

Navigation System Designs Based on Model Predictive and Sliding Mode Control for A Driverless Car's Trajectory Tracking

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This study designs modern controllers, SMC and MPC, for vehicle motion control to ensure accurate trajectory tracking at the desired speed. Longitudinal motion is regulated via throttle and brake using a PD-FLC controller, while lateral motion is controlled through steering angle with SMC and MPC. The novelty of SMC lies in using Decoupled Algorithm, addressing a model where the steering input simultaneously affects translational dynamics and rotational dynamics, which is not widely applied in autonomous vehicles. The MPC novelty lies in using a precomputed adjustable coefficient matrix based on the receding horizon principle, simplifying design compared to continuous optimization during operation. Simulations in Matlab Simulink/Carsim under "Handling Course Left Edge" and "Double-Lane Change" showed that in Case 1, MPC achieved better performance metrics, with a 15% lower average distance error than SMC, while in Case 2, SMC outperformed MPC across all metrics, particularly in trajectory adherence, with 20% improvement.

Keywords: Genetic Algorithm, Sliding-Mode Control, Model Predictive Control, Decoupled Algorithm, Carsim.

1. INTRODUCTION

The process of automation is increasingly developing and becoming widespread across various fields, particularly in transportation, with autonomous vehicles being a prime example. Autonomous vehicles are diverse, including ground-based vehicles (AGVs) [1,2], aerial vehicles (UAVs) [3,4], and underwater vehicles (AUVs) [5,6]. Among these, automobiles are one of the most common ground transportation modes and have attracted significant attention in the development of autonomous features. Research on self-driving cars began early and has been applied in practice [7,8]. This technology continues to advance rapidly due to the diverse real-world operating conditions and the growing emphasis on safety and stability to enhance human utility [9,10]. Autonomous vehicles are studied using control models for both lateral and longitudinal motion under a wide range of operating conditions [11–14]. Design evaluation methods are also diverse, ranging from simulation environments with specialized software such as Carsim to real-world testing [1,16].

In trajectory tracking control, vehicle control systems must ensure sufficient traction to achieve the desired speed while maintaining the steering wheel angle to follow the intended path. Penglei Dai and colleagues [17] applied an SMC-based controller (Sliding-Mode Control) to manage the steering angle and an online-optimized PID controller using PSO to

control the driving wheel torque in a 4×4 vehicle model. Raghavendra's team [18] designed and experimentally evaluated an SMC controller with a Proportional-Derivative sliding surface for vehicle motion control, integrating Fuzzy Logic to fine-tune the sliding surface parameters based on error and error rate, enhancing system stability. Baosen Ma's group [19] proposed an RITSMC (Recursive Integral Terminal Sliding Mode Control) design method, combining the advantages of RISM, TSM, and optimization algorithms, demonstrating superior performance under various test conditions compared to conventional SMC and ITSMC. Fengxi Xie and colleagues developed an adaptive SMC capable of simultaneously controlling longitudinal and lateral vehicle motion, with parameters optimized using PSO and improved GWO algorithms [20]. Ping Hu's team proposed an SMC design with parameters adjusted via fuzzy logic to reduce chattering in longitudinal motion control of autonomous vehicles [21].

In addition to SMC, MPC (Model Predictive Control) and its variants are widely applied in autonomous vehicle control. Specifically, Jiechao Liu and colleagues implemented an MPC controller on a high-center-of-gravity autonomous vehicle to reach a target position as quickly as possible without a reference trajectory. The system allows the vehicle to generate its own trajectory by optimizing a spatial objective function while ensuring wheel-road adherence through steering angle constraints using a 14-DOF dynamic model [22]. Chuanyang Sun and colleagues applied an improved MPC with fuzzy logic support to ensure lateral motion control in both steady-state and transient conditions, demonstrating higher trajectory tracking performance compared to non-adaptive systems [23]. Young-Min Choi and Jahng-Hyon Park proposed a novel game-based coupling

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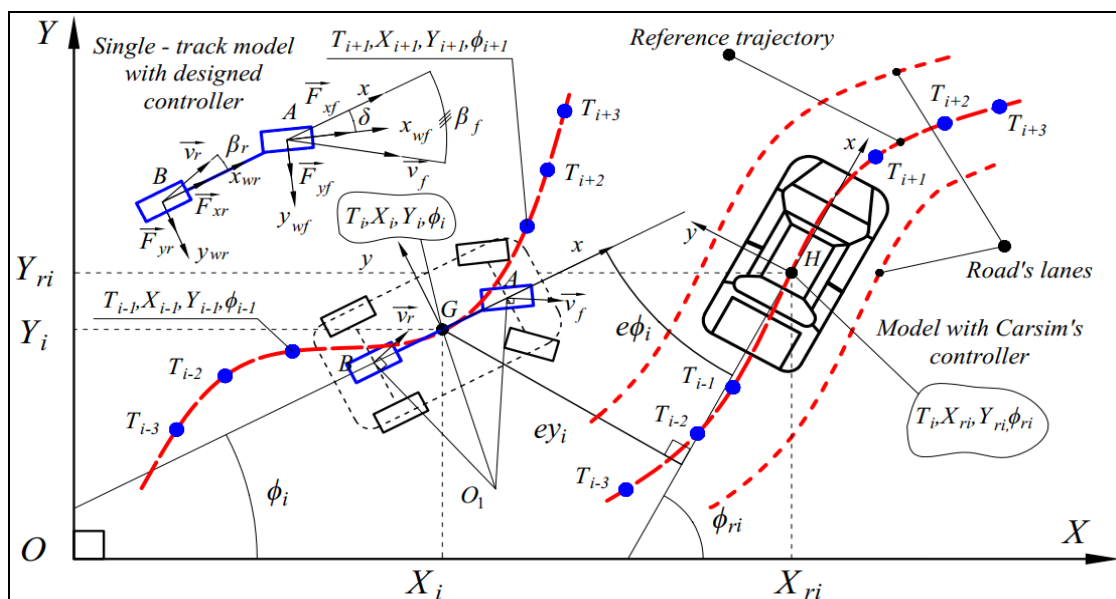
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In addition to *SMC* and *MPC*-based controllers, *FLC* has also been widely applied in autonomous vehicle control. While *SMC* and *MPC* systems require a vehicle dynamic model to compute control inputs based on the vehicle's state [18, 19, 27, 28], *FLC* design does not require explicit equations describing control input evolution over time [30,31]. Neil Eugene Hodge and colleagues proposed a modular *FLC*-based trajectory control system, integrating one *FLC* with four modules for steering angle control and another *FLC* with three modules for speed control [30]. Subramanian V and colleagues designed a sensor fusion navigation system using an enhanced Kalman Filter combined with fuzzy logic, integrating data from machine vision, *LIDAR*, *IMU*, and speed sensors [31]. Bhakti Yudho and colleagues proposed a Type-2 *FLC* for lateral vehicle motion control, enabling trajectory tracking and obstacle avoidance, demonstrating superior performance compared to traditional *PID* controllers [32]. Thinh and Phuong employed

The longitudinal vehicle speed is significantly influenced by lateral motion [34], requiring the driver to continuously adjust the throttle and brake pedals to maintain the desired speed. To capture this complex human behavior, previous studies have employed various control strategies, yet *PD-FLC* (Proportional Derivative) controllers, which can respond effectively to rapidly changing system states, have not been widely applied. While *SMC*-based lateral control methods are diverse, the use of a Decoupled Algorithm to simplify design and the incorporation of Hurwitz conditions to accurately calculate the correlation between control parameters have not been explored. *MPC* applications typically rely on continuous optimization over the control horizon, using the first computed value via *QP* (Quadratic Program), which complicates the design process. Moreover, systematic and efficient optimization of the state weighting matrices in the control signal has not been clearly addressed. Therefore, this study aims to contribute simplified *SMC* and *MPC* design methods for autonomous vehicles, capturing high correlations among control parameters to achieve improved operational performance. The main novelties and contributions of this research include:

- 1) A *PD-FLC* design method for longitudinal vehicle motion control, capable of effectively responding to rapid variations in vehicle speed state errors.
- 2) A simplified *SMC* design method using the Decoupled Algorithm, ensuring high accuracy in the correlation of sliding surface parameters based on Hurwitz conditions.
- 3) An *MPC* design method with control inputs directly determined according to the Receding Horizon Control principle, simplifying the computation process.

2.1 OF planar model



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Many types of simulation models can be used to describe vehicle dynamics with different degrees of freedom (DOF), such as single-track models with 3 DOF, two-track models with 4 DOF, or full 6-DOF models [34]. This study adopts a single-track, three-degree-of-freedom (3-DOF) vehicle dynamics model in the OXY plane, which includes translational motions along the X and Y axes and rotation about the vertical axis. The overall analysis framework consists of two models: the controlled model used for controller design and implementation, and the reference model (with Carsim's controller). Both models are simulated within the Carsim environment and integrated with Matlab Simulink.

The controlled vehicle model includes steering wheel, throttle, and brake controllers, while the reference model only has throttle and brake control. Each vehicle model has its own coordinate system with the origin at the vehicle's center of gravity, denoted as G_{xy} for the controlled vehicle and H_{xy} for the reference vehicle. At each time step T_i , both the controlled and reference models are operated, and signals such as velocity, acceleration, and angular rate are recorded to compute the vehicle coordinates $X_i - X_{ri}$, $Y_i - Y_{ri}$, and heading angle $\phi_i - \phi_{ri}$. From these, the lateral distance e_{y_i} and heading deviation e_{ϕ_i} between the controlled and reference vehicles are determined. The set of reference vehicle coordinates over the entire simulation period defines the desired trajectory that the controlled vehicle must follow.

