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ADJUSTMENT OF MORTALITY TABLES BY LIMITED FLUCTUATION METHOD

SCIENTIFIC PAPER

Abstract

Credibility theory provides tools to deal with the randomness of data that is used for predicting future events. The credibility procedure is used to improve the statistical significance of parameter estimates in a given task. Because it has been used since the very beginnings of actuarial practice, it is one of the most productive actuarial techniques, and is still being developed and adapted to solve contemporary problems. In this paper, the credibility procedure will be used to modify the standard population mortality table, more specifically the Serbian mortality table for females (Mortality Tables of the Republic Statistical Office 2010-2012), to reflect the experience with a particular subset of population and thus establish a more reliable estimate of mortality by including into analysis the experiential data and subgroups.

Key words: mortality tables, mortality rates, credibility

I Introduction

In recent years, regulators and pension funds have been paying greater attention to assumptions about mortality rates used in pension schemes. One of the reasons is an increasing awareness of the variability in mortality rates within different demographic groups and/or populations within the schemes, which results in the need to reconsider assumptions and better adapt to specific plans. Practical evidence suggests that mortality varies by industry, geographical area, and type of business. This has led to a renewed interest in credibility theory as the means of adapting standard mortality tables to specific pension schemes or to the population covered by those schemes. In this regard, in the USA and Canada, it is legally prescribed that the retirement plans use customized mortality tables

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which are prepared by using credibility theory.[3] Guidelines for the use of credibility procedure have been established and the Actuarial Standard of Practice no. 25 (Credibility Procedures) has been extended to include retirement plans.

The concept of credibility has been used in actuarial practice since the beginning of the actuarial science. One of the first recorded works on this subject is the work of Albert Mowbray[4] published in 1914. Early discussions of credibility were focused on estimating the mean incidence of damage using classical and empirical credibility procedures. Later, formulas were developed for Bayesian credibility procedures and limited fluctuation method[5]. The latest credibility methods include credibility estimation for generalized linear models or other multivariate techniques. Credibility procedures involve different approaches. Depending on the problem, an approach may be based on estimation, mathematical models, or their combination.[6]

This paper will explain the method of limited fluctuation and show how to use the credibility procedure when adjusting standard mortality tables to specific retirement plans.

II Credibility Procedure

Credibility theory provides tools for dealing with random variables (data) that are used to predict future events. Credibility is a measure of the predictive value in a given application that the actuary attaches to a particular set of data (predictive is used here in the statistical sense, and not in the sense of predicting the future).[7] Credibility procedure is a process that involves the evaluation of subject experience for potential use in setting assumptions without reference to other data, or includes the identification of relevant experience. Credibility procedure is also used to improve estimate of the parameter in a given task. Credibility can be used for insurance pricing, calculation of premium rates, determining future premium rates based on experience and reservations, and more.

The insurance company uses past claims data to estimate future costs of insurance coverage. However, insurance claims arise from accidental occurrences. Using the data on average annual claims in the past few years may result in a poor estimate of costs for the coming year. The expected accuracy of this estimate is a

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[3] Internal Revenue Service (IRS), www.irs.gov

[4] Albert H. Mowbray, „How extensive a payroll exposure is necessary to give a dependable pure premium? "

[5] Detailed formulas can be found in the papers of Gavin Benjamin, "*Selecting Mortality Tables: A Credibility Approach*", Research paper, Society of Actuaries, 2008 and American Academy of Actuaries, *Credibility Practice Note*, 2008.

[6] See Marija Kerkez, Nebojša M. Ralević, "Uncertainty analysis and risk modelling in insurance", *Insurance in the post-crisis era*, (Kočović, J. Baskakov. V. Boričić, B. et al. eds) University of Belgrade, Faculty of Economics Publishing Centre, 2018, Chapter 18, 309-326.

[7] Actuarial Standard of Practice No. 25. Credibility Procedures Applicable to Accident and Health, Group Term Life, and Property/Casualty Coverage.

function of the variability of claims. These data alone are not acceptable for the calculation of insurance premium rates.

When applying credibility methods, it is necessary to take into account the characteristics of both the company experience (past data) and the relevant experience (experience similar to that of a company). The actuary should use expert judgment when selecting, developing or using the credibility method. When making decisions, attention is paid to the extent to which the subject experience is incorporated into the relevant experience. If the subject experience is an essential part of the relevant experience, the actuary should use his or her expert judgment in deciding if he or she should use that relevant experience and in what way. In addition, homogeneity and predictive stability of these data should be taken into account, whereby segments that are not typical representatives of the experience as a whole can be excluded and thus better predictive value can be obtained. The actuary also considers the balance between the homogeneity of the data and the size of the data set.

The basic formula for calculating credibility weighted estimates is:

Estimate = Z x [Observation] + (1-Z) x [Other information], 0 <Z <1,
where Z is credibility factor assigned to observation, 1-Z is generally called complement of credibility factor.

1. Limited Fluctuation Credibility

Classical credibility procedures make assumptions as to the form of the underlying probability distribution. The corresponding number of claims, the amount of premium, etc. are calculated on the basis of this probability distribution, so that the probability of movement of subject claims is within the indicated percentage of expected value. One such approach that assumes that claims follow a normal distribution is Limited Fluctuation Credibility.

Limited fluctuation credibility method is less rigorous than the greatest fluctuation credibility[8], however, it requires subjective estimates of parameters. The method uses a linear combination of relevant experience $\hat{m}$ and subject experience $\hat{\alpha}$ to estimate:

$$\hat{E} = Z \cdot \hat{m} + (1-Z) \cdot \hat{\alpha},$$
where credibility factor Z needs to be established.

