

COMBINING THE DELAYED DIFFERENTIATION POLICY AND COMMON PARTS' PARTIAL OUTSOURCING STRATEGY INTO A MULTI-ITEM FPR-BASED SYSTEM

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This study investigates a multi-item finite production rate- (FPR-) based system incorporating a delayed product differentiation policy and common parts' outsourcing strategy. A two-stage fabrication scheme is proposed, wherein, in stage one, all common parts of the end products (assuming they have a known completion rate as compared with the finished products) are partially produced in-house and partially supplied by an outside contractor with an extra unit outsourcing; in stage two, all end products are finished in sequence, under a rotation fabrication cycle time discipline. An explicit model is developed to clearly represent the proposed problem. Through the optimization technique, the optimal rotation cycle decision is obtained. Thus, diverse characteristics of this particular multi-item, FPR-based system with postponement and outsourcing strategies can now be revealed. As demonstrated by numerical illustrations, these characteristics include the (i) convexity of the system cost function, (ii) impact of common parts' outsourcing strategy on the utilization, (iii) breakup of system cost components, (iv) combined impact of the outsourcing ratio and common parts' completion rate on the system cost function, and (v) effect of the outsourcing ratio on optimal rotation cycle decision. Our decision-support-type system can facilitate production managers in achieving their goals of reducing orders' response times and minimizing the overall system cost.

Key words: multi-item fabrication system, common intermediate part, delayed differentiation, outsourcing, finite production rate

INTRODUCTION

This study explores a multi-item FPR-based system incorporating a delayed product differentiation policy and common parts' outsourcing strategy. Operations in present-day manufacturing environments are turbulent. Therefore, production managers must constantly seek alternatives to (i) increase utilization, (ii) reduce orders' response times, and (iii) minimize the overall system cost. Classic FPR model [1] portrayed a manufacturing system that has a finite rate to manufacture a single product to meet its annual demand. The fabrication setup, variable, and stock holding costs were considered in a mathematical model, and through formulation along with optimization approach, the cost-minimization batch size was derived. To increase the utilization, an effective approach is to schedule a multi-item fabrication on a single machine. Bastian [2] showed that the economic order quantity (EOQ) formula is appropriate for reordering each single item in a multi-item inventory system under the following conditions: (i) the holding costs are linear, (ii) the combined setup costs for placing orders, (iii) the combined variable costs in the orders, and (iv) the same order cycle applies to the groups of items. As a result, through a heuristic for forming the groups, the author showed that the optimal grouping could be conditionally obtained. Perez and Zipkin [3] proposed a straightforward heuristic to solve a stochastic multiproduct fabrication-inventory system with capacity constraint. Their

proposed approach performed well demonstrated by a limited number of numerical tests. Balkhi and Foul [4] studied a multiproduct fabrication-inventory system with backlogging and within a finite time horizon, wherein for each product in each time period, shortages are permitted and backordered, and the demand, fabrication, and deterioration rates are known in advance. The problem was modeled and formulated; through the optimization techniques, the authors derived the optimal fabrication and cycle time that keep total system cost minimum. Extra studies [5-11] examined the diverse characteristics of multi-item production planning and controlling.

To smooth/optimize the manufacturing processes in terms of materials preparations, orders' response times, and/or total cost reductions, production managers often apply the postponement strategy to delay product differentiation by first fabricating the common parts of multiple end products. Gerchak and Henig [12] considered a few assemble-to-order-type systems featuring component commonality. The authors examined the stock status of product-specific components along with other components in both the static and dynamic models and commented on the optimality may be biased in terms of stock holding status. Van Hoek et al. [13] studied numerous companies that had experiences on implementing the postponement strategy. Evaluations included the (i) benefits gained from their specific business environments, (ii) related managerial characteristics, and (iii) bottle

necks encountered, during the implementation of the postponement policy. Accordingly, the authors suggested the ways of successfully carrying out such a strategy. Brun and Zorzini [14] explored the relationships between modularization and postponement in real companies in Italy, with the focuses on relating factors associated with product features. Multiple case studies along with statistical data analyses were employed to identify different degrees of complexity and customization on product/process under various real constraints. Kouvelis and Tian [15] proposed a three-stage decision process for deriving the optimal operating policy to meet uncertain aggregate demands but under the consideration of investing flexible capacity and postponement options. Extra studies [16-19] investigated diverse impact of postponement strategies on business operations and multiproduct fabrication systems.