Although roll motions around the Gx and Hx axes and pitch motions around the Gy and Hy axes affect vehicle behavior over time, the mathematical models for the designed controllers in this study are based on the 3-DOF model, which neglects these roll and pitch dynamics. However, the controller parameters are calibrated with the Carsim vehicle model, ensuring effective performance even in the presence of roll and pitch motions. Stability-related responses at the tire-road interface are represented using the built-in tire model provided by Carsim.

2.2 Mathematical model

The mathematical model describing the vehicle's lateral dynamics is established for the design of lateral motion controllers using *SMC* and *MPC*. The lateral forces exerted by the road on the front and rear wheels, F_{yf} and F_{yr} , during vehicle steering are calculated based on the Pacejka tire model [34]. However, within the scope of this study, the Pacejka model is linearized for small tire slip angles as expressed in (1) [34]:

$$\begin{cases} F_{yf} = F_{zf} \mu_y (\alpha_f) \approx C_{\alpha f} \alpha_f, \alpha_f = \delta - \beta_f \leq 10^\circ \\ F_{yr} = F_{zr} \mu_y (\alpha_r) \approx -C_{\alpha r} \alpha_r, \alpha_r = \beta_r \leq 10^\circ \end{cases} \quad (1)$$

Considering the vehicle speeds at point $A - v_f$ and point $B - v_r$, we have:

$$\begin{cases} \vec{v}_f = \vec{v}_{A/O_1} = \vec{v}_{A/G} + \vec{v}_{G/O_1} \Rightarrow \begin{cases} v_{fx} = v_{rx} = \dot{x} \\ v_{fy} = a_1 \dot{\phi} + \dot{y} \\ v_{ry} = -a_2 \dot{\phi} + \dot{y} \end{cases} \end{cases} \quad (2)$$

Considering the vehicle acceleration at point G , we have:

$$\begin{aligned} \vec{a}_G &= \vec{a}_{Gf} + \vec{a}_{Gr} \\ \begin{cases} a_{Gx} = \ddot{x} - \frac{v^2}{O_1 G} \cos\left(\frac{\pi}{2} - \beta\right) = \ddot{x} - \frac{v}{O_1 G} v \sin \beta \\ \quad \quad \quad = \ddot{x} - \dot{\phi} \dot{y} \\ a_{Gy} = \ddot{y} + \frac{v^2}{O_1 G} \sin\left(\frac{\pi}{2} - \beta\right) = \ddot{y} + \frac{v}{O_1 G} \dot{x} v \cos \beta \\ \quad \quad \quad = \ddot{y} + \dot{\phi} \dot{x} \end{cases} \end{aligned} \quad (3)$$

With the force vectors at the front wheels $F_{xf} - F_{yf}$ and rear wheels $F_{xr} - F_{yr}$, the dynamic equations describing the vehicle motion along the Gx , Gy , and vertical directions are formulated according to Newton's Second Law (4).

$$\begin{cases} \ddot{x} = \frac{1}{m} \left[-C_{\alpha f} \left(\delta - \arctan \frac{a_1 \dot{\phi} + \dot{y}}{\dot{x}} \right) \sin \delta + F_{xf} \cos \delta + F_{xr} \right] + \dot{\phi} \dot{y} \\ \ddot{y} = \frac{1}{m} \left[C_{\alpha f} \left(\delta - \arctan \frac{a_1 \dot{\phi} + \dot{y}}{\dot{x}} \right) \cos \delta + F_{yf} \sin \delta - C_{\alpha r} \arctan \frac{-a_2 \dot{\phi} + \dot{y}}{\dot{x}} \right] - \dot{\phi} \dot{x} \\ \ddot{\phi} = \frac{1}{I_z} \left\{ \left[C_{\alpha f} \left(\delta - \arctan \frac{a_1 \dot{\phi} + \dot{y}}{\dot{x}} \right) \cos \delta + F_{yf} \sin \delta \right] a_1 + C_{\alpha r} \arctan \frac{-a_2 \dot{\phi} + \dot{y}}{\dot{x}} a_2 \right\} \end{cases} \quad (4)$$

The dynamic model (4) is linearized by considering the steering angle δ to be below 10° .

$$\begin{cases} \ddot{x} = \frac{1}{m} \sum F_x + \dot{\phi} \dot{y} \\ \ddot{y} = \dot{\phi} \frac{(-a_1 C_{\alpha f} + a_2 C_{\alpha r})}{m \dot{x}} - \dot{y} \frac{(C_{\alpha f} + C_{\alpha r})}{m \dot{x}} + \delta \frac{C_{\alpha f}}{m} - \dot{\phi} \dot{x} \\ \ddot{\phi} = \dot{\phi} \frac{(-a_1^2 C_{\alpha f} - a_2^2 C_{\alpha r})}{I_z \dot{x}} - \dot{y} \frac{(a_1 C_{\alpha f} - a_2 C_{\alpha r})}{I_z \dot{x}} + \delta \frac{a_1 C_{\alpha f}}{I_z} \end{cases} \quad (5)$$

The vehicle coordinates in the OXY coordinate system are determined based on the integral of the vehicle's instantaneous velocity values v_x , v_y along the OX and OY directions (6).

$$\begin{cases} \vec{v}_X = \vec{v}_{x/X} + \vec{v}_{y/X} \Rightarrow X = \int_0^t (\dot{x} \cos \phi - \dot{y} \sin \phi) dt \\ \vec{v}_Y = \vec{v}_{x/Y} + \vec{v}_{y/Y} \Rightarrow Y = \int_0^t (\dot{x} \sin \phi + \dot{y} \cos \phi) dt \end{cases} \quad (6)$$

The controllers are designed based on the optimization of the distance error e_y and heading error e_ϕ . According to the analysis diagram in Figure 1, these values are determined through the vehicle coordinates X, Y and heading ϕ as in (7). X and Y denote the vehicle coordinates in the global XOY reference frame, whereas x and y represent the instantaneous displacement states of the vehicle in the local xGy coordinate frame.

$$\begin{cases} e\phi_i = \phi_i - \phi_{ri} \\ ey_i = (X_i - X_{ri})\sin\phi_{ri} - (Y_i - Y_{ri})\cos\phi_{ri} \end{cases} \quad (7)$$

Derivative of input signals:

$$\begin{cases} \dot{e}\phi_i = \dot{\phi}_i - \dot{\phi}_{ri} \\ \dot{e}y_i = \dot{x}\sin e\phi_i + \dot{y}\cos e\phi_i \approx \dot{x}e\phi_i + \dot{y} \end{cases} \quad (8)$$

2.3 Test cases' set up with Carsim software

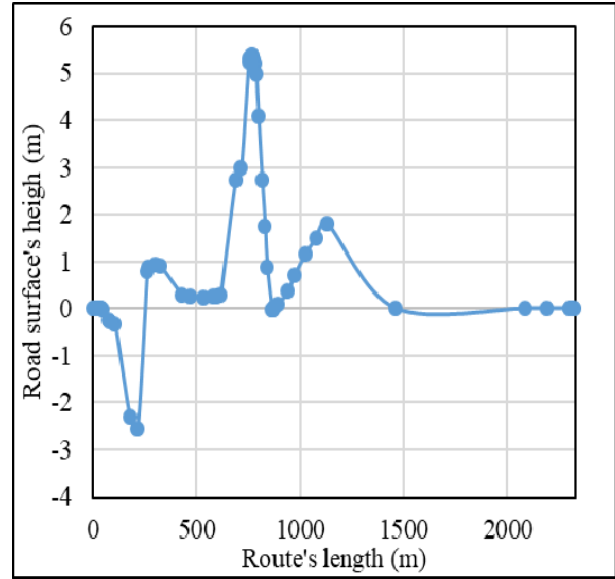
The study designs controllers to ensure the vehicle follows the desired speed and trajectory under operational scenarios including: starting on a long and complex trajectory from 0 km/h and maintaining stability at 40 km/h, Figure 2, which represents the permissible speed on urban roads in Ho Chi Minh City, Vietnam; and the “Double-Lane Change” scenario at 120 km/h, Figure 3, which corresponds to the maximum allowable speed on highways in Vietnam. These scenarios assume good road conditions with a static friction coefficient $\mu=0.9$. The controllers are implemented in Matlab Simulink and interact with the Carsim vehicle model. Furthermore, the controller parameters are primarily optimized to handle the first scenario while their obstacle avoidance capability at high speed is verified using the second scenario.

Case 1: Handling course left edge

The road is 2,328 m long within a test area of 470 m in length and 360 m in width, with 2 lanes, Figure 2a. The road elevation relative to the starting point varies continuously, with a maximum height of 5.2 m and a minimum of -2.5 m, Figure 2b.



a. Route's map



b. Route's height

Figure 2. Long route at constant speed 40 (km/h)

In Carsim, the environment and test model are set up in the “Road X-Y-Z edges” procedure, with the trajectory defined in the “Reference Path: X-Y Coordinates of Path” and “Handling Course Left Edge” modules. This trajectory is suitable for test scenarios with diverse curves and repetitive patterns. In this Carsim module, the user configures “Driver Controls” with the throttle mode set to “Do not specify speed control here” and the brake mode set to “No linked” allowing direct input of throttle and brake positions from the PD-FLC controller.

