bearing selection, and scuffing constraints (the main study's contribution). The main objectives of the optimization in the paper [13] are to minimize the mass of the planetary gear train while maximizing the gear ratio, taking into account constraints related to gear teeth bending stress, contact stress, and other side constraints. The selected design variables include the module, gear teeth width, number of teeth for the sun and planet gears, as well as the inner and outer diameters of the gears. To solve the optimization problem, the Genetic Algorithm (GA) is implemented using MATLAB optimization toolbox routings. Different types of materials for the planetary gears are studied. The kinematic analysis and evaluation of Ravigneaux gear pairs, including the effects of power flow and power loss, as well as the evaluation of energy utilization efficiency, were conducted in [14]. The optimization of Ravigneaux planetary gears, focusing on the safety coefficient through changes in materials, module, and gear width, was performed using the Taguchi method and ANOVA. The paper [15] focuses on optimizing the parameters of the central gears in a planetary gearbox using the Taguchi-Grey analysis, which combines the Taguchi orthogonal array and Grey relational analysis to address multi-performance optimization challenges. The study identifies the optimal parameter combination for maximizing the safety coefficient and determines that the module is the most influential factor, followed by gear width and material.

The multi-objective nonlinear optimization problem in planetary gear trains is presented in the paper [7]. The optimization process utilizes the genetic algorithm (GA) and a weighting method to approximate the Pareto set. By transforming the multi-objective optimization into a single-objective one, the GA-based approach yields satisfactory results, providing designers with preliminary design parameters for various gear ratios in planetary gear trains. The research focuses on addressing conflicting objectives, such as planetary gear train efficiency and the distance between centres of sun gear and planetary gear, which are essential considerations. The paper [16] addresses the challenging problem of multi-objective non-linear optimization in planetary gearboxes using a hybrid metaheuristic algorithm. Designing optimal planetary gear trains involves minimizing conflicting objectives simultaneously, such as gearbox volume, centre distance, contact ratio, and power loss. The authors propose a novel hybrid algorithm called MHPSODE, which combines particle swarm optimization (PSO) and differential evolution (DE) algorithms to tackle the complex multi-objective optimization problem. The proposed algorithm successfully obtains non-convex Pareto solutions, revealing key insights into weight reduction, efficiency improvement, and preventing premature gear failure. Numerical simulation results demonstrate the superiority of the MHPSODE algorithm compared to other well-known optimization algorithms, as it produces higher-quality Pareto solutions. The proposed MHPSODE algorithm leads to significant improvements in centre distance, gearbox volume, and efficiency. A study on the optimization of complex planetary gear trains consisting of two planetary gear sets of the basic type is presented in the paper [17]. The approach employs multicriteria optimization to achieve acceptable solutions, considering kinematic schemes, symbolic views, and power flow. Design parameters are evaluated by taking into account mass and manufacturing costs, using the Pareto optimization with a weighted coefficient method. The paper [18] discusses two concepts for evolutionary design optimization that rely on CAD-based design encodings of mechanical structures. The application of generative structural analysis for design optimization in CATIA allows engineers and designers to explore multiple design alternatives automatically, leading to more efficient and lighter structures [19]. By leveraging advanced algorithms and simulation capabilities, CATIA's generative structural analysis enables users to identify the best design options based on predefined objectives and constraints. The text [20] highlights how CATIA's

KnowledgeWare module enhances design efficiency, consistency, and quality while also reducing the risk of errors and rework. An important topic on gear simulations and potential optimizations in gear train design, with a focus on planetary transmissions is introduced in the paper [21]. It highlights the practical results obtained from structural analysis using the finite element method (FEM) and demonstrates the utility of CATIA V5 software for solving engineering problems related to gears. The paper [22] emphasizes the signi–ficance of optimization in machine design, particularly for achieving objectives such as minimum weight, energy loss, and other relevant design parameters. It highlights the relevance of optimization in addressing various tribology problems and presents a brief overview of investigations that use powerful optimi–zation methods for solving such problems. An approach to achieve design simplification and enhance mecha–nical power flow efficiency in an automotive planetary gearbox, developed through CAD modelling is pre–sented in [23]. The paper [24] presents a reliability assessment of planetary gearbox housing by conducting a comparative analysis of housings made from carbon fiber-reinforced composites, aluminum alloy, and steel. Using SolidWorks Simulation, the study evaluates stress (von Mises) and deformation to estimate competing risks associated with the operation of composite material housings.

The subsequent sections of this paper will delve deeper into the investigation and proposal of an app–roach for the structural optimization of planetary gear–boxes. This approach integrates an optimization process aimed at enhancing the initial transmission design. In this process, the reduction of mass is chosen as the objective function, with dimensions serving as variable parameters derived from structural analysis. This is coupled with a set of crucial inequality constraints. By presenting each of the transmission components in optimized form and shape, covered by comparing with the starting position, the paper underscores the significance of structural optimization. In doing so, the authors strive to contribute to the advancement of efficient engineering systems with extended service life.