To effectively shorten the common parts' fabrication time, the flexible capacity policy such as the outsourcing strategy can help. Spiegel [20] described horizontal-subcontract's practices and measured its possible benefits to a firm. The result indicated that by implementing the horizontal outsourcing policy, certain companies achieved more efficient planning in production, obtained mutual benefits with their external contractors, and boosted the industry outputs as a whole. Tan [21] proposed a simplified production system with a subcontractor to meet a random product demand, where the demand can switch exponentially between high and low levels in times. Due to limited capacity, to cope with high level demand, the producer must either in-house build up the inventories in advance or count on an outsider subcontractor to supply. The available time of the subcontractor is also random. A threshold kind of policy is used in the proposed Markovian model to determine the quantity to be produced in house and to be supplied externally. Kaya [22] analyzed and compared two distinct supply-chain facilities, one focuses on outsourcing, and the other makes an effort on in-house fabrication. The aim was to find the best contracts to be used in these cases by performing extensive numerical tests on contracts' parameters. The impact of system variables on the optimal contractor parameters such as demand, price, and effort was also explored. Extra studies [23-29] also investigated diverse effect of outsourcing strategies on business operations and fabrication-distribution systems. Since few prior works focused on exploration of combined impact of postponement and common parts' outsourcing strategy on multi-item FPR-based system, this work aims to bridge the gap.

MATERIALS AND METHODS

Problem description and modeling

The study considers a multi-item FPR-based system combining postponement and common parts' partial outsourcing strategy. We assume the following: (a) an extended multi-item FPR-based system; (b) known completion rate of common part (as compared with the end

product), (c) constant demand rates of end products; (d) in stage one of the fabrication, partial common parts are supplied by an outside contractor; and (e) in stage two of the fabrication: annual manufacturing rates of multi-item depend on the common part's completion rate γ , e.g., if $\gamma = 50\%$, then manufacturing rate of finished product becomes double the standard rate of a single-stage system. The proposed problem is explicitly described as follows: suppose there are L products with the existence of a common part, must be fabricated by a 2-stage production scheme. The annual demand rate is λ_i (where $i = 1, 2, \dots, L$), and in stage one, all needed common intermediate parts are fabricated, at a standard rate $P_{1,0}$, then in stage two, the customized L end products are produced under a common cycle time discipline, at a standard rate of $P_{1,i}$.

The fabrication of common parts in stage 1, takes up a large portion of uptime, to reduce the cycle time, a partial outsourcing strategy is employed in stage 1. A π_0 portion of required common parts (i.e., the sum of batch size Q_i of multi-item of stage 2) is outsourced, and the other $(1 - \pi_0)$ portion is fabricated in-house. Consequently, the following different fixed order cost K_{π_0} and a higher unit cost C_{π_0} are associated with outsourced parts:

$$K_{\pi_0} = (1 + \beta_{1,0}) K_0 \quad (1)$$

$$C_{\pi_0} = (1 + \beta_{2,0}) C_0 \quad (2)$$

where K_0 , C_0 , and $\beta_{i,0}$ denote in-house setup cost, unit manufacturing cost, and the linking factors between these outsourcing-relevant and in-house variables, respectively. For example, $\beta_{1,0} = -0.7$ stands for that K_{π_0} is 70% less than in-house setup cost, and $\beta_{2,0} = 0.3$ means unit outsourcing cost is 30% higher than the in-house unit cost, etc. It is noted that $0 < \pi_0 < 1$. If $\pi_0 = 0$, then all common parts are produced in-house. On the contrary, $\pi_0 = 1$ means all common parts are provided externally. The schedule of receipt of outsourced parts is at the end of the uptime of stage one (see Fig. 1).