Case 2: Double-lane change

The lane-change section is 75 m long, with a distance of 3.5 m between the two lanes, and the vehicle speed at the start of the lane change is 120 km/h, Figure 3. In Carsim, the environment and test model are set up in the “DLC @ 120 km/h” procedure, with the trajectory defined in the “Steering: Driver Path Follower” and “Double-Lane Change (Quick Start)” modules. In this scenario, the vehicle speed is maintained by default in Carsim, and only the steering angle controller is evaluated.

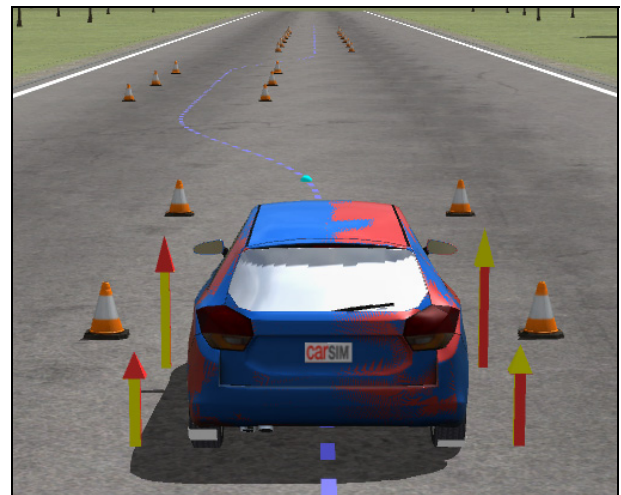


Figure 3. Double-lane change at 120 (km/h)

In this Carsim module for Case 2, the user configures “Driver Controls” with the throttle mode set to “Constant target speed – 120 km/h”. In this case, the *PD-FLC* is not applied for vehicle speed control; the Carsim controller is used instead. Only the steering angle controllers, *SMC* and *MPC*, are employed.

The specifications of the test tracks, such as total travel length, lane spacing, slope, and friction coefficient, are set according to Carsim’s standard testing conditions. Only the auxiliary *ABS* system is activated during the test scenarios, with operating parameters set to Carsim’s default values.

3. CONTROLLERS’ DESIGN

3.1. Longitudinal motion

The vehicle’s longitudinal motion (speed) is controlled by a *PD-FLC* (proportional–derivative fuzzy logic controller). The vehicle speed is regulated via throttle and brake positions in the “Driver Control” section of the “Road X-Y-Z edges” procedure. The control signal is the error between the desired vehicle speed $v_{ref}=40$ km/h and the actual vehicle speed v_{real} . However, the value applied for *PD-FLC* optimization according to the *ITAE* standard [35, 36] is the position error e_y between the controlled vehicle and the reference vehicle in Carsim.

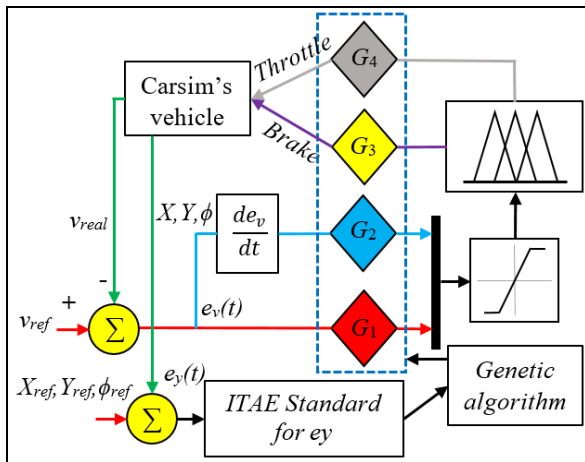
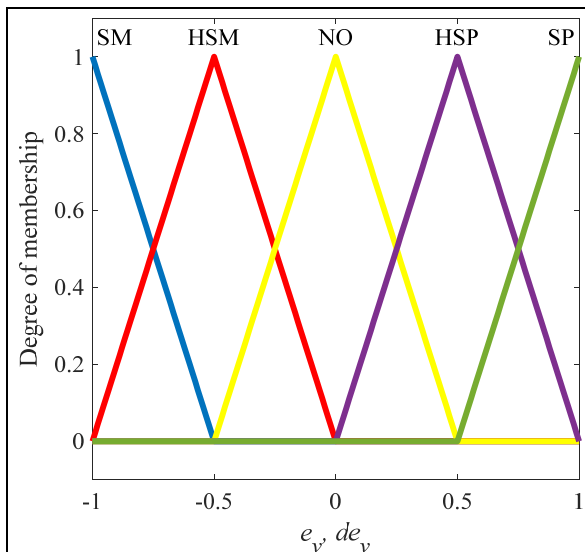
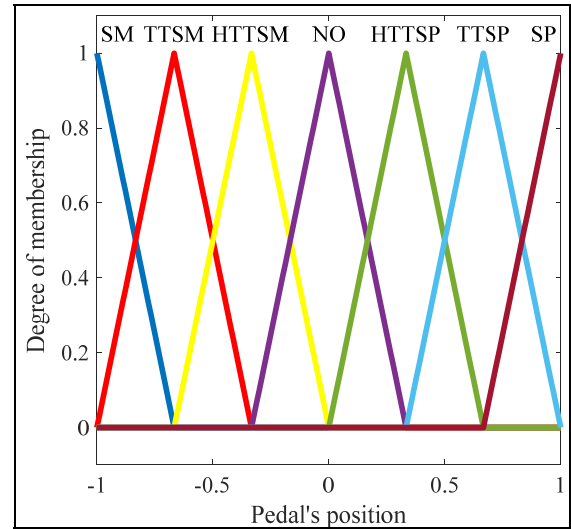


Figure 4. PD FLC's diagram



a. Membership function of e_v , de_v



b. Membership function of pedal's position

Figure 5. Membership function for e_v , de_v , pedal's position

The e_y value is calculated based on the controlled vehicle data X , Y , ϕ and the reference vehicle data X_{ref} , Y_{ref} , ϕ_{ref} according to equation (7). Applying e_y for *ITAE* instead of e_v is justified because when the vehicle follows the desired trajectory accurately, the vehicle speed naturally reaches the desired value. The *PD-FLC* controller parameters are tuned for optimal performance, including G_1 , G_2 , G_3 , G_4 . These parameters are determined by *GA* based on the minimum *ITAE* obtained.

The study applies a fuzzy logic controller with triangular membership functions for both input and output signals. The input signals e_v and de_v have five membership levels: “SM – Significant Minus”, “HSM – Half Significant Minus”, “NO – Non”, “SP – Significant Plus”, and “HSP – Half Significant Plus”. The output signals, representing throttle and brake positions, have seven membership levels: “SM – Significant Minus”, “TTSM – Two-Third Significant Minus”, “HTTSM – Half of Two-Third Significant Minus”, “NO – Non”, “SP – Significant Plus”, “TTSP – Two-Third Significant Plus”, and “HTTSP – Half of Two-Third Significant Plus”. The relationships between the input and output membership functions are defined by the design rules presented in Table 1. These relationships are visually represented in a 3D graph, Figure 6.

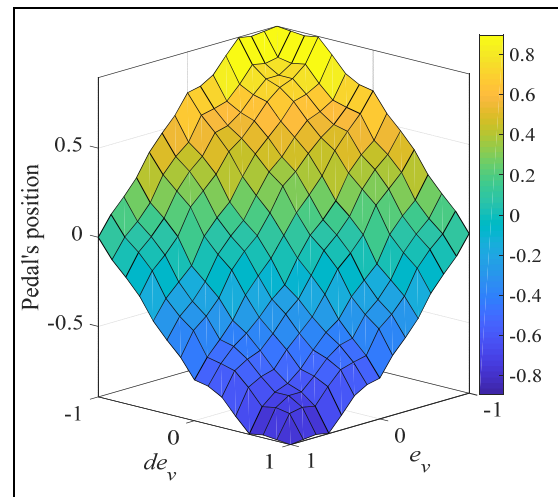


Figure 6. Control surface

Table 1. PD - FLC rules

Pedal's position	Error of velocity (e_v)					
	SM	HSM	NO	HSP	SP	
de _v	SM	SP	SP	TTSP	HTTSP	NO
	HSM	SP	TTSP	HTTSP	NO	HTTSM
	NO	TTSP	HTTSP	NO	HTTSM	TTSM
	HSP	HTTSP	NO	HTTSM	TTSM	SM
	SP	NO	HTTSM	TTSM	SM	SM

3.2 Lateral motion

3.2.1. Model predictive controller – MPC

The MPC controller for steering angle control of the front wheels is designed based on the 3-DOF dynamic

model according to (5) and (8). The steps for designing the MPC controller follow the procedure below [37].