2. PLANETARY GEARBOX COMPONENTS AND STRUCTURE ANALYSIS

The dynamic analysis of planetary gear components is critical for understanding their behaviour under different loads and operating conditions. In this regard, since the geometry of planetary gear components is complex and irregular and the designer is faced with complex boundary conditions, the FEM, as a widely employed numerical approach, where complex engineering pro–blems are discretized into sub-domains called elements, can be successfully employed to solve this problem. In FEM, each element is characterized by a set of equations that describe its behaviour. By assembling these element equations into a global matrix equation, the entire system can be represented and solved effi–ciently. FEM finds widespread use in various engi–neering disciplines for dynamic analysis. Some prominent areas where FEM is extensively applied include gear pairs [25], structural dynamics [26], vibrations [27], etc.

2.1 Gearbox components for analysis

Planetary gearboxes are complex mechanical systems that include many components, including gears, shafts, bearings, and housings. In this paper, we consider the single-stage planetary gearbox which consists of a carrier, three planet gears, a single sun gear with external gearing, an input shaft, an output shaft, and a ring gear with internal gearing. Due to the geometrical complexities of the abovementioned parts, it is not possible to analytically express equations for the frequency analysis. Therefore, the frequency analysis of each component is performed through FEM using the CATIA software. To perform FEM analysis, each of the components is first 3D modelled in CATIA, and then the appropriate mesh is generated based on the geo–metrical data. Once the mesh is generated, the material properties and boundary conditions are assigned to the model. Next, the FEM simulation is carried out, which involves solving the equations of motion for each component. The results of the simulation provide valuable information about the natural frequencies and mode shapes of the gears. Overall, the FEM analysis created in CATIA software provides a basis for the optimization model in this paper, which enables us to optimize the design of the gears and other components and ensure their reliability and performance.

The main parameters of the observed gearbox com–ponents are the number of teeth, for the sun central gear z_1 = 20, planet gear z_2 = 42 and ring gear with internal gearing z_3 = -105. Common gearing profile module has the value of m = 2.5 mm, where all gearing models are b = 40 mm wide. The properties of steel, as a material used in the calculation, were consistent with elasticity modulus E = 207 GPa, density ρ = 7860 kg/m^3 and Poisson ratio value ν = 0.3. Just to annotate, the results of the conducted frequency analysis of each component are presented together with optimization results in the next chapter, here follows just the theoretical background.

2.2 Theoretical background for gearbox dynamic behaviour

In developing appropriate equations for frequency analysis, we start from Langrage’s equations of motion in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\partial \dot{q}_j}+\frac{\partial V}{\partial \dot{q}_j}=Q_j, \quad j=1,2,...,n . \qquad (1)$$

where T is the kinetic energy of the system, V is the potential energy of the system, q_j is the j-th generalized coordinate, $\dot{q}$ is the first order derivative over time of the appropriate generalized coordinate, Q_j are genera–lized forces, and n is the total number of degrees of freedom of the system.

By introducing the vector of generalized coordinates $\mathbf{q}=\left[q_1,q_2,...,q_n\right]^T$ and the following expressions:

$$\left\{ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j} \right) \right\} = \mathbf{M}\ddot{\mathbf{q}}; \quad \left\{ \frac{\partial T}{\partial \dot{q}_j} \right\} = \mathbf{C}\dot{\mathbf{q}}; \quad \left\{ \frac{\partial V}{\partial \dot{q}_j} \right\} = \mathbf{K}\mathbf{q}, \tag{2}$$
$$j = 1, 2, ..., n.$$

we obtain the expression for Langrage's equations in matrix form as follows:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \{Q_i\} \tag{3}$$

where **M** is the mass matrix, **C** is the damping matrix, **K** is the stiffness matrix, and $\{Q_i\}$ is the vector of external forces applied to the system.

The kinetic energy for the single element can be expressed as:

$$T = \frac{1}{2}\int_V \mathbf{u}^T \mathbf{N}^T \mathbf{N}\mathbf{u}\rho dV, \tag{4}$$

where $\mathbf{u} = [u, v, w]^T$ is the displacement of the differential mass in the coordinate directions and **N** is the element displacement function. Thus, the element mass matrix **M** is determined as:

$$\mathbf{M} = \int_V \rho \mathbf{N}^T \mathbf{N} dV. \tag{5}$$

The appropriate stiffness matrix **K** is obtained from the principle of virtual work, where the equilibrium between internal and external forces is attained to minimize potential energy deviations from

$$\mathbf{M} = \int_V \mathbf{B}^T \mathbf{D}\mathbf{B} dV, \tag{6}$$

where **B** denotes the strain-displacement matrix, which depends on the derivatives of the shape functions concerning the global coordinates and **D** is the elasticity matrix, which characterizes the material's response to stress and strain.