No stock-out situations are permitted so in stage two, the manufacturing rate of finished product i must be greater than its demand rate, i.e., $P_{1,i} - \lambda_i > 0$ (where $i = 1, 2, \dots, L$). Cost parameters considered in our work include: setup cost K_i , unit holding cost $h_{1,i}$. Additional parameters also comprise the following:

Q_0 = the in-house batch size for common parts in stage 1,

γ = common part's completion rate (as compared with end product),

$t_{1,0}$ = uptime for fabricating common parts with the adoption of outsourcing option,

$t_{2,0}$ = time required to deplete all common parts in a cycle,

$H_{1,0}$ = the on-hand inventory level of common parts when uptime of stage one ends,

$H_{3,0}$ = the level of common parts when receipt of outsourced items,

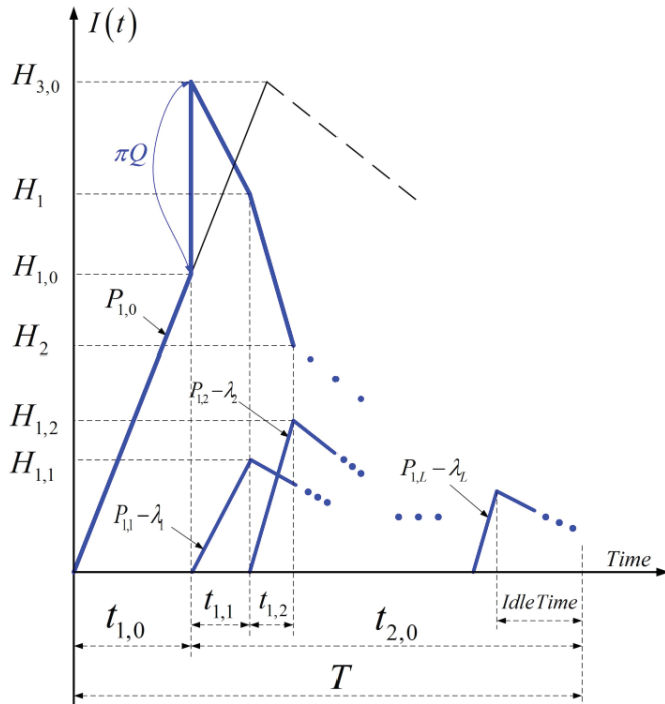


Figure 1: The on-hand inventory level in the proposed two-stage multi-item FPR-based system with delayed product differentiation and partial outsourcing strategy for common parts as compared to a system without outsourcing plan (in grey)

- λ_0 = annual demand rate of common parts,
- Q_i = batch size for each end product i (where $i = 1, 2, \dots, L$),
- C_i = unit manufacturing cost for the product i in stage 2,
- $t_{1,i}$ = uptime for product i in stage 2,
- $t_{2,i}$ = time needed to deplete all finished stocks of product i ,
- i_0 = holding cost relating ratio, i.e., $h_{1,i} = (i_0)C_i$ (where $i = 1, 2, \dots, L$),
- T = common cycle time - the decision variable,
- $H_{1,i}$ = the on-hand inventory level of product i when uptime ends,
- H_i = the level of common parts when uptime of product i ends,
- S_i = setup time for item i (where $i = 0, 1, 2, \dots, L$),
- T_{min} = the common cycle time which includes the summation of S_i ,
- $I(t)$ = the on-hand perfect inventory at time t ,
- t_0^* = the optimal uptime in stage one,
- t_1^* = the sum of optimal uptimes for end products in stage two,
- $I(t)_i$ = the on-hand inventory at time t for product i ,
- $TC(T)$ = total fabrication-inventory costs per cycle for the proposed system,
- $TCU(T)$ = the long-run average system cost per unit time for the proposed system.

