doi:10.5937/jaes16-15297 Paper number: 16(2018)2, 547, 410 - 415

SAGS AND FREQUENCIES OF NATURAL OSCILLATIONS OF COMPOSITE TWO-LAYER ISOTROPIC PLATES IN CASE OF CHANGE OF THICKNESS OF ONE OF THE LAYERS

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The article presents the results of a research of a composite isotropic two-layer plate of a square outline. We conduct the research for a number of plates with the variable thickness of layers and different conditions of fixing: pivotally supported and rigidly jammed on a circuit. By results of numerical researches we define the maximum sag of W0 from action of a static load and the frequency of natural cross oscillations ω of composite plates. By the results we define the proportionality coefficient K in a formula of the prof. V. I. Korobko. By the results of researches we construct the diagrams of change of the maximum sag and frequency of natural oscillations in case of change of thickness of layers of a composite plate.

Key words: A composite plate, A maximum sag, Frequency of natural oscillations, Proportionality coefficient K

INTRODUCTION

The composite plate represents the construction consisting of separate layers which are connected by means of compliant mechanical bindings. Such systems are often used in case of strengthening of structures by upbuilding of a layer that is dictated by increase in loading in case of reconstruction, change of operating conditions of the building, etc.

Design of modern buildings and constructions, and also their reconstruction and strengthening is inseparably connected with the analysis of their toughness, rigidness and stability under the influence of static and dynamic loads. The models of somestructures are presented in the form of composite plates with different boundary conditions and their researches are most fully reflected in [1, 2]. Such scientists as V. I. Korobko, A. V. Turkov and K. V. Marfin [3 … 11], E. V. Karpova [12], N. S. Abashina [13] were engaged in researches of correlation of the maximum sags and frequencies of natural oscillations of composite plates of different forms. In case of the solution of these problems bynumerical methods we often face the question of accuracy of the received results. For plate finite elements it depends, in particular, on density of a mesh spacing of the initial plate. Besides, the quantity of final elements of construction is often dictated by a problem of the best approximation of an outline of a plate.

There is a fundamental dependence of professor V. I. Korobko [3] connecting the maximum sag of W0 in case

$$W_0 \cdot \omega^2 = K \frac{q}{m}$$

of action of a uniformly distributed load q and frequency of natural cross oscillations of a solid isotropic plate ω:

As the expression standing in the right part doesn't depend on the stiffness propertiesof a plate, it is possible to make the assumption of applicability of this correlation to composite plates.

Use of analytical dependence about correlation of the maximum sag and oscillation frequencies simplifies the decision of designer tasks. Nowadays there have been made researches of two-layer composite plates of different outline with the identical thickness of layers [5 … 12] with use of the fundamental regularity offered by V. I. Korobko. At the same time the number of questions of deforming of composite plates in case of a static and dynamic load, in particular, the tasks of correlation of the maximum sag and frequencies of natural oscillations of two-layer plates at different thickness of layers and different boundary conditions, represents both theoretical, and practical interest.In this connection researches in this field of construction mechanics areof current interest.

RESEARCHES OF THE TWO-LAYER PLATE

A model structure is a two-layer square plate with the side of 1 m. At the first stage calculation of plates is performed (at first – pivotally supported, and then – rigidly jammed on a circuit), the low layer of each of which has constant thickness of 10 mm, and the upper one changes and makes 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 30, 50 and 100 mm. At the second stage plates with the similar thickness of layers and conditions of fixing are considered, at the same time the thickness of a high layer (10 mm) is invariable.

Both layers during calculation were broken into 400 finite elements of 50x50 mm in size, in each of them shift connection is established. Cross bindings and bindings of shift were put into construction nodes. Rigidness of

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cross bindings is constant and makes EAc = 644 kN that corresponds to a steel dowel with a diameter of 2 mm. All characteristics of layers are accepted for a plate of chipboard: average density ρ=720 kg/m3, elastic modulus in case of a bend of E=2600MPas. For dynamic calculation the weights in nodes were gathered according to volume weight of material of layers and the cargo area of a node taking into account plate layer thickness. In case of static calculation the uniformly distributed load of q=1 of kN/sq.m was applied to a high layer. The researches were conducted by the finite-element method. The distance between layers was accepted equal to distance between gravitational centersof layers. The diagram of setting of cross bindings and shift bindings is given below – "Figure 1", plates models – "Figure 2".