Establish the state variable matrix of the model X and transform equations (5) and (8) into the general form with X as in (9).

$$X = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8]^T$$

With:

$$\begin{aligned} x_1 &= y, x_2 = \dot{y}, x_3 = \phi, x_4 = \dot{\phi}, \\ x_5 &= ey, x_6 = \dot{ey}, x_7 = e\phi, x_8 = \dot{e\phi} \end{aligned}$$

Therefore:

$$\dot{X} = AX + BU + Dd$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-C_{\alpha f} - C_{\alpha r}}{m\dot{x}} & 0 & \frac{-a_1 C_{\alpha f} + a_2 C_{\alpha r}}{m\dot{x}} & -\dot{x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-a_1 C_{\alpha f} + a_2 C_{\alpha r}}{I_z \dot{x}} & 0 & \frac{-a_1^2 C_{\alpha f} - a_2^2 C_{\alpha r}}{I_z \dot{x}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{-C_{\alpha f} - C_{\alpha r}}{m\dot{x}} & 0 & \frac{-a_1 C_{\alpha f} + a_2 C_{\alpha r}}{m\dot{x}} & -\dot{x} & 0 & 0 & \dot{x} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{-a_1 C_{\alpha f} + a_2 C_{\alpha r}}{I_z \dot{x}} & 0 & \frac{-a_1^2 C_{\alpha f} - a_2^2 C_{\alpha r}}{I_z \dot{x}} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{C_{\alpha f}}{m} \\ 0 \\ \frac{a_1 C_{\alpha f}}{I_z} \\ 0 \\ \frac{C_{\alpha f}}{m} \\ 0 \\ \frac{a_1 C_{\alpha f}}{I_z} \end{bmatrix} \delta + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \ddot{\phi}_r \quad (9)$$

Establish the general discrete-time dynamic model by discretizing (9) using Matlab. The general discrete equation is given in (10).

$$\begin{cases} x_m(k+1) = A_m x_m(k) + B_m u(k) + D_m d(k) \\ y(k) = C_m x_m(k) \end{cases} \quad (10)$$

Transform the discrete-time model into the “Argument” form. We have:

$$\begin{cases} \circ x_m(k+1) - x_m(k) = A_m [x_m(k) - x_m(k-1)] + \dots \\ \bullet B_m [u(k) - u(k-1)] + D_m [d(k) - d(k-1)] \\ \circ y(k+1) - y(k) = C_m [x_m(k+1) - x_m(k)] \\ \begin{cases} \Delta x_m(k+1) = x_m(k+1) - x_m(k) \\ \Delta x_m(k) = x_m(k) - x_m(k-1) \\ \Delta u(k) = u(k) - u(k-1) \\ \Delta d(k) = d(k) - d(k-1) \end{cases} \end{cases} \quad (11)$$

Therefore:

$$\begin{cases} \bullet \Delta x_m(k+1) = A_m \Delta x_m(k) + B_m \Delta u(k) + \dots \\ D_m \Delta d(k) \\ \bullet y(k+1) = C_m A_m \Delta x_m(k) + C_m B_m \Delta u(k) + \dots \\ C_m D_m \Delta d(k) + y(k) \end{cases} \quad (12)$$

Control form:

$$\begin{cases} \bullet \begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} A_m & o_m^T \\ C_m A_m & 1 \end{bmatrix} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix} + \dots \\ \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \Delta u(k) + \begin{bmatrix} D_m \\ C_m D_m \end{bmatrix} \Delta d(k) \\ \bullet y(k) = [o_m \ 1] \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix} \end{cases} \quad (13)$$

$$\Leftrightarrow \begin{cases} \tilde{x}(k+1) = A\tilde{x}(k) + B\Delta u(k) + D\Delta d(k) \\ \tilde{y}(k) = C\tilde{x}(k) \end{cases}$$

Consider over the domain of control horizon N_c and prediction horizon N_p :

$$\begin{aligned} \bullet \tilde{x}(k_i+1 | k_i) &= A\tilde{x}(k_i) + B\Delta u(k_i) + D\Delta d(k_i) \\ \bullet \tilde{x}(k_i+2 | k_i) &= A^2 \tilde{x}(k_i) + AB\Delta u(k_i) + B\Delta u(k_i+1) + \dots \\ &\quad AD\Delta d(k_i) + D\Delta d(k_i+1) \\ \bullet \tilde{x}(k_i+3 | k_i) &= A^3 \tilde{x}(k_i) + A^2 B\Delta u(k_i) + AB\Delta u(k_i+1) + \dots \\ &\quad B\Delta u(k_i+2) + A^2 D\Delta d(k_i) + AD\Delta d(k_i+1) + D\Delta d(k_i+2) \\ &\vdots \\ \bullet \tilde{x}(k_i+N_p | k_i) &= A^{N_p} \tilde{x}(k_i) + A^{N_p-1} B\Delta u(k_i) + \dots \\ &\quad + A^{N_p-N_c} B\Delta u(k_i+N_c-1) + A^{N_p-1} D\Delta d(k_i) + \dots \\ &\quad + D\Delta d(k_i+N_p-1) \end{aligned} \quad (14)$$

Therefore:

$$\begin{aligned}
 \bullet y(k_i+1|k_i) &= CAx(k_i) + CB\Delta u(k_i) + CD\Delta d(k_i) \\
 \bullet y(k_i+2|k_i) &= CA^2x(k_i) + CAB\Delta u(k_i) + CB\Delta u(k_i+1) + \dots \\
 &\quad CAD\Delta d(k_i) + CD\Delta d(k_i+1) \\
 \bullet y(k_i+3|k_i) &= CA^3x(k_i) + CA^2B\Delta u(k_i) + CAB\Delta u(k_i+1) + \dots \\
 &\quad CB\Delta u(k_i+2) + CA^2D\Delta d(k_i) + CAD\Delta d(k_i+1) + CD\Delta d(k_i+2) \quad (15) \\
 &\vdots \\
 \bullet y(k_i+N_p|k_i) &= CA^{N_p}x(k_i) + CA^{N_p-1}B\Delta u(k_i) + \dots \\
 &\quad + CA^{N_p-N_c}B\Delta u(k_i+N_c-1) + CA^{N_p-1}D\Delta d(k_i) + \dots \\
 &\quad + CD\Delta d(k_i+N_p-1)
 \end{aligned}$$

Set:

$$\begin{cases} Y = [y(k_i+1|k_i) \ y(k_i+2|k_i) \ \dots \ y(k_i+N_p|k_i)]^T \\ \Delta U = [\Delta u(k_i) \ \Delta u(k_i+1) \ \dots \ \Delta u(k_i+N_c-1)]^T \\ \Delta D = [\Delta d(k_i) \ \Delta d(k_i+1) \ \dots \ \Delta d(k_i+N_p-1)]^T \end{cases} \quad (16)$$

Set:

$$Y = Fx(k_i) + \Phi\Delta U + \tau\Delta D \quad (17)$$

$$\begin{aligned}
 W_{1(N_p \times N_p)} &= \begin{bmatrix} I_w & 0 & 0 & \dots & 0 \\ 0 & I_w & 0 & \dots & 0 \\ 0 & 0 & I_w & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & I_w \end{bmatrix} \\
 I_{w(12 \times 12)} &= \begin{bmatrix} w_1 & 0 & 0 & \dots & 0 \\ 0 & w_2 & 0 & \dots & 0 \\ 0 & 0 & w_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & w_{12} \end{bmatrix}, \\
 W_{2(N_c \times N_c)} &= \begin{bmatrix} w & 0 & 0 & \dots & 0 \\ 0 & w & 0 & \dots & 0 \\ 0 & 0 & w & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & w \end{bmatrix}
 \end{aligned}$$

Determine the value of ΔU that minimizes J by taking the derivative of the quadratic equation of J with respect to ΔU (19).

$$\begin{aligned}
 \frac{\partial J}{\partial \Delta U} &= 2\Delta U(\Phi^T W_1 \Phi + W_2) - 2\Phi^T W_1(R_S - Fx(k_i)) \dots \\
 &\quad + 2\Phi^T W_1 \tau \Delta D \quad (19)
 \end{aligned}$$

With:

$$\begin{aligned}
 F_{N_p \times 1} &= \begin{bmatrix} CA & CA^2 & CA^3 & CA^4 & \dots & CA^{N_p} \end{bmatrix}^T \\
 \Phi_{N_p \times N_c} &= \begin{bmatrix} CB & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \dots & 0 \\ CA^2B & CAB & CB & \dots & 0 \\ CA^3B & CA^2B & CAB & \dots & 0 \\ CA^4B & CA^3B & CA^2B & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \dots & CA^{N_p-N_c}B \end{bmatrix} \\
 \tau_{N_p \times N_p} &= \begin{bmatrix} CD & 0 & 0 & \dots & 0 \\ CAD & CD & 0 & \dots & 0 \\ CA^2D & CAD & CD & \dots & 0 \\ CA^3D & CA^2D & CAD & \dots & 0 \\ CA^4D & CA^3D & CA^2D & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{N_p-1}D & CA^{N_p-2}D & CA^{N_p-3}D & \dots & CA^{N_p-N_p}D \end{bmatrix}
 \end{aligned}$$

Establish the quadratic objective function J of the state variable errors and control input changes ΔU with weighting matrices W_1 , W_2 according to (18).

$$\begin{aligned}
 J &= (R_S - Y)^T W_1 (R_S - Y) + \Delta U^T W_2 \Delta U \\
 &= (R_S - Fx(k_i) - \Phi\Delta U - \tau\Delta D)^T W_1 (R_S - Fx(k_i) - \Phi\Delta U - \tau\Delta D) + \Delta U^T W_2 \Delta U \quad (18)
 \end{aligned}$$

J reaches its minimum with the value of ΔU determined according to (20).

$$\begin{aligned}
 \frac{\partial J}{\partial \Delta U} &= 0 \\
 \Rightarrow \Delta U &= (\Phi^T W_1 \Phi + W_2)^{-1} \Phi^T W_1 (R_S - Fx(k_i) - \tau\Delta D) \quad (20)
 \end{aligned}$$