Taking into account that the problem of interest here is to determine the natural frequencies, we have $\delta W = 0$, therefore without externally applied nodal forces this leads to vector $\{Q_i\} = 0$, and since we consider an ideal elastic system, which contains no mechanism for energy dissipation, this leads to $\mathbf{C} = 0$. Therefore, the starting matrix system of Langrage's equations of motion (3) can be transformed to:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0. \tag{7}$$

Without externally applied nodal forces, equation (7) is a system of homogeneous second-order differential equations, with the independent variable - time. Hence, we have an eigenvalue problem in which the eigenvalues are the natural frequencies of oscillation of the structural system and the eigenvectors are the amplitude vectors (mode shapes) corresponding to the natural frequencies. Therefore, the frequency equation is represented as:

$$\det(\mathbf{K} - \lambda\mathbf{M}) = 0. \tag{8}$$

where $\lambda = \omega^2$. The equation (7) has n rootsλ_i ($i = 1,2,\ldots,n$) which are called eigenvalues. The natural frequencies of the considered mechanical system are then ω_i ($i = 1,2,\ldots,n$) among which the ω_i is called the fundamental frequency.

2.3 Analysis of the FEM mesh element size

To determine the optimal size of the FEM element mesh size in this study, an analysis was conducted on planetary gearbox components. The aforementioned components are of utmost importance in the assembly, since they facilitate most interactions between gear teeth. Moreover, the disparity in the dimensions of these components serves as a reliable indicator of the appropriate mesh element size for the rest of the gearbox parts. Fig. 1 displays a graph that depicts the variations in Von Misses stress values concerning the size of the finite mesh elements.

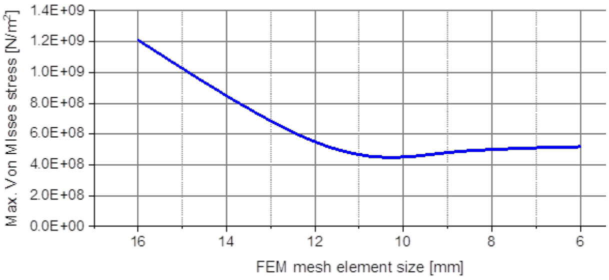

Figure 1. Analysis of the FEM mesh element size for gear with the internal gearing, planet gear and sun-central gear

Fig. 1 refers to the mesh size analysis of the internal gear, mesh size of the planet gear and also mesh size analysis for sun-central gear. Based on our analysis, it can be inferred that the results of stress analysis converge for a mesh size of 6mm is suitable and enough good for the parts taken for analysis in this paper. This conclusion is drawn from the observation that reducing the mesh element size does not yield substantial alterations in the strain tensor component values. Reducing the size of mesh elements would result in a higher computing demand.

2.4 Boundary conditions

The boundary conditions hold significant importance within the context of FEA, in which one aspect pertains to supports employed, while the other pertains to the type of load being considered. In this paper, the boun–dary conditions are determined based on the function and operational state of the gear. On the other hand, when we consider loads, in this paper, we are dealing with the frequency and stress analysis without exter–nally applied nodal forces, there are no external loads to apply. The boundary conditions that were chosen in this paper are as follows:

a. For the gear with internal gearing – On three points where internal gearing is in contact with planet gears the appropriate boundary conditions were intro–duced.
b. For planet gear – the boundary condition has been chosen on toothing on teeth that are in contact with central gear as well as on teeth that are in contact with internal gearing.

c. For sun-central gear – the boundary conditions are twofold, first on toothing, on three points where the gear interacts with planet gears, and secondly on parts of the central gear where it interacts with a coupling.

The selection of boundary conditions has been ch–osen appropriately to take into account the gearbox components and their respective influences on one another.

2.5 Natural frequencies of basic, non-optimized parts

In this paper, we have employed a FEM analysis to obtain the natural frequencies of each component of the planetary gearbox, intending to include this parameter as the constraint in the formulated optimization model. This FEM analysis has been applied and conducted for each planetary gearbox component separately, and the depiction of strain full tensor distribution for the first eigenmode of each considered component before optimization (initial gearbox parameters) is shown in Figures 2-4. Besides the first eigenmode, shown in those figures at the scale, for each gearbox component, others from the full list of ten natural frequencies are very important. Those frequencies are presented together with the list of frequencies for the optimized part, following the results of optimization (Tables 1-3).

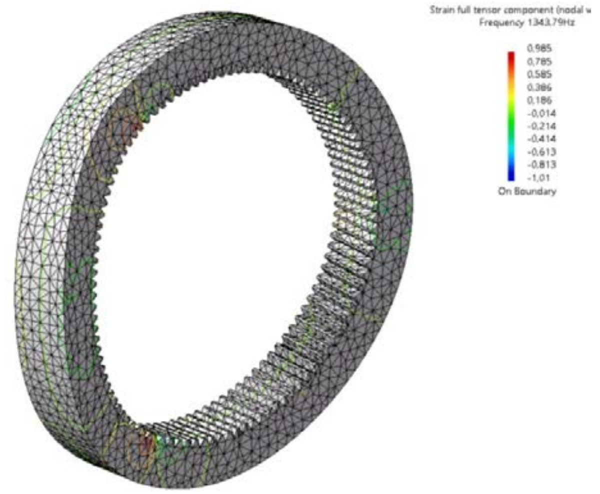

Figure 2. Strain full tensor distribution of the first eigenmode internal gear (before optimization)

