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On Application of Statistically-Based Mathematical Models for Power Prediction of High-Speed Craft

This paper presents a comprehensive framework for predicting the power requirements of high-speed craft, including semi-displacement, semi-planing, and planing hull forms, through the use of statistically-based mathematical models. These models belong to the class of reduced-order models, which streamline the inherently complex processes of modeling hull resistance, dynamic trim, and propeller efficiency, including the interactions of multiple design parameters. Model development has relied on various regression techniques and, more recently, artificial neural networks (ANN). These mathematical models often offer advantages over the CFD techniques that nowadays dominate professional journals and therefore deserve careful attention. This review presents the methodology in a structured way to facilitate its integration into power prediction and design-optimization workflows and adopts a user-focused perspective.

Keywords: *High-speed craft, Power prediction, Mathematical models, Regression analysis, Artificial neural network, Planing craft, Semi-displacement craft, Computational Fluid Dynamics*

1. INTRODUCTION

The designers of high-speed craft have the challenging task of using a very limited set of tools to predict the resistance and propulsion characteristics of a potential concept. Moreover, many design enterprises lack the financial resources to have a complete set of physical model tests or high-fidelity computational predictions made for their particular design. Failure to achieve the contract speed on trials may result in financial penalties for the designers and/or builders.

Some all-purpose design software packages incor–porate algorithms for computing hull resistance. These are frequently developed based on the statistically-based Mathematical Models (MMs) discussed in this paper. However, because they often operate more like a 'black box', designers may struggle to fully understand which parameters have the greatest impact on resistance. A solid grasp of the underlying principles empowers designers to create optimization routines that can identify effective hull forms.

Broadly speaking, powering predictions can be grouped into the following categories:

- *Towing Tank* tests on a physical model of the design.
- *Standard Series Charts and Tables*. A standard series is a set of towing tank test models based on a parent design, systematically varying parameters such as loading, length-to-beam ratio, or longitudinal center of gravity. There are also propeller standard series that vary the advance coefficient, number of blades, blade area ratio, pitch, etc. These methods do not require a three-dimen–sional hull geometry to run; only an understanding of the coefficients used to represent the hull is required. For many applications, the decades-old standard series, often considered archaic and hence frequently marginalized, suffice because better methods have not replaced them.
- *Computational Fluid Dynamics* (CFD) predictions are made on a 3D computer model of the design, often using a Reynolds-Averaged Navier-Stokes code, which solves the fluid equations around the hull with suitable approximations to reduce computational time. These methods have improved significantly over the last decade and are very useful for planing boat predictions. The methods are excellent for steady planing, but can require a lot of computer power for motions in waves, porpoising, or manoeuvring [1-3].
- *Potential Flow* is applied to a 3D computer model of the design. This method is like CFD but with the simplifying assumption of no viscosity. Generally, it is less accurate, especially for transom sterns and separated flow designs, but requires much less computational resources.
- *Strip Theory* is where the hull is considered as a series of two-dimensional planes, often transverse in the case of planing boats. It does not rely on a three-dimensional hull and is very fast to run. In some cases, it can provide very good results. While frequently used for water impact problems, it has gained popularity during the last decade for manoeuvring of planing boats [4].
- *Empirical or Semi-Empirical Methods* such as the Savitsky method [5], which is based on the equations of static equilibrium of prismatic planing hulls and estimates the forces due to the bottom pressures, propeller thrust, and friction. This is a method that can be applied by hand calculations, in a

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spreadsheet, or better yet, with a simple computer code. It accepts basic inputs like the deadrise, center of gravity position, displacement, and shaft angle. These are known before a 3D set of lines has been generated, and so this method is excellent for early-stage design.

- *Statistically based mathematical models* are derived from physical model tests and may use a single series, multiple series, or extensive datasets, some–times supplemented even with full-scale measure–ments. A reliable database is essential for accuracy, so combining model test data with full-scale measurements is challenging. For river vessels [6], for instance, though this is an extreme case, full-scale measurements, thus data too, are heavily inf–luenced by fairway conditions such as water depth and current, while model tests are conducted in con–trolled laboratory settings. Historically, these models were known as "Regressions", as were obtained through Regression Analysis (RA). Today, however, Artificial Neural Networks (ANNs) and other mac–hine-learning methods are increasingly employed, offering greater flexibility and predictive power.

It is important to recognize that several of the aforementioned methods, while fundamentally distinct, can be used in a complementary manner. Statistically-based Mathematical Models (MMs), for example, offer a fast, cost-effective, and reasonably accurate tool for preliminary design assessments. For more detailed analysis and refinement, such as optimizing hull form or propeller performance, computational fluid dynamics (CFD) simulations and towing tank experiments provide deeper insights. Similarly, although Standard Series charts offer valuable data and commentary from original sources, many designers prefer interactive software tools that allow them to input core design parameters and iterate rapidly through various configurations.

For a broader understanding, the mathematical mo–dels in this paper should be regarded as complementary to references that approach the subject from alternative perspectives. The models presented here focus specifi–cally on power prediction during the design phase and are derived from *Power Prediction Modeling of Con–ventional High-Speed Craft* [7], offering a streamlined and targeted interpretation of the material contained therein. Among the broader literature, *Performance by Design* by Blount [8] stands out as a foundational reference for naval architects working on high-speed craft, as it outlines a range of methodologies central to the vessel design process.

A notable and highly valuable review of planing hull hydrodynamics is presented by Tavakoli et al. [9], which, amongst others, discusses mathematical models developed using regression techniques and artificial neural networks (ANNs). Their work, however, does not delve deeply into the development or implementation of these models. This paper serves as a complementary resource to the Tavakoli et al. [9] study, providing professionals with practical guidance for integrating the presented mathematical models in routine engineering practice.

The rest of this paper is organized as follows. Section 2 addresses power prediction, highlighting key quantities for evaluation by the MMs addressed here. Section 3 presents the essential MMs, while those of secondary importance appear in Section 7. Sections 4–6 provide background on application, development, and common prediction mistakes. Section 8 considers the influence of hull and propeller parameters on powering and its estimation, Section 9 reviews the accuracy of statistically based MMs, and Section 10 discusses trade-offs with CFD predictions, supported by Appendices 1 and 2, which present a hypothetical numerical example. Finally, Section 11 offers concluding remarks.

2. POWER PREDICTION OF HIGH-SPEED CRAFT

A central objective in vessel design is to reduce power demand and enhance energy efficiency. Achieving this requires a holistic optimization of the entire propulsion system, including the hull, propeller, engine, and gearbox, as isolated improvements of individual com–ponents may not yield the best overall outcome (see [10] for example). Nonetheless, this article adopts the usual approach commonly used for conventional ships, where resistance and propulsion are treated as separate optimization tasks. As a result, the total installed power, or brake horsepower (P_B), is a function of the total hull resistance (R_T) and speed (v), along with the propulsive efficiency (η_P), expressed as:

$$P_B=P_E/\eta_P=R_T\cdot v/\eta_P$$

Among the components that contribute to overall propulsive efficiency – expressed as $\eta_P = \eta_O\cdot\eta_G\cdot\eta_S\cdot\eta_R\cdot\eta_H$ – the open-water efficiency (η_O) plays the most dominant role. While the other factors (gearing effici–ency (η_G), shaft efficiency (η_S), relative rotative effi–ciency (η_R), and hull efficiency (η_H)) are also relevant, but are typically less significant

Evaluating a propeller's open-water efficiency is a complex process requiring iterative calculations to reach the optimal result. Identifying the most suitable propeller for a specific set of operational requirements involves a distinct design task. The ideal propeller must generate sufficient thrust to overcome the vessel's resistance under both design and off-design conditions. As such, determining open-water efficiency requires a dedicated propulsion analysis, commonly referred to as propeller sizing or propeller matching.

Accordingly, the key parameters that must be accurately evaluated for power prediction include:

a) *Bare hull resistance* (R_T), which is a function of speed (or Froude number) and the representative hull parameters that depend on the hull form, such as Slenderness ratio ($L/\nabla^{1/3}$), Longitudinal Center of Buoyancy (LCB), etc.
b) *Propeller's open-water efficiency* (η_O), which de–pends on propeller advance coefficient (J), and thrust and torque coefficients (K_T and K_Q). These are functions of the Pitch-to-Diameter Ratio (P/D), Blade Area Ratio (A_E/A_O), Number of Blades (z), and propeller series.

Additionally, dynamic trim (τ) plays a critical role in both resistance and propulsion performance, making its evaluation an essential part of the design process. In practice, assessments of dynamic trim are typically conducted alongside calculations of total resistance (R_T), as the two are interrelated.

3. ESSENTIAL MATHEMATICAL MODELS FOR HSC POWER PREDICTION

Table 1 lists the key MMs used to estimate *bare hull resistance* (R_T) and, where available, *dynamic trim* (τ) for Semi-Displacement, Semi-Planing and Planing hull forms. MMs for evaluation of trust and torque coefficients (K_T and K_Q), necessary for the propeller's *open water efficiency* (η_O), are also listed in Table 1. Accordingly, Table 1 presents the MMs that play a primary role in predicting HSC power in deep and calm water, while those associated with less influential parameters are listed later in Table 2. Both tables originate from [7].

Note that a *user-centered approach* was consistently applied throughout. Consequently, the MMs used to evaluate resistance and propeller efficiency, typically addressed and presented separately, have been consolidated in both tables to enhance ease of reference.

Because MMs are derived from different hull series and by different methods, their structures and presen–tations vary; thus, a single uniform format has not been applied across all MMs.