3.2.2. Decoupled sliding-mode controller – SMC

Unlike the *MPC*, where the system model must integrate the influence of all state variables into a single control signal model, the *SMC* model is simpler, as it only varies according to the state variables that need to be stabilized, namely e_y and e_ϕ . The *SMC* controller is designed using the “Decoupled Algorithm” (*DA*). *DA* addresses the situation where the system control input δ directly affects multiple dynamic equations simultaneously according to (21) [38]. Accordingly, the *DA* method transforms the system of equations (21) into a new system as in (23), in which only one equation involves δ . This process simplifies and improves the efficiency of establishing the control signal model $\delta(t)$ as in (31).

$$\begin{cases} \ddot{e}_y = f_1(\dot{e}_\varepsilon, \dot{y}, \dot{\phi}) + g_1(e\phi)\delta \\ \ddot{e}_\phi = f_2(\dot{e}\phi, \dot{y}, \dot{\phi}) + g_2(e\phi)\delta + d(\ddot{\phi}_r) \end{cases} \quad (21)$$

where, f_1, f_2, g_1, g_2, d can be calculated as (11).

$$\left\{ \begin{array}{l} f_1 = \dot{x}\dot{e}\phi + \frac{1}{m\dot{x}}(-a_1C_{\alpha f} + a_2C_{\alpha r})\dot{\phi}... \\ -\frac{1}{m\dot{x}}(C_{\alpha f} + C_{\alpha r})\dot{y} - \dot{x}\dot{\phi} \\ f_2 = \frac{1}{I_z\dot{x}}(-a_1^2C_{\alpha f} - a_2^2C_{\alpha r})\dot{\phi}... \\ -\frac{1}{I_z\dot{x}}(a_1C_{\alpha f} - a_2C_{\alpha r})\dot{y} \\ g_1 = \frac{1}{m}C_{\alpha f}, g_2 = \frac{1}{I_z}a_1C_{\alpha f} \\ d = -\ddot{\phi}_r \end{array} \right. \quad (22)$$

Set:

$$\left\{ \begin{array}{l} x_1 = ey - \frac{I_z}{ma_1}e\phi \\ x_2 = \dot{e}y - \frac{I_z}{ma_1}\dot{e}\phi \\ x_3 = e\phi \\ x_4 = \dot{e}\phi \end{array} \right. \quad (23)$$

Therefore:

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = \left(\dot{x} + f_3 + \frac{I_z}{ma_1}f_4 \right)x_4 + f_4x_2 - \dot{x}f_4x_3 + f_3\dot{\phi}_r... \\ + \frac{I_z}{ma_1}\ddot{\phi}_r \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -\ddot{\phi}_r + \frac{1}{I_z\dot{x}}(-a_1^2C_{\alpha f} - a_2^2C_{\alpha r})\dot{\phi}... \\ - \frac{a_1C_{\alpha f} - a_2C_{\alpha r}}{I_z\dot{x}}x_2 + \frac{a_1C_{\alpha f} - a_2C_{\alpha r}}{I_z}x_3... \\ + \frac{ma_1(-a_1^2C_{\alpha f} - a_2^2C_{\alpha r}) - I_z(a_1C_{\alpha f} - a_2C_{\alpha r})}{I_z\dot{x}ma_1}x_4... \\ + \frac{1}{I_z}a_1C_{\alpha f}\delta \end{array} \right. \quad (24)$$

Set:

$$\left\{ \begin{array}{l} f_3 = \frac{a_2C_{\alpha r}l - ma_1\dot{x}^2}{ma_1\dot{x}} \\ f_4 = -\frac{C_{\alpha r}l}{ma_1\dot{x}} \end{array} \right. \quad (25)$$

Set:

$$\left\{ \begin{array}{l} \mu_1 = x_4 \\ \mu_2 = [x_1 \quad x_2 \quad x_3]^T \\ C = [c_1 \quad c_2 \quad c_3] \end{array} \right. \quad (26)$$

Sliding surface:

$$\sigma = \mu_1 - C\mu_2 = x_4 - c_1x_1 - c_2x_2 - c_3x_3 \quad (27)$$

Derive sliding surface:

$$\begin{aligned} \dot{\sigma} &= \dot{\mu}_1 - C\dot{\mu}_2 \\ &= -\ddot{\phi}_r \left(1 + \frac{c_2I_z}{ma_1} \right) + \dot{\phi}_r f_5 + (f_6 - c_1)x_2 - \dot{x}f_6x_3... \\ &\quad + \left(f_5 + \frac{I_zf_6}{ma_1} - c_2\dot{x} - c_3 \right)x_4 + \frac{a_1C_{\alpha f}}{I_z}\delta \end{aligned} \quad (28)$$

Set:

$$\left\{ \begin{array}{l} f_5 = \frac{-a_1^2C_{\alpha f} - a_2^2C_{\alpha r}}{I_z\dot{x}} - \frac{c_2a_2C_{\alpha r}l - c_2ma_1\dot{x}^2}{ma_1\dot{x}} \\ f_6 = \frac{-a_1C_{\alpha f} + a_2C_{\alpha r}}{I_z\dot{x}} + \frac{c_2C_{\alpha r}l}{ma_1\dot{x}} \end{array} \right. \quad (29)$$

According to quasi-sliding mode design with saturation function:

$$\left\{ \begin{array}{l} \dot{\sigma} = -\eta \text{sat}(\sigma) \\ \text{sat}(\sigma) = \begin{cases} 1, & \text{if } \sigma > \Delta \\ \frac{\sigma}{\Delta}, & \text{if } |\sigma| \leq \Delta \\ -1, & \text{if } \sigma < -\Delta \end{cases} \end{array} \right. \quad (30)$$

Therefore:

$$\delta = \frac{I_z}{a_1C_{\alpha f}} \left[\begin{array}{l} -\eta \text{sat}(\sigma) + \ddot{\phi}_r \left(1 + \frac{c_2I_z}{ma_1} \right) - (x_4 + \dot{\phi}_r)f_5... \\ - \left(x_2 - \dot{x}x_3 + \frac{I_z}{ma_1}x_4 \right) f_6 + c_1x_2 + c_2\dot{x}x_4 + c_3x_4 \end{array} \right] \quad (31)$$

Design the Lyapunov function as (35):

$$V = \frac{1}{2}\sigma^2 \quad (32)$$

Then:

$$\dot{V} = \sigma\dot{\sigma} = -\eta \text{sgn}(\sigma)\sigma \leq 0 \quad (33)$$

From $\sigma\dot{\sigma} \leq 0$, under controller (38), the system can reach and thereafter stay on the manifold $\sigma = 0$ in infinite time t_s (s). On the manifold we have:

$$\sigma = \mu_1 - C\mu_2 = 0 \Rightarrow \mu_1 = C\mu_2 \quad (34)$$

After t_s (s),

$$\left\{ \begin{array}{l} \dot{\mu}_2 = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1c_1G + x_2(c_2G + f_4) + x_3(c_3G - \dot{x}f_4)... \\ + \frac{I_z}{ma_1}\ddot{\phi}_r + f_3\dot{\phi}_r \\ c_1x_1 + c_2x_2 + c_3x_3 \end{bmatrix} \\ G = \dot{x} + f_3 + \frac{I_z}{ma_1}f_4 \end{array} \right. \quad (35)$$

Set:

$$\begin{aligned}\dot{\mu}_2 &= A_1\mu_1 + A_2\mu_2 + U = (A_1C + A_2)\mu_2 + U \\ &= A\mu_2 + U\end{aligned}\quad (36)$$

$$= A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + U$$

$$A_1 = [0 \ 0 \ 1]^T, C = [c_1 \ c_2 \ c_3],$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ c_1G & f_4 + c_2G & -\dot{x}f_4 + c_3G \\ 0 & 0 & 0 \end{bmatrix}, U = \begin{bmatrix} 0 \\ f_3\dot{\phi}_r + \frac{I_z}{ma_1}\ddot{\phi}_r \\ 0 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 0 & 1 & 0 \\ c_1G & f_4 + c_2G & -\dot{x}f_4 + c_3G \\ c_1 & c_2 & c_3 \end{bmatrix}$$

To guaranteed μ_2 become 0, A should be Hurwitz. The poles of A can be designed as:

$$\begin{aligned}|sI - A| &= \begin{vmatrix} s & -1 & 0 \\ -c_1G & s - f_4 - c_2G & \dot{x}f_4 - c_3G \\ -c_1 & -c_2 & s - c_3 \end{vmatrix} \\ &= s^3 - s^2(c_3 + f_4 + c_2G) + s(c_3f_4 + c_2\dot{x}f_4 - c_1G) + c_1\dot{x}f_4 \quad (37) \\ &= 0\end{aligned}$$

We have:

$$(s + p)^3 = s^3 + 3ps^2 + 3p^2s + p^3 = 0, p > 0 \quad (38)$$

With (37), (38), we have (39):

$$\begin{cases} c_1 = \frac{p^3}{\dot{x}f_4} \\ c_2 = \frac{3p^2 + c_1G + 3pf_4 + f_4^2}{f_4(\dot{x} - G)} \\ c_3 = -3p - f_4 - c_2G \end{cases} \quad (39)$$

4. GENETIC ALGORITHM – GA

The genetic algorithm is used in this study to determine the optimal parameters for the *PD-FLC*, *SMC*, and *MPC* controllers. The *ITAE*-based objective function J_{ITAE} according to the *ITAE* standard [35, 36] is selected (40) and optimized to identify the corresponding parameter values for the *PD-FLC*, *SMC*, and *MPC*. It should be noted that in this objective function, the distance error parameter e_y is calculated according to equation (7) rather than (8).