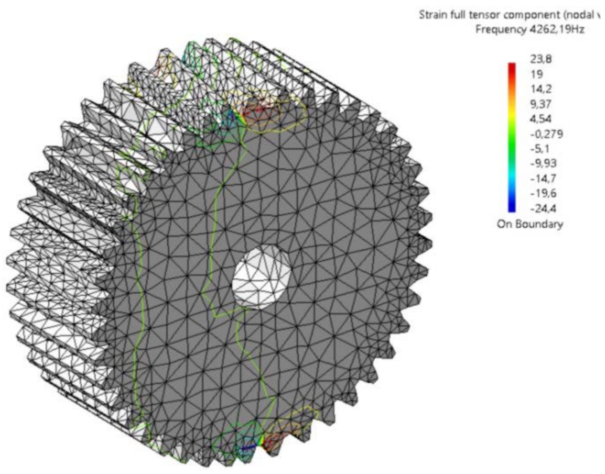

Figure 3. Strain full tensor distribution of the first eigenmode for planet gear (before optimization)

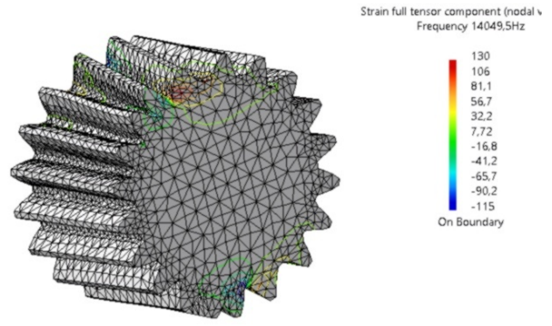

Figure 4. Strain full tensor distribution of the first eigenmode for sun-central gear (before optimization)

3. STRUCTURAL OPTIMIZATION FOR PLANETARY GEARBOX

Concerning planetary gearbox components, there are several optimization possibilities. As an introduction to structural optimization here will be presented an approach to optimization of planet gears number and also planetary gearbox ratio. This paper is mainly focused on the optimum design of transmission components targeting their minimum weight, which makes lighter-than-possible gearbox assembly useful for its final design.

3.1 Natural frequencies of basic, non-optimized parts

The main advantage of a planetary gear train is reflected in its transmission the power through several parallel branches at the same time, which also distributes the corresponding gear load on many more surfaces. This advantage, on another hand requests increasing the number of planet gears, which are positioned in one gear plane, but from the design side limited by the clearance. Regarding this distribution of several planet gears in one plane, the constraint can be expressed in the form of the following inequality:

$$f\left(n_w, z_a, u_{a-H}^b\right) = \left(\frac{z_a - 2.5}{z_a u_{a-H}^b}\right)^{\frac{1}{2}} - \sin\left(\frac{\pi}{4} - \frac{\pi}{2n_w}\right) \geq 0 \quad (9)$$

where n_w denotes the number of planet gears in a single plane of planetary gearbox, for the analysis in Fig. 5 taken as$n_w \in \{3,5\}$, $z_1$is the number of teeth of the central gear and $u^b_{a-H}$is the overall gear ratio.

Based upon the graphic representation of the results obtained in [8], it follows that the given functional constraint is exceptionally sensitive to the change of the planetary gear ratio. It gives the conclusion that gear trains with threeplanet gears can be used for the overall gear ratio values not greater than 13 and with five planets not greater than 4.4 (Fig. 5).

3.2 Structure optimization methods in CATIA

An approach in optimization procedure is conducted in this paper based on early mentioned dynamic analysis of gear train components. For this complex structural optimization problem authors used CATIA software, with specialized Product Engineering Optimizer modulus useful for solving every one-dimensional optimization in the design process. For solving different optimization problems, in this software tool couple of methods are available to the constructor. The Product Engineering Optimizer can operate with two algorithms: the *Conjugate Gradient* and the *Simulated Annealing*. A designer can select one or the other to run an optimization depending on the function to analyse.

In the field of gearbox optimization, various meta-heuristic algorithms have been utilized to address intri–cate design challenges, including hybrid Particle Swarm Optimization and Differential Evolution algorithm [16], Genetic Algorithm [7][13], Non-Dominated Sorting Genetic Algorithm (NSGA-II)[12], Taguchi method[15], etc. In this paper, the Simulated Annealing algorithm has been employed, as a stochastic, meta-heuristic opti–mization technique inspired by the annealing process in metallurgy. By analysing the considered planetary gear–box optimization problem presented in this paper, we have concluded that this problem is complex, non-linear, and multi-modal. Consequently, in order to address these complexities of the considered problems, the SA is selected as it is well-suited for the intricate nature of planetary gearbox optimization. Its ability to escape local optima by accepting occasional inferior solutions enables a more comprehensive exploration of the design space, thereby increasing the probability of identifying a global optimum. By employing SA, our approach effectively balances these competing factors, resulting in optimized designs that meet performance requirements while minimizing weight and cost.

