Statistical Modelling of Rolling Bearing Cage Fault Vibrations through Conditional Heteroscedastic Time-Series Analysis

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Rolling element bearings play an important role in rotating machinery and are the source of non-stationary and heteroscedastic vibration responses caused by localized defects which are impulsive. This study proposes and tests a conditional-volatility modelling approach for characterizing bearing cage-faults based on autoregressive (AR), generalized autoregressive conditional heteroskedasticity (GARCH) and exponential generalized autoregressive conditional heteroskedasticity (EGARCH) models. The tri-axial vibration measurements were taken on a bearing test rig provided by Spectra Quest and were analysed under healthy and cage-fault operating conditions. The modelling procedure starts by modelling the deterministic mean response with the axial and tangential vibration components and testing if there is any time-varying conditional variance in the remaining stochastic structure. 40,000 sync–hronized observations were assessed using a comparative model assessment based on parameter significance, likelihood-based crite–ria,regression error and volatility diagnostics. The AR baseline fit the best, with an R^2 of 0.9220, but it does not capture the conditional vari–ance dynamics. The GARCH (1,1) model seemed to have captured the shock sensitivity and high volatility persistence with the highest value of $\alpha + \beta = 0.9564$ as well as the best likelihood-based statistics among the volatility models: AIC = -0.4994, BIC = -0.4981. In addition, the EGARCH (1,1) model exhibited substantial magnitude and persistence effects; although the sign of the persistence effect was opposite for positive and negative shocks, it was not statistically significant (γ = -0.0025, p = 0.7766) for the dataset of tested cages. The results show that the GARCH family conditional variance modelling is a useful model to describe the bearing vibration when there is a fault, but currently, no evidence suggests that EGARCH is the best model for the present data over GARCH.

Keywords: *Bearing Fault Diagnosis; AR; GARCH; EGARCH; Conditional Variance Modeling; Volatility Clustering; Predictive Maintenance; Cond–ition Monitoring; Vibration Analysis; Rolling Element Bearings*

1. INTRODUCTION

The rolling element bearings are one of the most impor–tant bearings used in rotating machinery and are widely applied in industrial machinery including turbines, pu–mps, compressors, gearboxes, electric motors, marine propulsion systems, etc. The operational reliability of these systems is dependent on the health status of the bearings because faults in bearings are one of the most frequent causes of unplanned downtime, making it essential for condition-based maintenance and predic–tive maintenance systems to have both sensitive indi–cators and the ability to differentiate between normal operating vibration and stochastic disturbances caused by defects in bearings.

Conventional vibration-based diagnosis often relies on deterministic or summary features, such as spectral peaks, root mean square (RMS), kurtosis, crest factor, skewness, wavelet energy and envelope demodulation. The characteristics of vibration signals generated by bearings become increasingly complex as the defects develop, such as the introduction of nonlinearity, non-stationarity, impulsiveness, and even time-varying variance. Although traditional vibration analysis met–hods are commonly based on the deterministic charac–teristics of the signals, this approach might not be sufficient for capturing the stochastic nature of vibration fluctuations related to defects in the various stages. Thus, there is a need for advanced statistical models that describe the evolution of variance as well as the mean behavior for reliable fault diagnosis and prognosis. Tra–ditional statistical methods and degradation trend ana–

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lysis were the main methods that were used in early studies on bearing prognostics. Many smoothing met–hods, such as Exponential Moving Average (EMA) and regression, were employed to estimate the trend of degradation in vibration signals. In these methods, failure progression is tracked by health indicators, and once the health indicators exceed the failure thresholds, the Remaining Useful Life (RUL) is predicted by Support Vector Regression (SVR) and Kernel Regre–ssion (KR)[1]. These methods showed good prediction ability in controlled operating conditions but were not always able to model the nonlinear degradation beha–viors. Later studies added recursive regression models where parameters are updated as the data moves through the system. Such adaptive models allowed for conti–nuous monitoring of the health conditions of bearings and to increase the accuracy of estimating RUL by online monitoring[2]. Moreover, statistical degradation modeling was developed to recognize different stages of degradation of the bearings, to distinguish between a healthy, incipient fault, and severe defect conditions [3]. These techniques proved to be very useful in mainte–nance planning as they gave a structured representation of the progression of degradation[4]. As high frequency vibration data became available, then signal processing became a part of bearing health assessment[5]. Wavelet-based methods were used to solve the non-stationarity problem of vibrators. Specifically, the Discrete Wavelet Packet Transform (DWPT) was employed to develop new health indices that were able to capture transient fault signatures and localized frequencies information [6]. These indicators showed better correlation to bear–ing degradation than traditional statistical indicators [7].