4. APPLICATION OF MATHEMATICAL MODELS

Over fifty years of research have produced a suite of ready-to-use, statistically-based MMs for high-speed craft design. Reference [7] brought these disparate met–hods together in a single volume, complete with step-by-step examples. At its core are MMs derived from extensive model-tank experiments on various hull forms and propeller series. Their development spans classic Regression Analyses (RA) through to modern Artificial Neural Network (ANN) techniques. Because these formulations can be easily coded into design software, they enable both power prediction and rapid parametric studies – capabilities that are especially valuable during early-stage design optimization.

Reference [7] tabulates each resistance and propeller-efficiency MM with:

- *MM name* and citation
- *Input parameters* (normalized when required) – Hull dimensions for resistance and dynamic trim; J, P/D, EAR/DAR, z, $\sigma/\sigma_{0.7R}$ for propellers
- *Output metrics* (dependent or target variables) – R_T, C_R, and τ; K_T and K_Q for propellers
- Reference *geometry of a series* MM is based on – Hull body plan or propeller blade outline
- *Applicability limits*, shown as graphs and/or equa–tions
- *Regression or ANN equations* with their coefficients for evaluation of resistance and dynamic trim; K_T and K_Q (needed for the evaluation of η_O) for pro–pellers
- *Supplementary MMs* (optional), such as wetted-surface area and its length.

Some of the MMs span over 200 terms, with coefficients up to 16 digits long that must remain exact. To streamline implementation, Reference [7] provides all MM coefficients in Excel format as *Electronic Supplementary Material* (ESM), enabling designers to import and code the models directly.

5. HOW MATHEMATICAL MODELS ARE DEVELOPED

To effectively utilize statistically-based mathemati-cal models, it is essential first to understand how they are developed. Constructing mathematical models (MMs) for evaluating resistance and open-water efficiency requires significant expertise and meticulous attention to detail to ensure reliable performance across their intended range. Even seemingly simple tasks, such as fitting a polynomial to an X-Y dataset in Excel, can present challenges. Depending on the chosen trendline, the model may oscillate between data points, overfit noisy inputs, or fail to capture key patterns. For standard series modeling, this process usually involves working in *multi-dimensional space*, where careful calibration is necessary to maintain validity within the range of applicability.

The notably *broad speed spectrum* of high-speed craft, far wider than that of conventional ships, must be highlighted here, since it makes mathematical modeling significantly more complex. A single vessel can operate in four distinct regimes based on its Froude numbers:

- displacement ($Fn_L < 0.40$)
- semi-displacement ($0.40 < Fn_L < 0.65$)
- semi-planing ($Fn_L > 0.65$ but $Fn_\nabla < 3.0$)
- pure planing ($Fn_\nabla > 3.0$)

In each regime, the shape of the resistance-versus-speed curves differs markedly.

The simplest MMs are developed using the stati–stical data modeling tool called Regression Analysis (RA), while the more complex ones are derived through the application of Artificial Neural Networks (ANN). Although the techniques for extracting MMs differ, all statistically based models follow a similar structure. In their simplest general form, these MMs can be expressed as follows:

$$Y = a_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4 + \ldots + a_nX_n$$

where Y is called the *target variable,* and here is either $(R_T/\Delta)_{100000}$, C_R, τ, K_T, or K_Q.

On the right-hand side of the equation are:

- *Independent variables* $X_1, X_2, \ldots, X_n$, which consist of *basic independent variables* (e.g., for resistance evaluation X_1=L/B, $X_2=Fn_L$, etc., while for the propellers it might be X_1=J, X_2=P/D, etc.) and some *transformations of basic independent variables* (e.g. $X_3=X_1^2$, $X_4=X_2^2$, $X_5=X_1X_2$, …), and
- *Coefficients* $a_0, a_1, \ldots, a_n$, which are determined by the regression analysis or ANN.

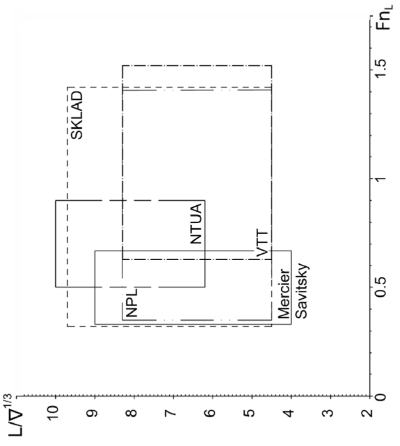
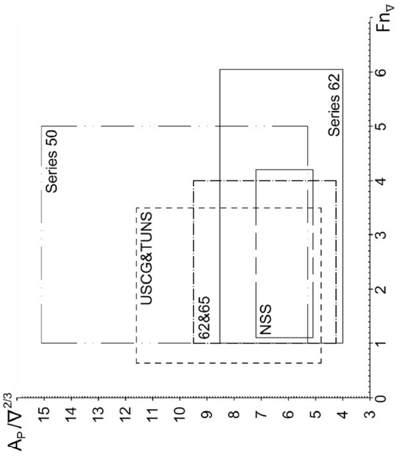
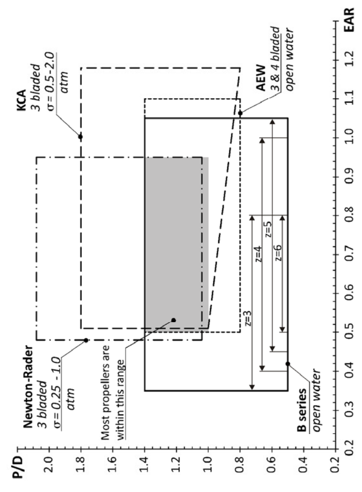
Two main equation types are used for resistance prediction, each with pros and cons: a) *Speed-Independent* models (SI MMs), which omit speed as a variable and use separate equations for each target speed; and b) *Speed-Dependent* models (SD MMs), which include speed as an independent variable and cover multiple speeds within a single formulation.

During the development phase, it is not unusual to develop a hundred or so different MMs before choosing the best one. The established and proper way to judge MM's quality is for the developer to compare its *predicted* values with the *actual* measurements. The MM is not good enough if:

a) Most commonly known evaluation metrics, such as Root Mean Square Error (RMSE), Coefficient of determination (R^2), and other statistics, are inadequate,
b) It does not satisfactorily represent the experimental results it is based on, and/or
c) There is an unexpected behavior between the original data points.

Errors in cases a) and b) are straightforward, but those in case c) are more involved. Consequently, merely reviewing the relevant statistics and comparing measured versus predicted values isn't enough; confirming the MM's behavior between data points is an essential part of its development.

Table 1. List of MMs for evaluation of resistance and dynamic trim of high-speed craft, and K_T and K_q of high-speed propellers; the applicavility range for each MM is also depicted.

RESISTANCE AND DYNAMIC TRIM PREDICTIONS		HIGH-SPEED PROPELLERS
Semi-Displacement Hull Forms	**Semi-Planing and Planing Hull Forms**	
• *Mercier & Savitsky* [11] - random hull forms, SI MM for $(R_T/\Delta)_{100000}$ • *VTT* [12] - random hull forms, SD MMs for round bilge and hard chine forms for $(R_T/\Delta)_{100000}$ • *NPL* [13] - systematic round bilge series, SI MM and SD MM for $(R_T/\Delta)_{100000}$ and τ • *SKLAD* [14] - systematic round bilge series, SI MMs for C_R and τ • *NTUA* [15] - systematic double chine series, SD MMs for C_R and τ.	• *62 & 65* [16] - systematic hard chine series, SI MMs for $(R_T/\Delta)_{100000}$ and τ • *USCG & TUNS* [17] - systematic hard chine series, simple and complex SD MMs for $(R_T/\Delta)_{100000}$ • *Series 50* [18] - systematic hard chine series, SD MMs for $(R_T/\Delta)_{100000}$ and τ • *Series 62* [19] - systematic hard chine series (β=12.5 - 30 deg), single and multiple outputs, SD MMs for $(R_T/\Delta)_{100000}$ and τ • *NSS* series [20] - systematic hard chine series, multiple outputs, SD MMs for $(R_T/\Delta)_{100000}$ and τ	• *B Series* [21] - RA MM for non-cavitating conditions for speeds up to 25-30 kn • *AEW Series* [22] - RA MM for 3- and 4-bladed propellers for non-cavitating and cavitating conditions • *KCA Series* (RA) [23] - RA MM for 3-bladed propellers for non-cavitating and cavitating conditions • *KCA Series* (ANN) [24] - ANN MM for 3-bladed propellers for transition and cavitating conditions • *Newton-Rader Series* [25] - ANN MM for non-cavitating, and transition and cavitating conditions for speeds of 50+ kn.
		

Note: SI MM - Speed Independent MM
SD MM - Speed Dependent MM
RA - Regression Analysis-based MM
ANN - Artificial Neural Network-based MM

Some propeller series suit high-speed applications better; e.g., segmental section propellers (AEW, KCA), with maximal blade thickness farther from the leading edge, are more prone to cavitation than propellers with hydrofoil sections (e.g., B Series). Newton-Rader Series is designed particularly for high-speed applications. Commercially available propellers are typically B-series, or flat-faced designs such as the AEW and KCA series, whereas hollow-faced blade sections, like those of the Newton-Rader series, are usually custom-engineered to meet specific performance requirements.

The end user, however, doesn't need to understand how the mathematical models were developed – they only need to know how to use the MMs and be familiar with the hull forms from which they originate and to be careful about the boundaries of applicability.