If Z = 1, the data meet the criterion for full credibility, whereas for Z = 0 the analysed data have no credibility for the observed process. If 0 < Z < 1, the observed body of data is not sufficient, and that is a so-called partial credibility.

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[8] For more information about this method refer to the papers: Stuart Klugman, Thomas Rhodes, Marianne Purushotham, Stacy Gill, MIB Solutions. *Credibility Theory Practices*, Society of Actuaries, 2009 and Stuart Klugman, Sample size selection for multiple samples-*A brief introduction to credibility theory and an example featuring rate-based insurance premiums*, Drake University, for presentation at NYU-March 4, 2004.

The model is designed to ensure that the error around the experience of the company is minimised to an acceptable level, where the acceptability of the error level is assessed by the actuary. Namely, subjective judgement is required.[9]

In this model, for Z=1 there is full credibility if $Pr\left(\left|\hat{m}-m\right|\leq rm\right)\geq p,$ which represents a relative error, where $\hat{m}$ is the estimate based on the subject experience, and m is the true, unknown value. The choice of confidence level (p) and margin of error (r) is subject to judgment. This is one of the main disadvantages of this method. In addition, a subjective judgment is required to determine the quantity of data necessary for full credibility. A 90% confidence error and 5% margin of error (p = 90% and r = 5%) are frequently cited as minimum levels required for full credibility; however, since there is no theoretical basis for this threshold, other assumptions may be just as valid. Thus, a 3% margin of error with 90% confidence level is recommended in Canada for purposes of setting mortality assumptions, and full credibility is achieved at 3.007 deaths. On the other hand, the proposed US mortality regulations define full credibility at 1,082 deaths, which is based on a 5% margin of error with a 90% confidence level.[10]

Let n be the number of observed lives at a given age (x), let d be the number of observed deaths, and let q be the true mortality probability.

Full credibility is assigned if the following is true

$$\Pr\left(\left|\frac{d}{n}-q\right|\leq 0,05q\right)\geq 0,9.$$

If n is large enough, using de Moivre-Laplace theorem, the binomial distribution with n and q or $B(n,q)$ can be approximated by a normal distribution, with the parameters $\mu=nq$ and $\sigma^2=nq(1-q)$, or $N(nq,nq(1-q))$.

$$\Pr\left(0,95nq\leq d\leq 1,05nq\right)\approx\Pr\left(\frac{0,95nq}{\sqrt{nq(1-q)}}\leq z\leq\frac{1,05nq-nq}{\sqrt{nq(1-q)}}\right)\geq 0,9.$$

This formula can be simplified by noting that $1-q\rightarrow 1$

$$\Pr\left(-0,05\sqrt{nq}\leq z\leq 0,05\sqrt{nq}\right)\geq 0,9.$$

For a standard normal random variable $Pr(-1,645\leq z\leq 1,645)=0,9,$ so the equation is satisfied when $0,05\sqrt{nq}\geq 1,645$ or $nq\geq 1.082.$ Thus, the expected number of deaths must be at least 1,082 for the full credibility requirement to be met.

Generalizing this for any p and r leads to the conclusion that full credibility is achieved when the number of observed deaths is greater than or equal to $\left(z_{(1+p)/2}/r\right)^2.$

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[9] For more information regarding subjective actuarial estimates refer to Marija Paunović, *Uncertainty Measures and Actuarial Application*, Doctoral Dissertation, Faculty of Technical Sciences, University of Novi Sad, 2019.

[10] Irina Pogrebivsky, Credibility Educational Resource for Pension Actuaries, *Application of Credibility Theory to Mortality Assumption*, Society of Actuaries (SOA), 2017.

In case the full credibility is not met, some portion of credibility is assigned to relevant data.

$$\sigma^2\left(\hat{E}\right) = \sigma^2\left(\hat{m}\right) = \sigma^2\left(\frac{d}{n}\right) = \frac{q(1-q)}{n^{cr}} \approx \frac{q}{n^{cr}}$$

$$\sigma^2\left(\hat{E}\right) = \sigma^2\left(\hat{m}\right) = \sigma^2\left(Z\cdot\hat{m}+(1-Z)\cdot\hat{\alpha}\right) = Z^2\sigma^2\left(\frac{d}{n}\right) \approx \frac{Z^2 q}{n^a}$$

$$\sigma^2\left(\hat{E}\right) = \sigma^2\left(\hat{m}\right) = \sigma^2\left(Z\cdot\hat{m}+(1-Z)\cdot\hat{\alpha}\right) = \sigma^2(Z\cdot\hat{m}) + \sigma^2((1-Z)\cdot\hat{\alpha})$$

$$= Z^2\sigma^2(\hat{m}) + (1-Z)^2\sigma^2(\hat{\alpha}) = Z^2\sigma^2\left(\frac{d}{n}\right) \approx \frac{Z^2 q}{n^a}$$

where n^a is the number of actual deaths and n^{cr} is the number necessary for full credibility.

Matching the two variances, we obtain

$$Z = \left(\sqrt{\frac{n^a}{n^{cr}}}, 1\right).$$

This means the credibility factor is the square root of the ratio of the observed (actual) deaths (the expected deaths are not available) to the number of deaths required for full credibility.

2. Adjustments of Standard Mortality Tables

Mortality tables are an essential demographic tool for analysing population mortality, and represent a tabular overview of mortality by age and sex. Mortality tables, that are created through observation of the entire population of a particular nation, are commonly referred to as population tables or standard mortality tables. Tables formed on the basis of mortality data derived from a collection of insurance portfolios or certain retirement plans are referred to as market mortality tables.

