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GEOMETRICALLY NONLINEAR PLATE BENDING UNDER THE ACTION OF MOVING LOAD

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Considerable scientific interest is the development of mathematical models that describe the behavior of materials that are sensitive to deformation rate and can improve the accuracy of analytical calculations of their deformation in the region of noticeable changes of loading rates. Nonetheless, in most works, the problems were solved under the assumption of small displacements (geometrically linear statement of the problem). Meanwhile, in practice, this is not always true and bending of cover can be commensurable with its thickness, this article approximately solves the problem of geometrically nonlinear deformation of a thin elastic plate in quasistatic setting under the action of an infinite normal uniformly distributed load moving along its surface at a constant speed. In the article, the methods of mathematical modeling, the analytical method, as well as the methods of spatial characteristics and bicharacteristics are used. The problem is solved in the quasistatic formulation and is reduced to a system of two nonlinear differential equations for deflections of the plate and the stress function, which include the speed of the load as a parameter. The results of methodological calculations are presented; based on these solutions of linear and nonlinear problems, they were compared, and the influence of finiteness of displacements on the critical speeds of the forces was determined. Materials of the article can be useful in the study of wave dynamics, aircraft, mechanics, and engineering.

Key words: thin plate, quasistatic solution, linear approximation, critical velocity, finite deflections

INTRODUCTION

When constructing mathematical models of deformation of thin-walled structures, it becomes necessary to take into account the nonlinear nature of the processes associated with physical and geometric factors, which leads to the construction of more accurate and meaningful models, the analysis of which allows us to detect new phenomena and effects. The development of methods of mathematical modeling and computer technology led to an increase in the role of the numerical experiment. With its help, it was possible to investigate complex systems, increasingly getting closer to real ones.

The problem of the action of pressure wave on elements of thin-walled structures is encountered, for example, when calculating wing cover, as well as elements of vertical and horizontal control of an aircraft in supersonic flight. The deformed state of such structures in the form of plates or shells was considered in monographs [1-3], as well as in journal publications [4, 5]. Nonetheless, in all these sources, the problems were solved under the assumption of small displacements (geometrically linear statement of the problem). In practice, this is not always true and the deflection of cover can be commensurable with its thickness [6-8]. Without studying the complete spatio-temporal picture of wave processes in bodies of finite dimensions arising under the action of unsteady dynamic loads, it is impossible to evaluate their performance [9-11]. Thus, the study of the features of the

propagation of three-dimensional waves in bodies of finite dimensions and identification of some patterns of unsteady processes in them is currently an important and urgent problem of both scientific value and applied interest. This brings us to the need to solve the problem in the geometrically nonlinear formulation.

In the present work, an attempt was made to take into account in quadratic form the finiteness of displacements of the deformable structural element in form of a plate. When investigating the problem as a whole, it is most interesting to find the influence of finiteness of displacements of the plate on its deformed state and to compare the results with the solution of its linear part (small displacements), as well as to establish the effect of the finiteness of displacements on critical speeds of the forces. Comparing these results, we can determine the need to take into account nonlinear effects when solving such problems.

MATERIALS AND METHODS

The proposed solution is based on the use of equations of non-linear bending of the plate (Margherequations) [12], obtained under the assumption of non-linear displacements in quadratic form. Moving load is considered as an infinite normal uniformly distributed force moving with constant speed on the plate surface. The problem is solved in the quasistatic setting, according to which the deflections of the plate depend only on spatial co-

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ordinates and do not depend on time. The equations are solved using the Bubnov method [13] in a one-term approximation, and the problem is reduced to the cubic equation for deflection of the plate, which contains the speed of the load as a parameter. The sufficiency of such an approximation is determined by the nature of acting uniform load and is confirmed by calculations. The essence of this method is a direct transition to a discrete calculation scheme, bypassing the stage of differential equations. The benefit of the method lies in its physical perceptibility and convenience in choosing the geometric shape and size of the elements. From the obtained solution, as a special case, a solution to the linear problem can be obtained and the critical speed of the load can be determined. Thus, Bubnov's method is based on the principle of possible displacements. The approximating function must be chosen so that it satisfies the geometric boundary conditions. Static conditions are not required.

The problems of the action of moving load (pressure waves) on elements of thin-walled structures are encountered, for example, when calculating the cover of an airplane's wings during supersonic flight. A review of the work in this direction is given in [14-16]. In journal publications, similar problems were considered in articles [17, 18]. Nonetheless, practically in all the above sources of the problem was solved in a geometrically linear setting. In the proposed work, an attempt was made to take into account the finiteness of displacements of the deformable structural element in the form of a plate.