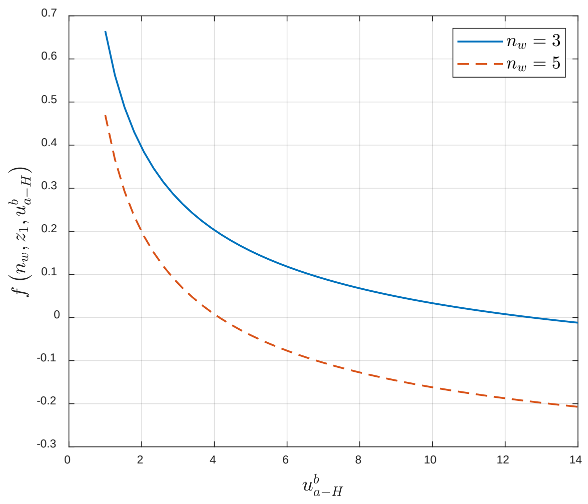

Figure 5. Graphical illustration of the trade-off between the advantage of having more planet gears in a single gear plane for increased performance and the design limitation imposed by clearance constraints.

3.3 Minimum component weight as the objective function

In this optimization model, the variable vector should include gear dimensions to get optimal geometric parameters which will result in minimum mass, as an objective function:

$$\min_{\overline{x} \in D} m(\overline{x}) = \min_{\overline{x} \in D} m(x_1, x_2, \ldots x_n)\,. \qquad (10)$$

The variable vector includes gear dimensions, where two important constraints are included in the optimization process. The safety factor of the tooth root strength and natural frequencies of every planetary gear train component are constraints that are aimed at getting a proper optimal solution. An admissible set of solutions *D* for this constrained optimization problem can be also written in the form of an equation:

$$D = \left\{ x \in R^n \left| \frac{[\sigma_F]}{\sigma_F} - S_F > 0 \wedge \omega_1 - \omega_z^* > 0 \right. \right\}. \qquad (11)$$

where $[\sigma_F]$ is the ultimate bending tooth root stress, σ_F is bending tooth root stress under certain geometrical and operational conditions, S_F is the safety factor ($S_F = 2$), ω_1 is the corresponding natural frequency for a new optimized solution, and ω_z is natural frequency.

This complex structural optimization problem was solved by CATIA modulus for one-dimensional optimi–zation, for this problem using the *Simulated Annealing* (SA) method [9]. This method is used to approximate the solution of very large combinatorial optimization problems. This technique originates from the theory of statistical mechanics and is based upon the analogy between the annealing of solids and solving opti–mization problems. A *Simulated Annealing* algorithm is slow but more robust than compared to other conventional algorithms. The appropriate pseudocode of the *Simulated Annealing* algorithm has been presented in Fig. 6.

Algorithm 1 Pseudocode of Simulated Annealing Algorithm

Generate initial solution $\mathbf{x}^c$
Initialize parameters G_{max} and T
$G = 0$
for $G = 1$ to G_{max} **do**
  while Stopping criteria not met **do**
    Generate randomly a neighbouring solution $\mathbf{x}^n$
    Compute $\Delta = f(\mathbf{x}^n) - f(\mathbf{x}^c)$
    Generate random variable r
    if $\Delta < 0$ or $e^{-\frac{\Delta}{T}} > r$ **then**
      Assign $\mathbf{x}^c = \mathbf{x}^n$
    end if
  end while
  Reduce T
  $G{+}{+}$
end for

Figure 6. Pseudocode of the Simulated Annealing algorithm.

To control the numerical accuracy of the obtained solution in the SA algorithm, the termination criterion is employed, whereas as the stopping criterion, the maximum number of iterations as well as the number of consecutive updates without improvements in the objective function are employed. The optimization process targeting minimum mass was conducted in the CATIA module (Product Engineering optimizer) using *Simulated Annealing* (SA) method, already described) following with the appropriate pseudocode (Fig. 6). This method leads to the minimal mass of each component, converging to the optimal solution after 50 iterations (shown in Fig. 7).