The credibility procedure is used to modify a standard population mortality table to reflect the experience of a subgroup for which a more appropriate mortality estimate can be determined by including the experiential data for that subgroup. When evaluating a retirement plan, the goal is to include the experiential data of participants covered by the evaluated plan. An employer may have contracts concluded with multiple retirement plans, however, a single plan may include multiple employee subgroups. In such cases, an actuarial assessment is used to decide whether different subgroups require different mortality tables, that is, produce mortality tables for a particular plan based on specific experiential data for each subgroup covered by the plan. The period that an actuary analyses is usually 3 to 5 years. The analysis covers employees and beneficiaries older than 50[11]. Namely, the minimum requirement for pension is observed (minimum age and service requirements).

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[11] IRS recommendation.

To compile a mortality table, it is necessary to calculate death probability q_x for each age class. As stated in the previous chapter, the analysis needs to include a sufficient number of deaths, namely, 1.082 of deaths in each age class to achieve full credibility (assuming that r = 5% and p = 90%). For example, if $q_{72} = 0,031$, it takes 1,082 / 0,031 = 34.903 live persons aged 72 to achieve full credibility of mortality rate at such age, which is a large quantity of data. In addition, the obtained values of qx must then be levelled, which requires additional analyses and approximations. Therefore, in most cases it is more acceptable to fit standard mortality tables to a specific subset of the insured.

The first step in adjusting mortality rates is to find the weight $\hat{\xi}$ which represents the ratio between actual deaths and expected number of deaths from all age groups. The ratio is applied to all insured persons aged x, and is determined as an aggregate value through all age groups,

$$\hat{\xi} = \frac{\sum_x d_x}{\sum_x q_x^{st}},$$

where mortality q_x^{st} is obtained from standard mortality tables.

Full credibility is achieved when the ratio $\hat{\xi}$ is within the margin of error (r) of actual ratio ξ, with probability of at least p:

$$\Pr\left(\left|\hat{\xi} - \xi\right| \le r\xi\right) \ge p.$$

Standard credibility formula is constructed as follows

$$q_x^{est} = Z \cdot \left(\hat{\xi} \cdot q_x^{st}\right) + (1 - Z) \cdot q_x^{st} = \left(Z \cdot \hat{\xi} - Z + 1\right) \cdot q_x^{st},$$

where q_x^{est} is estimated death rate of a plan.

III Numerical Example

Let us assume that a three-year data on women who are the members of a pension scheme show that there were 1,230 deaths (Table 1). The actuary will need to modify mortality rates shown in the standard mortality tables to the population of women in Serbia (Mortality Tables of the Statistical Office of the Republic of Serbia 2010-2012) and generate new mortality tables adjusted to the particular plan, assuming that the confidence interval is 95% and the margin of error is 5%.

According to the Normal Distribution tables $N(0,1)$ for p = 0,95 value is $z_\alpha = 1,96$, whereas for the margin of error of 5% full credibility is reached for $(1,96 / 0,05\ )^2 = 1.537$ deaths. As the number of actual deaths is 1,230, this number is not sufficient for full credibility and therefore, the credibility factor is calculated as the square root of the ratio of the actual deaths to the number of deaths required for full credibility $Z = \sqrt{\dfrac{1.230}{1.537}} = 0,895.$

Expected number of deaths is 2,097 (column 6). Further, the weight $\hat{\xi}$ is calculated which represents the ratio of the actual deaths to the expected number of deaths for all age groups $'\hat{\xi} = 1.230 / 2.097 = 0,59,$ and thus
Weight $= 0.895 \cdot 0,59 + (1 - 0.895) \cdot 1 = 0,63$ (column 8).

The calculation shows that every mortality rate q_x^{st} from the standard mortality table needs to be adjusted with the weight 0.63 in order to obtain the mortality rate q_x^{est} estimated for the plan.

Table 1 Formation of adjusted mortality table

x	lx	d_x	q_x^{st}	$q_x^{\exp}$	Expected number of deaths	Adjusted rate q_x^{est}	Weight	Ratio
1	2	3	4	5=2/4	6=2*5	7	8=7*2/6	9=3/6
50	503		0,003455	-	1,739287	0,002177	0,630	-
51	891	3	0,003738	0 ,003922	3,332705	0,002355	0,630	1,049
52	1.484	12	0,004224	0,008245	6,268780	0,002661	0,630	1,952
53	2.192	14	0,004653	0,006380	10,198316	0,002931	0,630	1,371
54	2.459	12	0,005171	0,004975	12,718530	0,003258	0,630	0,962
55	2.858	19	0,005730	0,006728	16,375410	0,003610	0,630	1,174
56	3.540	24	0,006180	0,006914	21,874295	0,003893	0,630	1,119
57	3.655	37	0,006681	0,010043	24,418500	0,004209	0,630	1,503
58	3.493	42	0,007267	0,012012	25,380635	0,004578	0,630	1,653
⋮								
93	26	2	0,335770	0,066667	8,803892	0,211535	0,630	0,199
94	5	1	0,372676	0,190694	1,954310	0,234786	0,630	0,512
95	16	2	0,413793	0,111111	6,509793	0,260690	0,630	0,269
96	10	2	0,460268	0,166667	4,827294	0,289969	0,630	0,362
97	10	3	0,510516	0,333333	5,354294	0,321625	0,630	0,653
Total	1.358	1.230			2.097			

A graphical representation of mortality tables, that is, mortality rates, is useful before and after adjusting the tables. One way to assess whether an adjustment is appropriate is to compare the actual mortality rates with the corresponding standard mortality table, as shown in the Chart 1. It can be seen that after the rate correction for the calculated value of 0.63, the shape of the curve is similar to the one obtained from the standard mortality table. It can be also noted that after the age of seventy, the deviations of the actual rates of the observed plan are higher than those of the standard mortality tables and therefore, in this example, it is justified to apply the presented method.