Let us consider a thin elastic plate of rectangular shape in plan, referred to the orthogonal system of coordinates $Oxyz$ (Fig.1) on the surface of which in the direction of axis x with a constant speed V moves the endless normally distributed load with intensity p .

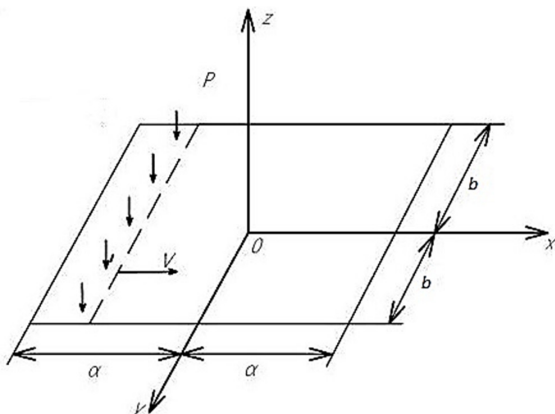


Figure 1: The plate under influence of the moving load

Figure 1 conventionally displays it in the form of forces acting along the line. The problem is solved in a quasi-static setting, following which the deflection of plate varies only longitudinal coordinates x , y and does not depend on time. Since the speed of the load is constant, then over time t its element will travel the distance (Eq. 1):

$$x = Vt \tag{1}$$

Then the inertial forces caused by this load will be (Eq. 2):

$$(p/g)\partial^2 w / \partial t^2 = (p/g)V^2 \partial^2 w / \partial x^2 \tag{2}$$

The inertia of the motion of the plate itself is neglected in comparison with the inertia of the moving load. To solve this problem, we use the equations of geometrically nonlinear deformation of the plate in the mixed form (Margherre equations) [12] concerning its deflection and stress functions F [19-21], which in the case under consideration take the form (Eqs. 3-4):

$$\frac{D}{h} \nabla^2 \nabla^2 w = \frac{p}{h} - \frac{pV^2}{gh} \frac{\partial^2 w}{\partial x^2} + L(w, F) \tag{3}$$

$$\frac{1}{E} \nabla^2 \nabla^2 F = -\frac{1}{2} L(w, w) \tag{4}$$

Where ∇^2 is Laplace operator, D and h are the cylindrical stiffness and plate thickness, respectively, g is the gravitational acceleration. Nonlinear differential operators $L(w, F)$ and $L(w, w)$ have the form [22, 23] (Eqs. 5-6):

$$L(w, F) = \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 w}{\partial y^2} \frac{\partial^2 F}{\partial x^2} - 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 F}{\partial x \partial y} \tag{5}$$

$$L(w, w) = 2 \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \tag{6}$$

Equations (3-4) represent a system of nonlinear differential equations with partial derivatives. To solve them, we use the Bubnov method [13] in a one-term approximation; the sufficiency of this approach for the problem under consideration will be substantiated below. Following this, we represent the deflection of the plate w and stress function F in the form (Eq. 7):

$$w = A\phi(x, y), \quad F = B\psi(x, y) \tag{7}$$

Where A and B are unknown constants, ϕ and ψ are the specified coordinate functions that satisfy the boundary conditions at the edges of the plate. Applying the Bubnov method [13] to the second of equations (4) we express B through A (Eq. 8):

$$B = -\frac{E A^2 \int_S L(\phi, \phi) \psi dS}{2 \int_S \nabla^2 \nabla^2 (\psi) \psi dS} \tag{8}$$

Further applying the Bubnov method [13], taking into account (8), also to the first of equations (3), we obtain the cubic equation with respect to constant A from the approximation (Eq. 9):

$$A^3 c + A d(V) + k = 0 \tag{9}$$

Where (Eqs. 10-12):

$$c = -\frac{E \int_S L(\phi, \psi) \phi dS \cdot \int_S L(\phi, \phi) \psi dS}{2 \int_S \nabla^2 \nabla^2 (\psi) \psi dS} \tag{10}$$

$$k = -\frac{p}{h} \int_S \phi dS \tag{11}$$

$$d(V) = \frac{D}{h} \int_S \nabla^2 \nabla^2(\phi) \phi dS + V^2 \frac{p}{gh} \int_S \frac{\partial^2 \phi}{\partial x^2} \phi dS \quad (12)$$

Here S is the area of plate. A particular point of this equation is that the load speed V enters into it as a parameter. The solution of equation (9) for a fixed value of speed can be obtained analytically in closed form.

