OPTIMIZATION OF AN INTRA-SUPPLY CHAIN SYSTEM WITH UNRELIABLE PRODUCTION FACILITY

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Optimization of a manufacturer-retailer integrated type of intra-supply chain system considering unreliable production facility is explored in this study. It is assumed that a producer of consumer goods has its own sales offices, and during the fabrication process, both equipment breakdown and production of nonconforming goods seem inevitable. To explicitly address such a real intra-supply chain problem, we incorporate end products transportation cost and sales office's stock holding cost into a prior study (Chiu [01]) with the objective of deriving the optimal fabrication runtime. Solution to the problem is obtained through the use of mathematical modeling, optimization techniques, and a proposed recursive algorithm. Applicability of research result and sensitivity analyses are demonstrated thru numerical example.

Key words: Optimizer, Manufacturer-retailer integrated system, Optimal fabrication runtime, Unreliable facility, Intra-supply chain system, Rework, Multiple shipments

INTRODUCTION

This study investigates the optimal fabrication runtime for a manufacturer-retailer integrated intra-supply chain system with stochastic machine breakdown and the reworking of defective items. Economic production quantity (EPQ) based model has been constantly utilized in manufacturing sector to help derive the most economical batch size (or runtime) that minimizes total fabrication and stock holding costs [02-04]. Assumption of EPQ model [02] is simplified, it includes (i) a continuous inventory issuing policy of end products, (ii) all products made are of perfect quality, and (iii) the manufacturing equipment is always in perfect operating condition during a production run. However, in the existing supply chain environments, the assumption of a continuous issuing policy of end products is somewhat impractical because of the enormous transportation costs incurred. Consequently, the periodic multi-shipment policy is often implemented. Goyal [05] proposed a vendor-buyer integrated supply-chain system. Numerical illustrations were used to demonstrate applicability of his proposed solution process to the problem. The fill-rate of a one-warehouse multi-retailer system with was investigated via an existing approximation method by Schwarz et al. [06]. Several properties of the fill-rate policy were presented to help managerial decision makings. Banerjee and Banerjee [07] used an electronic data interchange (EDI) approach to explore a one-vendor multi-buyer inventory system under the policy of rotation fabrication cycle length. Computations of values of diverse system parameters were executed by a proposed algorithm to demonstrate how their research results are beneficial to all parties in the system. Tyan and Wee [8] used real data from Taiwanese grocery industry to investigate characteristics of the supplier-retailer partnership. The result indicated that the vendor managed inventory (VMI) case can help improve service level and cut down costs for both parties in the system. Additional studies [09-17] explored diverse aspects of supply-chain management.

Contrasting to conventional EPQ model which implicitly assumes a perfect manufacturing condition, in real-life fabrication systems owing to process deterioration and diverse unpredictable reasons, manufacturing equipment failure and output of nonconforming products are inevitable. Henig and Gerchak [18] analyzed structure of a general periodic review production-inventory system considering variable yield. Groenevelt et al. [19] explored the effects of stochastic machine breakdown under two distinct inventory control policies. One is the no-resumption (NR) policy and the other is the abort resume (AR) policy; the former policy assumed that if an equipment failure happens, fabrication of the interrupted batch will not be resumed, while the latter (i.e., AR policy) assumed that if the level of on-hand inventories falls below a preset threshold point at the time of machine breakdowns, then fabrication of the interrupted batch is resumed immediately once the production equipment is repaired. Based on these policies, the exact optimal and the approximate batch size formulae were derived, respectively. Boone et al. [20] utilized the mathematical modeling technique to investigate impact of imperfect processes on fabrication runtime for a manufacturing system with defectives and equipment failure. Lin and Gong [21] explored an EPQ model subject to an exponential decay and under a no resumption stock controlling policy. As a result, they derived a near optimal runtime under the conditions of deterministic demand, no stock-out permitted, and continuous review policy. Sana and Chaudhuri [22] investigated economic manufacturing quantity (EMQ) system wherein the fabrication processes are not perfect. Both the NR

and AR policies were taken into account on breakdown incident. Accordingly, they determined the optimal manufacturing lot-size, fabrication rate, and safety stock. Chiu [01] solved a production runtime problem featuring rework of nonconforming items and Poisson breakdown under an AR inventory control policy. They first derived the expected cost function for their proposed system and then presented theorems on convexity as well as bounds of production runtime. With the help of a recursive algorithm they were able to locate the optimal fabrication runtime. Other studies [23-39] also addressed diverse aspects of fabrication systems featuring defective products, rework, and stochastic failure of equipment.

This paper extends a prior study (Chiu [01]) by incorporating the shipping cost of end products and stock holding cost of sales offices into Chiu's model and re-solves the optimal replenishment cycle time for such an intra-supple chain system. Since this particular area has not been a focus of attention, the present work aims to bridge the research gap.