Image 1. Comparative view of mortality tables
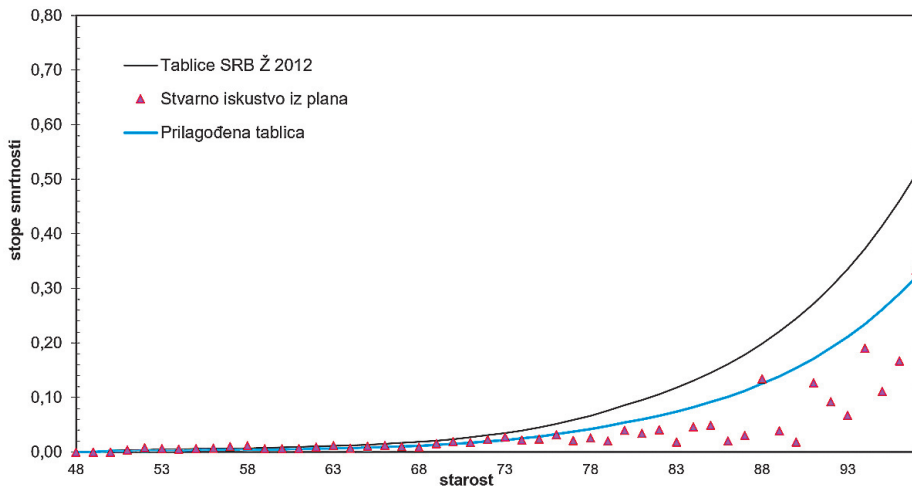
IV Conclusion

Credibility theory is nowadays widely used in life and non-life insurance. The main goal of applying the theory of credibility is to minimise the errors between the statistical evaluation of various parameters and their actual value. Credibility theory enables the application of different methods and models in the estimation of certain elements of an observed population subgroup, combining the outputs for such a particular subgroup with those obtained for the population as a whole. In contemporary actuarial life insurance practice, credibility serves as a tool for adapting standard mortality tables to specific retirement plans, that is, to the population covered by those plans. When evaluating a retirement plan, credibility theory allows the incorporation of experiential data of participants covered by the evaluated plan. Various models of credibility theory are used to modify the standard population mortality tables in order to implement quantitative experiences related to population subgroups, which contributes to increasing the reliability of estimates. In a highly competitive insurance market, a sound knowledge of credibility theory is an essential tool in assessing insurance risk.

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PRILAGOĐAVANJE TABLICA MORTALITETA KORIŠĆENJEM METODE OGRANIČENE FLUKTUACIJE

ORIGINALNI NAUČNI RAD

Apstrakt

Teorija kredibiliteta pruža alat za bavljenje slučajnim promenljivama koje se koriste za predviđanje budućih događaja. Procedura kredibiliteta koristi se da se poboljša procena statističkih parametara u datom zadatku. Budući da se koristi od samih početaka aktuarske prakse, to je jedna od najproduktivnijih aktuarskih tehnika, koja se i dalje razvija i prilagođava u cilju rešavanja savremenih problema. U ovom radu procedura kredibiliteta biće korišćena za modifikaciju standardne tablice mortaliteta stanovništva, konkretnije Tablice mortaliteta za žensko stanovništvo Srbije (Tablice mortaliteta RZS 2010-2012), kako bi nova tablica odražavala iskustvo određene podgrupe populacije stanovništva i kako bi se time utvrdila pouzdanija procena smrtnosti uključivanjem iskustvenih podataka te podgrupe u analizu.

Ključne reči: tablice mortaliteta, stopa smrtnosti, kredibilitet

I Uvod

Poslednjih godina regulatori i penzijski fondovi posvećuju veću pažnju pretpostavkama o stopama smrtnosti koje se koriste u penzijskim planovima. Jedan o razloga jeste taj što raste svest o varijabilnosti stope smrtnosti unutar različitih demografskih grupa i/ili populacija unutar planova, što rezultira potrebom da se pretpostavke preispitaju i bolje prilagode konkretnim planovima. Dokazi iz prakse upućuju na to da smrtnost varira u zavisnosti od delatnosti, geografskog područja i vrste posla. To je dovelo do ponovnog interesovanja za teoriju kredibiliteta kao sredstva za prilagođavanje standardnih tablica mortaliteta konkretnim penzijskim

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planovima, odnosno populaciji obuhvaćenoj tim planovima. U tom smislu u SAD i Kanadi zakonski je određeno da penzijski planovi koriste prilagođene tablice smrtnosti za čiju izradu se koristi teorija kredibiliteta.[3] Utvrđene su smernice za korišćenje procedure kredibiliteta, a Standard aktuarske prakse br. 25 (Procedure kredibiliteta) proširen je na penzijske planove.

