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INFLUENCE OF THE CALCULATED LENGTH OF ELEMENTS ON THE CRITICAL PARAMETERS OF STABILITY OF FRAME-BAR STRUCTURAL SYSTEMS

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This article examines the effect of changing the design length of the elements of frame-bar structural systems operating in a constrained bifurcation.

For the criterion of the form of buckling of an element of a frame-bar structural system, the sign of the work of end moments and shear forces is taken. Using this criterion, an expression was obtained to assess the effect of changing the calculated length of the frame struts on the critical parameters of the entire system, which allows varying the geometry of the structure to influence its performance.

A two-span frame is considered, in which the central pillar is loaded with a concentrated force $P_{cr}$, and the outer pillars are loaded with forces $\alpha R_{cr}$. The type of bifurcation of the rods (constrained or forced) and the critical parameters of the stability of the system before and after changing the calculated length of its elements are determined.

Changes in the design length of elements that passively lose stability do not have a significant effect on the critical stability parameters of the frame-bar structural system. At the same time, a 30% decrease in the calculated length of the struts in an active bifurcation leads to a decrease in the critical force by 50%.

The presented rather simple algebraic dependencies allow obtaining a qualitative and quantitative assessment of the effect of the calculated length coefficient on the stability of frame-rod structural systems made of wood. It has been established that the conditions for securing elements that passively lose stability do not have a significant effect on the critical parameters of the system.

Key words: stability, frame-rod system, active bifurcation, passive bifurcation, calculated length

INTRODUCTION

It is known that the reason for the loss of stability of an entire structural system such as a truss, frame, structure is often one element or a small group of them. In this regard, an important issue in solving problems of stability of structural systems is the identification of the most dangerous elements or parts of a structure with low resistance to buckling [1-4, 9-13, 17-19]. Therefore, it is necessary to investigate the deformation of the rods under constrained bifurcation conditions and establish the criteria for the type of rod bifurcation (constrained or forced).

MATERIALS AND METHODS

Let the bar separated from the system (Fig. 1) at some critical value of the load $P_{cr}$. (w, t) loses stability.

Its deformation energy $U_i$ added during bifurcation will be equal to the sum of the work of end forces and moments:

$$U_i = A_i(N_i) + A_i(M_i, Q_i),$$

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The criterion $A_i(M_i, Q_i) < 0$ becomes a sign of active buckling of the bar in the structural system, i.e. the work of one longitudinal force $N_i$ is sufficient to compensate for the increment in the potential energy of deformation of the rod $U_i$ and the environment of the rod (in the form of support reactions) to resist its bifurcation.

The criterion $A_i(M_i, Q_i) > 0$ becomes a sign of passive buckling, i.e. surrounding the rod helps its bifurcation.

Figure 1: Bar extracted from the system
The calculation of the work of end moments and shear forces is performed using special functions of the displacement method [5-8, 14-16].

We take for unknown angles of rotation and displacement of nodes $Z_1, Z_2, Z_n$, then the homogeneous system of equations will take the form:

$$\begin{align*}
    r_{11} \cdot Z_1 + r_{12} \cdot Z_1 + \cdots + r_{1n} \cdot Z_n &= 0 \quad (i = 1, 2, 3, \ldots n) \\
    r_{n1} \cdot Z_1 + r_{n2} \cdot Z_1 + \cdots + r_{nn} \cdot Z_n &= 0
\end{align*}$$

In formula (3) $B_{red}$ is the reduced stiffness of the bar section; $v_i$ - parameter of the secular equation.

Determine the determinant of the system (2):

$$\text{Def} = 0$$

Using the above equations, we find the critical parameters and forms of loss of stability of the considered structural system. Of interest is the assessment of the influence of the loading scheme on the nature of the bifurcation of individual struts.

It is also necessary to analyze the change in the value of the critical force in the event of a change in the calculated length of each of the frame struts. In fact, it can be expected that, under the influence of force factors, a change in the calculated length of the struts, which are losing stability passively and actively, will have a different effect on the overall bifurcation of the system.

To solve this problem, let us determine the parameter of the secular equation $v_{icr}$, determined after the occurrence of an emergency situation associated with a change in the estimated length of the racks:

$$v_{icr} = l \cdot \sqrt{\frac{P}{B_{red}}}$$

The parameter of the secular equation at a given constant value of the load and a fixed value of wood moisture will be respectively equal to:

$$v_{i0} = l \cdot \sqrt{\frac{P}{B_0}}$$

The ratio of the squares of the critical and initial parameters of the secular equation is directly proportional to the ratio of the corresponding critical forces:

$$\frac{v_{icr}^2}{v_{i0}^2} = \frac{P_{cr}}{P_{cr_0}}.$$  

The parameters $v_{i0}$ and $v_{icr}$ are determined when calculating the frame for stability according to the well-known rules of structural mechanics before and after changing the calculated length of the racks.

Using relation (7), it is possible to estimate the effect of changing the calculated length of the frame struts on the critical parameters of the entire system, which in turn allows, by varying the geometry of the structure, to influence its performance. The larger the value $v_{icr}^2$, the more likely the frame as a whole will lose stability.

Results. As an example, consider a wooden two-span frame in which the central pillar is loaded with a concentrated force $P_{cr}$ and the outer pillars are loaded with forces $\alpha P_{cr}$ (Fig. 2).