MATERIALS AND METHODS

Problem statement

Manufacturers of consumer goods operating in present-day supply chain environments often have their own sales offices to put their end products on the market. To specifically address such an intra-supply chains situation, we extend Chiu's model [01] by incorporating the shipping cost of end products and stock holding cost of sales offices into Chiu's model, and resolves the optimal replenishment cycle time for such an intra-supple chain system (see Figure 1).
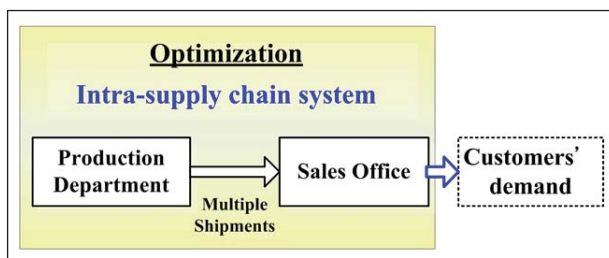

*Figure 1: Extension of Chiu's model [01]
to an intra-supply chain system*

Description of the proposed model is as follows. Considering the annual demand rate for a manufactured item is *λ* and this product can be fabricated at an annual rate of *P1*. In the fabrication process of this item, *x* proportion of nonconforming item is produced randomly. Hence, production rate of nonconforming products *d1=P1x*. Shortage is not allowed in this study, so (*P1 - d1 - λ*) > 0 is assumed. To guarantee conformance of quality, all manufactured products are inspected and this relevant cost is considered a part of unit manufacturing cost *C*. With the intention of lowering cost and improving quality, all nonconforming products can be reworked and repaired,

at a rate of *P2* when regular fabrication process ends. In the finishing point of rework process, end products are distributed to the sales office, under a discontinuous multi-shipment policy, whereas *n* fixed amount installments of the completed batch are delivered at a fixed interval of time during product delivery time of each replenishment cycle.

It is further assumed that in fabrication process the production equipment is subject to a stochastic failure that obeys the Poisson distribution. An AR policy is adopted once a machine failure happens. Under the AR policy, production equipment will be repaired right away, and the interrupted batch is resumed immediately once the equipment is restored. In this study, we assume the repair time *g* for breakdown is constant and in the case that the repair time exceeds *g*, a spare equipment will be used immediately to avoid further delay of the remaining fabrication process. To ease readers' comparison efforts, the same notation (see Appendix A) is adopted as that in Chiu [01]. Let *β* denote the mean of Poisson machine failure rate and t be the fabrication time to a machine failure occurrence. Two separate cases as shown in the following subsections should to be explored, respectively.

Mathematical modeling & analysis

Case 1: A machine failure happens in fabrication uptime

Case one describes that the time to a machine failure is shorter than fabrication uptime (i.e., *t < t1*). Because the A/R is used, the machine goes under repair at once, and fabrication of the interrupted batch resumed instantaneously when machine is restored. Figure 2 illustrates the on-hand inventory level of perfect quality products during a replenishment cycle at the production unit.

By observing Figure 2, it can be seen that when an equipment breakdown happens, the on-hand stock level stays at *H2'* for a period of time *t_r* (i.e., the repair time) till machine is restored. Then, the level of inventory continues to pile up during fabrication of the remaining batch to a level of *H1'* in the end of regular fabrication process. It follows that the reworking of nonconforming products during *t2'* brings the level of inventory to *H'*. So, the replenishment cycle length *T'* is as follows:

$$T' = t_1 + t_r + t_2' + t_3' \tag{1}$$

Upon completion of rework, the total nonconforming products, cost for rework, and cost for holding reworked products are given as follows:

$$d_1 t_1 = x P_1 t_1 \tag{2}$$

$$C_R \left(t_1 P_1 x \right) \tag{3}$$

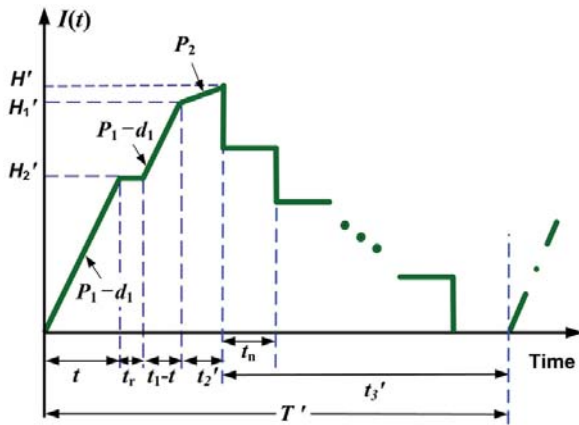$$h_1 \frac{d_1 t_1}{2} t_2' \tag{4}$$

Figure 2: The on-hand stock level during a replenishment cycle [01]

Equation (5) exhibits total fabrication-inventory-transportation cost per replenishment cycle in the production unit, it consists of (i) variable fabrication cost, (ii) setup cost, (iii) repair cost for failure equipment, (iv) cost for rework, (v) cost for holding reworked products, (vi) fixed and variable shipping costs, (vii) holding cost for safety stocks; and (viii) inventory holding costs for perfect quality and nonconforming products in the entire replenishment cycle.

$$
\begin{aligned}
&C\left(P_1 t_1\right)+K+M+C_R\left(xP_1 t_1\right)+h_1\frac{d_1 t_1}{2}t_2' \\
&+nK_1+C_T\left(P_1 t_1\right)+h_3(\lambda t_r)T' \\
&+h\left[\frac{H_1'+d_1 t_1}{2}t_1+(H_2'+d_1 t)t_r+\right. \\
&\left.+\frac{H_1'+H'}{2}(t_2')+\frac{n-1}{2n}H't_3'\right]
\end{aligned} \tag{5}
$$

Figure 3 shows the status of sales office's on-hand stock level in replenishment cycle, where the product demand between two consecutive shipments is λt_n, and leftover products in each fixed time interval t_n (after λt_n being satisfied) is as follows:

$$
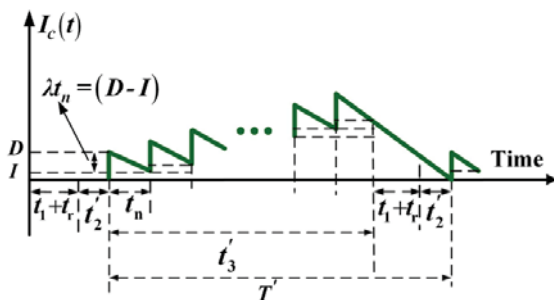I = D - \lambda t_n \tag{6}
$$

Figure 3: The sales office's on-hand inventory level during a replenishment cycle