However, many of the traditional methods were mainly based on amplitude and trend features of the vibration signals without paying attention to the dyna–mic variance behavior of the signals [8]. In recent years, volatility clustering of bearing vibration signals has been increasingly identified. From this observation a generalized autoregressive conditional heteroskedas–ticity (GARCH) model was suggested as a way to diag–nose bearing faults[9]. The GARCH-based approach offers a way to model the time varying conditional variance and has been shown to have improved ability to model stochastic fluctuations related to bearing defects[10]. GARCH derived health indicators can well capture the cyclo-stationary characteristics of vibration signals which are often overlooked from the conven–tional autoregressive approach [11], AR-GARCH and ARX-GARCH models allowed the modelling of the deterministic vibration dynamics and stochastic vola–tility behavior simultaneously. These methods resulted to be more effective at fault detectionthan the traditional autoregressive model [12]. Furthermore, the addition of GARCH components into machine learning models such as multi-class Support Vector Machines and hybrid diagnostic system showed enhanced classification accuracy and estimation of fault severity. The results of the experimental tests based on actual bearings data also showed that the GARCH models are superior in fault prediction and RUL estimation than many of the con–ventional statistical approaches [13,14]. ARX-GARCH models have proved successful, but there are some drawbacks. The first is the assumption of symmetry in the GARCH models, that is, they assume the same response in variance to both positive and negative shocks. In reality, these defects in bearings can induce asymmetric vibration response as a result of impacts, discontinuities in the surfaces, crack propagation and load changes [15]. Conventional GARCH models are not suitable for such asymmetric behavior. Second, be–cause the ARX–GARCH model is a linear model, its general form does not allow it to model the nonlinear degradation mechanisms that are found in actual bearing systems [16]. Third, there are several sources of external noise, such as impulsive outliers, measurement noise and operational uncertainty, which can have a signi–ficant impact on the performance of the model and on the estimation of the parameters [17]. In addition, most of the studies on ARX–GARCH model are concerned mainly with fault classification and/or RUL prediction with only limited consideration given to the modelling of asymmetric volatility patterns that can appear at different stages in the development of the defect [18]. These drawbacks have been circumvented by the Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) model [19]. EGARCH models the log of the conditional variance and includes the asymmetric effects by the leverage term unlike conventional GARCH models [20]. The advantage of this structure is that the model can be used to explain both positive and negative shocks on volatility, and the variance of the volatility does not need to be constrained to be positive, but the volatility model parameters are not required to be non-negative [21]. Therefore, bearing vibration signals with non-linear dynamics, impulsive excitations, asymmetric variance responses and volati–lity clustering are suitable for analysis using EGARCH [22]. The logarithmic variance formulation also helps to make EGARCH numerically stable and robust to subtle variations in the vibrations, enabling it to be a valuable tool for fault identification [23].

Autoregressive models provide a simple represen–tation of temporal dependence in vibration response, but it is assumed that the innovation variance is constant. GARCH-family models ease this assumption by allo–wing the conditional variance while resulting as a func–tion of past shocks and past variance [24]. This makes them attractive for bearing vibration data, where loca–lized cage, race or rolling-element defects can generate clustered energy bursts [25]. EGARCH extends the GARCH framework by modelling the logarithm of con–ditional variance and including an asymmetry term, which can potentially distinguish different volatility responses to positive and negative standardized shocks [26]. Despite the potential of GARCH-family models, a common weakness in bearing fault studies is that model claims are sometimes based on a single fitted volatility model without rigorous comparison against simpler alternatives [27]. EGARCH should only be preferred over standard GARCH when the asymmetry mechanism is statistically supported or when likelihood-based mo–del selection favorsfurther flexibility.

The objective of this study is to investigate whether conditional heteroscedastic time-series models can cha–racterize the time-varying variance and volatility clus–

tering present in rolling-bearing cage-fault vibration signals. In this study AR/least-squares baseline with GARCH(1,1) and EGARCH(1,1) models using 40,000 synchronized radial, axial and tangential vibration ob–servations obtained from an MB-ER16K bearing have been compared. A common conditional-mean equation is employed in which the radial response is modelled as the dependent variable and the axial and tangential res–ponses are included as explanatory variables, thereby allowing the comparison to focus on differences in variance modelling. Model performance is assessed using coefficient significance, likelihood-based infor–mation criteria and residual, conditional-volatility, news-impact and gradient diagnostics. The scope is limited to the statistical characterization of the inves–tigated bearing containing a single 0.4 mm-long notch on its machined metallic cage ring under the available test condition. The study does not develop a fault-clas–sification or remaining-useful-life prediction model, and its fitted parameters are not assumed to be directly transferable to other defect geometries, bearings or ope–rating condition

2. EXPERIMENTAL SETUP AND DATA DESCRIPTION

The experimental vibration data were obtained from a SpectraQuest bearing prognostic simulator[28]. The bearing tested was an MB-ER16K rolling element bea–ring. Vibration signals were collected in three ortho–gonal directions (radial, axial and tangential) from acce–lerometers installed on the bearing housing, which had a cage defect of 0.4 mm as specified by the manufacturer. The MB-ER16K bearing was equipped with a machined metallic cage. The cage defect investigated was a localized notch formed on the cage ring. The manu–facturer-specified defect dimension of 0.4 mm repre–sents the length of the notch, rather than its width or depth. The coupled dynamic res–ponse of the bearing housing under healthy and cage-fault condition is described by these three directional components.

A total of 250,000 samples per operating condition were acquired in raw form. In the comparative AR-GARCH-EGARCH modelling in this manuscript, we have used the same 40,000 synchronized observations in each of the models, and the fit statistics and likelihood-based criteria are thus directly comparable. The variable response radial direction was taken as the dependent vibration response and the variable response in the axial and tangential directions were set as explanatory vib–ration responses in the conditional mean equation. This modelling choice is based on the physical assumption that the measured response is not due to a single isolated sensor direction, but due to a coupled directional vibration because of localized cage instability.