$$J_{ITAE} = \int_0^{+\infty} t \left[|X(t) - X_r(t)| \sin \phi_n - |Y(t) - Y_r(t)| \cos \phi_n \right] dt \quad (40)$$

The constraints of the algorithm include ensuring that all wheels remain in contact with the road ($F_{zi} > 0$) and that the maximum lateral acceleration a_y along the

G_y axis at any time does not exceed 8.83 m/s². Additionally, the study evaluates the impact of the control process on the overall passenger perception according to ISO 2631:1-1997 [39], using the total acceleration a_v across all three axes G_x , G_y , and G_z .

$$a_v = \left(k_x^2 a_{wx}^2 + k_y^2 a_{wy}^2 + k_z^2 a_{wz}^2 \right)^{1/2} \quad (41)$$

$$a_w = \sqrt{\frac{1}{T} \int_0^T a_w^2(t) dt} \quad (42)$$

$$a_w(t) = \sqrt{\sum_i (w_i a_i(t))^2} \quad (43)$$

$$a_i(t) = \sqrt{\frac{1}{N} \sum_{j=1}^N a_j^2} = \sqrt{\frac{a_1^2 + a_2^2 + \dots + a_N^2}{N}} \quad (44)$$

A step-by-step breakdown on controller optimization [40]:

Step 1: Define the system's technical parameters to be searched and the fitness function to be optimized

- The technical parameters of *PD-FLC*, *SMC* and *MPC* to be determined are in Table 3.
- The objective function is J_{ITAE} calculated as (40).

Step 2: Choose the Genetic Algorithm's technical parameters and create initial values of searched parameters

- The population's size (number of individuals), the generation N as well as the indexes of crossover, mutation are chosen as in Table 3.
- Randomly initialize the searched parameters' values of each individual.

Step 3: Calculate the fitness function value J_{ITAE} for each individual with the chosen system's technical parameters

Step 4: Select the individuals as parents with rank-based selection method

Step 5: Create mutation by applying small random changes to offspring

Step 6: Repeat this process N times and get the results

In this study, the total number of variables to be determined is large, ranging from 7 (*PD-FLC*, *SMC*) to 12 variables (*MPC*), so the population size and the number of generations for crossover also need to be correspondingly large. The selected parameters are summarized in Table 2.

Table 2. Parameters' search region

Value	<i>GA-SMC</i>	<i>GA-MPC</i>
Variables' name	$p, \eta, \Delta, G_1, G_2, G_3, G_4$	$w_1, w_2, w_3, \dots, w_{12}$
Generation	100	200
Population size	10	20
Crossover	0.8	0.8
Mutation	0.1	0.1
Fitness function	J_{ITAE}	J_{ITAE}
<i>GA-SMC</i> 's search region		
Value	Lower bound	Upper bound
p	0.01	0.1
η	500	1000
Δ	0.1	1
G_1	0.1	5

G_2	0.01	0.1
G_3	0.1	1
G_4	1	300
<i>GA-MPC's search region</i>		
Value	Lower bound	Upper bound
$w_{1...12}$	0	1

5. RESULTS AND DISCUSSIONS

The results of the controller parameter optimization process are summarized in Table 3.

Table 3. Optimal parameters summary

GA's search results			
Value	<i>GA-SMC</i>	Value	<i>GA-MPC</i>
P	0.0445	w_1	0.0015
H	973.4083	w_2	0.3626
Δ	0.6124	w_3	0.0152
G_1	1.3187	w_4	0.6179
G_2	0.1000	w_5	0.2788
G_3	0.5225	w_6	0.2453
G_4	161.5577	w_7	0.1527
--	--	w_8	0.8068
--	--	w_9	0.0012
--	--	w_{10}	0.0207
--	--	w_{11}	0.7908
--	--	w_{12}	0.7904

5.1 Case 1

The study evaluates the performance of the controllers in Case 1 according to the criteria of the root mean square (RMS) of the distance error e_y , heading error e_ϕ , lateral acceleration a_y , and vehicle body roll angle θ . These criteria follow (45), (46), (47), (48), and (49).

$$RMS \quad \delta = \sqrt{\frac{1}{T} \int_0^T \delta^2(t) dt} \quad (45)$$

$$RMS \quad e_y = \sqrt{\frac{1}{T} \int_0^T e_y^2(t) dt} \quad (46)$$

$$RMS \quad e_\phi = \sqrt{\frac{1}{T} \int_0^T e_\phi^2(t) dt} \quad (47)$$

$$RMS \quad a_y = \sqrt{\frac{1}{T} \int_0^T a_y^2(t) dt} \quad (48)$$

$$RMS \quad \theta = \sqrt{\frac{1}{T} \int_0^T \theta^2(t) dt} \quad (49)$$

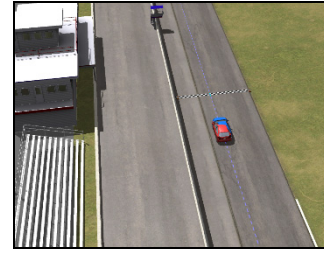
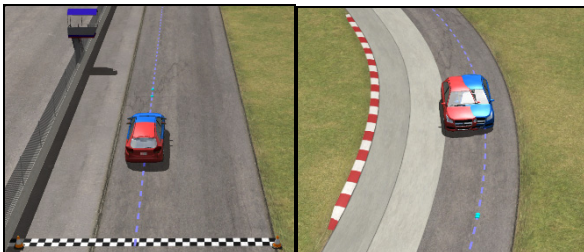


Figure 7. Simulation's results in Carsim

The simulation results in Carsim, shown in Figure 7, illustrate the vehicle states with the *MPC* controller (blue vehicle) and *SMC* controller (red vehicle) at the beginning, middle, and end of the trajectory. The results over the entire trajectory are presented in Figure 8, showing that the vehicle with the *MPC* controller follows the reference trajectory more closely than the vehicle with the *SMC* controller.

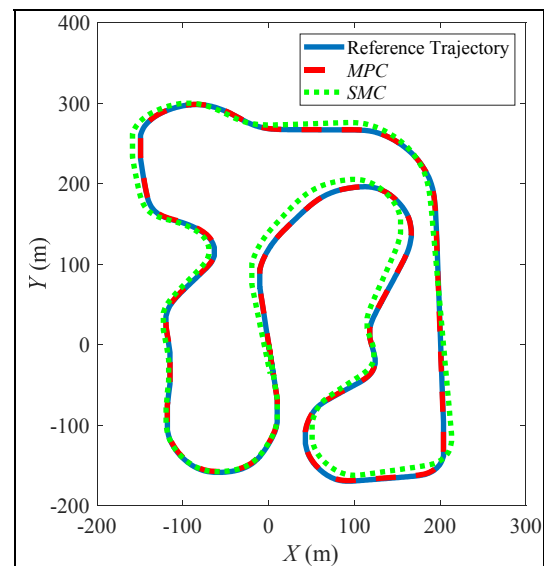
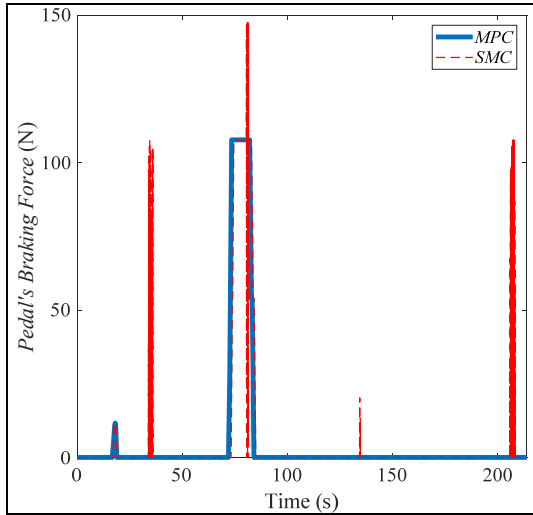


Figure 8. Trajectories with controllers

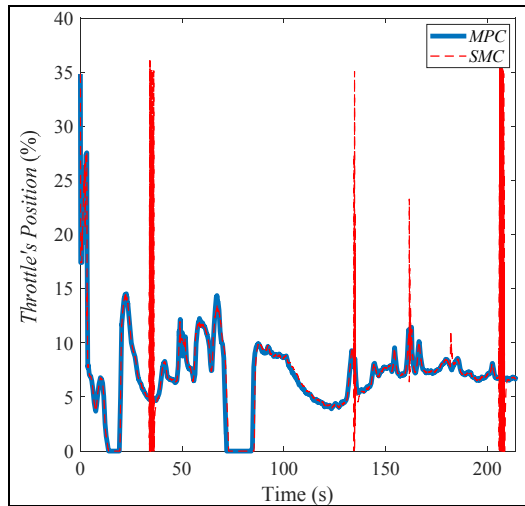
Corresponding to the trajectory motion in Figure 8, the braking force (pedal force) and traction force (throttle position) controlled by the *PD-FLC* are shown in Figure 9. Although the *PD-FLC* controller parameters used in both the *MPC* and *SMC* cases are identical, the responses differ. The brake pedal force in Figure 9a and throttle position in Figure 9b for the vehicle with the *MPC* controller exhibit more stable responses compared to the *SMC* controller. The timing of the maximum pedal force generated by the *PD-FLC* is the same for both *SMC* and *MPC* vehicles in the operational regions at 10 s and 60–80 s. However, there are moments when the *SMC* vehicle generates instantaneous braking forces to respond to the instantaneous speed errors of the system, such as at 30 s, 135 s, and 208 s.