RESULTS AND DISCUSSION

Let us consider a square plate ($a = b$) freely supported on all edges by flexible inextensible ribs. We consider that the shape of its deformed surface in all the examples given below is symmetric concerning the x and y coordinate axes. Then, the approximating functions in (7) take the form (Eq. 13):

$$\phi(x, y) = \psi(x, y) = \cos(\mu_m \frac{x}{a}) \cos(\lambda_n \frac{y}{b}) \quad (13)$$

Where (Eqs. 14-16):

$$\mu_m = m\pi/2 \quad (14)$$

$$\lambda_n = n\pi/2 \quad (15)$$

$$(m = n = 1, 3, 5 \dots) \quad (16)$$

At first, we solve the linear problem.

Its resolving equation is obtained from (9) when the first term A^3 is equal to zero. Then, starting from the condition $(V=0)$ we define the square of the critical speed of the load movement (Eq. 17):

$$V_{CR}^2 = - \frac{Dg \int_S \nabla^2 \nabla^2(\phi) \phi dS}{p \int_S \frac{\partial^2 \phi}{\partial x^2} \phi dS} \quad (17)$$

When choosing approximating functions in the form (13), this velocity coincides with the corresponding velocity value from the work [24-26]. Considering (17), the deflection w of the center of the plate ($x = y = 0$) in the linear approximation can be represented as (Eqs. 18-19):

$$w = w_{ST} \frac{1}{1 - V^2/V_{CR}^2} \quad (18)$$

$$w_{ST} = \frac{ph \int_S \phi dS}{D \int_S \nabla^2 \nabla^2(\phi) \phi dS} \quad (19)$$

Where V_{CR}^2 is the square of critical speed of the moving load, determined by the formula (17), w_{ST} is the static deflection of the center of the plate under the action of the uniformly distributed normal load of p intensity equal to constant A in approximation (7). For the plate with the relative thickness (Eq. 20):

$$a/h = 20 \quad (20)$$

At $m = n = 1$ in approximation (13) dimensionless deflection (Eq. 21) will be $w_{ST}^* = 1.168 \times 10^{-5}$ and in the works [27, 28] the same value is equal to $w_{ST}^* = 1.136 \times 10^{-5}$. Thus, we can assume that the use of one-term approximations of unknowns at solving this problem is justified.

$$w_{ST}^* = w_{ST}(p/Eh) \quad (21)$$

Figure 2 shows the dependence of the dimensionless deflection of the center of the plate $w^* = 10^5$ on the square of the relative velocity of the load V^2/V_{CR}^2 . At $V \rightarrow V_{CR}$ V_{CR}^2 deflections increase unlimitedly.

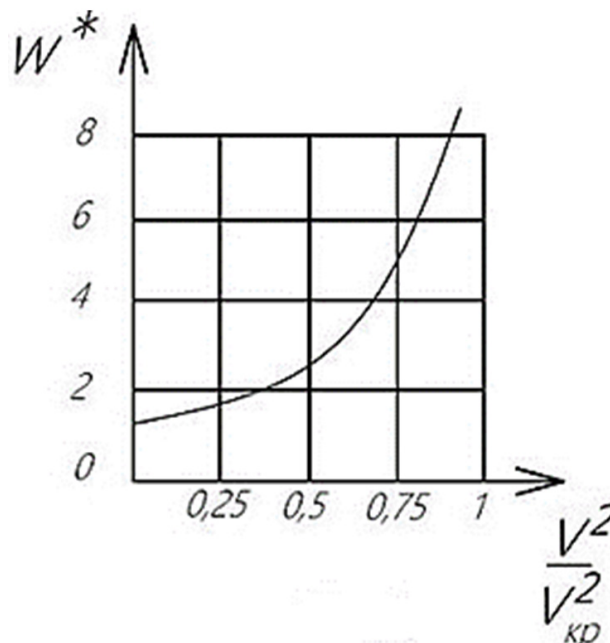


Figure 2: The dependence of linear deflection of the center of the plate on the speed of movement of the load

When solving a geometrically nonlinear problem, we first consider the problem of static deformation of the plate under the action of normal load p . The resolving equation is obtained from (9) at $V = 0$. It allows an analytical solution and gives two complex conjugate roots and one real, which determines the deflections of the center of the plate ($A = w$) [29-31]. For the plate with $a/h=20$. Figure 3 demonstrates the dependence of dimensionless deflection of its center (Eq. 22):

$$w^* = w(p/Eh) \cdot 10^5 \quad (22)$$

from dimensionless linear load (Eq. 23):

$$p^* = (p/E) \cdot 10^4 \quad (23)$$

The dashed line indicates a solution to the same problem, but in a geometrically linear formulation.