Equation (7) shows total stock holding cost in the sales office.

$$
h_2\left[\frac{n(n+1)}{2}It_n+n\frac{(D-I)}{2}t_n+\frac{nI}{2}\left(t_1+t_r+t_2'\right)\right] \tag{7}
$$

Incorporating the aforementioned sales office's stock holding cost into Chiu's model [01], we obtain the total system cost $TC_1(t_1)$ for our proposed model as follows:

$$
\begin{aligned}
TC_1(t_1) &= C\left(P_1 t_1\right)+K+M+C_R\left(xP_1 t_1\right)+ \\
&+h_1\frac{d_1 t_1}{2}t_2'+nK_1+C_T\left(P_1 t_1\right)+h_3(\lambda t_r)T' \\
&+h\left[\frac{H_1'+d_1 t_1}{2}t_1+\frac{H_1'+H'}{2}(t_2')+(H_2'+d_1 t)t_r+\frac{n-1}{2n}H't_3'\right] \\
&+h_2\left[\frac{n(n+1)}{2}It_n+n\frac{(D-I)}{2}t_n+\frac{nI}{2}\left(t_1+t_r+t_2'\right)\right]
\end{aligned} \tag{8}
$$

For the reason that x is a random variable, we use expected value to cope with its randomness, and by substituting relevant system variables in Equation (8) and with additional derivations, $E[TC_1(t_1)]$ is gained as shown in Equation (9).

$$
\begin{aligned}
E[TC_1(t_1)] &= K+M+nK_1+htP_1 g \\
&+\left[CP_1+C_R P_1 E[x]+C_T P_1+h_3 P_1 g-\frac{P_1 g(h-h_2)}{2}\left(1-\frac{1}{n}\right)\right]\cdot t_1 \\
&+\left[\begin{array}{l}\dfrac{hP_1^2}{2\lambda}+\dfrac{hP_1^2 E[x]}{P_2}(1-E[x])+\dfrac{h_1 P_1^2 E[x]^2}{2P_2}+\dfrac{h_2 P_1}{2}\\[6pt]+\dfrac{h_2 P_1^2 E[x]}{2P_2}+\dfrac{(h-h_2)P_1}{2n}\left[\dfrac{P_1 E[x]}{P_2}+1-\dfrac{P_1}{\lambda}\right]\end{array}\right]t_1^2
\end{aligned} \tag{9}
$$

Case 2: No machine failure happens in fabrication uptime

Case two expresses that the time to a machine failure is longer than fabrication uptime (i.e., $t > t_r$). In other words, machine failure does not take place in fabrication process (Figure 4).
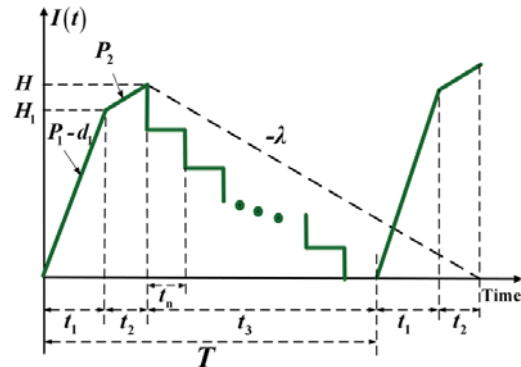

Figure 4: The on-hand stock level during a replenishment cycle when machine failure does not take place

The fabrication cycle length *T* in Case 2 is as follows:

$$T = t_1 + t_2 + t_3 \qquad (10)$$

Equation (11) shows total fabrication-inventory-transportation cost per replenishment cycle in the production unit, it comprises (i) variable fabrication cost, (ii) setup cost, (iii) repair cost for failure equipment, (iv) cost for rework, (v) cost for holding reworked products, (vi) fixed and variable shipping costs, (vii) holding cost for safety stocks; and (viii) inventory holding costs for perfect quality and nonconforming products in the entire replenishment cycle.

$$\begin{aligned}
& C\left(P_1 t_1\right) + K + C_R\left(xP_1 t_1\right) + h_1 \frac{d_1 t_1}{2} t_2 + nK_1 + C_T\left(P_1 t_1\right) \\
& + h_3\left(\lambda t_r\right)T + h\left[\frac{H_1 + d_1 t_1}{2} t_1 + \frac{H_1 + H}{2}\left(t_2\right) + \frac{n-1}{2n}Ht_3\right]
\end{aligned} \qquad (11)$$

Equation (12) gives the total stock holding cost in the sales office.

$$h_2\left[\frac{n(n+1)}{2}It_n + n\frac{(D-I)}{2}t_n + \frac{nI}{2}\left(t_1 + t_2\right)\right] \qquad (12)$$

Incorporating the aforementioned sales office's stock holding cost into Equation (11), we obtain the total system cost $TC_2(t_1)$ for our proposed model as follows:

$$\begin{aligned}
TC_2(t_1) = & \ C\left(P_1 t_1\right) + K + C_R\left(xP_1 t_1\right) + \\
& + h_1 \frac{d_1 t_1}{2} t_2 + nK_1 + C_T\left(P_1 t_1\right) \\
& + h_3\left(\lambda t_r\right)T + h\left[\frac{H_1 + d_1 t_1}{2} t_1 + \frac{H_1 + H}{2}\left(t_2\right) + \frac{n-1}{2n}Ht_3\right] \\
& + h_2\left[\frac{n(n+1)}{2}It_n + n\frac{(D-I)}{2}t_n + \frac{nI}{2}\left(t_1 + t_2\right)\right]
\end{aligned} \qquad (13)$$