The bearing characteristic ratio shown in Table 1 are assumed to be shaft-order multipliers. These ratios have to be multiplied by the shaft frequency to get the chara–cteristic frequencies in Hertz for the final deployment under different rotation speeds.

The time domain vibration responses of the normal and defective bearing states are shown in Figure 1. The healthy bearing shows a relatively localized and stable response, and the bearing with cage-fault case shows definite amplitude modulation as well as high-energy spikes at irregular intervals. The bursts are the physical basis of volatility-based modelling since the vibration response variance is not constant. The defective con–dition corresponds to high-volatility sections being fol–lowed by more high-volatility sections, which iscon–sistent with clustered defect interactions and repeated impact excitation.

Table 1. Bearing Specification

S. No	Bearing Specification	MB-ER 16K
1	Outer diameter, D (mm)	125
2	Inner diameter, d(mm)	70
3	Pitch diameter, dp (mm)	38.5
4	Ball diameter, db (mm)	7.9375
6	Number of Balls, Nb	9
7	Contact Angle (Degree)	00
9	FTF (rpm)	0.402
10	BPFO (rpm)	3.572
11	BPFI (rpm)	5.43
12	BSF (rpm)	2.322

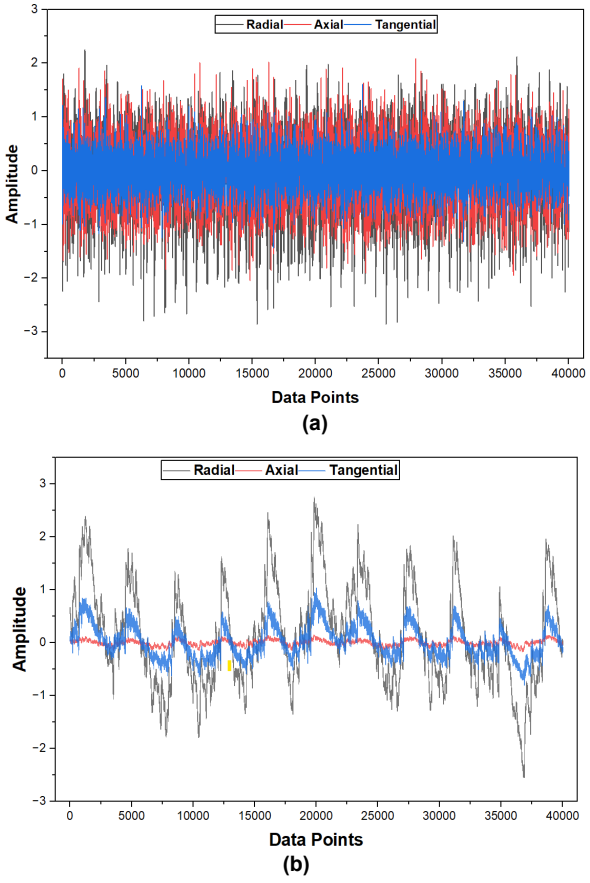


Figure 1. Acquired Vibration Signal (a) Normal Bearing (b) Bearing with cage defect

3. METHODOLOGY

3.1 Conditional Mean Modelling

To enable consistent comparison among the AR, GARCH, and EGARCH modelling frameworks, a common conditional mean specification was adopted. The radial vibration component was selected as the dependent variable, while the axial and tangential

vibration components were independent variables. The conditional mean is expressed as:

$$y_t = c + b_1 x_{1,t} + b_2 x_{2,t} + \varepsilon_t \quad (1)$$

where:
y_t denotes the radial vibration signal at time t;
$x_{1,t}$ and $x_{2,t}$ represent the axial and tangential vibration components, respectively;
c is the intercept term;
b_1 and b_2 are regression coefficients describing the directional coupling effects among the vibration components;
ε_t is the innovation or residual term.

The baseline AR/least-squares model estimates the deterministic relationship between the vibration components under the assumption of homoscedastic residuals, i.e., constant error variance over time. In contrast, the GARCH and EGARCH frameworks retain the same conditional mean structure while explicitly modelling the variance as a time-varying stochastic component. This formulation enables the models to capture volatility clustering and heteroskedastic behavior frequently observed in bearing vibration signals, particularly under defective operating conditions. By maintaining a common mean equation across all model structures, differences in model performance can be attributed primarily to their respective variance modelling capabilities rather than to changes in the deterministic component of the model.

3.2 GARCH (1,1) Conditional Variance Model

A time-varying conditional variance model, called the Generalized Autoregressive Conditional Heteroskedasticity (GARCH (1,1)) model is used to describe the variance of the vibration signal innovations. A GARCH model can explain volatility clustering, in contrast to constant variance models, in which spans of high volatility are followed by spans of low volatility and vice versa. The conditional variance process is expressed as:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \quad (2)$$

where:
h_t is the conditional variance at time t;
$\omega > 0$ represents the constant or long-run variance component;
$\alpha \varepsilon_{t-1}^2$ is the squared innovation (shock) from the previous time step;
$\alpha \geq 0$ is the ARCH coefficient, measuring impact of recent shocks;
$\beta \geq 0$ is the GARCH coefficient, representing the persistence of past volatility.