Numerical researches were conducted by means of the

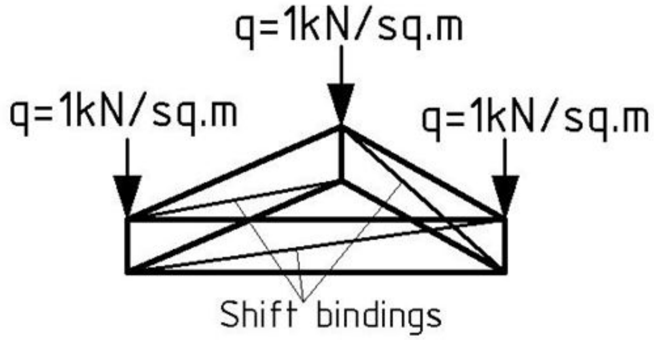

Figure 1 – Plate finiteelement

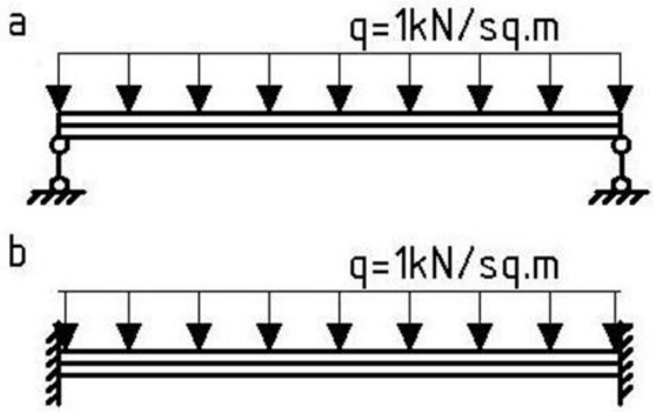

Figure 2 – Conditions of fixing of a plate: hinge fixing (a) and rigid fixing (b)

Table 1: Results of calculation of hinge-supported plates with different thickness of layers (the first stage of calculation)

Thickness of a high layer h_{high}, mm	Thickness of a low layer h_{low}, mm	h_{high}/h_{low}	Maximum sag W_0, mm	Frequency of the main tone of natural oscillations ω, 1/c	Coefficient K in FEM	Theoretical value Ktheor	Divergence of K from K_{theor}, %
10	10	W0, mm	Frequency of the main tone of natural oscillations ω, 1/c	Coefficient K in FEM	Theoretical value	1.582	-1.0
11	10	Ktheor	Divergence of K from Ktheor,	247.7138	1.56554	1.582	-1.04
12	10	%	1.503294	256.3577	1.56492	1.582	-1.1
13	10	1.3	1.344859	265.0514	1.56013	1.582	-1.6
14	10	1.4	1.207723	273.7952	1.56458	1.582	-1.1
15	10	1.5	1.088417	282.5877	1.56450	1.582	-1.1
16	10	1.6	0.9841364	292.4268	1.57542	1.582	-0.4
17	10	1.7	0.8925969	300.3102	1.56492	1.582	-1.1
18	10	1.8	0.811921	309.2350	1.56524	1.582	-1.1
19	10	1.9	0.7405551	318.1986	1.56561	1.582	-1.0
20	10	2	0.6772047	328.1292	1.57494	1.582	-0.4
30	10	3	0.310998	418.7003	1.57021	1.582	-0.7
50	10	5	0.099168	606.4788	1.57575	1.582	-0.4
100	10	10	0.0165543	1098.145	1.58109	1.582	-0.1

program SCAD. During calculation the maximum sags of W0 from action of a permanent load of q=1kN/m2 uniformly distributed on plate surface, and the frequency of the free cross oscillations ω in the no-load condition were defined.

RESULTS OF RESEARCHES

Results of calculation of plates are given in Tables 1 – 4. According to the tables the diagrams of change of sags and oscillation frequencies in the plates under study, "Figure 3", "Figure4 " and coefficient of proportionality K of Professor V. I. Korobko fundamental dependence (1) "Figure 5" were constructed.