Applying expected value of *x* to cope with its randomness, and substituting relevant system variables in Equation (13) and with extra derivations, $E[TC_2(t_1)]$ can be obtained as shown in Equation (14).

$$\begin{aligned}
E[TC_2(t_1)] = & \left[CP_1 + C_R P_1 E[x] + C_T P_1 + h_3 P_1 g\right]t_1 + K + nK_1 \\
& + \left[\begin{array}{l}
\dfrac{hP_1^2}{2\lambda} + \dfrac{hP_1^2 E[x]}{P_2}\left(1 - E[x]\right) + \dfrac{h_1 P_1^2 E[x]^2}{2P_2} + \dfrac{h_2 P_1}{2} \\
+ \dfrac{h_2 P_1^2 E[x]}{2P_2} + \dfrac{(h - h_2)P_1}{2n}\left[\dfrac{P_1 E[x]}{P_2} + 1 - \dfrac{P_1}{\lambda}\right]
\end{array}\right]t_1^2
\end{aligned} \qquad (14)$$

Integration of cases 1 and 2

Since fabrication process is subject to a Poisson distribution equipment failure rate. So, time to an equipment failure follows the exponential distribution with density function $f(t) = \beta e^{-\beta t}$ and the cumulative density function $F(t) = 1 - e^{-\beta t}$. To calculate $E[TCU(t_1)]$, we need to integrate cost

functions in both case one and two, and consider the expected cycle length to cope with its variation as follows:

$$E[TCU(t_1)] = \frac{\left\{\int_0^{t_1} E[TC_1(t_1)]f(t)dt + \int_{t_1}^{\infty} E[TC_2(t_1)]f(t)dt\right\}}{E[T]} \qquad (15)$$

where

$$E[T] = \int_0^{t_1} E[T']f(t)dt + \int_{t_1}^{\infty} E[T]f(t)dt \qquad (16)$$

Substitute Equations (9), (14), and (16) into Equation (15), and with extra derivations we obtain $E[TCU_2(t_1)]$ as shown in Equation (17).

$$E[TCU(t_1)] =$$

$$= \lambda\left\{
\begin{array}{l}
\left(C + C_R E[x] + C_T + h_3 g\right) + \dfrac{(K + nK_1)}{t_1 P_1} - hg\left(e^{-\beta t_1}\right) \\[1em]
+ \dfrac{1}{2}\left[\begin{array}{l}
\dfrac{hP_1}{\lambda} + \dfrac{hP_1 E[x]}{P_2}\left(1 - E[x]\right) + \dfrac{h_1 P_1 E[x]^2}{P_2} + h_2 \\[0.5em]
+ \dfrac{(h - h_2)}{n}\left[\dfrac{P_1 E[x]}{P_2} + 1 - \dfrac{P_1}{\lambda}\right] + \dfrac{h_2 P_1 E[x]}{P_2}
\end{array}\right]\cdot t_1 \\[2em]
+ \left[\dfrac{M}{P_1} + \dfrac{hg}{\beta}\right]\left(\dfrac{1 - e^{-\beta t_1}}{t_1}\right) - (h - h_2)\left[\dfrac{g}{2}\left(1 - \dfrac{1}{n}\right)\right]\left(1 - e^{-\beta t_1}\right)
\end{array}
\right\} \qquad (17)$$

or

$$E[TCU(t_1)] =$$

$$= \lambda\left\{
\begin{array}{l}
\pi_1 + \dfrac{(K + nK_1)}{t_1 P_1} + \pi_2 t_1 + \left[\dfrac{M}{P_1} + \dfrac{hg}{\beta}\right]\left(\dfrac{1 - e^{-\beta t_1}}{t_1}\right) \\[1em]
- hg\left(e^{-\beta t_1}\right) - (h - h_2)\left[\dfrac{g}{2}\left(1 - \dfrac{1}{n}\right)\right]\left(1 - e^{-\beta t_1}\right)
\end{array}
\right\} \qquad (18)$$

where

$$\pi_1 = \left[C + C_R E[x] + C_T + h_3 g\right]$$

and

$$\begin{aligned}
\pi_2 = & \ \frac{1}{2}\left[\frac{hP_1}{\lambda} + \frac{hP_1 E[x]}{P_2}\left(1 - E[x]\right) + \frac{h_1 P_1 E[x]^2}{P_2} + h_2 + \right. \\
& \left. + \frac{(h - h_2)}{n}\left[\frac{P_1 E[x]}{P_2} + 1 - \frac{P_1}{\lambda}\right] + \frac{h_2 P_1 E[x]}{P_2}\right]
\end{aligned} \qquad (19)$$

RESULTS AND DISCUSSION

Convexity and optimal fabrication runtime

Once $E[TCU(t_1)]$ is obtained, we apply second-derivative of Equation (18) with respect to t_1 to test for convexity as follows:

$$\frac{d^2 E[TCU(t_1)]}{d^2 t_1^2} =$$

$$= \lambda\left\{
\begin{array}{l}
\dfrac{2(K + nK_1)}{t_1^3 P_1} - (h - h_2)\left[\dfrac{g}{2}\left(1 - \dfrac{1}{n}\right)\right]\left(\beta^2 e^{-\beta t_1}\right) - hg\left(\beta^2 e^{-\beta t_1}\right) \\[1em]
+ \left[\dfrac{M}{P_1} + \dfrac{hg}{\beta}\right]\left(\dfrac{2\left(1 - e^{-\beta t_1}\right)}{t_1^3} - \dfrac{2\beta e^{-\beta t_1}}{t_1^2} - \dfrac{\beta^2 e^{-\beta t_1}}{t_1}\right)
\end{array}
\right\} \qquad (20)$$