The term α measures the sensitivity of the conditional variance, while β measure the extent to which past volatility influences future volatility. The sum (α+β) is an indicator of volatility persistence. As the sum gets closer to 1, the shocks have long tails, which suggests that the variance process has volatility clustering and long-memory. On the other hand, the lower value indicates that the volatility returns more quickly to its long-run average level.For the GARCH (1,1) process to remain covariance-stationary, the condition

$$\alpha + \beta < 1 \quad (3)$$

In the context of bearing vibration analysis, a large persistence parameter implies that the impulses generated by the bearing fault persist for longer periods of time, which makes the GARCH model appropriate for modelling a stochastic process involving the evolution of a bearing fault.

3.3 EGARCH (1,1) Conditional Variance Model

The Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH (1,1)) model is an extension of the GARCH model which models the log of the conditional variance and explicitly includes possible asymmetric effects of shocks on volatility. The conditional variance equation is given by:

$$\ln(h_t) = \omega + \alpha \left| \frac{\varepsilon_{t-1}}{\sqrt{h_t - 1}} \right| + \gamma \left(\frac{\varepsilon_{t-1}}{\sqrt{h_t - 1}} \right) + \beta \ln(h_{t-1}) \quad (4)$$

where:
h_t denotes the conditional variance atparticular time t;
ω is the constant term;
α measures the effect of the magnitude of standardized shocks on volatility;
γ is the asymmetry parameter, capturing the differential impact of positive and negative shocks;
β represents the persistence of volatility through the lagged log-variance term.

The log specification of the EGARCH model ensures that the conditional variance is positive, and it avoids the need to impose the usual non-negativity constraints on the variance parameters. The coefficient α measures how responsive volatility is to the size of past shocks, while β measures the persistence of volatility over time. The asymmetry parameter γ indicates whether positive and negative values have different effects on future volatility. If γ is statistically significant, asymmetric volatility dynamics are supported; if γ is not statistically significant, volatility is mainly driven by shock magnitude and persistence rather than shock direction. Based on this structure, the EGARCH model is a flexible model for explaining volatility clustering and potential nonlinear asymmetric effects in bearing vibration signals related to fault development.

3.4 Model Comparison Criteria

For comparison of models the Coefficient significance, coefficient of determination, standard error of regression, log likelihood, Akaike information criterion (AIC), Schwarz criterion/Bayesian information criterion (BIC/SC), Hannan-Quinn criterion and Durbin-Watson statistic were utilized. Lower AIC, BIC and Hannan-Quinn values suggest a more efficient model for penalizing number of parameters. Since GARCH and EGARCH are likelihood-based volatility models, information criteria and residual diagnostics are more suitable than just using R^2. Since it is reported as sample error of the

model, the standard error of regression was adopted as the RMSE-equivalent error measure.

4. RESULTS

4.1 Conditionalmean parameter comparison

The mean-equation parameter estimates between the different models (AR/least-squares, GARCH and EGARCH) which have been compared in Table 2. The axial and tangential vibration components are statistically significant in all the three models which mean that the radial response has strong deterministic coupling with the other vibration directions measured. The meaning of these coefficients justifies the application of a directional mean equation prior to modelling the rest of the stochastic variance structure.

Table 2. Mean-equation parameter comparison for the defective bearing dataset.

Parameter	AR / LS	GARCH(1,1)	EGARCH(1,1)	Interpretation
Axial coefficient	2.8544	3.4551	3.3420	Directional vibration coupling
Axial standard error	0.0349	0.0199	0.0160	Lower uncertainty in volatility models
Axial t/z-statistic	81.6778	173.7711	209.0339	Highly significant in all models
Tangential coefficient	2.6067	2.3776	2.3855	Directional vibration coupling
Tangential standard error	0.0064	0.0038	0.0030	Low coefficient uncertainty
Tangential t/z-statistic	409.2853	632.1509	808.2023	Highly significant in all models
Constant	0.0526	0.0621	0.0661	Baseline response level
Constant standard error	0.0013	0.0008	0.0006	Small uncertainty

4.1 Variance equation parameters

Table 3 reports the estimates of variance-equations for GARCH and EGARCH. The GARCH magnitude coefficient, 0.6232, is statistically significant and the persistence coefficient while 0.3332, is also statistically significant. Together, they add up to 0.9564, which shows effective persistence in conditional variance. This indicates that the irregular bearing signal has more clusters of volatility than it has independent constant variance fluctuations.

The shock magnitude and shock persistence effects are also statistically significant in the EGARCH model. The magnitude coefficient is 1.0006, and the persistence coefficient is 0.8460. But the EGARCH asymmetry coefficient is -0.0025 and p = 0.7766. The main interpretation is that the existing data does not support the hypothesis that positive and negative peaks have different impacts on subsequent volatility. Hence, EGARCH models are informative from the perspective of being a nonlinear model of log-variance, but not for this case of cage faults because the asymmetric component is not substantiated as the dominant physical cause of this defect.