While RA has traditionally dominated statistical model development, ANNs offer greater flexibility in capturing nonlinear relationships from large datasets. However, ANN-based models also come with known limitations such as sensitivity to training data quality, risk of overfitting, limited interpretability, and insta–bility outside the training range. In the authors' practice, these limitations were mitigated by validating each ANN architecture (including different numbers of layers and neurons) against experimental data and expected physical trends. Only networks demonstrating stable, physically plausible behavior were retained and pub–lished in fixed mathematical form with associated coefficients preventing further modification or retra–ining. This ensured reproducibility and reliability, while leveraging the generalization strength of ANNs. In many cases, ANN-based models showed smoother ext–rapolation behavior than polynomial regressions, espe–cially when the training domain was carefully selected and informed by domain knowledge in hydrodynamics and planing hull resistance.

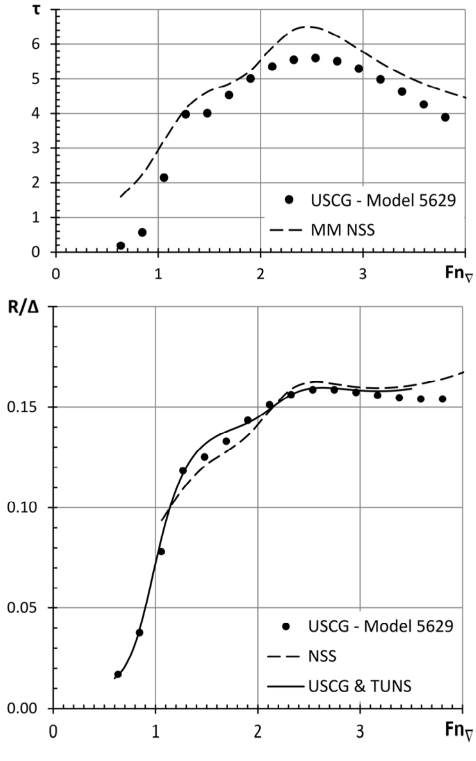


Figure 1. Comparison of two hard chine, wide-transom MMs with the USCG (Model 5629) measurements (β=23°, $L_P/\nabla^{1/3}$=5.15, LCG/L_P=0.373, L_P/B_{PX}=4.09) [7]

Comparing predicted results with the test results of similar hull forms (see Figure 1) is more demanding than comparing them with the same data MM is based on (which the MM developer did anyway). Figure 1 shows resistance and dynamic trim predicted with the USCG & TUNS [17] and NSS [20] MMs, while test results are for USCG Model 5629, whose characteristics are given in [26].

Similarly, a comparison with the full-scale tested commercial propeller, mentioned in the discussion of [23], with three MMs (AEW [22], KCA (RA) [23], and KCA (ANN) [24]) is shown in Figure 2. Note that the MMs are based on laboratory propellers, whereas commercial propellers aren't produced to such precise tolerances.

Accurate power predictions depend entirely on the quality of the data used to develop the mathematical model. Ideally, MMs are built from both resistance and self-propulsion test results, but such comprehensive datasets are seldom available. In the case of the high-speed Catamaran Series '89, however, full test data did exist, and a suite of highly reliable models for predicting resistance, trim, propulsion coefficients, and delivered power is detailed in [27] (and reproduced in [7]).

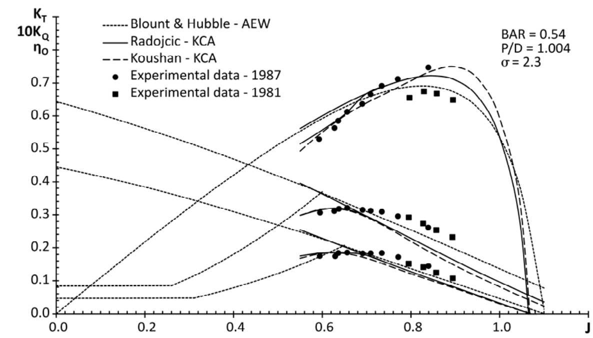


Figure 2. Comparison of three MMs with a commercial propeller EAR=0.54 ($EAR_{nominal}$=0.50), P/D=1.004 ($P/D_{nominal}$=1.00), σ=2.3 [7]

6. COMMON PREDICTION MISTAKES

Two principal mistakes commonly undermine the appli–cation of statistical methods:

a) *An incorrect prediction model* (MM) is selected (the vessel under analysis has different characteristics from those upon which the MM is based), and
b) *Violation of the boundaries of applicability* of MM.

The root of error (a) lies in model misapplication. Namely, even when a target hull's *primary input para–meters* fall within an MM's stated bounds, that alone doesn't guarantee validity. *Secondary characteristics*, i.e. the detailed hull form not captured by primary variables, must also resemble the series on which the MM was built. For instance, a semi-displacement yacht might meet the L/B, $L/\nabla^{1/3}$, and other limits of a traw–ler-based MM, yet her hull geometry departs so far from trawler lines that using that MM would produce wrong results.

7. MATHEMATICAL MODELS WITH SECONDARY IMPACT ON HSC POWER PREDICTION

Bare hull resistance dominates the powering require–ments, often by an order of magnitude over other fac–tors. However, a more accurate power prediction must also account for other resistance components, such as appendage and air resistance, that various empirical methods can approximate. Table 2 compiles the prin–cipal references for estimating additional resistance components related to bare-hull resistance in deep and calm water.

Environmental factors, such as wind, waves, and shallow-water effects, are typically treated as additional resistance. Depending on their intensity, these influ–ences can substantially affect a vessel's in-service resistance and, consequently, the power required during operation. However, since the objective here is to predict power requirements in deep and calm water, environmental effects are excluded from consideration.

The *propulsive coefficients* represent the hull-propeller interaction. As mentioned earlier, relative rotative efficiency (η_R) and hull efficiency (η_H) are important but typically less significant than open-water efficiency (η_O). These coefficients are difficult to determine because high-speed craft tests usually rely on small towing-tank models limited by speed; although these models yield reliable resistance and trim data, their tiny propellers cannot develop fully turbulent flow. Full-scale trials bring their own logistical and cost chal–lenges, so obtaining accurate η_R and η_H values is problematic. In contrast, gearing efficiency (η_G) and shaft efficiency (η_S) can be readily estimated as percentages of transmitted power.

Propulsive coefficients can also be estimated using empirical equations. Useful references for assessing propulsive coefficients applicable to high-speed craft are listed in Table 2. Consequently, Table 2 (amended from [7]) presents MMs used to evaluate secondary parameters, while the MMs that primarily determine HSC power are given in Table 1. Reference [7] also offers several approximate methods for predicting these factors.

When maximum accuracy is required, the "long method" is typically employed, i.e., each coefficient term (η_G, η_S, η_R, η_H, η_O) is calculated separately. Alternatively, the "short method" skips individual components and directly yields the overall propulsive efficiency η_P in one step.

8. INFLUENCE OF HULL AND PROPELLER PARA–METERS ON POWERING AND ITS ESTIMATION

The mathematical model for power prediction is usually assembled from standalone modules – bare-hull resistance, appendage resistance, propulsive coeffici–ents, and so on – each derived independently and usable on its own. Contemporary high-speed craft methods, by contrast, perform *system-wide optimization* by minimizing delivered power (min P_D) rather than treating resistance (*min* R_T) and propulsive efficiency (*max* η_D) as separate objectives; see the thorough treatment in [10].

Table 2 Useful references for evaluation of additional resistance components and propulsive coefficients

Additional Resistance Components	
Appendages	
Struts, Shafts etc.	Hadler [28], Lasky [29]
Rudders*	Gregory and Dobay [30]
Sterndrives, Outboards	Scherer and Patil [31]
Model experiments	Gregory and Beach [32]
Spray rails	Mikloš et al. [33] Ebrahimi et al. [34]
Flaps and wedges**	Brown [35] – planing hulls Millward [36] – semi-displacement Sakaki et al. [37]
Interceptors**	De Luca and Pensa [38] Gultekin Avci and Barlas [39] Sakaki et al. [37]
*) Maneuverability and Turning characteristics: - Denny and Hubble [40], Lewandowski [41]	
**) Reduce dynamic trim and may cause dynamic instability: - Transverse (dynamic trim too low) – Blount and Codega [42], Müller-Graf [43] - Longitudinal, porposing (dynamic trim too high) – Celano [44]	
Air Resistance	Blount [6], Fossati et al. [45]
Correlation Allowance and Margins	Blount [6]
Propulsive coefficients	
	Bailey [46] – for round-bilge hulls Blount [47] – for tunnel-propeller installations Katayama et al. [48] – for outboard and stern-drive units Blount and Bjarne [49] – for various high-speed craft

Here, we revert to the simpler, *conventional app–roach*: resistance and propulsion are optimized sepa–rately, and propulsive efficiency is calculated either via the "long" or "short" method. All necessary MMs for power prediction in deep and calm water are listed in Tables 1 and 2. A hypothetical example in Appendix 1 demonstrates the evaluation of power by using the MM modules step by step.

Environmental conditions can significantly affect *in-service powering.* Namely, HSC performance is strongly influenced not only by added resistance in a seaway but also by excessive acceleration, pitch, and heave, which often necessitate speed reduction. While speed reduction might be considered as a voluntary measure, power is influenced only by the added resis–tance in waves. These effects, however, are complex; thus, seaway influences on HSC performance are usu–ally treated separately. In contrast, shallow water powering can be estimated with reasonable reliability using empirical equations (see References [7] and [50]).

From a powering perspective, the key hull form parameters for high-speed craft (HSC) vary with both the mathematical model (MM) and the speed range. Primary input variables differ across MMs (see Table 1 and [7]), and each MM has its own speed limits of applicability. Yet displacement, or the vessel's weight, remains the single parameter that consistently governs resistance and thus power across the full speed range in which HSC operate. Even in the displacement regime (Fn_L < 0.40) typical of conventional ships, vessel mass, especially lightweight, plays an important role (see [51] for instance), though far less than for HSC. In this sense, a perfect HSC is one from which nothing can be removed without harming safety or performance; perfection is not "nothing could be added," but rather "nothing could be taken away."