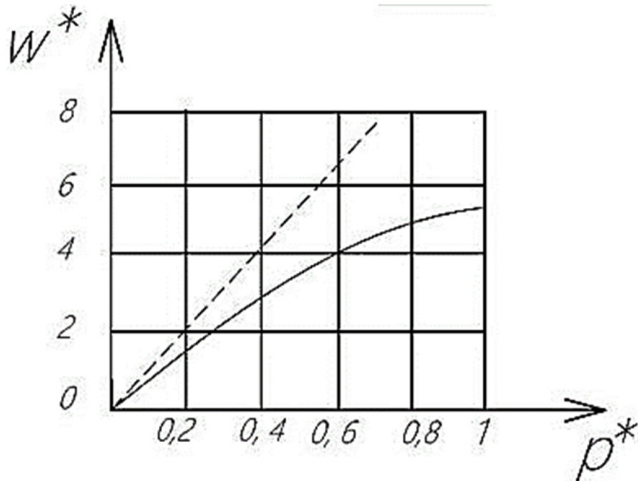


Figure 3: The dependence of deflection of the center of the plate from the magnitude of moving load

In the case of motion of an infinite linear load p , the solution of equation (9) also has two complex conjugate roots and one real, which determines the deflection of the center of the plate ($A = w$). In Figure 4 at $m = n = 1$ in (13), its dimensionless dependence (22) on the ratio of squares of velocities at a dimensionless load is provided. The dashed line in this figure displays the solution to the static geometrically nonlinear problem considered in the previous example. Considering the speed of movement of the load weakly affects the deflection of the plate. However, at $V = V_{CR}$ they are finite, in contrast to the solution of the linear problem, in which the deflections increase unlimitedly (Fig. 3).

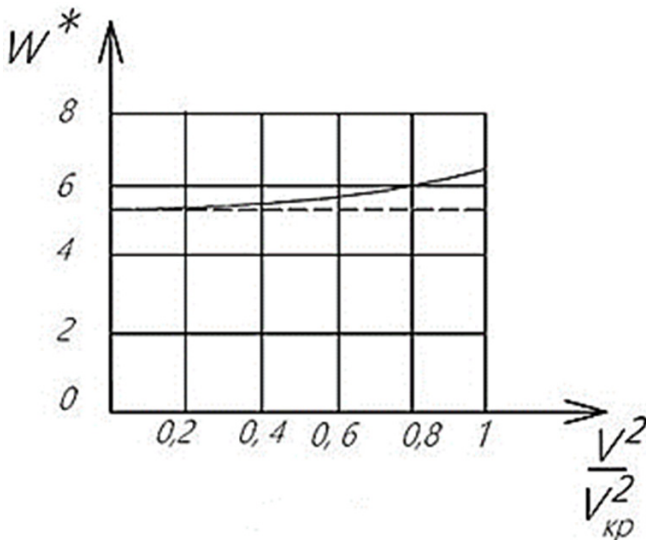


Figure 4: The dependence of nonlinear deflection of the center of the plate on the speed of the load

Thus, consideration of the finiteness of displacements (Figs. 3-4) shows a strong influence of nonlinear effects on the results. This is especially noticeable in Fig. 3, showing the effect of nonlinearity on deflections of the center of the plate. It should be noted that there is a qualitative difference in the determination of critical ve-

locities in linear and nonlinear formulations. In the first case, when the velocity tends to a critical value, the plate deflections increase unlimitedly (Fig. 3) [32, 33]. Though in a nonlinear statement when reaching critical values, the rates remain finite. This result fully coincides with the data for solving stability problems under the action of static forces (Euler problems), which indicates the correct choice of the research method.