Table 3. GARCH and EGARCH variance-equation parameter comparison

Variance parameter	GARCH estimate	p-value	EGARCH estimate	p-value	Technical meaning
Ω / Constant	0.0031	<0.001	-1.4050	<0.001	Baseline conditional variance level
α / Shock magnitude	0.6232	<0.001	1.0006	<0.001	Immediate effect of defect-induced shocks
β / Persistence	0.3332	<0.001	0.8460	<0.001	Persistence of conditional volatility
γ / Asymmetry	Not included	-	-0.0025	0.7766	No significant asymmetric shock effect
$\alpha + \beta$	0.9564	-	Not directly comparable	-	High GARCH volatility persistence

4.2 Overall model fit comparison

Table 4. presents comparative fit statistics. The AR baseline has the lowest standard error of estimation and the highest in sample R^2. However, AR only models the conditional mean and fails to consider conditional heteroskedasticity. The results for volatility modelling from the likelihood-based criteria are different, i.e GARCH (1,1) has the highest log likelihood, AIC, BIC/SC and Hannan-Quinn criterion among the volatility models.

Table 4. Comparative model fit statistics for AR, GARCH and EGARCH models.

Fit statistic	AR / LS	GARCH (1,1)	EGARCH (1,1)	Best value
R-squared	0.9220	0.9191	0.9190	AR / LS
Adjusted R-squared	0.9220	0.9191	0.9190	AR / LS
S.E. of regression / RMSE-equivalent	0.2640	0.2688	0.2689	AR / LS
Sum squared residuals	2787.2282	2890.1842	2893.1333	AR / LS
Log likelihood	-3480.9046	9993.5201	9703.1512	GARCH
AIC	0.1742	-0.4994	-0.4848	GARCH
BIC / Schwarz criterion	0.1748	-0.4981	-0.4833	GARCH
Hannan-Quinn criterion	0.1744	-0.4990	-0.4843	GARCH
Durbin-Watson statistic	0.1605	0.1217	0.1220	None fully removes serial dependence

The present data set shows that the standard GARCH model is statistically more efficient than EGARCH as shown by the information criteria, but this does not negate the correctness of EGARCH method. However, it illustrates that the extra asymmetric parameter does not add enough to the model's adequacy in this case. This outcome is in line with the near-symmetric news impact curve, and the negligible EGARCH asymmetry coefficient.

4.3 Actual-fitted-residual behavior

Figure 2 plots the actual-fitted-residuals to visually assess the fit of the mean-variance specification to the defective bearing response. The fitted series generally captures the general shape of the oscillations and major amplitude shifts, with the residual close to zero for most of the observations. The residual still displays individual impulses, as it is found in localized bearing faults and as a motivation for additional residual diagnostic testing in future extensions.

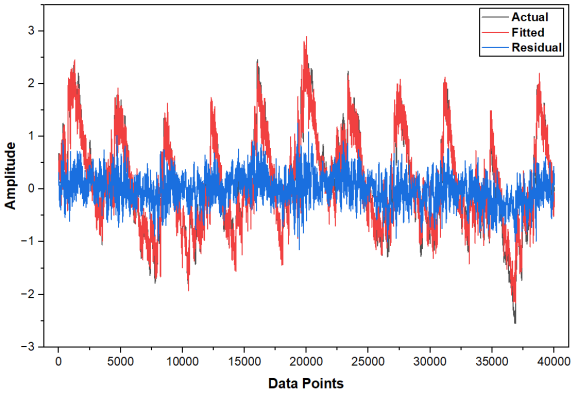


Figure 2. Actual, fitted and residual response for the defective bearing model.

4.4 Conditional volatility diagnostics

The conditional standard deviation and conditional vari–ance plots for EGARCH are shown in Figures 3 and 4 respectively. The diagnostics of both confirm there is no constant variance for the defective bearing vibration response. Relatively, high conditional volatility spans are preceded by subsequent high volatility spans, and relatively low volatility periods also tend to cluster. It is consistent with the physical process of cage fault impacts, which involves an irregular moving path of rolling elements and a sudden disturbance of the contact, which results in vibration energy being released in bursts.

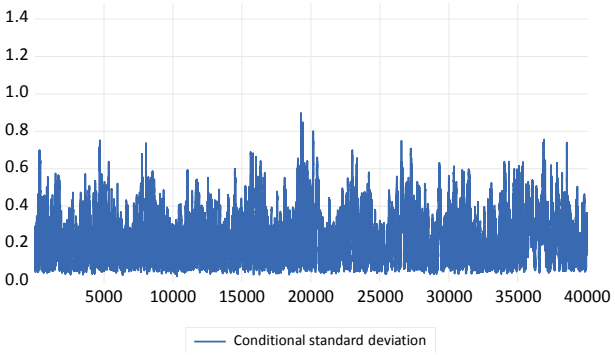


Figure 3. EGARCH model estimated Conditional standard deviation.

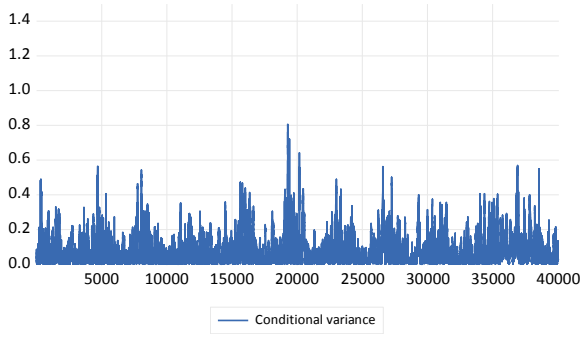


Figure 4. EGARCH estimated model Conditional variance.