9. ENHANCING PREDICTIVE ACCURACY OF STA–TISTICALLY-BASED MATHEMATICAL MODELS

Established design firms often possess model test data or trial results specific to their hull designs. This infor–mation plays a crucial role in enhancing MM's pre–diction accuracy – first, by identifying which mathe–matical models best suit their designs, and second, by learning how to fine-tune model input parameters to more accurately reflect those designs.

To improve the MM's predictive capability, it is sometimes necessary to slightly *modify the input parameters* of the target hull. This process, known as "mapping of the input parameters," involves substituting the *required* MM's inputs with suitable adjusted *effective values* (see, for example, [19]). However, mapping is not a straightforward task; it demands an understanding of the underlying principles and is therefore often misapplied or overlooked. Accurate interpretation of both input parameters and prediction results is essential.

Because results are sensitive to input parameters, designers should understand the variables that signifi–cantly influence performance. Relying solely on soft–ware to measure the hull and generate inputs can cause errors, and automatic transitions to power evaluation after hull design may produce misleading results.

To build confidence in the results, a general recommendation is to apply as many different predictive methods as possible (see Figures 1 and 3, for instance).

10. EVALUATING MMS AND CFD: TRADE-OFFS IN ACCURACY, EFFICIENCY, AND APPLICABILITY

The increasing availability of CFD tools has prompted many naval architects to consider switching from clas–sical statistically-based prediction methods (MMs) to full numerical simulations; some of the latest investi–gations are presented in [1-3 and 52-55]. However, both approaches carry specific advantages and limitations, especially when assessed through the lens of practical design processes. Follow the strengths of MMs that highlight CFD's weaknesses; also, CFD's strengths reveal the limitations of MMs.

Mathematical Models (MMs):

- Provide fast, reliable power estimates in early-stage design using basic hull parameters (length, beam, draft, deadrise, displacement, etc.).
- Well-suited for concept evaluation, variant comparison, and feasibility studies.
- Inherently constrained by the scope of the datasets used in their development.
- Accuracy is generally strong within interpolation zones but limited when extrapolating to novel hull forms.
- Cannot capture detailed geometry or complex flow phenomena around appendages or hull inflections.

Computational Fluid Dynamics (CFD):

- Offers full control of hull geometry and flow domain.
- By simulating the bare hull resistance using Rey–nolds-averaged Navier–Stokes (RANS) equations and turbulence models, designers can observe detailed flow patterns and pressure distributions.
- Indispensable for unconventional hull forms or late-stage refinement.
- Requires significant time and expertise: 20–40 hours per speed range for resistance predictions (see Appendix 2), with propulsion estimates adding further complexity.
- Demands careful attention to mesh quality, boundary conditions, and numerical stability.

Summarizing, MMs and CFD should be seen as complementary rather than competing. MMs remain indispensable in early-stage design for speed and simp–licity, while CFD refines and validates predictions as accuracy becomes critical, especially for novel geomet–ries. Effective design depends on knowing when to apply each method and how to interpret their outputs together. This study confirms that MMs remain highly effective for high-speed semi-displacement and planing hulls, provided designers respect their statistical boun–daries and supplement them with CFD when necessary.

11. CONCLUDING REMARKS

Careful consideration of the hull, propeller, and their interaction can reduce overall in-service power requi–rements. The most critical elements for reliable power prediction are the assessment of hull resistance and the propeller's open-water efficiency; therefore, these mathematical models (MMs) are the primary focus here. Moreover, for high-speed craft (HSC), dynamic trim strongly influences both resistance and propulsion, so MMs that estimate dynamic trim are provided alongside the resistance MMs.

Among the reduced-order mathematical models used to evaluate resistance for HSC, Savitsky's method [5] is the most widely adopted, although its applicability is inherently limited to planing conditions. This paper presents a broader set of reduced-order MMs designed to assess resistance for semi-displacement [11–15], and semi-planing and planing hull forms [16–20]. These MMs offer improved predictive performance over a wide range of operating speeds, making them more versatile for contemporary design and analysis.

For accurate power assessment, evaluating the propeller's open-water efficiency is equally critical. The corresponding MMs are detailed in references [21–25]. Consequently, Table 1 compiles the resistance, dynamic trim, and propeller open-water efficiency MMs mentioned above, which all play a primary role in HSC power prediction.

The MMs for assessing less influential parameters are systematically categorized in Table 2. These are additional resistance components, cited in references [28–45], while references [46–49] address propulsive coefficients, completing the framework for comprehensive in-service deep and calm water power prediction.

Despite differences in extraction methods, target variables, and the structure and complexity of polynomial forms, the MMs presented exhibit fundamental similarities. This consistency supports efficient implementation and facilitates integration into optimization routines. While users are not expected to understand the derivation of these models – a task reserved for developers – they must be aware of the limitations and assumptions each MM imposes to ensure proper application and interpretation.

Among the various MMs presented, the most recent ones, derived using artificial neural network techniques (ANN), particularly those employing multiple outputs [19, 20], show considerable promise. These models leverage the simultaneous use of all available resistance and dynamic trim data during training, resulting in a unified MM capable of predicting both resistance and trim. This integrated approach inherently captures the physical interdependence between resistance and dynamic trim, offering a more holistic and efficient modeling framework.

The comparative analysis (Appendix 2) shows that MMs and CFD are complementary rather than competing approaches. MMs deliver fast, reliable results for preliminary design within their boundaries of applicability, while CFD, though more time- and expertise-intensive, enables geometry-sensitive refinement and validation in later stages. A workflow that starts with MMs and integrates CFD ensures both efficiency and accuracy in power prediction for semi-displacement and planing craft.

Finally, the core takeaways for designers are summarized below:

- *Focus on essentials*: Hull resistance, propeller efficiency, and dynamic trim drive power prediction.
- *Use MMs first*: Reduced order and ANN based MMs give fast, versatile preliminary results.
- *Add CFD later*: Combine MMs with CFD for refinement and validation, ensuring both efficiency and accuracy.

REFERENCES

[1] Lee, E.J, Mousaviraad, M., Weil, C.R, Jiang, M.J, Fullerton, A., Stern, F.: Assessment of experiments and CFD for the semi-planing R/V Athena Model in calm water. Ocean Engineering, Vol. 236, 109254, ISSN 0029-8018, 2021. https://doi.org/10.1016/j.oceaneng.2021.109254

[2] Park, S., Wang, Z., Stern, F., Husser, N., Brizzolara, S., Morabito, M., Lee, E.: Single- and two-phase CFD V&V for high-speed stepped planing hulls. Ocean Engineering, Vol. 261, 112047, ISSN 0029-8018, 2022. https://doi.org /10.1016 /j.oceaneng.2022.112047

[3] Matveev, K., Morabito, M.: Hydrodynamics of planing surfaces with negative deadrise angles. Ocean Engineering, Vol. 212, 107601, ISSN 0029-8018, 2020. https://doi.org/10.1016/j.oceaneng.2020.107601

[4] Morabito, M.: Prediction of planing hull side forces in yaw using slender body oblique impact theory. Ocean Engineering, Vol. 101, pp. 47-57, ISSN 0029-8018, 2015. https://doi.org/10.1016/j.oceaneng. 2015.04.014

[5] Savitsky, D.: Hydrodynamic design of planing hulls. Marine Technology, Vol. 1, No. 1, 1964.

[6] Simić, A., Radojčić, D.: On Energy Efficiency of Inland Waterway Self-Propelled Cargo Vessels. FME Transactions, Vol. 41, No. 2, 2013.

[7] Radojčić, D., Kalajdžić, M., Simić, A.: Power Prediction Modeling of Conventional High-Speed Craft. Springer, ISBN 978-3-030-30606-9, 2019.https://doi.org/10.1007/978-3-030-30607-6

[8] Blount, D.L.: Performance by design. ISBN 0-978-9890837-1-3, 2014.

[9] Tavakoli, S., Zhang, M., Kondratenko, A.A., Hirdaris, S.: A review on the hydrodynamics of planing hulls. Ocean Engineering, Vol. 303, 117046, ISSN 0029-8018, 2024. https://doi.org/10.1016/j.oceaneng.2024.117046

[10] Radojčić, D.: An engineering approach to predicting the hydrodynamic performance of planing craft using computer techniques. RINA Trans., Vol. 133, 1991.

[11] Mercier, J.A., Savitsky, D.: Resistance of Transom-Stern Craft in the Pre-Planing Regime. Davidson Laboratory, Report 1667, 1973.

[12] Lahtiharju, E., Karppinen, T., Hellevaara, M., Aitta, T.: Resistance and Seakeeping Characteristics of Fast Transom Stern Hulls with Systematically Varied Form. SNAME Trans., Vol. 99, 1991.

[13] Radojčić, D., Rodić, T., Kostić, N.: Resistance and Trim Predictions for the NPL High Speed Round Bilge Displacement Hull Series. RINA Conf. On Power, Performance and Operability of Small Craft, Southampton, 1997.

[14] Radojčić, D., Prinčevac, M., Rodić, T.: Resistance and Trim Predictions for the SKLAD Semidisplacement Hull Series. Oceanic Engineering International, Vol. 3, No. 1, 1999.

[15] Radojčić, D., Grigoropoulos, G.J., Rodić, T., Kuvelić, T., Damala, D.P.: The Resistance and Trim of Semi-Displacement, Double-Chine, Transom-Stern Hull Series. Proceedings of 6th Int. Conf. on Fast Sea Transportation (FAST 2001), Southampton, 2001.