CONCLUSION

As a result of a numerical research it is defined that with the increase in thickness of one of the layers of a composite plate, the frequency of its free oscillations increases and the sag decreases that proves the increase of rigidness of a plate. The method of fixing of a plate gives different results in the conducted experiment: more rigid are the plates which are rigidly jammed on a circuit – that is well noticeable in the diagrams –"Figure 3", "Figure 4" (the values of frequencies are bigger, and the number of sags are less, than at pivotally supported ones). The relative positioning of plate layers, different in thickness, slightly influences the results; however, it is possible to mark that the smaller sag characterizes plates which layer of a bigger thickness is located on top, and this difference is more noticeable when calculating plates which have a changeable thickness of a layer much more than constant: 50 mm, 100 mm (the difference reaches 1.09% in case of a hinge fixing and 3,04% in case of rigid jamming). The proportionality coefficients K defined in the calculation differ from analytical values Ktheor. within 1.6% in case of a hinge fixing and within 5% in case of a rigid jamming.

Table 2: Results of calculation of hinge-supportedplates with different thicknessof layers (the second stage of calculation)

Thickness of a high layer h_{high}, mm	Thickness of a low layer h_{low}, mm	h_{high}/h_{low}	Maximum sag W_0, mm	Frequency of the main tone of natural oscillations ω, 1/c	Coefficient K in FEM	Theoretical value Ktheor	Divergence of K from K_{theor}, %
10	10	1	1.902618	239.1168	1.56651	1.582	-1.0
10	11	0.909	1.687378	247.7138	1.56554	1.582	-1.0
10	12	0.833	1.503297	256.3577	1.56492	1.582	-1.1
10	13	0.769	1.344863	265.0515	1.56458	1.582	-1.6
10	14	0.714	1.20773	273.7952	1.56446	1.582	-1.1
10	15	0.667	1.088425	282.5877	1.56451	1.582	-1.1
10	16	0.625	0.9841465	291.4268	1.56468	1.582	-1.1
10	17	0.588	0.8926088	300.3102	1.56494	1.582	-1.1
10	18	0.556	0.8119348	309.235	1.56527	1.582	-1.1
10	19	0.526	0.7405708	318.1986	1.56565	1.582	-1.0
10	20	0.5	0.6772224	327.1984	1.56606	1.582	-1.0
10	30	0.333	0.3110359	418.7003	1.57040	1.582	-0.7
10	50	0.2	0.0992484	606.4788	1.57702	1.582	-0.3
10	100	0.1	0.0167369	1098.145	1.59853	1.582	1.0

Table 3: Results of calculation of rigidly jammed plates with different thickness of layers (the first stage of calculation)

Thickness of a high layer h_{high}, mm	Thickness of a low layer h_{low}, mm	h_{high}/h_{low}	Maximum sag W_0, mm	Frequency of the main tone of natural oscillations ω, 1/c	Coefficient K in FEM	Theoretical value Ktheor	Divergence of K from K_{theor}, %
10	10	1	1.259987	296.3286	1.59322	1.610	-1.0
11	10	1.1	1.104124	308.9617	1.5936	1.610	-1.0
12	10	1.2	0.9694215	322.2326	1.59443	1.610	-1.0
13	10	1.3	0.8530879	336.0734	1.5956	1.610	-0.9
14	10	1.4	0.7526142	350.4212	1.59697	1.610	-0.8
15	10	1.5	0.6657761	365.2191	1.59848	1.610	-0.7
16	10	1.6	0.5906274	380.4161	1.60006	1.610	-0.6
17	10	1.7	0.5254822	395.9671	1.60167	1.610	-0.5
18	10	1.8	0.4688918	411.8326	1.60326	1.610	-0.4
19	10	1.9	0.4196182	427.9777	1.60482	1.610	-0.3
20	10	2	0.3766072	445.6363	1.61549	1.610	0.3
30	10	3	0.1473327	617.5049	1.61798	1.610	0.5
50	10	5	0.0388027	985.7546	1.62886	1.610	1.2
100	10	10	0.0054736	1942.727	1.63615	1.610	1.6