Koncept kredibiliteta u aktuarskoj praksi koristi se od početka postojanja aktuarske nauke. Jedan od prvih zabeleženih radova na tu temu jeste rad Alberta Moubraja (Albert Mowbray),[4] objavljen 1914. godine. Rane diskusije o kredibilitetu fokusirale su se na procenu srednje učestalosti šteta pomoću klasičnih i empirijskih procedura kredibiliteta. Kasnije su razvijene formule za Bajesove procedure kredibiliteta i metod ograničene fluktuacije.[5] Najnovije metode kredibiliteta uključuju procenu kredibiliteta u generalizovane linearne modele ili druge multivarijantne tehnike modelovanja. Različiti pristupi upotrebljavaju se u procedurama kredibiliteta. U zavisnosti od problema, pristup se zasniva na proceni, matematičkim modelima ili njihovoj kombinaciji.[6]

U ovom radu biće objašnjena metoda ograničene fluktuacije i prikazan način korišćenja procedure kredibiliteta prilikom prilagođavanja standardnih tablica mortaliteta konkretnim penzijskim planovima.

II Procedura kredibiliteta

Teorija kredibiliteta pruža alat za bavljenje slučajnim promenljivama (podacima) koji se koriste za predviđanje budućih događaja. Kredibilitet je procena prediktivne vrednosti u datoj primeni koju aktuar dodaje određenom skupu podataka (prediktivno se ovde koristi u statističkom smislu, a ne u smislu predviđanja budućnosti).[7] Procedura kredibiliteta predstavlja proces koji obuhvata procenu iskustva društva za potencijalnu upotrebu u određivanju pretpostavki bez referenci na druge podatke ili identifikaciju relevantnog iskustva. Procedura kredibiliteta koristi se i da se poboljša procena parametra u datom zadatku. Kredibilitet se može koristiti za određivanje cene osiguravajućeg pokrića, izračunavanje premijske stope osiguranja, utvrđivanje buduće premijske stope na osnovu iskustva i rezervisanja i dr.

Društvo za osiguranje koristi podatke o prošlim štetama za procenu budućih troškova za pružanje osiguravajućeg pokrića. Međutim, štete u osiguranju proizlaze

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[3] Internal Revenue Service (IRS), *www.irs.gov*

[4] Albert H. Mowbray,„How extensive a payroll exposure is necessary to give a dependable pure premium?"

[5] Detaljne formule mogu se naći u radovima autora Gavina Benjamina, „Selecting Mortality Tables: A Credibility Approach", Research paper, Society of Actuaries, 2008 i American Academy of Actuaries, *Credibility Practice Note*, 2008.

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[7] Actuarial Standard of Practice No. 25. Credibility Procedures Applicable to Accident and Health, Group Term Life, and Property/Casualty Coverage.

iz slučajnih pojava. Korišćenje prosečnog godišnjeg podatka o štetama u proteklih nekoliko godina može rezultirati lošom procenom troškova za narednu godinu. Očekivana tačnost te procene je funkcija varijabilnosti šteta. Ti podaci sami po sebi nisu prihvatljivi za izračunavanje premijskih stopa u osiguranju.

Prilikom primene metoda kredibiliteta, potrebno je uzeti u razmatranje karakteristike kako iskustva društva (prošli podaci) tako i relevantnog iskustva (iskustvo slično iskustvu društva). Aktuar treba da koristi stručnu procenu pri izboru, razvoju ili upotrebi metode kredibiliteta. Prilikom odlučivanja, vodi se računa o obimu u kojem je iskustvo društva uključeno u relevantno iskustvo. Ako je iskustvo društva bitan deo relevantnog iskustva, aktuar na osnovu stručne procene treba da odluči da li i kako koristiti to relevantno iskustvo. Takođe, potrebno je uzeti u obzir i homogenost i prediktivnu stabilnost tih podataka, pri čemu se mogu isključiti segmenti koji nisu tipični predstavnici iskustva u celini i tako dobiti bolja prediktivna vrednost. Aktuar takođe razmatra ravnotežu između homogenosti podataka i veličine skupa podataka.

Osnovna formula za kredibilitet:

Procena = Z x [Opservacija] + (1-Z) x [Ostale informacije], 0 <Z <1,
gde je Z faktor kredibiliteta dodeljen posmatranju, 1-Z se generalno naziva komplementom faktora kredibiliteta.

1. Kredibilitet ograničene fluktuacije

U klasičnim procedurama kredibiliteta postavljaju se pretpostavke u vezi s oblikom osnovne raspodele verovatnoće. Na osnovu te funkcije raspodele verovatnoća, izračunavaju se odgovarajući broj šteta, iznos premije i drugo, tako da je verovatnoća kretanja šteta društva u okviru naznačenog procenta očekivane vrednosti. Jedan takav pristup koji pretpostavlja da štete prate normalnu raspodelu jeste kredibilitet ograničene fluktuacije.

Metod ograničene fluktuacije manje je rigorozan od metode najveće preciznosti,[8] ali zahteva subjektivne procene parametara. Metod koristi linearnu kombinaciju relevantnog iskustva $\hat{m}$ i iskustva društva $\hat{\alpha}$ za procenu:

$$\hat{E} = Z \cdot \hat{m} + (1-Z) \cdot \hat{\alpha},$$
gde treba utvrditi faktor kredibiliteta Z.

Ako je Z = 1, podaci zadovoljavaju kriterijum za potpun kredibilitet, dok pri Z = 0, podaci koji su razmatrani nemaju kredibilitet za posmatrani proces. Ukoliko je 0 < Z < 1, posmatrana količina podataka nije dovoljna i to je tzv. parcijalni kredibilitet.

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[8] Više o ovoj metodi naći u radovima: Stuart Klugman, Thomas Rhodes, Marianne Purushotham, Stacy Gill, MIB Solutions. *Credibility Theory Practices*, Society of Actuaries, 2009 i Stuart Klugman, Sample size selection for multiple samples-*A brief introduction to credibility theory and an example featuring rate-based insurance premiums*, Drake University, for presentation at NYU-March 4, 2004.