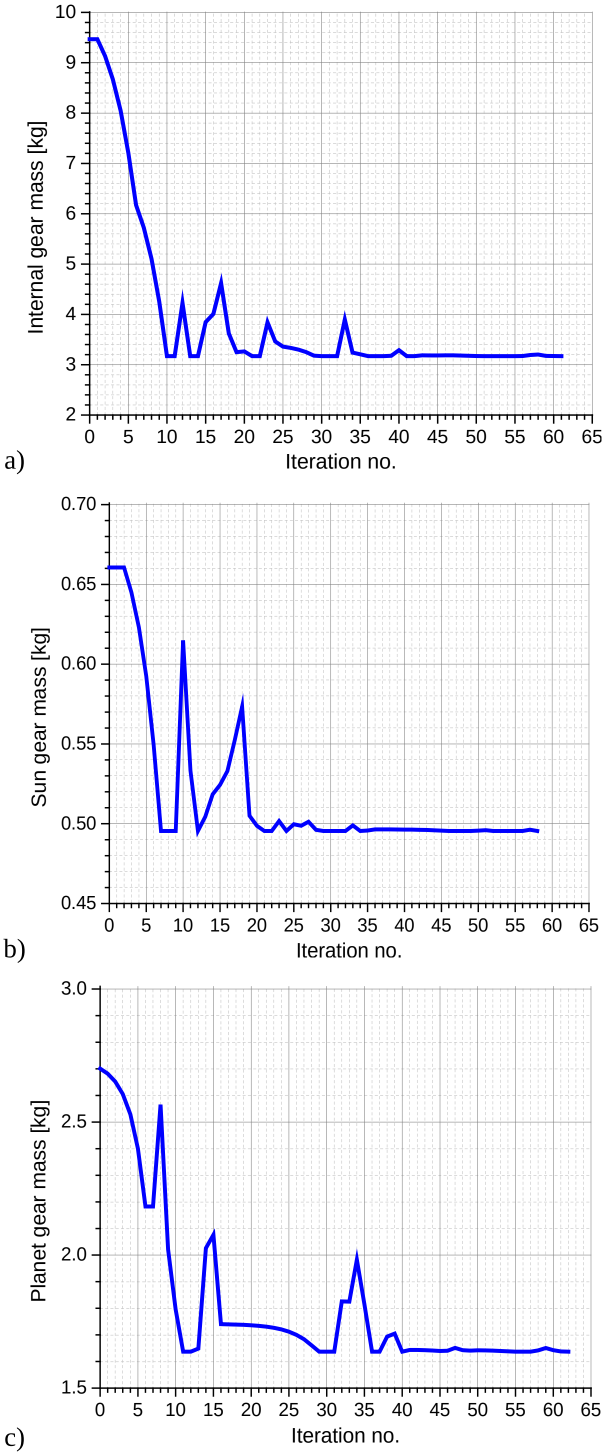

Figure 7. SA iteration process for each gearbox component

This optimization procedure and algorithm have been applied and conducted for each planetary gearbox component separately. For the frequency analysis, more important are strain full tensor values, compared with Von Mises stress which was taken into account just for FEM mesh size analysis (Figure 1) and as a constraint in the optimization process, by calculation of bending tooth root safety factor. The final solution after opti–mization for every planetary transmission component with a new full tensor component distribution is shown in Figures 8-10. Every figure is followed by a corres–ponding table (Tables 1-3). The first row in the table for each geartrain component presents natural frequencies for its original dimensions and corresponding original mass value. The second row in the table presents natural frequencies for an optimized solution with corres–ponding easily observed reduced component mass.

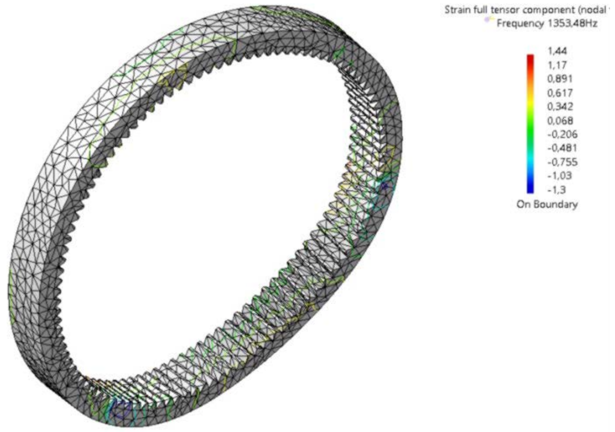

Figure 8. Illustration of strain full tensor for internal gear (optimized dimensions)

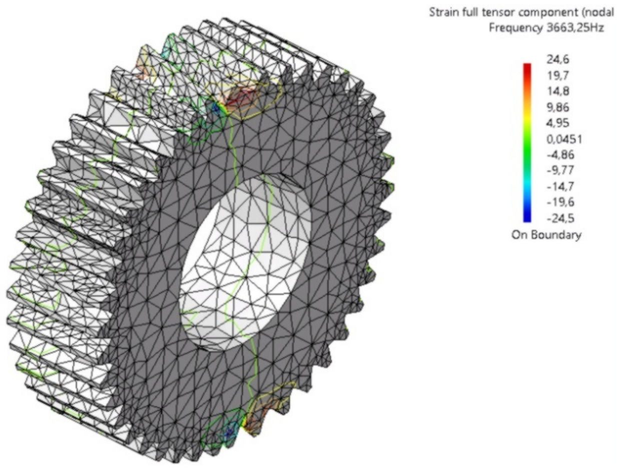

Figure 9. Illustration of strain full tensor for planet gear (optimized dimensions)

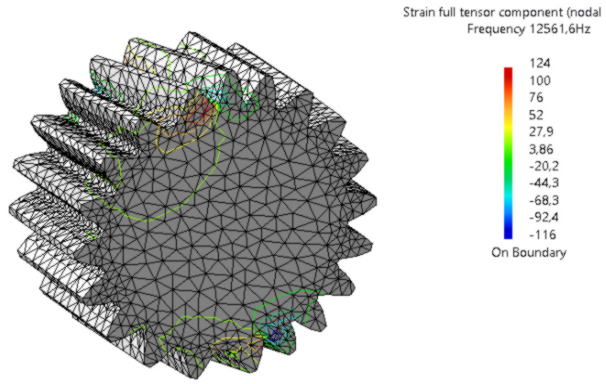

Figure 10. Illustration of strain full tensor for sun-central gear (optimized dimensions)