The estimated conditional standard deviation and variance of the defective bearing vibration signal based on the GARCH(1,1) model are shown in figures 5 and 6. The diagnostics clearly show the presence of time varying variance as well as strong volatility clustering, which means that the vibration response is not of constant variance. High conditional volatility periods are followed by additional high volatility periods and low conditional volatility periods show persistence over time as well. This behavior shows how the magnitude of energy fluctuations of current vibrations is dependent on the magnitude of previous shocks and the level of variance in shocks at previous times.

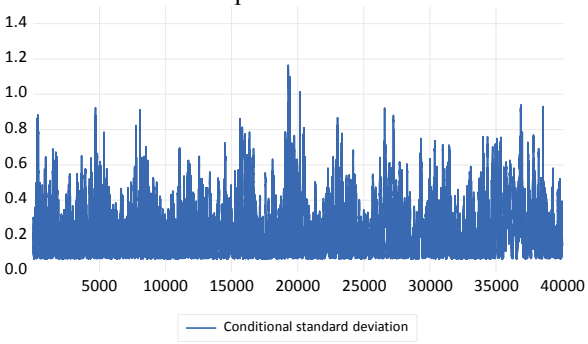


Figure 5. GARCH estimated model Conditional Standard deviation.

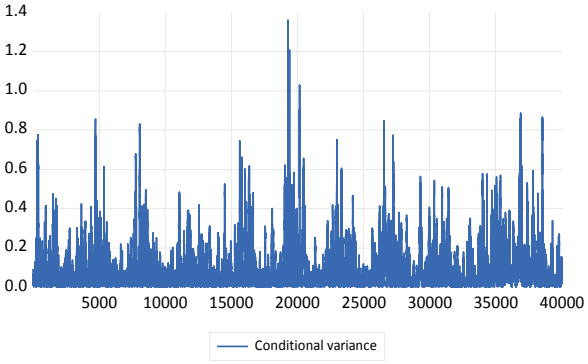


Figure 6. GARCH estimated model Conditional variance.

The volatility clustering observed is in line with the fundamental physics of bearing cage defects. Cage damage causes rolling elements to move and guide in an irregular manner, causing intermittent impacts, unstable contact conditions and irregular energy transfer in the bearing assembly. These are dynamic interactions that cause the vibrations to release in bursts, which are represented as spikes in the conditional variance esti–mates from the GARCH model. These findings hence validate the fact that the vibration signal from the de–

fective bearing is heteroscedastic and the GARCH is appropriate for the modelling and characterization of the stochastic volatility of the cage-fault vibration behavior.

4.5 News impact curve and asymmetry interpretation

News impact curve in Figure 7 refers to the change in the conditional variance that is induced by the previous standardized innovation. The curve is close to being symmetrical, which suggests that big positive shocks and big negative shocks are equally important in deter–mining future volatility. These graphical pieces of evi–dence are consistent with the estimated EGARCH asy–mmetry coefficient, which is not statistically significant. So, the main mechanism in the tested cage-fault data is shock magnitude.

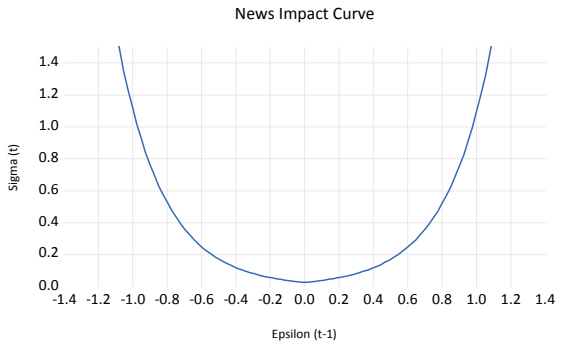


Figure 7. News impact curve obtained from the EGARCH model.

Figure 8 presents the News Impact Curve (NIC) for the bearing dataset, estimated from the fitted GARCH model. In the bearing dataset, the NIC is approximately symmetric around zero. This indicates that positive and negative shocks of equal magnitude have a nearly identical effect on future volatility. This symmetry is consistent with the estimated asymmetry coefficient (e.g., the leverage term in EGARCH or the indicator term in GARCH), which is statistically insignificant in the bearing data. Therefore, the primary driver of future volatility in the bearing fault condition is the magnitude of the shock, not its sign.

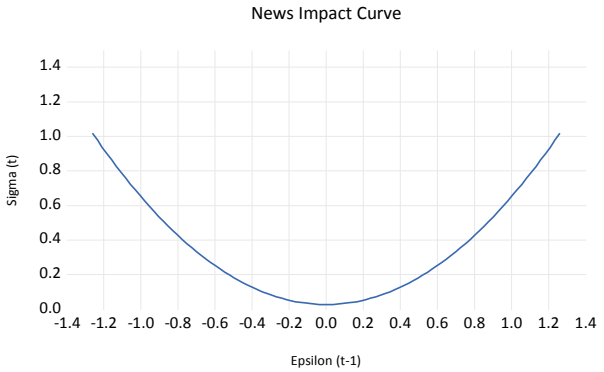


Figure 8. News impact curve obtained from the GARCH model.