Because annual demand $\lambda > 0$, to show that Equation (20) > 0 is equivalent to demonstrate that Equation (21) > 0.

$$\left\{ \begin{array}{l} \dfrac{2(K+nK_1)}{t_1^3 P_1} - \left[(h-h_2)\left[\dfrac{g}{2}\left(1-\dfrac{1}{n}\right)\right]-hg\right]\left(\beta^2 e^{-\beta t_1}\right) \\ + \left[\dfrac{M}{P_1}+\dfrac{hg}{\beta}\right]\left(\dfrac{2\left(1-e^{-\beta t_1}\right)}{t_1^3}-\dfrac{2\beta e^{-\beta t_1}}{t_1^2}-\dfrac{\beta^2 e^{-\beta t_1}}{t_1}\right) \end{array} \right\} > 0 \qquad 21)$$

That is

if
$$\left\{ \begin{array}{l} 2(K+nK_1)\beta-\left[(h-h_2)\left[\dfrac{g}{2}\left(1-\dfrac{1}{n}\right)\right]+hg\right]\left(\beta^2 e^{-\beta t_1}\right)\left(t_1^3 P_1\beta\right) \\ +(M\beta+hgP_1)\left[2\left(1-e^{-\beta t_1}\right)-\left(2\beta e^{-\beta t_1}\right)t_1-\left(\beta^2 e^{-\beta t_1}\right)t_1^2\right] \end{array} \right\} > 0 \qquad 22)$$

then $\dfrac{d^2 E\left[TCU(t_1)\right]}{dt_1^2} > 0$

Let

$$\pi_3 = \left[(h-h_2)\left[\dfrac{g}{2}\left(1-\dfrac{1}{n}\right)\right]+hg\right] \qquad 23)$$

and $\pi_4 = M\beta + hgP_1$ \qquad 24)

Substitute Equations (23) and (24) in Equation (22), it becomes as follows:

if
$$\left\{ \begin{array}{l} 2(K+nK_1)\beta-\pi_3\left(\beta^2 e^{-\beta t_1}\right)\left(t_1^3 P_1\beta\right) \\ +\pi_4\left[2\left(1-e^{-\beta t_1}\right)-\left(2\beta e^{-\beta t_1}\right)t_1-\left(\beta^2 e^{-\beta t_1}\right)t_1^2\right] \end{array} \right\} > 0 \qquad 25)$$

then $\dfrac{d^2 E\left[TCU(t_1)\right]}{dt_1^2} > 0$

The left-hand side of Equation (25) can be rearranged as

if
$$\left\{ 2(K+nK_1)\beta+2\left(1-e^{-\beta t_1}\right)\pi_4 - \\ -\left[\pi_3\beta^2 P_1 t_1^2+\pi_4(2+\beta t_1)\right]\left(\beta e^{-\beta t_1}\right)t_1 \right\} > 0 \qquad 26)$$

or

if
$$\left[2(K+nK_1)\beta+2\left(1-e^{-\beta t_1}\right)\pi_4\right] > \\ > \left[\pi_3\beta^2 P_1 t_1^2+\pi_4(2+\beta t_1)\right]\left(\beta e^{-\beta t_1}\right)t_1 \qquad 27)$$

Let

$$\gamma(t_1) = \dfrac{2(K+nK_1)\beta+2\left(1-e^{-\beta t_1}\right)\pi_4}{\left[\pi_3\beta^2 P_1 t_1^2+\pi_4(2+\beta t_1)\right]\left(\beta e^{-\beta t_1}\right)} \qquad 28)$$

Equation (25) becomes as follows:

if $\quad 0 < t_1 < \gamma(t_1)$ \qquad 29)

then $\dfrac{d^2 E\left[TCU(t_1)\right]}{dt_1^2} > 0$

Once we proved $E[TCU(t_1)]$ is convex, then setting first-derivative of $E[TCU(t_1)] = 0$, we can find the optimal fabrication runtime t_1^*. Let the following be the upper and lower bounds for fabrication runtime t_1^*:

$$t_{1U}^* = \sqrt{\dfrac{(K+nK_1)\beta+\pi_4}{P_1\beta\pi_2}} \qquad 30)$$

$t_{1L}^* =$ the positive root of

$$\left\{ \dfrac{-\pi_4 \pm \sqrt{\pi_4^2+4P_1\left(\pi_2+\pi_3\beta\right)(K+nK_1)}}{2P_1\left[\pi_2+\pi_3\beta\right]} \right\} \qquad 31)$$

Theorem 3.1: $t_1 L^* < t_1^* < t_1 U^*$

Setting the following first-derivative of $E[TCU(t_1)] = 0$:

$$\dfrac{dE[TCU(t_1)]}{dt_1} = \\ = \lambda\left\{ \begin{array}{l} \pi_2+\pi_3\left(\beta e^{-\beta t_1}\right)-\dfrac{(K+nK_1)}{t_1^2 P_1} \\ +\left[\dfrac{M}{P_1}+\dfrac{hg}{\beta}\right]\left(\dfrac{-\left(1-e^{-\beta t_1}\right)}{t_1^2}+\dfrac{\beta e^{-\beta t_1}}{t_1}\right) \end{array} \right\} = 0 \qquad 32)$$