For the probably most important component, rep–resented by internal gearing, an optimization process targeting minimum mass as the objective function, was conducted with two variable dimension parameters. Those parameters for the ring gear with internal gearing are represented by: gear width (starting from the original 40mm to a minimal value of 30mm) and gear rim thickness (starting from the original 35mm and reducing to a minimal 18mm, aimed to meet the safety factor S_F = 2 against bending tooth root stress. Planet gears with external gearing were optimized by varying also two parameters: gear width (40 to minimal 30mm) followedby inner diameter for connection with shaft (20 to 50mm). The last optimized component of the planetary gearing was the sun-central gear, where a single variable parameter in the form of gear width (40 to minimal 30mm) makes a much simpler optimization process, of course, targeting minimum mass as an objective function. Next Figures 8-10 represent strain full tensor distribution for the first eigenmode of each considered component, following with corresponding tables (1-3). It could be easy to observe and compare the original starting mass with the optimized one, as well as the list of natural frequencies in the corresponding tables for each planetary gearbox component.

Table 1. Natural frequency series for internal gear (original and optimized dimensions)

Mod	*Q* [Hz]	
	m = 9.47 kg	*m* = 3.17 kg
1	1344	1353
2	1440	1444
3	1747	1523
4	1953	1583
5	2898	1923
6	3032	2062
7	3653	3074
8	3865	3206
9	3910	3675
10	4007	3698

Table 2. Natural frequency series for planet gear (original and optimized dimensions)

Mod	*Q* [Hz]	
	m = 2.70 kg	*m* = 1.64 kg
1	4262	3663
2	6026	5222
3	8579	9657
4	10741	10006
5	12380	10125
6	12712	10442
7	13534	11485
8	17301	13655
9	19312	15001
10	22610	15708

Table 3. Natural frequency series for sun-central gear (original and optimized dimensions)

Mod	*Q* [Hz]	
	m = 0.661 kg	*m* = 0.495 kg
1	14050	12562
2	16914	16068
3	19314	19250
4	25884	25758
5	27945	27811
6	28480	28611
7	36578	34661
8	46004	43025
9	46095	48195
10	48175	49299

4. RESULTS AND DISCUSSION

To prevent the work from becoming overly extensive, the authors of the paper in the first step decided to present the results of frequency analysis for each com– ponent of the planetary gearbox (Figures 2-4). These figures display strain full tensor distribution values for natural frequencies, focusing specifically on the first and most significant (Mod 1) frequency. Later in Tables 1-3, comparison of basic model frequency list could be easily compared with the final reduced mass and the primary self-frequency for this optimized solution.

As described in Chapter 3, the optimization pro– cedure involves varying the main dimensions of each component to achieve a minimum mass, as defined by the objective function in relation (10), while adhering to the constraints outlined in relation (11). Probably the most important component of the transmission is ring gear with internal gearing, the optimization results of which are shown in Fig. 8 and Table 1. The optimal solution for this component shows that it is possible to reduce the mass by much more than 2.5 times compared to the original construction, where the request of targeting the reduced size of the outer ring, calculating with the thickness of the toothing rim meets the strength conditions from the limitations.

In the case of the planet gear, a crucial transmission component, the presented results in Fig. 9 and Table 2 indicate a twofold mass reduction for a single gear. Given that there are three such gears in the assembly, this outcome significantly contributes to overall energy savings.

The sun-central gear as a single and the smallest component (values in Fig. 10 and Table 3) exhibits a mass reduction of around 25% while considering the constraint related to the safety factor of tooth root stress.

The authors of the paper would like to add that the whole described analysis and structural optimization process is conducted for each of the transmission com– ponents separately.

As the first step in further research directions in structural optimization of gear transmission as a whole planetary gear train, the authors make an effort to complete assembly analysis, and simplify the gearbox, without input/output shafts, corresponding keys, satellite carrier and connections. Figures 11-12 show simplified planetary gearbox assembly, basic and optimized with corresponding strain full tensor distribution, all for the first natural frequency.