Formulas in stage two of the proposed model

During stage two, the common parts are gradually consumed to meet the needs of production of each end item i ; its stock status is as shown in Fig. 2. By the assumption of the proposed problem and from the observation of Figs. 1 and 2, the following formulas (for $i = 1, 2, \dots, L$) can be gained:

$$t_{1,i} = \frac{Q_i}{P_{1,i}} = \frac{H_{1,i}}{P_{1,i} - \lambda_i} \tag{3}$$

$$t_{2,i} = \frac{H_{1,i}}{\lambda_i} \tag{4}$$

$$T = t_{1,i} + t_{2,i} = \frac{Q_i}{\lambda_i} \tag{5}$$

$$H_{1,i} = (P_{1,i} - \lambda_i)t_{1,i} \tag{6}$$

$$\sum_{i=1}^L Q_i = \sum_{i=1}^L \lambda_i T \tag{7}$$

$$H_{3,0} = \sum_{i=1}^L Q_i \tag{8}$$

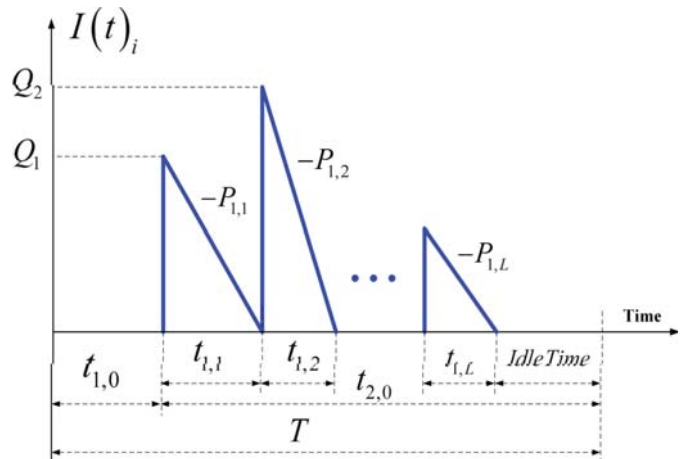


Figure 2: Inventory level of common parts in the proposed two-stage multi-item system

Formulas in stage 1

By the assumption of the proposed problem and from the observation of Figs. 1 and 2, the following formulas (for $i = 1, 2, \dots, L$) can be gained in stage 1:

$$\lambda_0 = \frac{\sum_{i=1}^L Q_i}{T} \tag{9}$$

$$Q_0 = (1 - \pi_0) \left(\sum_{i=1}^L Q_i \right) \tag{10}$$

$$H_{3,0} = H_{1,0} + \pi_0 \left(\sum_{i=1}^L Q_i \right) \tag{11}$$

$$T = t_{1,0} + t_{2,0} \tag{12}$$

$$t_{1,0} = \frac{Q_0}{P_{1,0}} = \frac{H_{1,0}}{P_{1,0}} \quad (13)$$

$$H_{1,0} = (P_{1,0})t_{1,0} \quad (14)$$

$$H_1 = H_{3,0} - Q_1 \quad (15)$$

$$H_i = H_{(i-1)} - Q_i, \text{ for } i = 2, 3, \dots, L \quad (16)$$

or

$$H_i = H_{3,0} - \sum_{j=1}^i Q_j, \text{ for } i = 2, 3, \dots, L; j = 2, 3, \dots, i \quad (17)$$

$$H_L = H_{(L-1)} - Q_L = 0 \quad (18)$$

Total cost in a production cycle

Total fabrication-inventory cost in a production cycle comprise common parts' outsourcing relevant variable and setup costs, and in-house setup, variable fabrication, and holding costs (in stage one); and the sum of L finished products' variable fabrication, setup, and holding costs (in stage two). Thus, the following TC(T) can be obtained:

$$TC(T) = C_{\pi_0} \left[\pi_0 \left(\sum_{i=1}^L Q_i \right) \right] + K_{\pi_0} + K_0 + C_0 Q_0 + h_{1,0} \left[\frac{H_{1,0} t_{1,0}}{2} + \sum_{i=1}^L \left[\frac{Q_i}{2} (t_{1,i}) + H_i (t_{1,i}) \right] \right] \\ + \sum_{i=1}^L \left\{ C_i Q_i + K_i + h_{1,i} \left[\frac{H_{1,i} t_{1,i}}{2} + \frac{H_{1,i}}{2} (t_{2,i}) \right] \right\} \quad (19)$$