Table 4: Results of calculation of rigidly jammed plates with different thickness of layers (the second stage of calculation)

Thickness of a high layer h_{high}, mm	Thickness of a low layer h_{low}, mm	h_{high}/h_{low}	Maximum sag W_0, mm	Frequency of the main tone of natural oscillations ω, 1/c	Coefficient K in FEM	Theoretical value Ktheor	Divergence of K from K_{theor}, %
10	10	1	1.259987	296.3286	1.59322	1.610	-1.0
10	11	0.909	1.104128	308.9617	1.5936	1.610	-1.0
10	12	0.833	0.969429	322.2326	1.59445	1.610	-0.9
10	13	0.769	0.8531	336.0733	1.59562	1.610	-0.8
10	14	0.714	0.75263	350.4212	1.597	1.610	-0.7
10	15	0.667	0.665795	365.2191	1.59853	1.610	-0.6
10	16	0.625	0.59065	380.4161	1.60012	1.610	-0.5
10	17	0.588	0.525507	395.9671	1.60175	1.610	-0.4
10	18	0.556	0.468921	411.8326	1.60336	1.610	-0.3
10	19	0.526	0.41965	427.9777	1.60494	1.610	-0.2
10	20	0.5	0.376641	444.3721	1.60648	1.610	-0.2
10	30	0.333	0.147393	617.5049	1.61864	1.610	0.5
10	50	0.2	0.038906	985.7546	1.63321	1.610	1.4
10	100	0.1	0.005671	1942.727	1.69524	1.610	5.0

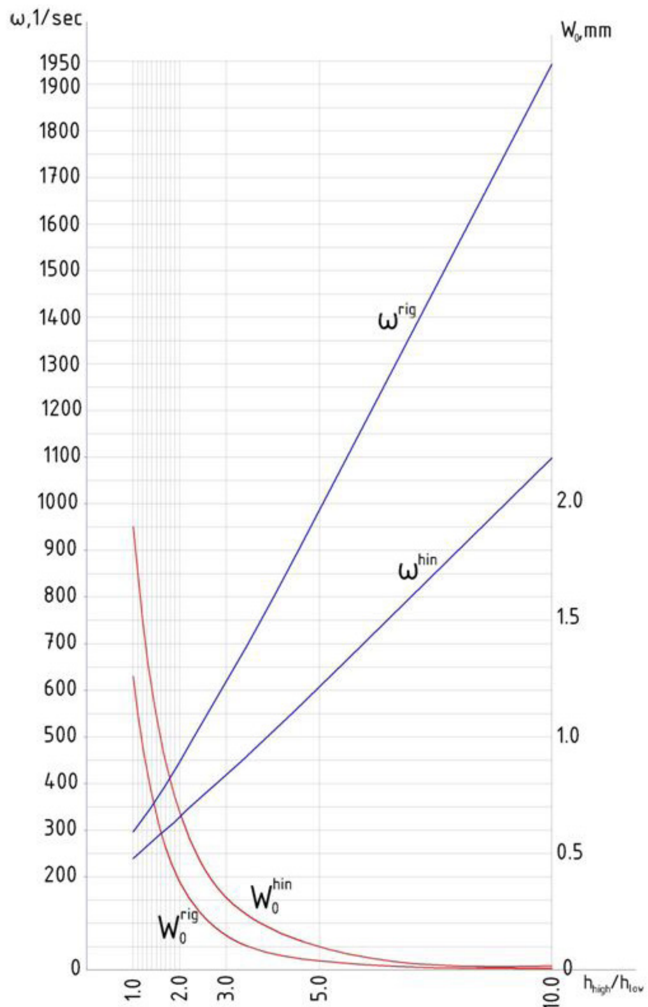

Figure 3: The diagram of change of the maximum sags and frequencies of natural oscillations depending on change of thickness of a high layer of a two-layer composite plate at a constant thickness of a low layer (10 mm) in the conditions of a hinge fixing and rigid jamming