That is

$$\left\{ \begin{array}{l} \pi_2+\pi_3\left(\beta e^{-\beta t_1}\right)-\dfrac{(K+nK_1)}{t_1^2 P_1}+ \\ +\left[\dfrac{M}{P_1}+\dfrac{hg}{\beta}\right]\left(\dfrac{-\left(1-e^{-\beta t_1}\right)}{t_1^2}+\dfrac{\beta e^{-\beta t_1}}{t_1}\right) \end{array} \right\} = 0 \qquad 33)$$

Multiplying Equation (33) by $\left(P_1 t_1^2 \beta\right)$, we obtain

$$\left\{ \begin{array}{l} \left[\pi_2+\pi_3\left(\beta e^{-\beta t_1}\right)\right]P_1 t_1^2\beta-(K+nK_1)\beta+ \\ +\pi_4\left[-\left(1-e^{-\beta t_1}\right)+\beta e^{-\beta t_1}t_1\right] \end{array} \right\} = 0 \qquad 34)$$

or

$$\left[\pi_2+\pi_3\left(\beta e^{-\beta t_1}\right)\right]P_1\beta t_1^2+\left(\pi_4\beta e^{-\beta t_1}\right)t_1 - \\ -\left[(K+nK_1)\beta+\left(1-e^{-\beta t_1}\right)\pi_4\right] = 0 \qquad 35)$$

It can be seen from Equation (35) that t_1^* is the positive root of the following

$$t_1^* = \dfrac{-\left(\beta e^{-\beta t_1}\pi_4\right) \pm \sqrt{\left[\beta e^{-\beta t_1}\pi_4\right]^2-4P_1\beta\left[\pi_2+\pi_3\left(\beta e^{-\beta t_1}\right)\right]\cdot\left[-\left[(K+nK_1)\beta+\left(1-e^{-\beta t_1}\right)\pi_4\right]\right]}}{2\left[\pi_2+\pi_3\left(\beta e^{-\beta t_1}\right)\right]P_1\beta} \qquad 36)$$

Further rearrange Equation (35) as follows:

$$P_1\beta\pi_2 t_1^2-\left[(K+nK_1)\beta+\pi_4\right]+ \\ +\left[P_1\beta^2\pi_3 t_1^2+\pi_4(1+\beta t_1)\right]\left(e^{-\beta t_1}\right) = 0 \qquad 37)$$

or

$$\left[P_1\beta^2\pi_3 t_1^2 + \pi_4\left(1+\beta t_1\right)\right]\left(e^{-\beta t_1}\right) = \\ = \left[\left(K+nK_1\right)\beta + \pi_4\right] - P_1\beta\pi_2 t_1^2 \qquad (38)$$

or

$$e^{-\beta t_1} = \frac{\left[\left(K+nK_1\right)\beta + \pi_4\right] - P_1\beta\pi_2 t_1^2}{P_1\beta^2\pi_3 t_1^2 + \pi_4\left(1+\beta t_1\right)} \qquad (39)$$

Since $e^{-\beta t_1}$ is the complement of cumulative density function, so $0 \le e^{-\beta t_1} \le 1$, and $e^{-\beta t_1} = 0$ and $e^{-\beta t_1} = 1$ are the upper and lower bounds of $e^{-\beta t_1}$, respectively. Now, substituting its upper and lower bounds into Equation (36), we obtain the following:

$$t_{1U}^* = \sqrt{\frac{\left(K+nK_1\right)\beta + \pi_4}{P_1\beta\pi_2}}$$

$t_{1L}^* = $ the positive root of

$$\cdot\left\{\frac{-\pi_4 \pm \sqrt{\pi_4^2 + 4P_1\left(\pi_2 + \pi_3\beta\right)\left(K+nK_1\right)}}{2P_1\left[\pi_2 + \pi_3\beta\right]}\right\}$$

and $t_1 L^* < t_1^* < t_1 U^*$.

It is noted that t_1^* falls within the bounds, although we cannot derive it as a closed form solution.

Let $\delta(t_1) = e^{-\beta t_1} = \frac{\left[\left(K+nK_1\right)\beta + \pi_4\right] - P_1\beta\pi_2 t_1^2}{P_1\beta^2\pi_3 t_1^2 + \pi_4\left(1+\beta t_1\right)}$ (from Equation (39)),

since $0 \le e^{-\beta t_1} \le 1$, so $0 \le \delta(t_1) \le 1$. The following proposed recursive algorithm enables us to get the t_1^*:

1. Initially let $\delta(t_1) = 0$ and $\delta(t_1) = 1$ and first compute upper and lower bounds of t1*, respectively (that is to calculate initial values of $[t_1 L^*, t_1 U^*]$).
2. By substituting current values of [t1L*, t1U*] in $e^{-\beta t_1}$ and re-compute the new values for bounds (i.e., δL and δU) for $e^{-\beta t_1}$, where $\delta L < \delta(t_1) < \delta U$.
3. Let $\delta(t_1) = \delta L$ and $\delta(t_1) = \delta U$ and re-calculate and update values of bounds for t_1^*, respectively (that is $[t_1 L^*, t_1 U^*]$).
4. Repeat steps 2 and 3, until there is no significant difference between $t_1 L^*$ and $t_1 U^*$ (the criterion here is no significant difference in terms of their effects on the expected system cost function $E[TCU(t_1^*)]$).
5. Stop. Optimal fabrication runtime t_1^* found.