[16] Radojčić, D.: An Approximate Method for Calculation of Resistance and Trim of the Planing Hulls. University of Southampton, Ship Science Report No. 23. Paper presented on SNAME Symposium on Powerboats, 1985.

[17] Radojčić, D., Zgradić, A., Kalajdžić, M., Simić, A.: Resistance Prediction for Hard Chine Hulls in the Pre-Planing Regime. Polish Maritime Research, Vol. 21, No. 2(82), Gdansk, 2014.

[18] Radojčić, D., Morabito, M., Simić, A., Zgradić, A.: Modeling With Regression Analysis and Artificial Neural Networks the Resistance and Trim of Series 50 Experiments with V-Bottom Motor Boats. Journal of Ship Production and Design, Vol. 30, No. 4, 2014.

[19] Radojčić, D., Kalajdžić, M., Zgradić, A., Simić, A.: Resistance and Trim Modeling of Systematic Planing Hull Series 62 (With 12.5, 25 and 30 Degrees Deadrise Angles) using Artificial Neural Networks, Part 2: Mathematical Models. Journal of Ship Production and Design, Vol. 33, No. 4, 2017.

[20] Radojčić, D., Kalajdžić, M.: Resistance and Trim Modeling of Naples Hard Chine Systematic Series. RINA Trans., Int. J. of Small Craft Technology, 2018.

[21] Oosterveld, M.W.C., van Oossanen, P.: Further Computer-Analyzed Data of the Wageningen B-Screw Series. International Shipbuilding Progress, Vol. 22, No. 251, 1975.

[22] Blount, D.L., Hubble, E.N.: Sizing Segmental Section Commercially Available Propellers for Small Craft. Propellers ’81 Symposium, SNAME, Virginia Beach, 1981.

[23] Radojčić, D.: Mathematical Model of Segmental Section Propeller Series for Open-Water and Cavitating Conditions Applicable in CAD. Propellers ’88 Symposium, SNAME, Virginia Beach, 1988.

[24] Koushan, K.: Mathematical Expressions of Thrust and Torque of Gawn-Burrill Propeller Series for High Speed Crafts Using Artificial Neural Networks. Proceedings of 9th Int. Conf. on Fast Sea Transportation (FAST 2007), Shanghai, 2007.

[25] Koushan, K.: Mathematical Expressions of Thrust and Torque of Newton-Rader Propeller Series for High Speed Crafts Using Artificial Neural Networks. Proceedings of 8th Int. Conf. on Fast Sea Transportation (FAST 2005), St. Petersburg, 2005.

[26] Kowalyshyn, D.H., Metcalf, B: A USCG Systematic Series of High-Speed Planing Hulls. SNAME Trans., Vol. 114, 2006.

[27] Müller-Graf, B., Radojčić, D., Simić, A.: Resis–tance and Propulsion Characteristics of the VWS Hard Chine Catamaran Hull Series ’89. SNAME Trans., Vol. 110, 2002.

[28] Hadler, J.B.: The prediction of power performance of planing craft. SNAME Trans, Vol. 74, 1966.

[29] Lasky, M.P.: An investigation of appendage drag. DTNSRDC Report SPD-458-01, 1980.

[30] Gregory, D.L., Dobay, G.F.: The performance of high-speed rudders in a cavitating environment. SNAME Spring Meeting, Florida, 1973.

[31] Scherer, J.O., Patil, S.K.R.: Hydrodynamics of surface piercing outboard and sterndrive propulsion systems. Proceedings of 11th International Conference on fast sea transportation, FAST, Honolulu, 2011.

[32] Gregory, D., Beach, T.: Resistance measurements of typical planing boat appendages. DTNSRDC, Report SPD-0911-01, 1979.

[33] Lakatoš, M., Sahk, T., Andreasson, H., Tabri, K.: The effect of spray rails, chine strips and V-shaped spray interceptors on the performance of low planing high-speed craft in calm water. Applied Ocean Research, Vol. 122, 103131, 2022. https://doi.org/10.1016/j.apor.2022.103131

[34] Ali, E., Amin, N., Alireza, N., AliAsghar, M.: Investigating the effect of underlying angle, width, and number of spray rails on the performance of planing hull and their optimization using the Taguchi method. Journal of Ocean Engineering and Marine Energy, Vol. 10, 987–1009, 2024. https://doi.org/10.1007/s40722-024-00338-w

[35] Brown, P.W.: An experimental and theoretical study of planing surfaces with trim flaps. Davidson Laboratory, Report 1463. Stevens Institute of Technology, 1971.

[36] Millward, A.: Effect of wedges on the performance characteristics of two planing hulls. Journal of Ship Research, Vol. 20, 1976.

[37] Sakaki, A., Ghassemi, H., Keyvani, S.: Evaluation of the hydrodynamic performance of planing boat with trim tab and interceptor and its optimization using genetic algorithm. J. Marine. Sci. Appl. 18, 131–141, 2019. https://doi.org/10.1007/s11804-018-0040-6

[38] De Luca, F., Pensa, C.: Experimental investigation on conventional and unconventional interceptors. RINA Trans, vol. 153, Part B2, Int. J. Small Craft Technol, July–Dec, 2012.

[39] Gultekin Avci, A., Barlas, B.: An experimental investigation of interceptors for a high-speed hull. International Journal of Naval Architecture and

Ocean Engineering, Vol. 11, No. 1, 2019 https://doi.org/10.1016/j.ijnaoe.2018.05.001

[40] Denny, S.B., Hubble, E.N.: Predicting of craft turning characteristics. Marine Technology, Vol. 28, No. 1, 1991.

[41] Lewandowski, E.: Maneuverability of high-speed power boats. 5th Power boat symposium, SNAME Southeast Section, 1993.

[42] Blount, D.L., Codega, LT.: Dynamic stability of planing boats. Marine Technology and SNAME News, Vol. 29, No. 1, 1992. https://doi.org/10.5957/mt1.1992.29.1.4

[43] Müller-Graf, B.: Dynamic stability of high speed small craft. 25th WEGEMT School, Small Craft Technology, NTUA, Athens. ISBN I 900 453 053, 1997.

[44] Celano, T.: The prediction of porpoising inception for modern planing craft. USNA Trident Report No. 254, Annapolis, 1998.

[45] Fossati, F., Muggiasca, S., Bertorello, C.: Aerodynamics of high speed small craft, Proceedings of 12th international conference on fast sea transportation, FAST, Amsterdam, 2013.

[46] Bailey, D.: A statistical analysis of propulsion data obtained from models of high speed round bilge hulls. RINA symposium on small fast warships and security vessels, London, 1982.

[47] Blount, D.L.: Design of propeller tunnels for high speed craft. Proceedings of the 4th international conference on fast sea transportation, FAST, Sydney, 1997.

[48] Katayama, T., Nishihara, Y., Sato, T.: A study on the characteristics of self-propulsion factors of planing craft with outboard engine. SNAME's 3rd Chesapeake power boat symposium, Annapolis, 2012.

[49] Blount, D.L., Bjarne, E.: Design and selection of propulsors for high speed craft. 7th Lips propeller symposium, Nordwijk-on-Sea, 1989.

[50] Radojčić, D., Bowles, J.: On high-speed Monohulls in shallow water. SNAME's 2nd Chesapeake Power Boat Symposium, Annapolis, 2010.

[51] Vasilev, M., Kalajdžić, M.: Influence of Lightweight Change on Ship Performance. FME Transactions, Vol. 50, No. 4, 2022.

[52] Gatin, I.: CFD validation study: Calm water resistance of the Series 62 planing hull (Validation report). Cloud Towing Tank, Zagreb, Croatia. 2025. https://cloudtowingtank.com/validation-study-series-62-planing-hull-resistance/

[53] Sulman, M., Mancini, S., Niazmand Bilandi, R. : Numerical investigation of single and double steps in planing hulls. Journal of Marine Science and Engineering, 12(4), 614, 2024.https://doi.org/10.3390/jmse12040614

[54] Sulman, M., Mancini, S., Niazmand Bilandi, R., Vitiello, L.: Hydrodynamic performance of high-speed craft: A CFD study on spray rails. Journal of Marine Science and Engineering, 13(3), 438, 2025. https://doi.org/10.3390/jmse13030438

[55] Lakatoš, M., Mancini, S., Niazmand Bilandi, R., Tabri, K.: Verification and validation of hard-chine hull performance in calm water: Effects of numerical setups and hull configurations. Ocean Engineering, Volume 329, 2025. https://doi.org/10.1016/j.oceaneng.2025.121056

[56] MacPherson, D., Harries, S., Walker, A, Arapakopoulos, A.: Early-Stage System-Level Optimization of a Fast Planing Monohull, The SNAME International Conference on Fast Sea Technology (FAST 2025), Norfolk, VA, USA, Oct. 2025. doi: 10.5957/FAST-2025-022.

[57] Gamulin, A.: A semidisplacement series of ships. Int Shipbuilding Progress 43(43), 1996.

[58] Vasilev, M.: Estimation of total resistance and dynamic trim of semi-displacement hull forms using CFD analysis [Seminar paper]. Faculty of Mechanical Engineering, University of Belgrade. Institutional Repository, 2024.