Furthermore, at the same braking instant, the *SMC* vehicle exhibits a larger brake force fluctuation (148 N), which is over 36% higher than the *MPC* vehicle (110 N) at 80 s. The throttle response in Figure 9b shows that the 0% points correspond to the braking instances in Figure 9a. Over the entire operating time, the throttle responses of the *SMC* and *MPC* vehicles are nearly identical, but the *SMC* vehicle shows several larger throttle fluctuations at 30 s, 135 s, and 208 s. At these moments, the

braking and throttle actions occur alternately; however, the fluctuation values remain low compared to the system's practical capacity, so the overall response ensures operation until the end of the trajectory. At moments of large fluctuations, the throttle response of the *SMC* vehicle is nearly six times higher than that of the *MPC* vehicle.



a. Pedal's braking force



b. Throttle's position

Figure 9. PD-FLC' results

Corresponding to the trajectory in Figure 8, the steering angle δ , lateral acceleration a_y , and vehicle body roll angle θ over the entire operating time for the vehicles are summarized in Figures 10, 11, and 12. Overall, throughout the operating period, the instantaneous values of these parameters remain within the vehicle's allowable limits. Similar to the results in Figure 9, Figures 10, 11, and 12 show that the vehicle with the *SMC* controller exhibits more instances of fluctuations with higher amplitudes compared to the vehicle with the *MPC* controller.

Apart from the fluctuation periods, the responses of the vehicles are nearly identical. The vehicle with the *MPC* controller has a maximum steering angle below 200° , which is approximately 40% lower than that of the *SMC* vehicle (over 500°). In addition, the *MPC* vehicle's maximum lateral acceleration is below $0.7g$,

whereas the *SMC* vehicle exhibits lateral acceleration fluctuations with a peak amplitude above $0.85g$. The vehicle body roll angle θ remains small ($<10^\circ$), but the *MPC* vehicle shows a maximum amplitude approximately 44% higher than the *SMC* vehicle.

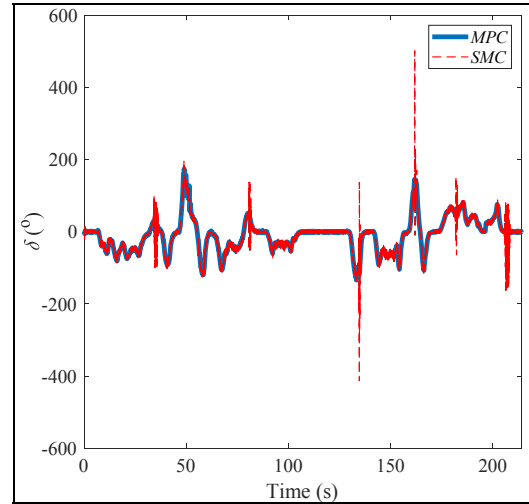


Figure 10. Handwheel's angle

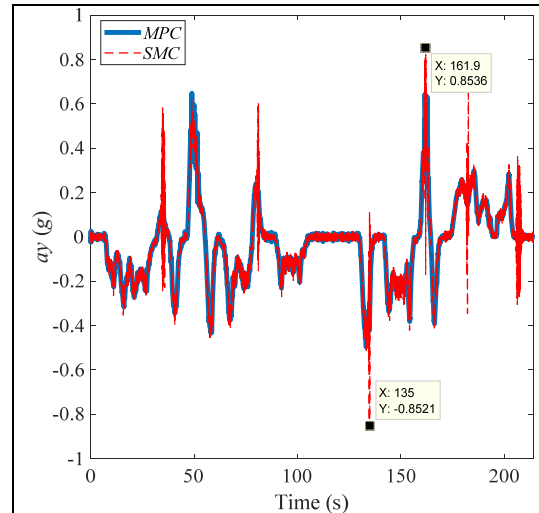


Figure 11. Lateral acceleration

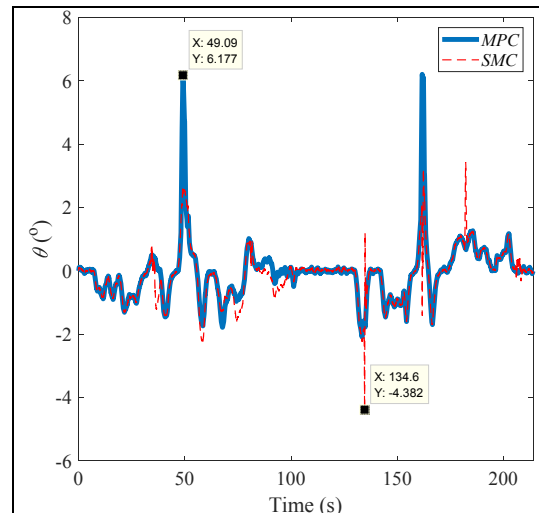


Figure 12. Vehicle body's roll angle

The evaluation indices of the *SMC* controller according to (43), (44), (45), and (46) are summarized

and compared with the *MPC* controller in the graph of Figure 13. The results show that, on average over the entire operating trajectory, the *SMC* controller consistently exhibits higher values for δ , a_y , e_y , and e_ϕ compared to the *MPC* controller. However, the differences are small - below 5% for *RMS* δ and below 1% for *RMS* a_y . The remaining indices, such as *RMS* e_y , show a high difference above 15%, and *RMS* e_ϕ nearly 10%. Only the vehicle body roll angle *RMS* θ of the *SMC* controller is lower than that of the *MPC* by 15%.

It can be observed that the vehicle with the *MPC* controller has significantly lower average errors in e_y and e_ϕ compared to the *SMC* vehicle, but exhibits considerably higher body roll responses, while the other indices show minimal differences.

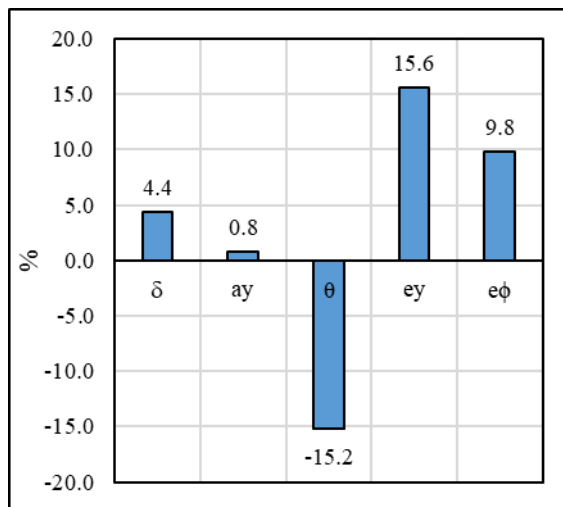


Figure 13. Deviation rate of SMC compared to MPC

5.2 Case 2

The *SMC* and *MPC* controllers, after design, are evaluated in the double-lane change scenario with simulation results in Carsim shown in Figure 14. The study assesses the performance of the controllers in Case 2 according to the criteria of maximum steering angle $\text{Max } |\delta|$, maximum distance error $\text{Max } |e_y|$, maximum heading error $\text{Max } |e_\phi|$, maximum lateral acceleration $\text{Max } |a_y|$, and maximum vehicle body roll angle $\text{Max } |\theta|$.

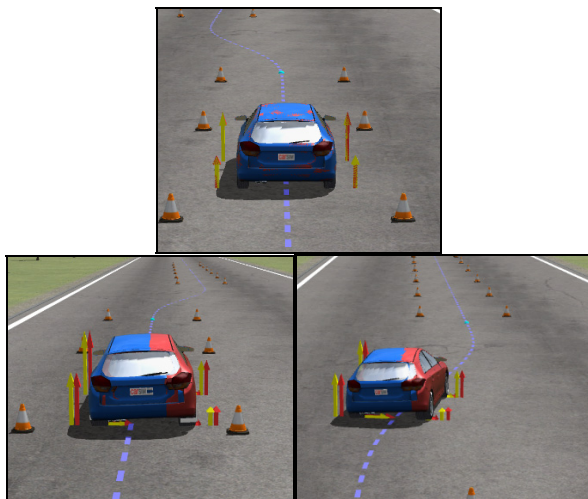


Figure 14. Simulation's results in Carsim

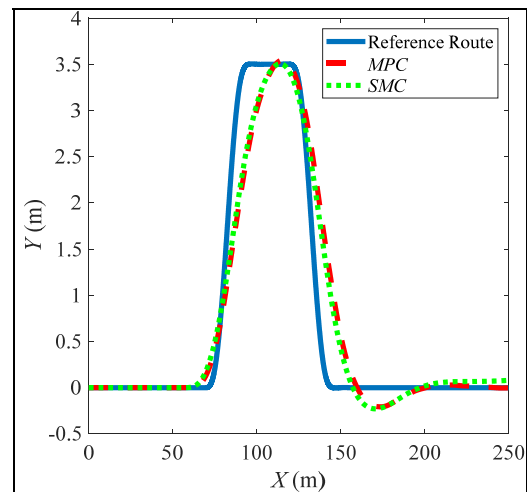


Figure 15. Trajectories with controllers

The results in Figure 15 show that the trajectories of the vehicles with *SMC* and *MPC* controllers are nearly identical and both satisfy the requirements for the “Double-Lane Change” trajectory. In the area where the trajectory merges back into the original lane, the maximum lateral deviation is below 0.25 m from the predefined *Y*-direction trajectory. Although the trajectories are similar, the steering angle responses generated by the controllers differ significantly, as shown in Figure 16. Accordingly, the *SMC* controller provides a more stable steering angle response compared to the *MPC* controller. The *MPC* controller produces δ values consistently higher than those of the *SMC* across the entire trajectory, but these δ values remain within the normal operational limits of the vehicle steering system.