Substitute Q_i with T (i.e., Eq. (9)), and Eqs. (1) to (18) in Eq. (19), the following TC(T) can be gained:

$$TC(T) = \left[(1 + \beta_{2,0}) C_0 \right] \pi_0 (\lambda_0 T) + \left[(1 + \beta_{1,0}) K_0 \right] + K_0 + C_0 (1 - \pi_0) (\lambda_0 T) + \frac{h_{1,0}}{2} \left[(1 - \pi_0) \lambda_0 T \right]^2 \left(\frac{1}{P_{1,0}} \right) \\ + h_{1,0} \sum_{i=1}^L \left(\frac{\lambda_i^2 T^2}{2 P_{1,i}} \right) + h_{1,0} \sum_{i=1}^L \left[\left(\frac{\lambda_i T}{P_{1,i}} \right) \left(\sum_{i=1}^L \lambda_i T - \sum_{j=1}^i \lambda_j T \right) \right] + \sum_{i=1}^L \left[C_i \lambda_i T + K_i + \frac{h_{1,i}}{2} (\lambda_i T)^2 \left(\frac{1}{\lambda_i} - \frac{1}{P_{1,i}} \right) \right] \quad (20)$$

RESULTS AND DISCUSSION

Solution procedure

The long-run average cost per unit time TCU(T) can be gained by computing $[TCU(T) / T]$ as follows:

$$TCU(T) = \left[(1 + \beta_{2,0}) C_0 \right] \pi_0 \lambda_0 + \frac{\left[(1 + \beta_{1,0}) K_0 \right] + K_0}{T} + C_0 (1 - \pi_0) \lambda_0 + \frac{h_{1,0} \lambda_0^2 (1 - \pi_0)^2 T}{2} \left(\frac{1}{P_{1,0}} \right) \\ + h_{1,0} \sum_{i=1}^L \left(\frac{\lambda_i^2 T}{2 P_{1,i}} \right) + h_{1,0} \sum_{i=1}^L \left[\left(\frac{\lambda_i T}{P_{1,i}} \right) \left(\sum_{i=1}^L \lambda_i - \sum_{j=1}^i \lambda_j \right) \right] + \sum_{i=1}^L \left[C_i \lambda_i + \frac{K_i}{T} + \frac{h_{1,i} \lambda_i^2 T}{2} \left(\frac{1}{\lambda_i} - \frac{1}{P_{1,i}} \right) \right] \quad (21)$$

The 1st and 2nd derivatives of TCU(T) can be derived as follows:

$$\frac{d[TCU(T)]}{dT} = \frac{-\left[(1 + \beta_{1,0}) K_0 \right]}{T^2} - \frac{K_0}{T^2} + \frac{h_{1,0} \lambda_0^2 (1 - \pi_0)^2}{2} \left(\frac{1}{P_{1,0}} \right) + h_{1,0} \sum_{i=1}^L \left(\frac{\lambda_i^2}{2 P_{1,i}} \right) \\ + h_{1,0} \sum_{i=1}^L \left[\left(\frac{\lambda_i}{P_{1,i}} \right) \left(\sum_{i=1}^L \lambda_i - \sum_{j=1}^i \lambda_j \right) \right] + \sum_{i=1}^L \left[-\frac{K_i}{T^2} + \frac{h_{1,i} \lambda_i^2}{2} \left(\frac{1}{\lambda_i} - \frac{1}{P_{1,i}} \right) \right] \quad (22)$$

$$\frac{d^2[TCU(T)]}{dT^2} = \frac{2\left[(1 + \beta_{1,0}) K_0 \right]}{T^3} + \frac{2K_0}{T^3} + \sum_{i=1}^L \left(\frac{2K_i}{T^3} \right) \quad (23)$$

From Eq. (23), since the setup costs K_{π_0} (i.e., $[(1 + \beta_{1,0})K_0]$), K_0 , and K_i are all positive, and T is also positive; thus, TCU(T) is convex. It follows that the optimal T^* can be solved by letting Eq. (22) = 0. Therefore, the following optimal

T* can be gained:

$$T^* = \sqrt{\frac{(2 + \beta_{1,0})K_0 + \sum_{i=1}^L (K_i)}{h_{1,0}\lambda_0^2(1-\pi_0)^2\left(\frac{1}{P_{1,0}}\right) + h_{1,0}\sum_{i=1}^L\left(\frac{\lambda_i^2}{2P_{1,i}}\right) + h_{1,0}\sum_{i=1}^L\left[\left(\frac{\lambda_i}{P_{1,i}}\right)\left(\sum_{j=1}^L\lambda_j - \sum_{j=1}^i\lambda_j\right)\right] + \sum_{i=1}^L\left[\frac{h_{1,i}\lambda_i^2}{2}\left(\frac{1}{\lambda_i} - \frac{1}{P_{1,i}}\right)\right]} \quad (24)$$