Model je smišljen tako da osigura da se greška vezana za iskustvu društva svede na prihvatljiv nivo, gde prihvatljivost nivoa greške procenjuje aktuar, što znači da je to predmet subjektivne procene.[9]

U ovom modelu, za Z=1 imamo pun kredibilitet ako je $Pr\left(\left|\hat{m}-m\right|\leq rm\right)\geq p$, što predstavlja relativnu grešku, gde je $\hat{m}$ procena na osnovu predmetnog iskustva, a m je stvarna nepoznata vrednost.

Izbor nivoa pouzdanosti (p) i marginalna greška (r) podložan je proceni, što je i jedan od glavnih nedostataka metode. Takođe, za utvrđivanje veličine skupa podataka potrebnih za potpun kredibilitet potrebna je subjektivna procena. Nivo poverenja od 90% i 5% greške (p = 90% i r = 5%) često se navode kao minimalan nivo potreban za pun kredibilitet, ali s obzirom na to da ne postoji teorijska osnova za odbir ovih parametara, druge pretpostavke mogu biti jednako validne. Tako se u Kanadi preporučuje korišćenje marginalne greške od 3% sa nivoom pouzdanosti od 90% za potrebe postavljanja pretpostavki za smrtnost, a potpun kredibilitet postiže se za 3.007 smrtnih slučajeva. S druge strane, predloženi propisi o smrtnosti u SAD definišu potpun kredibilitet pri 1.082 smrtna slučaja, uz korišćenje marginalne greške od 5% sa nivoom pouzdanosti od 90%.[10]

Neka je n broj posmatranih živih lica starosti (x), d broj umrlih lica, a q stvarna verovatnoća smrtnosti.

Potpun kredibilitet se postiže pri

$$Pr\left(\left|\frac{d}{n}-q\right|\leq 0,05q\right)\geq 0,9.$$

Za dovoljno veliko n, koristeći Moivere-Laplaceovu teoremu, binomna raspodela sa parametrima n i q odnosno $B(n,q)$ može se aproksimirati normalnom raspodelom, sa parametrima $\mu = nq$ i $\sigma^2 = nq(1-q)$, odnosno $N\left(nq, nq(1-q)\right)$.

$$Pr\left(0,95nq\leq d\leq 1,05nq\right)\approx Pr\left(\frac{0,95nq}{\sqrt{nq(1-q)}}\leq z\leq\frac{1,05nq-nq}{\sqrt{nq(1-q)}}\right)\geq 0,9.$$

Formula se dalje može pojednostaviti koristeći $1-q\rightarrow 1$

$$Pr\left(-0,05\sqrt{nq}\leq z\leq 0,05\sqrt{nq}\right)\geq 0,9.$$

Za standardnu slučajnu promenljivu je $Pr\left(-1,645\leq z\leq 1,645\right)=0,9,$ pa je uslov zadovoljen kada je $0,05\sqrt{nq}\geq 1,645$ ili $nq\geq 1.082$. Odatle zaključujemo da očekivani broj umrlih lica mora biti najmanje 1.082 da bi bio ispunjen uslov za potpun kredibilitet.

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[9] Za više informacija u vezi sa subjektivnosti u aktuarskim procenama pogledati Marija Paunović, *Mere neodređenosti i primena u aktuarstvu*, doktorska disertacija, Fakultet tehničkih nauka, Univerzitet u Novom Sadu, 2019.

[10] Irina Pogrebivsky, Credibility Educational Resource for Pension Actuaries, *Application of Credibility Theory to Mortality Assumption*, Society of Actuaries (SOA), 2017.

Uopštavanjem prethodnog za bilo koje vrednosti p i r zaključuje se da se pun kredibilitet postiže kada je posmatrani broj umrlih lica veći ili jednak $(z_{(1+p)/2} / r)^2$.

U slučaju da nije ispunjen potpun kredibilitet, određeni deo kredibiliteta dodeljuje se relavantnim podacima.

$$\sigma^2\left(\hat{E}\right) = \sigma^2\left(\hat{m}\right) = \sigma^2\left(\frac{d}{n}\right) = \frac{q(1-q)}{n^{cr}} \approx \frac{q}{n^{cr}}$$

$$\sigma^2\left(\hat{E}\right) = \sigma^2\left(\hat{m}\right) = \sigma^2\left(Z\cdot\hat{m} + (1-Z)\cdot\hat{\alpha}\right) = Z^2\sigma^2\left(\frac{d}{n}\right) \approx \frac{Z^2 q}{n^a}$$

$$\sigma^2\left(\hat{E}\right) = \sigma^2\left(\hat{m}\right) = \sigma^2\left(Z\cdot\hat{m} + (1-Z)\cdot\hat{\alpha}\right) = \sigma^2(Z\cdot\hat{m}) + \sigma^2((1-Z)\cdot\hat{\alpha})$$

$$= Z^2\sigma^2(\hat{m}) + (1-Z)^2\sigma^2(\hat{\alpha}) = Z^2\sigma^2\left(\frac{d}{n}\right) \approx \frac{Z^2 q}{n^a}$$

gde je n^a broj stvarno umrlih lica i n^{cr} broj potreban za potpuni kredibilitet. Izjednačavanjem tih dveju formula za varijansu dobijamo

$$Z = \left(\sqrt{\frac{n^a}{n^{cr}}}, 1\right).$$

To znači da je faktor kredibiliteta kvadratni koren odnosa posmatrane (stvarne) stope smrtnosti (očekivane stope smrtnosti nisu dostupne) i broja smrtnih slučajeva potrebnih za potpun kredibilitet.