NOMENCLATURE

A_T	Transom area (m^2)
A_X	Maximum section area (m^2)
$A_P/\nabla^{2/3}$	Planing area coefficient
B_{OA}	Beam overall (m)
B_{PX}	Maximal projected chine beam (m)
B_{WL}	Beam of hull on designed waterline at rest (m)
C_A	Correlation allowance
C_{AA}	Air resistance (allowance) coefficient
C_B	Block coefficient
C_R	Residuary resistance coefficient ($C_R=R_R/(\rho/2 \cdot S \cdot v^2)$)
C_X	Maximum section area coefficient
D	Propeller diameter (m)
DAR	Developed area ratio
EAR	Expanded area ratio
Fn_L	Length Froude number ($Fn_L=v/(g \cdot L_{WL})^{1/2}$)
Fn_∇	Volumetric Froude number ($Fn_\nabla=v/(g \cdot \nabla^{1/3})^{1/2}$)
i_e	Half-angle of entrance of waterline at bow (deg)
J	Advance coefficient ($J=v_A/n \cdot D$)
K_T	Thrust coefficient ($K_T=T/(\rho \cdot n^2 \cdot D^4)$)
K_Q	Torque coefficient ($K_Q=Q/(\rho \cdot n^2 \cdot D^5)$)
L_{OA}	Length overall (m)
L_P	Projected chine length (m)
L_{WL}	Waterline length (m)
$L/\nabla^{1/3}$	Slenderness ratio
n	Propeller rotational speed (1/sec)
P_B	Brake power (kW)
P_D	Delivered power (kW)
P_E	Effective power (kW)
P/D	Pitch–diameter ratio
Q	Propeller torque (kNm)
R_F	Frictional resistance (kN)
R_R	Residuary resistance (kN)
R_T or R	Total bare hull resistance (kN)

R_T*	Total in-service resistance (kN)
R/Δ	Resistance-to-weight ratio
S	Wetted surface area (m^2)
T	Hull draught (m)
T	Propeller thrust (kN)
v	Velocity of craft (m/s)
v_A	Speed of advance of propeller (m/s)
z	Number of propeller blades
β	Deadrise angle at midship ($L_P/2$) (deg)
Δ	Displacement, mass (t)
∇	Displacement volume (m^3)
η_D	Propulsive efficiency (quasi-propulsive efficiency)
η_O	Propeller open-water efficiency ($\eta_O=(J/2\pi)\cdot(K_T/K_Q)$)
σ	Cavitation number based on advance velocity
$\sigma_{0.7R}$	Cavitation number based on resultant water velocity at 0.7 radius
τ	Dynamic trim (deg)
ν	Kinematic viscosity of water (m^2/s)

APPENDIX 1 – HYPOTHETICAL EXAMPLE

A shipyard producing hard-chine motor yachts capable of reaching speeds of 30 knots is preparing to introduce a new design with a semi-displacement or semi-planing hull form, optimized for more economical cruising at approximately 20 knots. The updated design embraces contemporary styling with a vertical plumb bow, enhancing both aesthetics and functionality. To improve hydrodynamic efficiency at lower speeds, the traditional hard-chine hull should be replaced by a round-bilge configuration, while preserving the vessel's principal dimensions and displacement. Performance assessments are focused on calm water conditions.

The main dimensions of an existing hard-chine yacht are the following:

L_{OA}=16.7 m, L_P=13.8 m,

B_{OA}=4.65 m B_{PX}=3.96 m, Δ=24 t

The main dimensions of a new round-bilge design are the following:

L_{OA}=16.7 m, L_{WL}=16.4 m,

B_{OA}=4.65 m B_{WL}=4.0 m, Δ=24 t

Consequently, the input variables for the MMs that will be applied are the following main non-dimensional parameters: (M)=$L_{WL}/\nabla^{1/3}$=5.733 and L_{WL}/B_{WL}=4.1. Other assumed variables are A_T/A_X=0.56 and C_B=0.396 (according to the NPL series, thus T=0.9 m), C_X=0.573 (according to the VTT series), and i_e=15 deg. The idea is to keep the stern sections of considered round-bilge forms unchanged and adapt the bow sections with raked stem according to the requirements. The Longitudinal Centre of Gravity (LCG) is assumed to be optimal for each MM.

All subsequent calculations are performed using the MMs described in the main text. The tabulated results are provided to help users verify the correctness of their own calculations when applying these models.

Bare Hull Resistance and Dynamic Trim Evaluations for the Semi-Displacement Hull Forms

Tables 3, 4, and Figure 3 present the final results for the new design. A detailed evaluation procedure for a similar numerical example is provided in [7]. Calculated R/Δ, shown in Figure 3, is valid for the displacement of 24 t. For evaluation of total resistance, the following values were adopted: ρ = 1.025 t/m^3, g = 9.81 m/s^2, ν = 1.1907E-6 m^2/s, C_A= 0.2·10^{-3}, and C_F according to ITTC-1957.

Note that:

- The discrepancies among the VTT and NPL results are negligible (due to the NPL hull form, which is dominant in both MMs). Mercier and Savitsky's results are marginally below the previous two.
- SKLAD's results are slightly different, though discrepancies among the MMs are expected as they represent different hull forms (i.e. their secondary hull form parameters are different, but their input variables are identical).
- As expected, the hard chine resistance curve is well above all round bilge curves for Fn_L values of up to around 1 (here corresponding to Fn_∇≈2.5 or speed of around 25 kn). For higher speeds, the hard-chine resistance is below that of round-bilge hulls (hence this form was justified for 30 kn).

Resistance of a Virtual 24 t Motor Yacht

The SKLAD Series was selected for further calculations due to a) its reliable results, and b) this series and its corresponding mathematical models deserve more attention from professionals than they currently receive. Hence, more detailed results for this method are presented in Table 4.

So, for the prediction of the total in-service resis–tance in calm and deep water (approximately corres–ponding to trial conditions), the total appendage resis–tance for the tween screw installation of 7% R_T is assu–med. Air resistance (assumed above water area of A_W= 20 m^2 and C_{AA} = 0.65) is estimated according to recom–mendations given in [7] and increases from 2% to 4% within the considered speed range. Thus, resistance components for a 24 t semi-displacement/semi-planing motor yacht having the SKLAD hull form and 2-screw propulsion are shown in Table 4 and in Figure 4. Figure 4 shows total in-service resistance (R_T*), Bare hull re–sistance (R_T), Residuary resistance (R_R), and Frictional resistance (R_F) for a speed range between 10 and 26 kn.

Propulsion of a Virtual 24 t Motor Yacht

The focus here, concerning the propulsion, is on propeller performance; thus, the characteristics of the engine with a gearbox are ignored. Other assumptions are that the propellers are optimized for a speed of 20 kn. As propeller sizing techniques per se are not the subject of this article, further elaboration regarding this topic is omitted and can be found elsewhere; the finally chosen propellers (according to KCA (RA) MM) are the three-bladed KCA propellers with D=0.65m, P/D=1.25, and area ratio DAR=0.8, see Table 5.

Powering of a Virtual 24 t Motor Yacht

Off-design calculations were carried out, leading to the diagram shown in Figure 5. Numerical results are summarized in Table 6, and detailed computations for a similar case can be found in [7]. According to the conventional long method, the estimated propulsive efficiency (η_D) for a speed of 20 kn is relatively high, i.e. around 0.60, while the open-water efficiency (η_O) is almost 0.66.

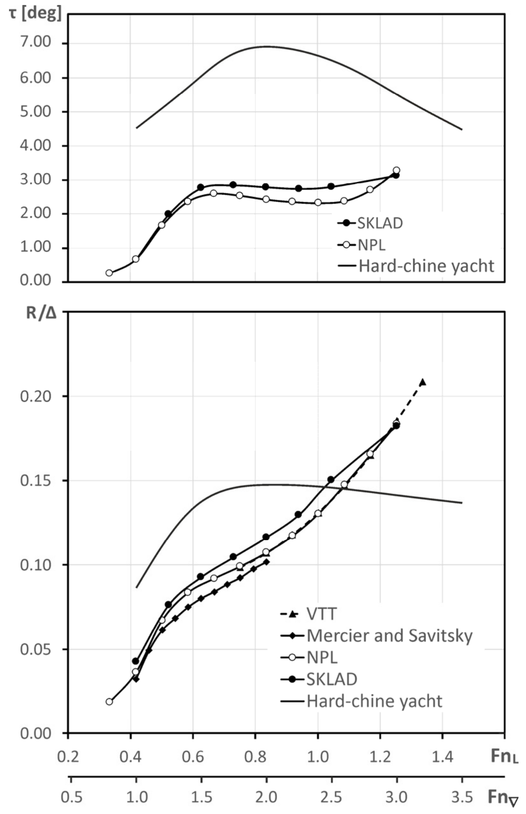


Figure. 3 The Predictions of four MMs applicable for a typical 24 t motor yacht (L_{OA}=16.7 m, B_{OA}=4.65 m). Note: Fn_L scale is valid only for the round-bilge hulls

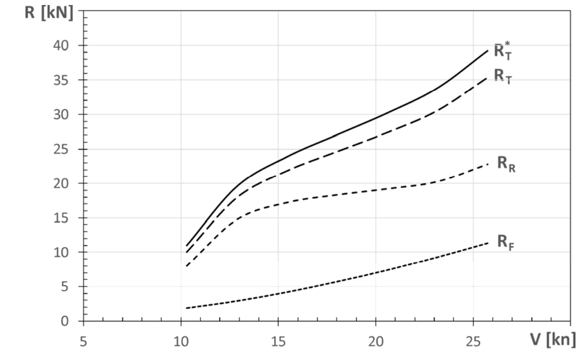


Figure. 4 Components of in-service resistance (deep and calm water)

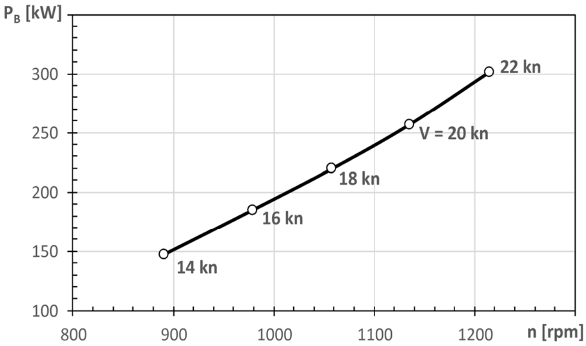


Figure. 5 Propeller curve in deep and calm water

Table 3 Bare hull resistance of the new design, according to VTT, Mercier&Savitsky and NPL MMs