The effect of the setup times on the proposed system

If the sum of the setup times S_i is larger than the idle time (see Fig. 1) in T^* , then, the T_{min} (as indicated by Nahmias [30]) must be calculated and the maximum of (T^* , T_{min}) should be selected as "the final cycle length" for the proposed system to guarantee that there is adequate time for the setup and the fabrication.

$$T_{min} = \frac{\sum_{i=0}^L (S_i)}{1 - \sum_{i=0}^L \left[\frac{\lambda_i}{P_{1,i}}\right]} \quad (25)$$

Numerical illustration

Consider that the demands of 5 distinct end products must be met, and they have the common intermediate part in their fabrication processes. A two-stage multi-item manufacturing plan with postponement and common parts' partial outsourcing option is established to first make all necessary common parts in stage 1, and then fabricate the distinct finished products in sequence under a rotation cycle time discipline in stage 2. Table 1 shows the parameters' values for 5 distinct products as they are produced using the one-stage fabrication plan.

Table 1: Parameters' values used under the one-stage fabrication plan

Product number	P_{1i}	K_i	λ_i	C_i	h_{1i}
1	58000	\$17000	3000	\$80	\$16
2	59000	\$17500	3200	\$90	\$18
3	60000	\$18000	3400	\$100	\$20
4	61000	\$18500	3600	\$110	\$22
5	62000	\$19000	3800	\$120	\$24

For the proposed delayed differentiation two-stage fabrication scheme with a partial outsourcing option for the common parts, we assume the following parameters' values (as exhibited in Tables 2 and 3):

Table 2: Parameters' values used in stage 1 of the proposed study

$P_{1,0}$	K_0	C_0	i_0	$h_{1,0}$	λ_0	π_0	$\beta_{1,0}$	γ	δ	$\beta_{2,0}$
120000	\$8500	\$40	0.2	\$8	17000	0.4	-0.7	0.5	50%	0.4

Table 3: Parameters' values used in stage 2 of the proposed study

Product i	P_{1i}	K_i	λ_i	C_i	h_{1i}
1	112258	\$8500	3000	\$40	\$16
2	116066	\$9000	3200	\$50	\$18
3	120000	\$9500	3400	\$60	\$20
4	124068	\$10000	3600	\$70	\$22
5	128276	\$10500	3800	\$80	\$24

Compute Eqs. (25) and (22), $T^* = 0.5696$ (year) and $TCU(T^*) = \$2,034,365$ are obtained our proposed delayed differentiation two-stage multi-item FPR-based system.

The convexity of TCU(T)

The analytical result on the convexity of $TCU(T)$ is demonstrated in Fig. 3. It confirms that $TCU(T)$ goes up significantly, as the cycle length deviates from T^* .

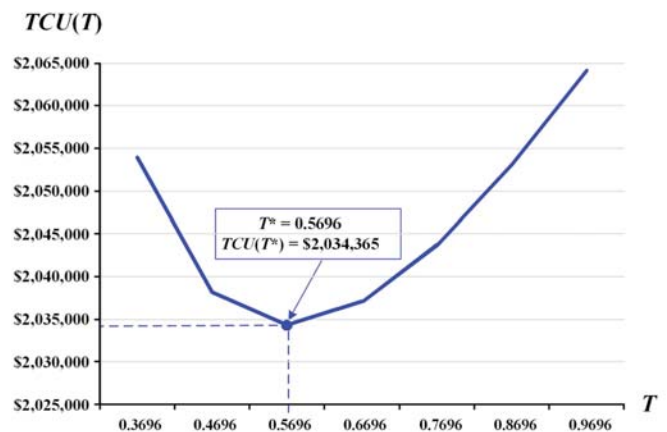


Figure 3: The analytical result on the convexity of TCU(T)

The impact of common parts' outsourcing strategy on machine utilization

Since a π_0 portion of required common parts (see Eqs. (10) and (11)) is outsourced, the impact of common parts' outsourcing strategy on machine utilization was explicitly analyzed, and the outcome is depicted in Fig. 4. For $\pi_0 = 0.4$, the in-house fabrication uptime in stage one t_0^* declines from 0.0776 to 0.0484 (year), i.e., a 37.63% reduction for stage one.