2. Prilagođavanje standardnih tablica mortaliteta

Tablice mortaliteta su osnovni demografski alat za analiziranje smrtnosti stanovništva i predstavljaju tabelaran prikaz smrtnosti po starosti i polu. Tablice mortaliteta formirane na osnovu posmatranja koje uključuje celu populaciju jedne nacije obično se nazivaju populacione ili standardne tablice mortaliteta. Tablice formirane na osnovu podataka o smrtnosti koji potiču iz kolekcije portfelja osiguranja ili određenih penzijskih planova nazivaju se tržišnim tablicama mortaliteta.

Procedura kredibiliteta koristi se za modifikaciju standardne tablice mortaliteta stanovništva kako bi odražavala iskustvo podgrupe za koju se može utvrditi pogodnija procena smrtnosti uključivanjem iskustvenih podataka te podgrupe. Pri vrednovanju penzijskog plana, cilj je da se uključe iskustveni podaci učesnika obuhvaćenih planom koji se vrednuje. Poslodavac može imati zaključene ugovore sa više penzijskih planova, ali i jedan plan može obuhvatiti više podgrupa zaposlenih. U takvim slučajevima koristi se aktuarska procena prilikom odlučivanja o tome da li su za različite podgrupe potrebne različite tablice mortaliteta, odnosno izrada tablica mortaliteta za određeni plan zasnovanih na specifičnim iskustvenim podacima za svaku podgrupu koja je obuhvaćena planom. Period koji aktuar analizira obično je od 3 do 5 godina. Analiza obuhvata zaposlene, odnosno

korisnike starije od 50 godina,[11] ili se posmatra minimalni uslov za odlazak u penziju (starosna penzija ili stažni uslov).

Da bi se sačinila tablica mortaliteta, potrebno je da se za svaku klasu starosti izračuna verovatnoća smrtnosti q_x. Analiza treba da obuhvati dovoljno velik broj umrlih lica, kao što je naglašeno u prethodnom poglavlju – 1.082 umrla lica u svakoj starosnoj klasi, da bi se postigao pun kredibilitet (pod pretpostavkom da je r = 5% i p = 90%). Na primer, ako je $q_{72} = 0{,}031$, potrebno je 1,082 / 0,031 = 34.903 živih lica starosti 72 godine da bi se postigao pun kredibilitet za stopu smrtnosti u toj životnoj dobi, što je velika količina podataka. Pored toga, dobijene vrednosti q_x moraju se zatim izravnati, što iziskuje dodatne analize i aproksimacije. Zbog toga je u većini slučajeva prihvatljivije da se standardne tablice mortaliteta prilagode konkretnoj podgrupi osiguranika.

Prvi korak u prilagođavanju stopa smrtnosti jeste pronaći ponder $\hat{\xi}$ koji predstavlja racio stvarnog broja umrlih lica i očekivanog broja umrlih lica za sve starosne dobi. Racio se primenjuje na sve osiguranike starosti x, pa se određuje kao agregirana vrednost kroz sve starosne dobi,

$$\hat{\xi} = \frac{\sum\limits_{x} d_x}{\sum\limits_{x} q_x^{st}},$$

gde se smrtnost q_x^{st} dobija iz standardnih tablica mortaliteta.

Potpun kredibilitet postiže se kada je racio $\hat{\xi}$ unutar marginalne greške (r) stvarnog racija ξ, sa verovatnoćom najmanje p:

$$\Pr\left(\left|\hat{\xi} - \xi\right| \le r\xi\right) \ge p.$$

Standardna formula kredibiliteta postaje

$$q_x^{est} = Z \cdot \left(\hat{\xi} \cdot q_x^{st}\right) + (1 - Z) \cdot q_x^{st} = \left(Z \cdot \hat{\xi} - Z + 1\right) \cdot q_x^{st},$$

gde je q_x^{est} procenjena stopa smrtnosti plana.

III Numerički primer

Pretpostavimo da za ženske članove nekog penzijskog plana podaci u poslednje tri godine pokazuju da je bilo 1.230 umrlih lica (Tabela 1), Potrebno je da aktuar prilagodi stope smrtnosti standardnih Tablica mortaliteta za žensko stanovništvo Srbije (Tablice mortaliteta RZS 2010-2012) i napravi nove tablice mortaliteta prilagođene konkretnom planu, pod pretpostavkom da je interval poverenja 95% uz marginalnu grešku od 5%.

Prema tablicama za Normalnu raspodelu $N(0,1)$ za p = 0,95 vrednost $z_\alpha = 1{,}96$, dok se za marginalnu grešku od 5% potpun kredibilitet postiže za $(1{,}96 / 0{,}05\ )^2 = 1.537$ umrlih lica. Kako je broj stvarno umrlih lica 1.230, broj nije

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[11] Preporuka IRS-e.

dovoljan za potpun kredibilitet, pa se faktor kredibiliteta izračunava kao kvadratni koren količnika broja stvarno umrlih lica i broja potrebnog za potpun kredibilitet

$$Z = \sqrt{\frac{1.230}{1.537}} = 0,895.$$

Očekivani broj umrlih lica je 2.097 (kolona 6). Dalje se računa ponder $\hat{\xi}$ koji predstavlja racio stvarnog broja umrlih lica i očekivanog broja umrlih lica za sve starosne dobi $\hat{\xi} = 1.230 / 2.097 = 0,59$, pa je

Ponder = 0.895 · 0,59 + (1 − 0.895) · 1 = 0,63 (kolona 8).