Let us define the type of the rod bifurcation (constrained or forced). The calculation of the frame is performed by a quasi-static method of displacement using a step-iterative procedure. The calculation of the work of the end moments and shear forces is performed using special functions of the displacement method. If we take as unknown angles of rotation of nodes $Z_1, Z_2, Z_3$ (Fig. 4), the homogeneous system of equations will take the form:
where $B_{red}$ is the reduced stiffness of the bar section; $v_i$ – parameter of the secular equation.

The determinant of system (8) is determined by the following expression:

$D\text{et} = (8 \cdot i + 4 \cdot i \cdot \varphi_2(v_1)) \cdot (8 \cdot i + 4 \cdot i \cdot \varphi_2(v_2)) \cdot (4 \cdot i + 4 \cdot i \cdot \varphi_2(v_3)) + 16 \cdot i^2$  \hspace{1cm} (9)

Using the above equations, we find the critical parameters and forms of loss of stability of the considered structural system.

It is also necessary to analyze the change in the value of the critical force in the event of a change in the calculated length of each of the frame struts. In fact, it can be expected that, under the influence of force factors, a change in the calculated length of the struts, which are losing stability passively and actively, will have a different effect on the overall bifurcation of the system.

To solve this problem, let us determine the parameter of the secular equation $v_{i_c}$, determined after the occurrence of an emergency situation associated with a change in the estimated length of the racks:

$v_{i_c} = l \cdot \frac{P_i}{B_{red}}$  \hspace{1cm} (10)

The parameter of the secular equation at a given constant value of the load and a fixed value of wood moisture will be respectively equal to:

$v_{i_0} = l \cdot \frac{P}{B_0}$  \hspace{1cm} (11)

The ratio of the squares of the critical and initial parameters of the secular equation is directly proportional to the ratio of the corresponding critical forces:

$\frac{v_{i_c}^2}{v_{i_0}^2} = \frac{P_{cr}}{P_{cr,0}}$ \hspace{1cm} (12)

The parameters $v_{i_c}$ and $v_{i_0}$ are determined when calculating the frame for stability according to the well-known rules of structural mechanics before and after changing the calculated length of the racks.

Using relation (12), it is possible to estimate the effect of changing the calculated length of the frame struts on the critical parameters of the entire system, which in turn allows, by varying the geometry of the structure, to influence its performance. The larger the value $v_{i_c}^2/v_{i_0}^2$ the more likely the frame as a whole will lose stability.

Consider the change in the calculated lengths of each of the frame struts, replacing the rigid support with a hinged-fixed one. The design diagram of the frame for this case is shown in Figure 5.

The based and equivalent systems of the displacement method are developed similarly to those presented above (see Fig. 3, 4). The homogeneous system of equations will take the form:

$r_{11} \cdot Z_1 + r_{12} \cdot Z_2 + r_{13} \cdot Z_3 = 0 \hspace{1cm} (13)$

$r_{11} = 8 \cdot i + 4 \cdot i \cdot \varphi_2(v_1); \hspace{1cm} r_{12} = 8 \cdot i + 4 \cdot i \cdot \varphi_2(v_2); \hspace{1cm} r_{13} = 4 \cdot i + 4 \cdot i \cdot \varphi_2(v_3);$

$r_{13} = r_{31} = 0; v_i = l \cdot \frac{P_i}{B_{red}(w,t)}, \hspace{1cm} (i = 1,2,3).$

The determinant of system (13) is determined by the expression:

$D\text{et} = (8 \cdot i + 4 \cdot i \cdot \varphi_4(v_1)) \cdot (8 \cdot i + 4 \cdot i \cdot \varphi_2(v_2)) \cdot (4 \cdot i + 4 \cdot i \cdot \varphi_2(v_3)) + 16 \cdot i^2 \hspace{1cm} (14)$

Similarly, the calculation of the change in the critical stability parameters of the considered frame-bar structural system (see Fig. 2) was performed when the calculated lengths of the racks 2 and 3 were changed.

The results of this calculation are shown in Figure 6. The vertical lines limit the zones of active and passive bifurcation of racks 1 and 3, rack 2 actively loses its stability, regardless of the value of the load application parameter $\alpha$.

From the analysis of Figure 6, it can be concluded that with a value of $\alpha<0.756$, the left and right struts of the structural system passively lose their stability. In the case when the
EXPERIMENTAL RESEARCH

In order to test the developed methodology on the basis of the Southwestern State University, experimental studies of the influence of the calculated length on the stability of frame-rod structural systems made of wood were carried out. To achieve this goal, three series of three frames in each were designed with the following parameters: material - spruce wood, grade I, height - 1 m; spans - 0.67 m (Fig. 7). The calculated length was varied due to the rigidity of the support nodes. In the first series, all frame units are...

Figure 7: Scheme (a) and general view (b) of the test setup
rigid; in the second series of hinged-fixed, the support of the left pillar was adopted, in the third - the right one. The tests were carried out with a loading coefficient α = 0.85, therefore, in the second group of structures, the calculated length of the rack, which loses its stability actively, changes, in the third - passively.

As a result of the experiment, after statistical data processing, it was found that the critical force Pcr. for frames of the first series it is 4.23 kN, for the second - 3.09 kN, for the third - 4.19 kN.

CONCLUSION

The presented rather simple algebraic dependences make it possible to obtain a qualitative and quantitative assessment of the influence of the calculated length coefficient on the stability of frame-rod structural systems made of wood. At the same time, it was found that the conditions for fixing elements experiencing passive bifurcation do not significantly affect the overall stability of the system.