Numerical example with discussion

To ease comparison efforts for readers, the same numerical example as in Chiu [01] is used. The corresponding values of system variables are listed as follows:

λ=4,000 items per year,

P_1=10,000 items per year,

P_2=5,000 items per year,

β=0.5 per year – the mean of Poisson breakdown rate,

M=\$500 per failure,

g=0.018 year – the fixed equipment repairing time (the same as t_r),

x=[0, 0.2] – interval of nonconforming rate which follows a uniform distribution,

C_R=\$0.50 – unit rework cost,

h_1=\$0.80 – unit holding cost per item reworked,

K_1=\$80 per shipment,

n =4 – the number of shipments per cycle,

C_T=\$0.001 per items shipped,

K=\$450 per cycle,

C=\$2 – unit fabrication cost,

h=\$0.6 – unit stock holding cost at the production unit,

h_2=\$1.5 – unit stock holding cost at the sales office.

First of all, the convexity of $E[TCU(t_1)]$ (Equation (29)) is tested. From computations of Equations (30) and (31), $t_1 U^*$ = 0.2834 and $t_1 L^*$ = 0.1941 are obtained, respectively. Substitute $t_1 U^*$ and $t_1 L^*$ in Equations (28) and (29), we confirm both $t_1 U^* < \gamma(t_1 U^*)$ = 2.1663 and $t_1 L^* < \gamma(t_1 L^*)$ = 0.9233. Therefore, $[TCU(t_1)]$ is convex.

Then, we apply proposed recursive algorithm (as stated in end of aforementioned subsection 3.1) to find the optimal fabrication runtime t_1^*. Table 1 exhibits the step-by-step iterative results of the algorithm. It can be seen that the initial values of $E[TCU(t_1 U^* = 0.2834)]$ = \$11,530.67 and $E[TCU(t_1 L^* = 0.1941)]$ = \$11,370.03. By applying algorithm further, at the fourth step it shows that $E[TCU(t_1 L^*)]$ = $E[TCU(t_1 U^*)]$ = \$11,366.29, at $t_1 U^* = t_1 L^*$ = 0.2040. That is the optimal replenishment runtime t_1^* = 0.2040, and $E[TCU(t_1^*)]$ = \$11,366.29.

Effects of variations in h_2 on total holding cost at sales office and on $E[TCU(t_1^*)]$ are explored and exhibited in Table 2. It is noted that as h_2 (or the ratio of h_2/h) increases, both total holding cost at sales office and $E[TCU(t_1^*)]$ increases notably.

Further analysis exposes effects of variations in h_2/h ratio on t_1^* (see Figure 5). It can be seem that that as h_2/h ratio increases, the optimal fabrication runtime t_1^* declines significantly.

Moreover, analytical results reveal the effects of variations in number of shipments n on $E[TCU(t_1)]$ and on optimal runtime t_1^* (see Figure 6). It can be seen that is noted that as n raises, $E[TCU(t_1)]$ goes higher slightly, but t_1^* goes up notably.

The effect of fabrication runtime t_1 on $E[TCU(t_1)]$ is explored and the result is depicted in Figure 7. It reconfirms that at t_1^* = 0.2040 the optimal $E[TCU(t_1)]$ = \$11,366.

In a highly competitive business environment, our research results for such an intra-supply chain system can offer valuable in-depth information of effects of different stock holding costs of diverse sales offices on system parameters and optimal operating policy, to facilitate managerial decision making.

Table 1: Results obtained from the proposed recursive algorithm for finding t_1*

β	Step #	t_1U*	$\delta_L = e^{-\beta t_{1U}}$	t_1L*	$\delta_U = e^{-\beta t_{1U}}$	Difference between t_1U* & t_1L*	[U] $E[TCU(t_1U*)]$	[L] $E[TCU(t_1L*)]$	Difference between [U] & [L]
0.5	initial	-	0.0000	-	1.0000	-	-	-	-
	1st	0.2834	0.8679	0.1941	0.9075	0.0893	$11,530.67	$11,370.03	$160.64
	2nd	0.2075	0.9014	0.2036	0.9032	0.0039	$11,366.73	$11,366.30	$0.43
	3rd	0.2042	0.9029	0.2040	0.9030	0.0002	$11,366.30	$11,366.29	$0.01
	4th	**0.2040**	**0.9030**	**0.2040**	**0.9030**	**0.0000**	**$11,366.29**	**$11,366.29**	**$0.00**

Table 2: Effects of variations in h_2 on total holding cost at sales office and on $E[TCU(t_1*)]$