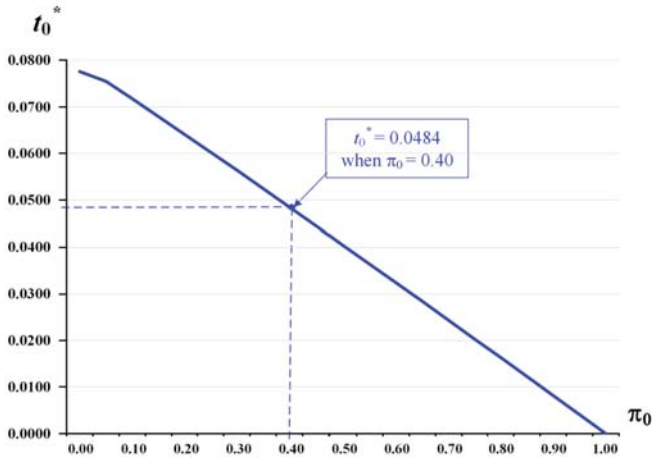


Figure 4: The impact of common parts' outsourcing strategy on machine utilization

The breakup of TCU(T*)

For the outsourcing ratio $\pi_0 = 0.4$ and the common part's completion rate $\gamma = 0.5$, the breakup of $TCU(T^*)$ is explicitly investigated, and the outcomes are exhibited in Fig. 5. It specifies that common parts' outsourcing relevant cost is 18.94% of $TCU(T^*)$, and the sum of in-house setup and variable fabrication costs for the remaining 60% of common parts is 21.16% of $TCU(T^*)$. For $\gamma = 0.5$ and $\pi_0 = 0.4$, it costs a total of 59.9% of $TCU(T^*)$ to produce the finished products in stage 2.

The combined impact of π_0 and γ on $TCU(T^*)$

The combined impact of outsourcing ratio π_0 and the common part's completion rate γ on $TCU(T^*)$ was studied, and the outcome is depicted in Fig. 6. It indicates that $TCU(T^*)$ increases drastically as both π_0 and γ rise noticeably.

The effect of outsourcing ratio π_0 on T^*

The result of further analysis on the effect of outsourcing

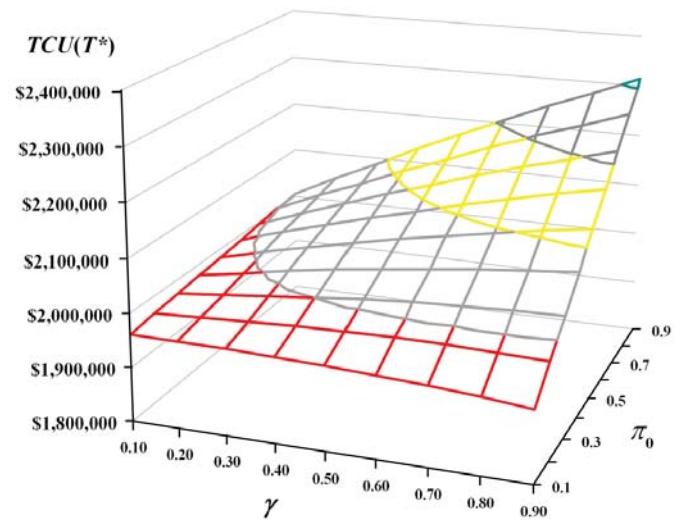


Figure 6: The combined impact of π_0 and γ on $TCU(T^*)$ ratio π_0 on T^* is illustrated in Fig. 7. It reveals that T^* increases considerably as π_0 goes up, and when $\pi_0 = 0.4$, it confirms our previous finding (i.e., $T^* = 0.5696$).