CONCLUSIONS

Therefore, the article approximately solved the problem of geometrically nonlinear deformation of a thin plate in the quadratic approximation under the action of a moving uniformly distributed linear normal load. For this purpose, four options for posing the problem were considered – a static and dynamic solution to a linear problem, as well as a static and dynamic solution to a nonlinear problem. The dynamic tasks used quasistatic formulation, in accordance to which the deflection plate is not dependent on time, and determined only geometry of the plate. The sufficiency of a one-term approach when using the method of Bubnov is shown on the examples of the solution of linear problems.

Taking into account that the finiteness of displacements demonstrates a strong influence of nonlinear effects on the results. It should be mentioned that there is a qualitative difference in determination of critical velocities in linear and nonlinear formulations. In the first case, when the velocity tends to a critical value, the plate deflections increase infinitely, while in the nonlinear formulation, when the linear critical values are reached, these velocities remain finite. Summarizing all the above it can be stated that taking into account the geometric nonlinearity of deformation load on the plate strongly influences on its deformed state, heavily dependent on the speed of the load. Solving the problem in a linear formulation gives unsatisfactory results, while it is less laborious from a computational point of view.

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REFERENCES

1. Konoplev, Yu. G., Yakushev, R. S. (2003). Lectures on the dynamics of structures with moving loads. Otechestvo, Kazan.
2. Igudisman, O. S., Abramovich, H. (2012). Loss of stability and supercritical behavior of shallow reinforced cylindrical panels. University News. Aeronautical Engineering, no. 4, 11-14.
3. Timoshenko, S. P. (1948). Plates and shells. GITTL, Moscow.

4. Yakushev, N. Z. (1990). Dynamics of deformable systems under the action of moving loads. Studies on the theory of plates and shells. KSU Publishing House, Kazan.
5. Zhigalko, Yu. P., Shigabutdinov, A. T. (2003). The buckling of elastic cylindrical shells and rods under the action of a rectangular pulse of force. University News. Aeronautical Engineering, no. 4, 7-10.
6. Skvortsov, A. A., Pshonkin, D. E., Luk'yanov, M. N. (2018). Influence of constant magnetic fields on defect formation under conditions of heat shock in surface layers of silicon. Key Engineering Materials, vol. 771, 124-129.
7. Blinov, D. G., Prokopov, V. G., Sherenkovskii, Yu. V., Fialko, N. M., Yurchuk, V. L. (2002). Simulation of natural convection problems based on low-dimensional model. International Communications in Heat and Mass Transfer, vol. 29, no. 6, 741-747.
8. Dinzhos, R., Lysenkov, E., Fialko, N. (2015). Simulation of thermal conductivity of polymer composites based on poly (methyl methacrylate) with different types of fillers. Eastern-European Journal of Enterprise Technologies, vol. 6, no. 11, 21-24.
9. Ryndin, V. V. (2019). Calculation of the nonequilibrium systems consisting of an aggregate of locally-equilibrium subsystems. Periodico Tche Quimica, vol. 16, no. 33, 289-303.
10. Mykhalevskiy, D. M., Kychak, V. M. (2019). Development of information models for increasing the evaluation efficiency of wireless channel parameters of 802.11 standard. Latvian Journal of Physics and Technical Sciences, vol. 56, no. 5, 22-32.
11. Sultanov, K., Khusanov, B., Rikhsieva, B. (2020). Underground pipeline strength under non-one-dimensional motion. IOP Conference Series: Materials Science and Engineering, vol. 883, no. 012023.
12. Kolpakova, E. V. (2010). On the solvability of the Margerr-Vlasov model in an unbounded domain. Kazan Publishing House Publishing House, no. 2, 6-11.
13. Awrejcewicz, J., Kryśko, A. V. (2003). Analysis of complex parametric vibrations of plates and shells using Bubnov-Galerkin approach. Applied Mechanics, no. 73, 495-504.
14. Kuznetsova, E. L., Rabinskiy, L. N. (2019). Numerical modeling and software for determining the static and linkage parameters of growing bodies in the process of non-stationary additive heat and mass transfer. Periodico Tche Quimica, vol. 16, no. 33, 472-479.
15. Danilin, A. N., Rabinskiy, L. N., Zhavoronok, S. I. (2019). Deformation of the helical type wire structures. Periodico Tche Quimica, vol. 16, no. 33, 583-601.
16. Kuznetsova, E. L., Makarenko, A. V. (2019). Mathematical model of energy efficiency of mechatronic modules and power sources for prospective mobile objects. Periodico Tche Quimica, vol. 16, no. 32, 529-541.
17. Egorova, O. V., Kurbatov, A. S., Rabinskiy L. N., Zhavoronok, S. I. (2019). Modeling of the dynamics of plane functionally graded waveguides based on the different formulations of the plate theory of I. N. Vekua type. Mechanics of Advanced Materials and Structures. <https://doi.org/10.1080/15376494.2019.1578008>.
18. Lurie, S. A., Rabinskiy, L. N., Solyaev, Y. O. (2019). Topology optimization of the wick geometry in a flat plate heat pipe. International Journal of Heat and Mass Transfer, no. 128, 239-247. <https://doi.org/10.1016/j.ijheatmasstransfer.2018.08.125>.
19. Antufev, B. A., Egorova, O. V. Rabinskiy L. N. (2019). Dynamics of a cylindrical shell with a collapsing elastic base under the action of a pressure wave. INCAS Bulletin, no. 11, 17-24. <https://doi.org/10.13111/2066-8201.2019.11.S.2>.
20. Rabinskiy, L. N. (2019). Non-stationary problem of the plane oblique pressure wave diffraction on thin shell in the shape of parabolic cylinder. Periodico Tche Quimica, vol. 16, no. 32, 328-337.
21. Antufev, B. A., Egorova, O. V., Medvedskii, A. L., Rabinskiy, L. N. (2019). Dynamics of shell with destructive heat-protective coating under running load. INCAS Bulletin, no. 11, 7-16. <https://doi.org/10.13111/2066-8201.2019.11.S.2>.
22. Antufev, B. A., Egorova, O. V., Rabinskiy, L. N. (2019). Quasi-static stability of a ribbed 14shell interacting with moving load. INCAS Bulletin, no. 11, 33-39. <https://doi.org/10.13111/2066-8201.2019.11.s.4>.
23. Getmanov, A. G., Rabinskiy, L. N. (2019). Assessment of durability of coatings in difficult stress conditions. Periodico Tche Quimica, vol. 16, no. 33, 490-497.
24. Dmitriev, V. G., Egorova, O. V., Rabinsky, L. N. (2015). Solution of nonlinear initial boundary-value problems of the mechanics of multiply connected composite material shells on the basis of conservative difference schemes. Composites: Mechanics, Computations, Applications: An International Journal, vol. 6, no. 4, 265-277.
25. Dmitriev, V. G., Egorova, O. V., Zhavoronok, S. I., Rabinsky, L. N. (2018). Investigation of buckling behavior for thin-walled bearing aircraft structural elements with cutouts by means of numerical simulation. Russian Aeronautics, vol. 61, no. 2, 165-174.