h_2	0.30	0.45	0.60	0.75	0.90	1.05	1.20	1.35	1.50	1.65	1.80	1.95
h_2/h ratio	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25
Total holding cost at sales office	$264	$374	$473	$564	$649	$727	$801	$871	$937	$1001	$1061	$1119
$E[TCU(t_1*)]$	$10489	$10617	$10739	$10854	$10965	$11071	$11173	$11271	$11366	$11459	$11548	$11636
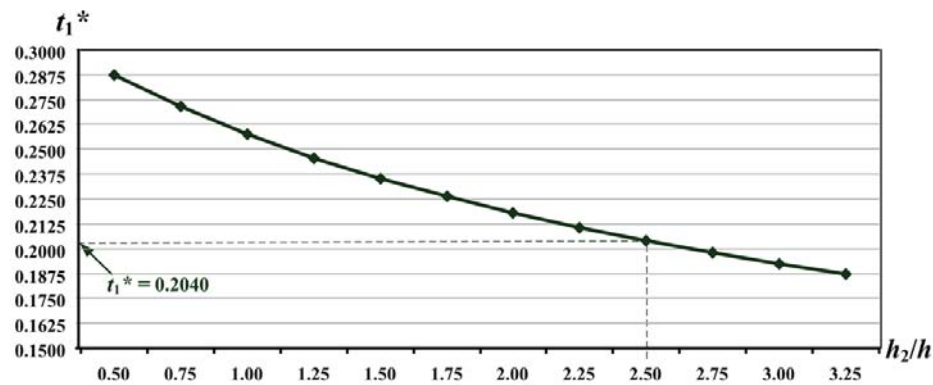
Figure 5: Effects of variations in h_2/h ratio on t_1*
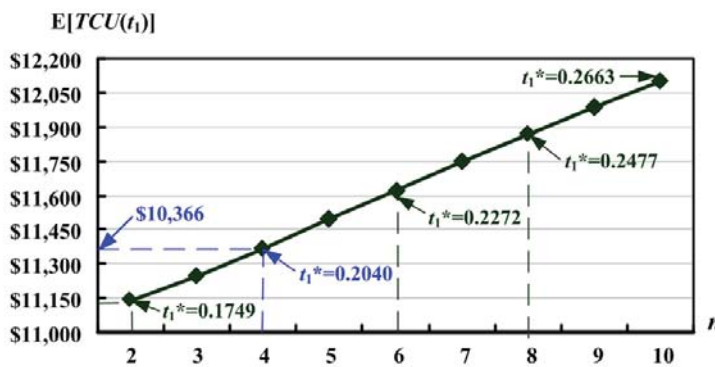
Figure 6: Effects of variations in number of deliveries n on $E[TCU(t_1)]$ and t_1*
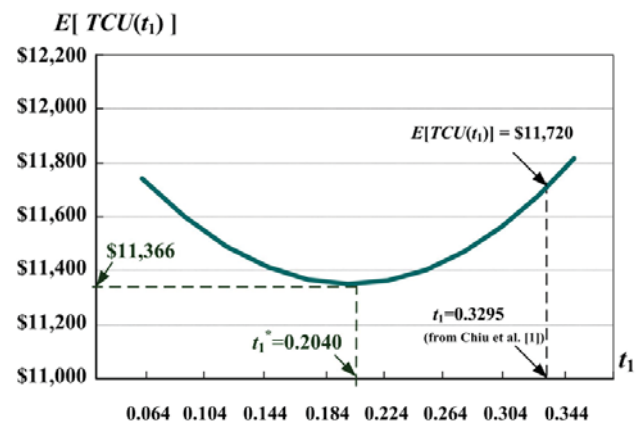
Figure 7: Behavior of $E[TCU(t_1)]$ with respect to the fabrication runtime t_1

CONCLUSION

This study explores the optimal fabrication runtime for an imperfect intra-supply chain system. With the help of mathematical modeling and optimization techniques, we are able to obtain system cost function, theorem on convexity, and bounds for fabrication runtime. By applying a recursive algorithm, we find the optimal fabrication runtime. A numerical example helps us show how the proposed solution procedure works. Without the in-depth exploration of such a realistic intra-supply chain system, the optimal fabrication runtime and various important facts of the system cannot be revealed (see Figures 5-7 and Table 2). This study fills the research gap in this specific area, and to investigate impact from a stochastic production rate on the same model may be an interesting subject for future research.

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APPENDIX - A

Notations

t_1=fabrication uptime,

t_2'= rework time (in the breakdown case),

t_r=a constant (or maximal allowable) equipment repair time, it is the same as g,

t_3'=delivery time (in the breakdown case),

t=time to equipment failure,

β=number of machine failure per year – a random variable obeys the Poisson distribution,

K=fabrication setup cost,

M=machine repair cost,

h=holding cost per product in production department,

C_R=unit rework cost,

h_1=unit holding cost per item reworked,

h_2=unit holding cost at the sales office,

K=fixed cost per delivery,

C_T=unit shipping cost,

H'=on-hand inventory level at the time when rework finishes (in the breakdown case),

H_2'=on-hand inventory level at the time when machine failure happens,

H_1'=on-hand inventory level at the time when fabrication process finishes (in the breakdown case),

T'=cycle length (in the breakdown case),

Q=batch size,

h_3=holding cost per safety stock,

$TC_1(t_1)$ = total system cost per cycle (in the breakdown case),

$E[TC_1(t_1)]$ = expected system cost per cycle (in the breakdown case),

t_2=rework time (in the case of no equipment failure taking place),

t_3=delivery time (in the case of no equipment failure taking place),

T=cycle length (in the case of no equipment failure taking place),

H=on-hand inventory level at the time when rework finishes (in the case of no equipment failure taking place),

H_1=on-hand inventory level at the time when fabrication process finishes (in the case of no equipment failure taking place),

$TC_2(t_1)$ = total system cost per cycle (in the case of no equipment failure taking place),

$E[TC_2(t_1)]$ = expected system cost per cycle (in the case of no equipment failure taking place),

D =quantity of end products (a fixed number) transported to retail store per shipment,

I = number of left over products per time interval tn, after satisfying demand in tn,

$I(t)$= on-hand inventory level at time t,

$I_d(t)$= on-hand inventory level of defective items at time t,

$I_c(t)$= on-hand stock level at sales office at time t,

T=cycle length (whether an equipment failure occurs or not),

$TCU(t_1)$ = total system cost per unit time (whether an equipment failure occurs or not),

$E[TCU(t_1)]$ = the long-run average system cost per unit time (whether an equipment failure occurs or not).

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