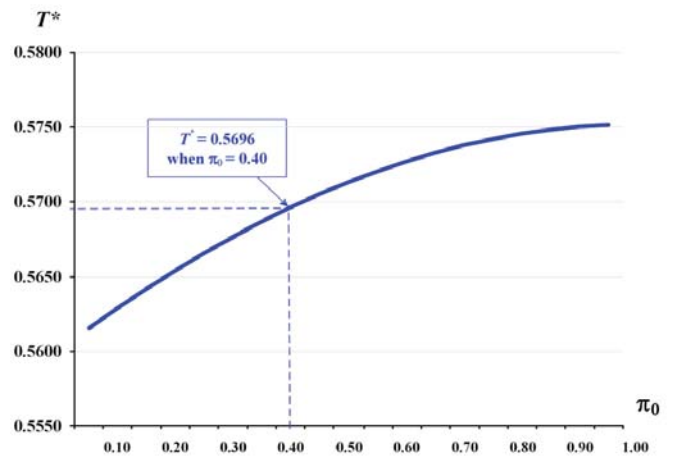


Figure 7: The effect of outsourcing ratio π_0 on T^*

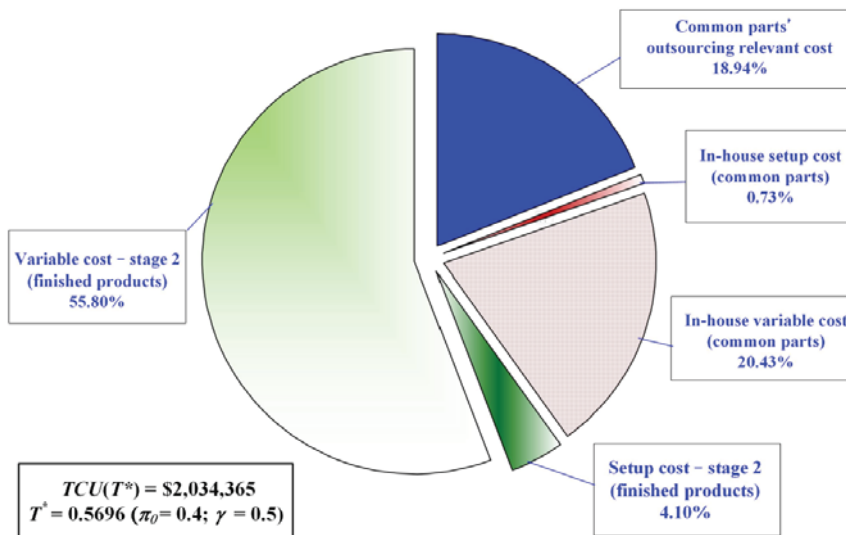


Figure 5: The breakup of $TCU(T^*)$

CONCLUSIONS

This study examines a multi-item FPR-based system incorporating the postponement policy and common parts' outsourcing strategy. An explicit model is developed to clearly represent the proposed problem. Through the optimization technique, the optimal rotation cycle decision is obtained. Thus, diverse characteristics of this particular multi-item, FPR-based system with postponement and outsourcing strategies can now be revealed. As demonstrated by the numerical illustration section, these characteristics include the (i) convexity of the system cost function, (ii) impact of common parts' outsourcing strategy on the utilization, (iii) breakup of system cost components, (iv) combined impact of the outsourcing ratio and common parts' completion rate on the system cost function, and (v) effect of the outsourcing ratio on the optimal rotation cycle decision. Our decision-support-type system can facilitate production managers in achieving their goals of reducing orders' response times and minimizing the overall system cost. For future study, an interesting subject will be to combine the real-life product quality factors into the same model.

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Appendix

The following prerequisite condition must hold to ensure the machine in the proposed system has sufficient capacity to produce the common parts in stage one and L distinct end products in stage two (Nahmias [30]):

$$\left[(t_{1,0}) + \sum_{i=1}^L (t_{1,i}) \right] < T \quad (\text{A-1})$$

or

$$\left[\left(\frac{\lambda_0 (1 - \pi_0)}{P_{1,0}} \right) + \sum_{i=1}^L \left(\frac{\lambda_i}{P_{1,i}} \right) \right] < 1 \quad (\text{A-2})$$

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