No.	1	2	3	4	5	6	7	8	9	10	11	12	13
MM	VTT				Mercier & Savitsky				NPL				
Fn_∇	$(R_T/\Delta)_{100000}$	(S)	Fn	R_T	$(R_T/\Delta)_{100000}$	(S)	Fn	R_T	$(R_T/\Delta)_{100000}$	τ	(S)	Fn	R_T
[-]	[-]	[-]	[-]	[kN]	[-]	[-]	[-]	[kN]	[-]	[deg]	[-]	[-]	[kN]
0.8									*0.0178*	*0.2483*		0.33	4.37
1.0					*0.0325*		0.42	7.92	*0.0350*	*0.6635*		0.42	8.52
1.2					*0.0590*		0.50	14.26	*0.0653*	*1.6564*		0.50	15.77
1.4					*0.0733*		0.58	17.76	*0.0813*	*2.3613*		0.58	19.67
1.6					*0.0802*	*6.8276*	0.67	19.52	*0.0890*	*2.5918*		0.67	21.63
1.8	*0.0954*		0.75	23.22	*0.0893*		0.75	21.81	*0.0954*	*2.5347*	7.0979	0.75	23.32
2.0	*0.1031*		0.84	25.20	*0.0982*		0.84	24.09	*0.1028*	*2.4207*		0.84	25.25
2.2	*0.1131*		0.92	27.74					*0.1119*	*2.3496*		0.92	27.57
2.4	*0.1254*		1.00	30.83					*0.1241*	*2.3184*		1.00	30.67
2.6	*0.1401*	*6.8276*	1.09	34.51					*0.1402*	*2.3763*		1.09	34.69
2.8	*0.1571*		1.17	38.76					*0.1570*	*2.7018*		1.17	38.91
3.0	*0.1767*		1.25	43.62					*0.1740*	*3.2712*		1.25	43.19
3.2	*0.1988*		1.34	49.09									

Note: values in italic font correspond to the MMs outputs.

Table 4 Yacht resistance for the new design according to SKLAD MM

No	Fr_∇	[-]	1.0	1.3	1.5	1.8	2.0	2.3	2.5	3.0
1	$C_R\ 10^3$	[-]	*9.2142*	*10.6769*	*8.4224*	*6.4428*	*5.1271*	*4.2362*	*3.8487*	*3.0445*
2	(M)	[-]	*5.7225*	*5.7305*	*5.6601*	*5.5592*	*5.5297*	*5.4960*	*5.6080*	*5.4432*
3	(S)	[-]	*7.3681*	*7.5142*	*7.7240*	*7.8997*	*7.9661*	*8.0370*	*8.0600*	*7.9335*
4	τ	[deg]	*0.67*	*1.99*	*2.76*	*2.84*	*2.79*	*2.74*	*2.80*	*3.13*
5	V	[kn]	10.30	12.87	15.45	18.02	20.60	23.17	25.75	30.89
6	L	[m]	16.369	16.392	16.191	15.902	15.818	15.721	16.042	15.570
7	$Re\ 10^{-8}$	[-]	0.7282	0.9116	1.0805	1.2381	1.4074	1.5737	1.7842	2.0781
8	$C_F\ 10^3$	[-]	2.1824	2.1115	2.0602	2.0204	1.9840	1.9530	1.9191	1.8791
9	$C_T\ 10^3$	[-]	11.5966	12.9884	10.6826	8.6632	7.3110	6.3892	5.9678	5.1236
10	S	[m^2]	60.29	61.48	63.20	64.64	65.18	65.76	65.95	64.92
11	R_F	[kN]	1.89	2.92	4.21	5.75	7.44	9.35	11.38	15.79
12	R_R	[kN]	7.99	14.75	17.22	18.34	19.22	20.28	22.81	25.58
13	R_T	[kN]	10.05	17.95	21.85	24.66	27.41	30.59	35.38	43.05
15	R_T*	[kN]	10.98	19.55	23.88	27.08	30.23	33.87	39.25	48.08

Table 5 Selected propeller: input and output data

INPUT		
Resistance per propeller	[kN]	14.64
Propeller diameter	[m]	0.65
Speed	[kn]	20
Number of blades	[-]	3
Propeller area ratio (DAR)	[-]	0.80
Thrust deduction fraction	[-]	0.0388
Wake fraction	[-]	-0.0223
Relative rotative coefficient	[-]	0.9680

OUTPUT		
Propeller rotational speed	[rpm]	1141
Pitch–diameter ratio	[-]	1.25
Propeller open–water efficiency	[-]	0.6614
Advance coefficient	[-]	0.8505
Thrust ceofficient	[-]	0.2357
Torque coefficient	[-]	0.0482
Propeller thrust	[kN]	15.23
Propeller torque	[kNm]	2.03

Table 6 Propeller power absorption vs. operating conditions, in deep and calm water

V	[kn]	14	16	18	20	22
n	[rpm]	891	979	1058	1135	1215
P_B	[kW]	147	185	220	257	302

APPENDIX 2 – IMPLEMENTATION OF COMPUTATIONAL FLUID DYNAMICS (CFD) FOR THE SCENARIO DESCRIBED IN APPENDIX 1

Appendix Overview: CFD Case Study

This appendix presents a CFD analysis that complements the statistically derived Mathematical Models (MMs) discussed earlier; it is based on the hypothetical example elaborated in Appendix 1. The goal is to compare the scope, efficiency, and trade-offs between MM and CFD methods for predicting the power requirements of a 24-ton semi-displacement yacht. A somewhat similar comparison for evaluating the power of planing hulls was recently published in [56].

MMs rely on primary hull (or propeller) parameters, and include secondary characteristics indirectly; i.e. the MMs discussed throughout this paper belong to reduced order MMs. In contrast, CFD uses exact 3D geometry (like bow shape, for instance), capturing all form-specific hydrodynamic effects. This allows secondary design parameters to directly influence the assessment of trim and resistance.

Methodology Applied for MM–CFD Comparison

In practical design scenarios, naval architects frequently modify hulls beyond the bounds of standard series forms (on which the MMs are based). In particular, the virtual 24-ton yacht discussed in Appendix 1 is defined only by its principal dimensions L_{WL}, B_{WL}, T, and Δ; which for the SKLAD MM are sufficient to form primary input parameters L_{WL}/BWL, B_{WL}/T, and C_B, needed for trim and resistance estimates. The yacht's secondary parameters follow from the SKLAD hull form, with the notable exception of a vertical stem (plumb bow), which significantly deviates from the original SKLAD series characterized by a raked (flared) bow. To assess the impact of this modification, a CFD-based comparison was conducted using two hull variants (see Figures 6 and 7):

- Original SKLAD form: Base geometry generated directly from MM input parameters (L_{WL}, B_{WL}, T, and Δ) and maintaining geometric similarity with the series.
- Modified SKLAD form with vertical stem: Slightly refined geometry retaining the same general proportions (L_{WL}, B_{WL}, T, and Δ), but incorporating new (required) design features, such as a vertical stem and optimized aft geometry.

Although we use the term Original SKLAD form, it refers only to our interpretation of the SKLAD series form, derived from the transverse section drawings originally published in Gamulin's paper [57].

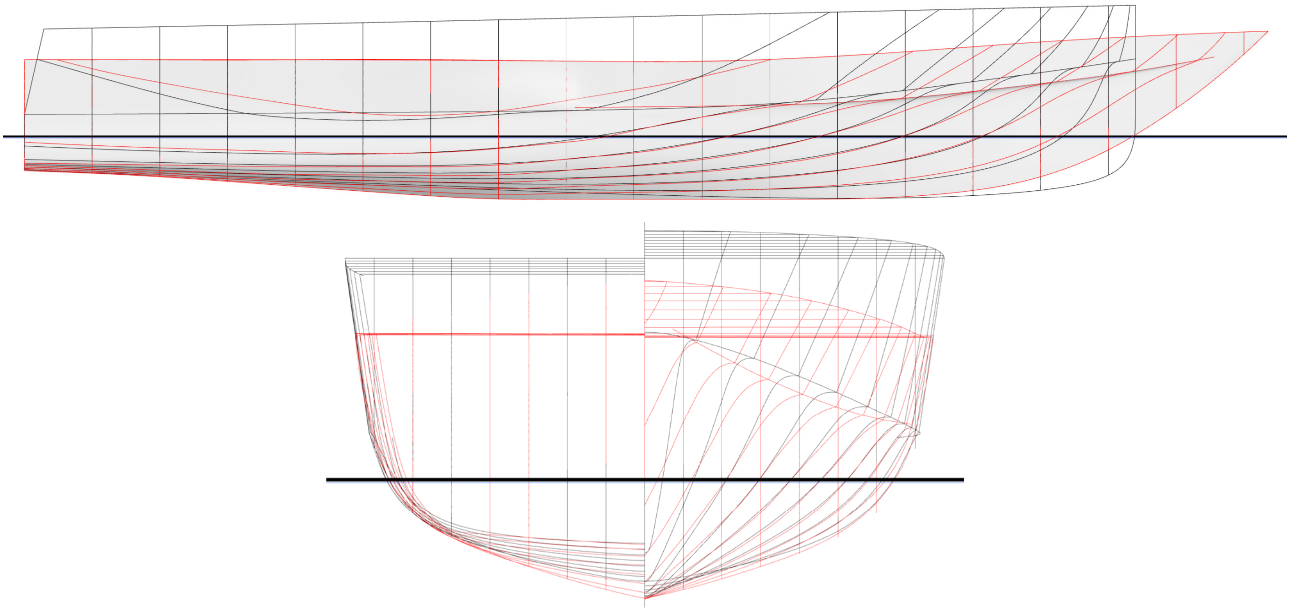


Figure. 6 Original SKLAD hull (grey, longitudinal and transverse sections red) and Modified SKLAD hull (unshaded, sections black)