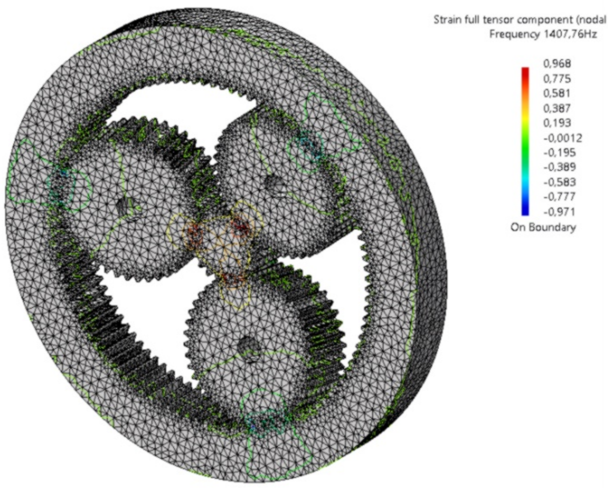

Figure 11. Illustration of strain full tensor for the original whole assembly

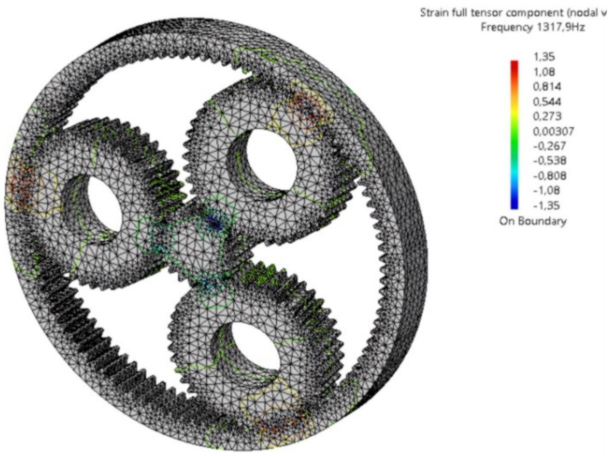

Figure 12. Strain full tensor distribution for planetary gear assembly (optimized dimensions)

The authors of the paper would like to add that the whole described analysis and structural optimization process is conducted for each of the transmission com–ponents separately. As the first step in further research directions in structural optimization of gear transmission as a whole planetary gear train, the authors make an effort to complete assembly analysis, and simplify the gearbox, without input/output shafts, corresponding keys, satellite carrier and connections. Figures 11-12 show simplified planetary gearbox assembly, basic and optimized with corresponding strain full tensor distribution, all for the first natural frequency.

A complete list of 10 natural frequencies fornon-optimized and final assembly with mass-reduced components is given in Table 4, where also starting and reduced gearbox mass is easy to observe.

Table 4. Natural frequency series for whole gear assembly (original and optimized dimensions)

Mod	*Q* [Hz]	
	m = 18.231 kg	*m* = 8.57 kg
1	1408	1318
2	2876	2525
3	2955	2591
4	2994	3059
5	3393	3101
6	3758	3305
7	3762	3480
8	3792	3495
9	4011	3523
10	6492	5849

5. CONCLUSION

In this part authors would like to outline primary objectives of the paper, summarizing key findings and proposing new ideas for further research to enhance and optimize planetary gear transmissions in practical applications.

Authors have conducted optimization procedure of each transmission component meets targeting the reduced size calculating, with main constraint following the tooth root strength conditions from the limitations. For the ring gear with internal gearing, as probably the most sensitive component of the transmission, results show that it is possible to reduce the mass by much more than 2.5 times compared to the original construction. In the case of the planet gear as crucial transmission component, because there are three such gears in the assembly, outcome significantly contributes to overall mass reduction. As a next step in structural optimization authors made an effort to complete assembly analysissimplifying the planetary gearbox, where double reduced gearbox mass could be easy observed.

The conclusion highlights that the results presented in this paper offer valuable insights and practical significance by elucidating the vibrational behaviour of the planetary transmission structure. Furthermore, the study underscores that the outcomes of structural analysis and optimization serve as an excellent foun–dation for redesigning existing products or creating entirely new designs. These endeavours aim to address various characteristics required for different gear trans–missions, ultimately striving for highly reliable and efficient gearbox solutions, while contributing to overall energy savings.

As an annotation the authors would like to add that those results represent just a proper basic idea for fur–ther complete planetary gearbox analysis, if possible, to compare and validate them by experiments in next coming research activities.

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СТРУКТУРАЛНА ОПТИМИЗАЦИЈА КОМПОНЕНТИ ПЛАНЕТАРНОГ ПРЕНОСНИКА

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Планетарни преносници представљају један од најзначајнијих типова механичких преносника снаге посебно у случајевима када је неопходно остварити велики степен преноса у компактној конструкцији, истовремено омогућавајући преношење већих обр–тних момената у односу на конвенционалне механичке преноснике са фиксним осама. Струк–турна анализа зупчастог склопа спроведена је коришћењем софтвера CATIA, уз примену анализе расподеле комплетног тензора деформација помоћу Методе коначних елемената (МКЕ). Истраживање сопствених учестаности и модова вибрација сваког појединачног елемента обезбеђује кључне инфор–мације за прецизно померање резонантних учеста–ности ван радних опсега брзина. На основу доби–јених резултата, димензије компоненти су оптими–зоване коришћењем CATIA оптимизационог модула у циљу остваривања компактне конструкције плане–тарног преносника. Минимизација масе преносника дефинисана је као функција циља, при чему су ди–мензије зупчаника усвојене за оптимизационе пара–метре. При томе, ограничења у облику неједнакости су задата у виду критичног напона у корену зубца, степена сигурности, као и критичних учестаности. Резултати структурне оптимизације приказани су

табеларно, уз поређење почетних сопствених учес–таности са онима добијеним након оптимизације. У завршном делу рада презентована су и разматрана коначна оптимизована решења за сваку компоненту зупчастог склопа, као и за склоп у целини. У закључку, истакнути су примарни циљеви рада, сумирани кључни резултати и предложене нове идеје за даља истраживања у циљу унапређења и оптимизације планетарних преносника у прак–тичним инжењерским применама.