Similarly, the vehicle with the *SMC* controller exhibits more stable lateral acceleration a_y and vehicle body roll angle θ , with lower peak values compared to the *MPC* controller. In this case study, the body roll angle θ is small (below 4°), while the lateral acceleration a_y is relatively high (approximately $0.81g$). Overall, the evaluation indices for both controllers in this scenario remain below the allowable limits, ensuring normal operational performance.

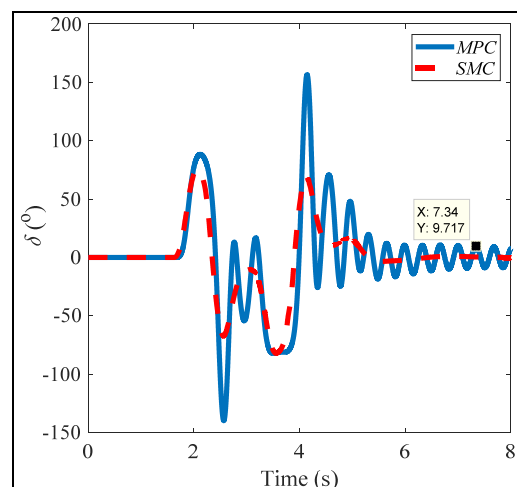


Figure 16. Handwheel's angle

According to the established evaluation criteria, the deviations of the *MPC* controller compared to the *SMC* controller are summarized in Figure 19.

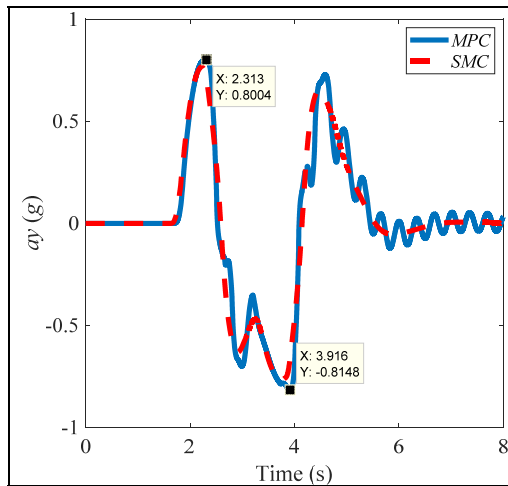


Figure 17. Lateral acceleration

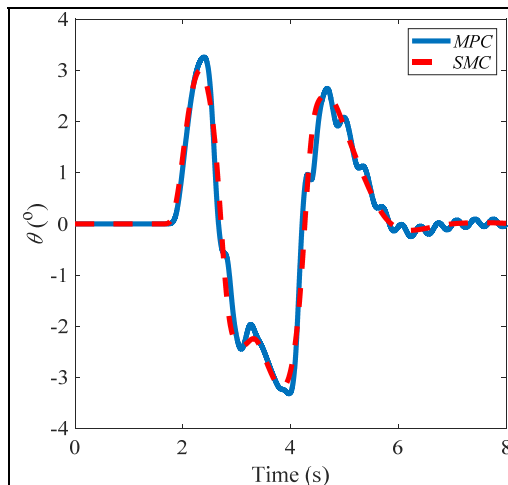


Figure 18. Vehicle body's roll angle

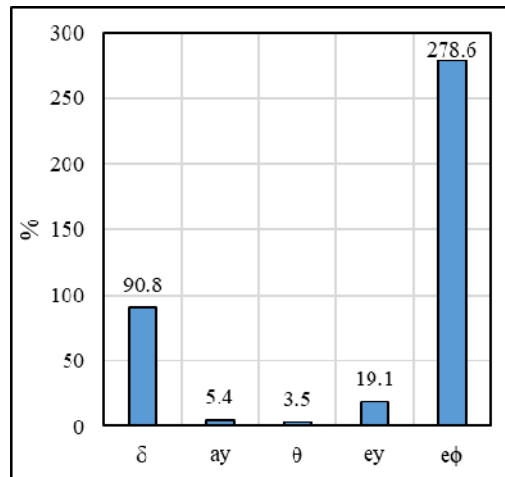


Figure 19. Deviation rate of MPC compared to SMC

The *SMC* controller performs better across all criteria. The maximum steering angle δ response of the *SMC* is 90.8% better, indicating that the *SMC* requires a smaller steering range to achieve a satisfactory trajectory. The heading error e_ϕ response of the *SMC*, although 278% better, is not significant since this deviation is calculated from a small error value (below 1°). For a_y and θ , the *SMC* performs better than the *MPC*, but the improvement is minor (below 5.5%). The *SMC*

produces a significantly better e_y response compared to the *MPC*, with an improvement of nearly 20%.

6. CONCLUSIONS

This study applied modern controllers to manage the longitudinal and lateral motions of the vehicle, ensuring adherence to various trajectories such as “*Handling Course Left Edge – 40 km/h*” and “*Double-Lane Change – 120 km/h*.” A linear 3DOF dynamic model was analyzed and applied to design the steering angle controllers. The designed controllers were then tested using Carsim software. The evaluation criteria were diverse, comprehensively reflecting different aspects of vehicle operation, including lateral stability, vehicle body roll motion, and passenger comfort. Accordingly, the main novelties and contributions of this study include:

- * A complete methodology for designing the *PD-FLC* controller was established, applied to simultaneously control the brake pedal force and throttle position to regulate the vehicle's longitudinal speed.

- * A complete methodology for designing the *MPC* controller was established, involving the pre-calculation of the steering angle based on the minimization of the objective function J . This approach simplifies the computation of the required control input.

- * A complete methodology for designing the *SMC* controller using the *Decoupled Algorithm* was established for a system in which the control input δ simultaneously appears in the dynamic equations describing the translational motion along G_y and the rotational motion around G_z . Additionally, the Hurwitz condition was applied to determine the relationship among the sliding surface coefficients, ensuring rapid computation and high stability.

- * A complete methodology was established for the simultaneous optimization of multiple complex parameters of the *PD-FLC-SMC* controllers (7 parameters) and *PD-FLC-MPC* controllers (12 parameters) using *GA*, with constraints on stability and passenger comfort according to ISO 2631:1-1997.

Simulation results show that the *MPC* controller enables the vehicle to follow the trajectory more accurately and achieve better response indices (except θ), particularly with *RMS* e_y improving by 15% compared to the *SMC* controller in “Case 1.” Evaluation results for Case 2 indicate that the *SMC* controller outperforms the *MPC* in all indices, notably achieving nearly 20% better trajectory tracking with a maximum steering angle value less than 90° .

The research results can be further developed by integrating simultaneous optimization with vehicle stability systems such as *ABS*, *ESC*, and *TCS* for autonomous vehicles under various road operating conditions.

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NOMENCLATURE

l	Wheel base	2.91	m
a_1	Distance of A and G	1.015	m

a_2	Distance of B and G	1.895	m
m	Vehicle mass	1413	kg
C_{af}	Front cornering stiffness	128662	N/rad
C_{ar}	Rear cornering stiffness	69890	N/rad
I_z	Yaw moment of inertia	1536.7	kg.m ²
g	Gravitational acceleration	9.81	m/s ²
v_x	Longitudinal velocity of reference model	11.11	m/s
v	Vehicle velocity	--	m/s
$v_{\phi} v_r$	Front and rear velocity at the contact patch	--	m/s
β	Vehicle's sideslip angle	--	°
δ	Front wheel's steering angle	--	°
$\square$	Vehicle's yaw angle	--	°
$F_{xf} F_{xr}$	Front and rear longitudinal force	--	N
$F_{yf} F_{yr}$	Front and rear lateral force	--	N
μ	Friction coefficient	0.9	--

ДИЗАЈН НАВИГАЦИОНИХ СИСТЕМА ЗАСНОВАН НА ПРЕДИКТИВНОМ И КЛИЗНОМ РЕЖИМУ УПРАВЉАЊА МОДЕЛОМ ЗА ПРАЋЕЊЕ ПУТАЊЕ АУТОМОБИЛА БЕЗ ВОЗАЧА

Н.Д. Фам

Ова студија пројектује модерне контролере, SMC и MPC, за управљање кретањем возила како би се осигурало прецизно праћење путање жељеном брзином. Уздужно кретање се регулише путем гаса и кочнице помоћу PD-FLC контролера, док се бочно кретање контролише углом управљања помоћу SMC и MPC. Новина SMC-а лежи у коришћењу раздвојеног алгоритма, који се бави моделом где улаз управљања истовремено утиче на транслациону динамику и ротациону динамику, што није широко примењено у аутономним возилима. Новина MPC-а лежи у коришћењу унапред израчунате подесиве матрице коефицијената засноване на принципу удаљеног хоризонта, што поједностављује дизајн у поређењу са континуираном оптимизацијом током рада. Симулације у Matlab Simulink/Carsim под условима „Руковање левом ивицом курса“ и „Промена двоструке траке“ показале су да је у случају 1, MPC постигао боље метрике перформанси, са 15% мањом просечном грешком у растојању од SMC, док је у случају 2, SMC надмашио MPC по свим метрикама, посебно у придржавању путање, са побољшањем од 20%.