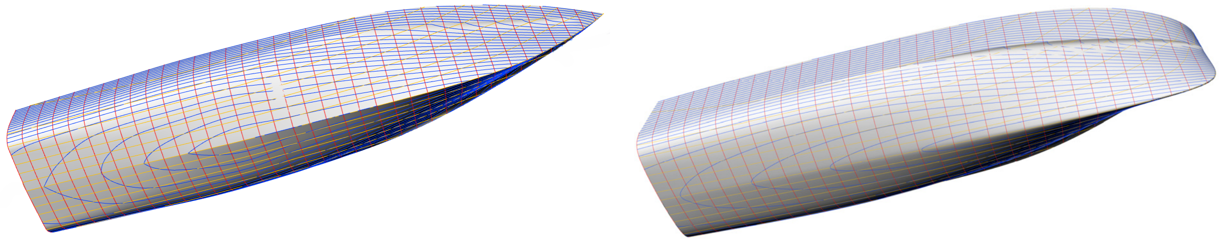


Figure. 7 3D representation of Original SKLAD hull (left) and Modified SKLAD hull (right)

The time required to generate the 3D form shown in Figure 7 depends on the software employed and the designer's level of experience, typically ranging from 2 to 4 hours. In a specific case involving a 24-ton yacht, the 3D model was completed in approximately 3 hours. However, when using MMs for resistance and trim assessments, 3D modeling is unnecessary, as the results rely solely on primary geometric parameters.

The 3D geometry of the Original and Modified SKLAD form (IGES) together with its corresponding parametric dataset will be available upon request from the corresponding author, for research and validation purposes.

Note that the reduced-order methods, i.e. the SKLAD MM discussed in Appendix 1, cannot distinguish between the Original and Modified SKLAD forms, as their results are based solely on the primary parameters, which are identical in both cases.

CFD Assessment of Resistance and Dynamic Trim of a 24 t Motor Yacht

Since CFD itself is not the primary focus of this study, detailed meshing and setup procedures are not shown nor discussed further, except for the brief overview provided below.

The meshing and CFD setup followed established marine best practices, with mesh refinement applied in the transom and free surface regions. The solver used was RANS (Reynolds-Averaged Navier-Stokes), with the k-ω SST turbulence model and VOF (Volume of Fluid) method for free surface modeling. Degrees of freedom included heave and pitch (trim), and simulations were conducted under calm water conditions using STAR-CCM+ software. Further details on the CFD setup, meshing procedures, and boundary conditions can be found in the seminar paper [58]. Simulations were performed at five different speeds, with each case requiring approximately 5 to 10 hours, depending on mesh size and hardware capabilities.

Consequently, CFD simulations were performed on the Original and Modified SKLAD under identical con–ditions and compared with the MM results, see Figure 8.

Key comparative observations include:

- Resistance and dynamic trim predictions showed good agreement in trend between MM and Original SKLAD in the interpolation region ($Fn_L \approx 0.4$–1.1).
- The Modified SKLAD hull demonstrated lower resistance, especially in transition regimes and particularly above $Fn_L = 0.8$, highlighting the potential of form optimization, where CFD captures all geometric effects (eg, local curvature, sectional area shifts, etc).
- LCB differed between the two hull forms, influencing dynamic trim and further contributing to divergence in predicted resistance, which is just another example of secondary parameters not addressed by MMs.

Note, however, that this CFD simulation isn't intended to validate MM or CFD methods, but rather to demonstrate their possibilities and application.

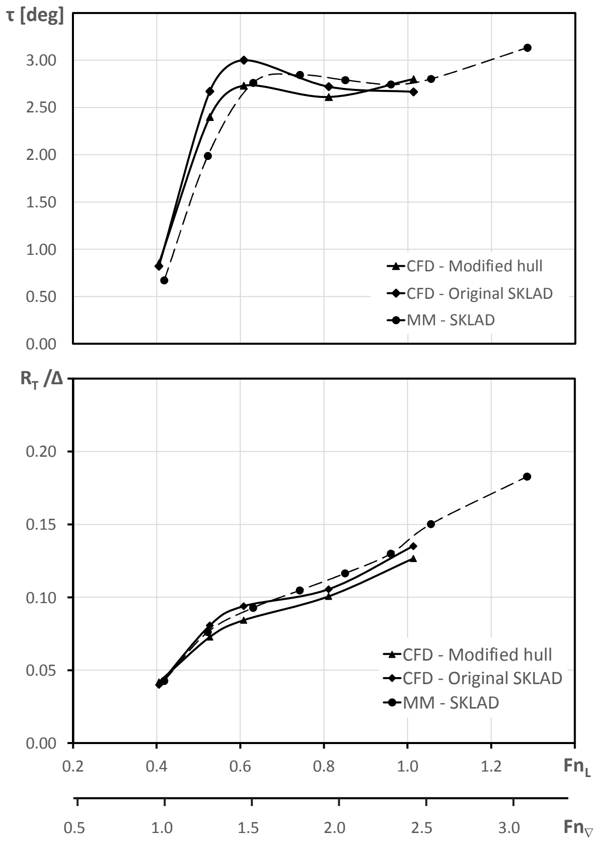


Figure. 8 Comparison of MM predictions with CFD results for Original and Modified SKLAD hull forms

CFD Plan for Estimating Power Demand

Designers usually seek the engine power required for a target speed, not just resistance. Since resistance is only a fraction (though significant) of the total required power, further modeling is needed to estimate brake power. For the case in hand, this can be done in a few ways:

1. *Using MM for propeller performance* assessment (as done in Appendix 1), in combination with the CFD resistance output. This method is fast and aligned with early design stages.
2. *Open-Water CFD simulation* of the propeller. For this, knowledge of a propeller geometry is necessary (this requires 6–10 h).
3. *Self-Propulsion CFD simulation* (comprises hull-propeller-rudder interaction) is much more deman–

ding in terms of expertise and resources (this requires 12–30 h and advanced modeling).

Considering the effort and complexity involved in CFD propulsion modeling, Approach (1) is recommended during the early design stages as a reliable and relatively efficient method for estimating HSC power.

Concluding Remarks on the CFD Case Study

When the hull geometry lies outside the MM's database, as demonstrated in this study, CFD becomes essential for accurate trim and resistance estimates. The key conclusions of this case study are:

- This study reaffirms the *complementary roles of MM and CFD* tools in the design process of high-speed craft (HSC). Rather than competing, these methods serve distinct yet synergistic purposes.
- *MMs remain highly practical* during the conceptual design phase, offering speed, simplicity, robustness, and ease of use; qualities that support rapid early-stage decision-making.
- *CFD becomes essential* in later design stages or when dealing with innovative hull geometries, providing accurate, geometry-sensitive predictions for trim and resistance.
- Discrepancies between MM and CFD arise primarily: a) from MM's abstraction from specific hull geometry, and b) its dependence on the underlying MM database.

Amongst the objectives of Appendix 2 was to evaluate the time demands of MM versus CFD. Table 7 summarizes the comparative durations for obtaining results with both methods.

Note that the time estimates provided in Table 7 are based on the authors' experience from various projects involving CFD-based performance analyses of high-speed craft. These represent approximate durations for an experienced engineer using typical commercial or open-source software. Time requirements may vary depending on tool availability, computational resources, and user expertise. For reference, estimated durations for open water tests and self-propulsion CFD simulations are also included to enable comparison across levels of modeling fidelity.

Table 7. Comparative computer time required to obtain results by CFD and MM methods

Task	CFD Time	MM Time	Comments
3D Hull modeling	2–4 h	–	Only needed for CFD
Mesh generation	4–8 h	–	Depends on refinement and experience
Bare hull resistance simulation	5–10 h	<1 min	For 3–5 speed points
Open water propeller simulation	6–10 h	–	Requires full propeller geometry
Self-propulsion CFD simulation	12–30 h	–	Advanced stage analysis, often omitted in the concept phase
Power prediction - as done in Appendix 1	–	<1 min	Fast and sufficient in early design

ПРЕДВИЂАЊЕ СНАГЕ БРЗИХ ПЛОВИЛА СТАТИСТИЧКИМ МАТЕМАТИЧКИМ МОДЕЛИМА

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У раду су представљене методе за предвиђање потребне снаге брзих пловила полудепласманске, полуглисерске и глисерске форме трупа, применом математичких модела заснованих на статистичкој обради резултата добијених моделским испитивањима. Ови модели припадају класи математичких модела нижег реда (тзв. редуковани модели – *reduced-order models*), који омогућавају поједностављено моделирање отпора, динамичког трима и ефикасности пропелера, узимајући у обзир међусобне интеракције појединих параметара. Развој ових модела заснива се на статистичким методама, најчешће регресионој анализи, док се у новије време све чешће примењују вештачке неуронске мреже (ANN). Статистички математички модели у многим случајевима пружају предности у односу на CFD методе, које доминирају савременом стручном литературом, те стога заслужују посебну пажњу. Овај прегледни рад, уз наглашену практичну примену усмерену ка кориснику, представља методологију на систематизован начин, са циљем да се олакша предвиђање снаге и оптимизација брзих пловила током процеса пројектовања.