Obračun pokazuje da svaku stopa smrtnosti q_x^{st} iz standardne tablice mortaliteta treba korigovati ponderom 0,63 kako bi se dobila procenjena stopa smrtnosti q_x^{est} za plan.

Tabela 1. Formiranje prilagođene tablice mortaliteta

x	lx	d_x	q_x^{st}	q_x^{exp}	Očekivani br. umrlih	Prilagođena stopa q_x^{est}	Ponder	Racio
1	2	3	4	5=2/4	6=2*5	7	8=7*2/6	9=3/6
50	503		0,003455	-	1,739287	0,002177	0,630	-
51	891	3	0,003738	0 ,003922	3,332705	0,002355	0,630	1,049
52	1.484	12	0,004224	0,008245	6,268780	0,002661	0,630	1,952
53	2.192	14	0,004653	0,006380	10,198316	0,002931	0,630	1,371
54	2.459	12	0,005171	0,004975	12,718530	0,003258	0,630	0,962
55	2.858	19	0,005730	0,006728	16,375410	0,003610	0,630	1,174
56	3.540	24	0,006180	0,006914	21,874295	0,003893	0,630	1,119
57	3.655	37	0,006681	0,010043	24,418500	0,004209	0,630	1,503
58	3.493	42	0,007267	0,012012	25,380635	0,004578	0,630	1,653
⋮								
93	26	2	0,335770	0,066667	8,803892	0,211535	0,630	0,199
94	5	1	0,372676	0,190694	1,954310	0,234786	0,630	0,512
95	16	2	0,413793	0,111111	6,509793	0,260690	0,630	0,269
96	10	2	0,460268	0,166667	4,827294	0,289969	0,630	0,362
97	10	3	0,510516	0,333333	5,354294	0,321625	0,630	0,653
Ukupno	1.358	1.230			2.097			

Grafički prikaz tablica mortaliteta, to jest stopa smrtnosti, korisno je uraditi pre i nakon prilagođavanja tablica. Jedan od načina da se proceni je li prilagođavanje prikladno jeste upoređivanje stvarnih stopa smrtnosti s odgovarajućom standardnom tablicom mortaliteta, prikazano na grafikonu 1. Primećuje se da je korekcijom stopa za izračunatu vrednost 0,63 zadržan sličan oblik krive dobijen

iz standardne tablice mortaliteta. Takođe se primećuje da su nakon sedamdesete godine života veća odstupanja stvarnih stopa posmatranog plana u odnosu na stope iz standardnih tablica mortaliteta, pa je opravdano primeniti prikazani metod u ovom primeru.

Slika 1. Uporedni prikaz tablica mortaliteta
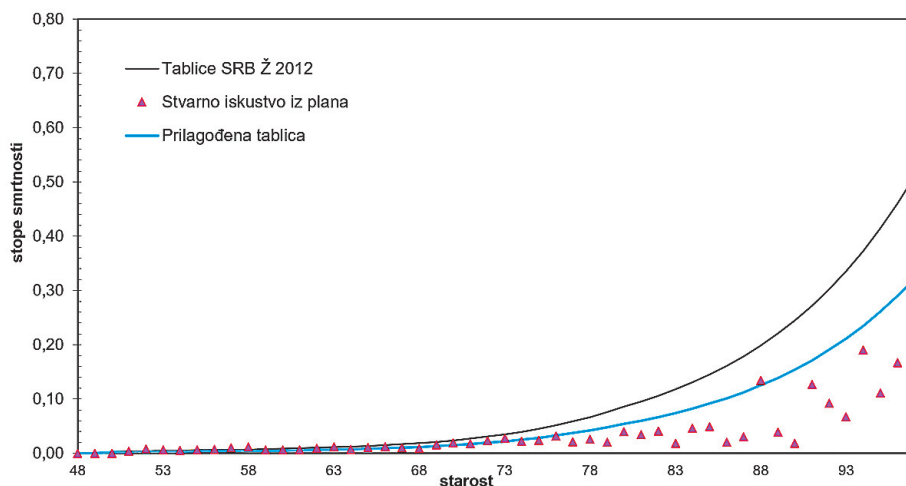
IV Zaključak

Teorija kredibiliteta danas se intenzivno koristi kod životnih i neživotnih osiguranja. Osnovni cilj primene teorije kredibiliteta jeste minimizacija grešaka između statističke ocene različitih parametara i njihove stvarne vrednosti. Teorija kredibiliteta omogućuje primenu različitih metoda i modela prilikom procene vrednosti određenih elemenata podskupa posmatrane populacije, kombinovanjem izlaznih rezultata za takav poseban podskup, s rezultatima dobijenim za populaciju u celini. U savremenoj aktuarskoj praksi životnih osiguranja kredibilitet služi kao alat za prilagođavanje standardnih tablica mortaliteta konkretnim penzijskim planovima, odnosno populaciji obuhvaćenoj tim planovima. Pri vrednovanju penzijskog plana, cilj je da se uključe iskustveni podaci učesnika obuhvaćenih planom koji se vrednuje, što teorija kredibiliteta omogućuje. Različiti modeli teorije kredibiliteta koriste se za modifikaciju standardne tablice mortaliteta stanovništva u cilju primene kvantitativnih iskustava vezanih za podgrupe stanovništva, što doprinosi povećanju pouzdanosti ocene parametara. U uslovima velike konkurencije na tržištu osiguranja, dobro poznavanje teorije kredibiliteta kao važnog alata u proceni rizika u osiguranju je od izuzetne važnosti.

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