26. Rabinskiy, L. N. (2019). Non-stationary problem of the plane oblique pressure wave diffraction on thin shell in the shape of parabolic cylinder. *Periodico Tche Quimica*, vol. 16, no. 32, 328-337.
27. Danilin, A. N., Rabinskiy, L. N., Zhavoronok, S. I. (2019). Deformation of the helical type wire structures. *Periodico Tche Quimica*, vol. 16, no. 33, 583-601.
28. Volmir, A. S. (1972). *Nonlinear dynamics of plates and shells*. Nauka, Moscow.
29. Toporovskiy, V., Kudryashov, A., Samarkin, V., Sheldakova, J., Rukosuev, A., Skvortsov, A., Pshonkin, D. (2019). Bimorph deformable mirror with a high density of electrodes to correct for atmospheric distortions. *Applied Optics*, vol.58, no. 22, 6019-6026.
30. Fedotov, A. M., Murzakhmetov, A. N., Dyusembaev, A. E. (2018). Expansion of ideas and processes in social and biological communities. *Eurasian Journal of Mathematical and Computer Applications*, vol. 6, no. 4, 17-28.
31. Durakovic, B., Demir, R., Abat, K., Emek, C. (2018). Lean manufacturing: Trends and implementation issues. *Periodicals of Engineering and Natural Sciences*, vol. 6, no. 1, 130-139.
32. Ryndin, V.V., Ivin, V.I. (1981). Investigation of multicylinder engine filling-up nonuniformity. *Izvestia vyssih ucebnyh zavedenij. Masinostroenie*, vol. 10, 71-75.
33. Sultanov, K., Khusanov, B., Rikhsieva, B. (2020). Underground pipeline reliability under longitudinal impact load. *IOP Conference Series: Materials Science and Engineering*, vol. 869, no. 052008.

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