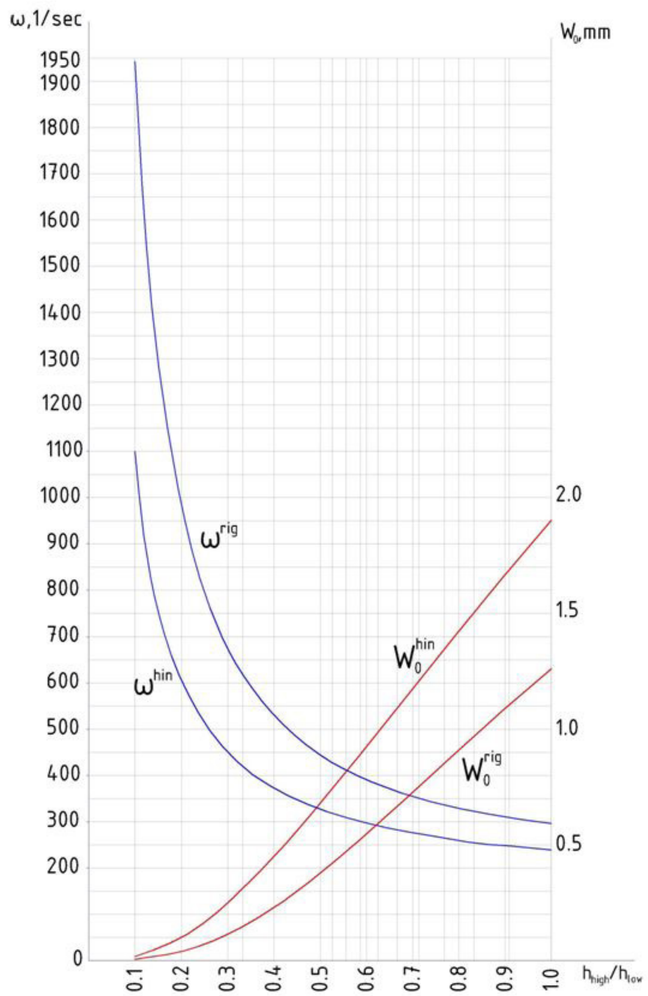

Figure 4: The diagram of change of the maximum sags and frequencies of natural oscillations depending on change of thickness of a low layer of a two-layer composite plate at a constant thickness of a high layer (10 mm) in the conditions of a hinge fixing and rigid jamming

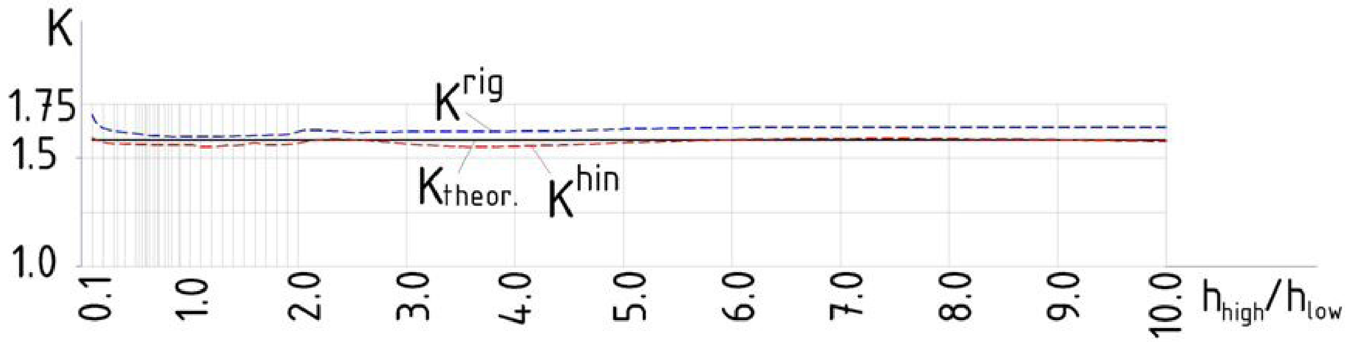

Figure 5: Diagrams of change of coefficient K depending on change of thickness of one of the layers of a two-layer plate in comparison with its theoretical value

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Paper submitted: 10.10.2017.
Paper accepted: 03.07.2018.