4.6 Gradient diagnostics

Thegradients of the objective function for the estimated parameters are plotted in Figure 9. The gradients have been rarely fluctuated around the fixed level with no apparent long-term tendency to drift, which indicates that a stable solution has been achieved in the likelihood optimization. These intermittent "gradient excursions" are associated with high-impact observations in the vibration signal. While these plots are helpful as a supplement to other convergence diagnostics, they do not stand on their own as a measure of the accuracy of fault detection.The maximum and minimum values for each gradient function are tabulated as follows:

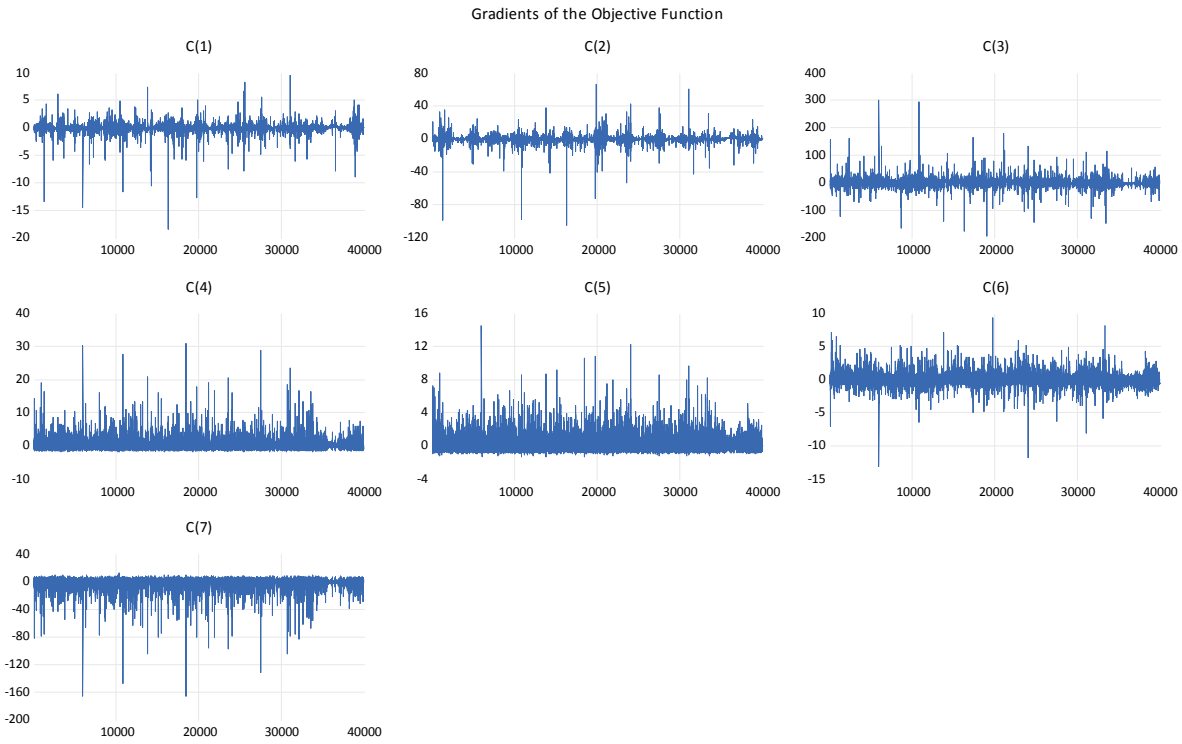


Figure 9. Gradient of the objective function;C(1), intercept; C(2), axial coefficient; C(3), tangential coefficient; C(4), variance constant ω; C(5), magnitude coefficient α; C(6), asymmetry coefficient γ; and C(7), persistence coefficient β

Table 5. Minimum and maximum objective-function gradi–ents for the parameters.

Series	Parameter	Approx. minimum	Approx. maximum
C(1)	Intercept	−18	10
C(2)	Axial coefficient	−105	65
C(3)	Tangential coefficient	−190	300
C(4)	Variance constant, ω	−2	31
C(5)	Magnitude coefficient, α	−1	15
C(6)	Asymmetry coefficient, γ	−13	9
C(7)	Persistence coefficient, β	−170	10

5. DISCUSSION

The AR/least-squares model is the best model in terms of in-sample deterministic fit measured by R^2 and standard error of regression. This is a result that is anticipated as the AR/LS baseline directly minimizes squared prediction error in the mean equation. Deter–ministic fit, however, is not enough to characterize bearing faults as it is often the case that defects are not simply seen as mean amplitude shifts but also as intermittent variance changes. The best volatility-model performance is for the GARCH (1,1) model. Both ARCH and GARCH are statistically significant with it indicating that the current variance is influenced by recent vibration shocks and past volatility. The persis–tence value $\alpha + \beta = 0.9564$ suggests that the effect of a highvolatility is not strong. This is similar in mechanical terms to repeated cage-fault interaction and concent–rated impact excitation.

An EGARCH (1,1) model provides helpful inter–pretation of the model diagnostics that examine the asy–mmetric reaction of the volatility. The estimated magni–tude and persistence effects are significant, but the asy–mmetry coefficient is insignificant. The present cage-fault data does not allow for the addition of the EGARCH asymmetry mechanism. Thus, the scientific claim is not that EGARCH is superior to GARCH, but rather GARCH-family conditional volatility modelling is useful to model the vibration of heteroscedasticity caused by defects in the data and that, among GARCH-family models, standard GARCH is the most stati–stically efficient one for the data at hand.

The principal practical application is the use of conditional variance or conditional standard deviation as an additional condition-monitoring indicator for rotating machinery such as motors, pumps, compressors, gear–boxes, and marine auxiliary equipment. Repeated high-volatility episodes may support early inspection or alarm decisions when combined with conventional vib–ration features. For field application, machine-specific thresholds must be calibrated for changes in speed, load, sensor position, and environmental noise, and the met–hod must be validated with independent bearing data.

The findings are limited to the tested bearing, operating condition, sensor arrangement, andthe analyses and estimated time-series models apply specifically to the investigated bearing containing a single 0.4 mm-long notch on the machined metallic cage ring. Different cage materials, constructions, defect sizes, geometries, and locations may change the timing and magnitude of rolling-element impacts. Accordingly, multiple cage defects may impact frequency, cause overlapping volatility bursts, and result in change of the persistence or apparent asymmetry of the conditional variance. The present results should therefore not be ge–neralized to other cage-fault configurations without ex–perimental validation.Future work should evaluate fully documented cage defects of different sizes, types, locations, and multiplicities under several speeds and loads; compare the volatility indicators with conventi–onal diagnostic features; test independent datasets; and investigate robust, heavy-tailed, higher-order, asym–metric, and regime-switching variance models.

6. CONCLUSIONS

This study was based on a modelling framework of bearing faults where three models (AR, GARCH and EGARCH) were compared using defective bearing vib–ration signals dataset. The AR/least-squares model best described the data, with $R^2 = 0.9220$ and the standard error of regression = 0.2640. This baseline is, however, not a model for the time varying variance which is physically relevant in an impulsive bearing fault vibration. WhereasGARCH (1,1) model had the highest likelihood-based performance standards in terms of the volatility models: log likelihood = 9993.5201 and AIC = -0.4994 and BIC/SC = -0.4981. Also $\alpha + \beta$ persistence value of 0.9564 suggests that response to the defective bearing is highly clustered around a high alpha value. This means that the shocks which cause the faults cause lasting conditional variance trend instead of independent constant variance noise. The EGARCH (1,1) model was evaluated to include the volatility persistence effect and shock magnitude effects from the bearing dataset and the model results indicated that it can be applied well with no statistically significant asymmetry coefficient. In the cage-fault type, the estimated gamma value is -0.0025, and the value of p is 0.7766, which shows that the differences between the positive and negative shocks is not statistically significant within the volatility of the test signal in the future. This thus does not gua–rantee the claim that EGARCH is better than GARCH for this data set. The overall comparative results suggest that conditional volatility modelling from the GARCH family is a valuable tool for the bearing cage-fault cha–racterization. The volatility-based measures can reflect the variance behavior that is dynamic but not reflected by the constant variance AR baseline.

DATA AVAILABILITY

The vibration data and model output tables used in this study are available upon request.

DECLARATION OF CONFLICTING INTERESTS

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СТАТИСТИЧКО МОДЕЛОВАЊЕ ВИБРАЦИЈА УСЛЕД КВАРА КАВЕЗА КОТРЉАЈУЋЕГ ЛЕЖАЈА ПРИМЕНОМ АНАЛИЗЕ ВРЕМЕНСКИХ СЕРИЈА СА УСЛОВНОМ ХЕТЕРОСКЕДАСТИЧНОШЋУ

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Лежајеви са котрљајућим елементима играју важну улогу у ротирајућим машинама и извор су нестаци–онарних и хетероскедастичних вибрационих одзива изазваних локализованим кваровима импулсног карактера. Ова студија предлаже и тестира приступ моделовања условне волатилности ради каракте–ризације кварова кавеза лежаја, заснован на ауто–регресивним (AR) моделима, као и моделима гене–рализоване ауторегресивне условне хетероскедас–тичности (GARCH) и експоненцијалне генерали–зоване ауторегресивне условне хетероскедастич–ности (EGARCH). Мерења вибрација у три осе вр–шена су на уређају за испитивање лежајева компа–није Spectra Quest, а анализирани су услови рада са исправним лежајем и са лежајем код ког постоји квар кавеза. Поступак моделовања започиње моде–ловањем детерминистичког средњег одзива помоћу аксијалних и тангенцијалних компоненти вибрација и провером постојања временски променљиве условне варијансе у преосталој стохастичкој струк–тури. Анализирано је 40.000 синхронизованих мерења путем упоредне процене модела засноване на значајности параметара, критеријумима заснова–ним на вероватноћи (likelihood), грешци регресије и дијагностици волатилности. Основни AR модел је показао најбоље поклапање, са вредношћу R^2 од 0,9220, али не обухвата динамику условне вари–јансе. Чини се да је GARCH (1,1) модел успешно обухватио осетљивост на ударце и високу посто–јаност волатилности, уз највишу вредност $\alpha + \beta = 0{,}9564$, као и најбоље статистичке показатеље зас–нованe на вероватноћи међу моделима волатил–ности: AIC = -0,4994, BIC = -0,4981. Поред тога, EGARCH (1,1) модел је показао значајне ефекте интензитета и постојаности; иако је знак ефекта постојаности био супротан за позитивне и негативне ударце, тај ефекат није био статистички значајан ($\gamma = -0{,}0025$, $p = 0{,}7766$) за скуп података испитиваних кавеза. Резултати показују да је моделовање условне варијансе помоћу модела из породице GARCH ко–рисно за опис вибрација лежаја у случају квара, али тренутно нема доказа који би указивали на то да је EGARCH модел бољи од GARCH модела за ана–лизиране податке.