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THE INFLUENCE OF EXPEDITED FABRICATION RATE, UNRELIABLE MACHINES, SCRAP, AND REWORK ON THE PRODUCTION RUNTIME DECISION IN A VENDOR-BUYER COORDINATED ENVIRONMENT

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Transnational manufacturing firms operate in highly competitive marketplaces. This means that they are continuously seeking ways to reduce order response and fabrication cycle times, maintain the desired product quality, manage unanticipated machine failures, and provide timely delivery to effectively minimize overall operating cost and maintain a competitive advantage over their intra-supply chains. To assist firms in achieving these operational goals, we examine a buyer-vendor coordinated system with an expedited fabrication rate, unreliable machines, scrap, and rework, with the objective of minimizing the overall operating costs. An imperfect manufacturing process is assumed, which arbitrarily produces repairable and scrap items, with the latter being reworked in each fabrication cycle. Additionally, the process is subject to a Poisson-distributed machine breakdown. The corrective action is undertaken immediately when the machine fails, and the production of unfinished/interrupted lot resumes when the process is restored. The expedited fabrication rate option is used at an extra cost to reduce the cycle length. We built a fabrication-shipment model to characterize the problem's features explicitly. Mathematical and optimization approaches assist us in determining the optimal fabrication runtime policy. A numerical example illustrates the capability/applicability of our outcomes. Furthermore, it exposes a diverse set of information relating to the collective/individual effect of differences in the expedited rates, mean-time-to-breakdown, frequency of shipment, and rework/disposal rates of defective items on the optimal policy, utilization, total operating cost, and various cost contributors. This information can contribute to facilitating better decision-making.

Key words: production management, unreliable machine, expedited fabrication rate, rework, scrap, multi-shipment

INTRODUCTION

The present work investigates the influence of expedited fabrication rate, unreliable machines, rework, and scrap on the production runtime decision in a vendor-buyer collaborative environment. To reduce order response and fabrication cycle times, the expedited rate is an effective option at an extra cost. Gershwin [1] analyzed transfer lines with a series of machines that can take different processing rates/times to perform operations on components. A simulation approach was used to provide the research outcomes from numerical examples. Makino and Tominaga [2] examined a flexible assembly system that comprises several sub-systems. Their capacity/production rate is pre-estimated, and the standardized, flexible assembly systems are built to meet the required cycle length. The authors discussed the effect of the cycle time and the consequent fabrication rate on a flexible assembly system's performance. Freiheit et al. [3] built several reserve-capacity models for pure serial and parallel-serial fabrication lines to estimate their productivities. The authors applied combinatorial mathematics to decide the fabrication quantities, and the system states' occurrence probabilities. They also demonstrated how to quantify the relevant productivity improvements and to use reserve capacity in place of buffers. Singh and Sharma [4] developed a manufacturing-delivery integrated model incorporating inflation, a time-dependent demand rate, and a fabrication rate that links to the demand rate. The authors provided a numerical example to illustrate their model's concept and purpose and conducted a sensitivity analysis of system parameters. Chiu et al. [5] explored a multiproduct inventory refilling system featuring expedited fabrication rate, rework, and multi-delivery policy. The authors built a model to portray the system's features, derived its cost function, and simultaneously determined the optimal refilling cycle time and frequency of shipments. They demonstrated via a numerical example how their results work and crucial managerial decision-related information is accessible via their system. Other works [6-9] investigated the effects of diverse aspects of expedited fabrication rates on the production and operations planning.

The imperfect fabrication process comprising the production of random nonconforming items and failures of the machine. Such instances must be carefully managed to avoid the undesirable quality and unanticipated delay

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of a production schedule. They will increase the total operating cost and decrease the customer's satisfaction. Moinzadeh and Lee [10] considered an inventory system featuring Poisson demand arrival rate, continuous-review, known resupply time, and defective items. An approximation structure with wide-ranging numerical tests was proposed to save the computation efforts on system characteristics and simplify the derivation of the optimal or near-optimal ordering policies for such a system. Hong et al. [11] analyzed a production line with multiple unreliable machines and arbitrary processing times. By using the decomposition concept and simulation techniques, the authors were able to conduct numerical experiments to explore the problem efficiently. Geren and Lo [12] studied the requirements for automating the rework process of the printed circuit board assembly (PCBA). The authors indicated that to meet the economic and crucial rework requirements, an industrial robot along with proper hardware and appropriate methods are essential. Accordingly, they developed/integrated a robotic rework cell with suggestions of the required tooling and techniques. Kyriakidis and Dimitrakos [13] examined a fabrication system with a nonstationary deteriorating installation, an intermediate buffer, and a production unit. Under the control-limit disciplines, a Markov algorithm was proposed to decide the optimal preventive maintenance policy for the system. Widyadana and Wee [14] considered rework as one of the core issues in the green supply chain for it can lower operating costs and decrease the environmental-related problem. Specifically, the authors examined the effect of rework on an economic production quantity (EPQ)-based system for deteriorating stocks. A (m, 1) fabrication scheme was explored in particular, i.e., m fabrication setups plus one rework set up in a production cycle. Through numerical illustration and sensitivity analysis, they found that the deteriorating rate's effect on the optimal cost is not significant. Still, the impact of demand rate, fabrication setup, and serviceable holding costs on the optimal cost is significant. Nasr et al. [15] studied an EPQ model with defective items where the quality of products made in the same fabrication run is correlated. The authors explored the scheduled maintenance and fabrication policies for such a correlated binomial fabrication system. Both the impact of correlation on certain system performance measures and the maintenance and fabrication policies were investigated. They further showed that the use of their proposed approach could also analyze the interrupted geometric fabrication systems. Other works [16-19] examined the effects of diverse aspects of the imperfect manufacturing process and random failures on fabrication systems and production and operations planning.

In real vendor-buyer collaborative environments, the finished goods' delivery policy is usually scheduled by a periodic/discontinuous multi-shipment discipline. Aderohunmu et al. [20] considered a buyer-vendor cooperation policy on cost information exchange in just-in-time manufacturing environments. The authors examined the influence of exchanging setup, ordering, stock holding, and delivering costs on the supply relationship in the long run. They found that joint optimization between vendor and buyer was not necessarily the common batch quantity. Sensitivity in various exchanges of operating parameters on the overall cost savings was analyzed. Thomas and Hackman [21] developed models with simulation approximation techniques to explore the impact of a distributor's committed shipping strategy on a price-sensitive demand supply-chain environment, wherein a fixed-quantity fixed-frequency shipping discipline over a finite time horizon and under the normal-distribution demand was considered. As a result, the optimal ordering policy and the resale price solution were derived. Shaw et al. [22] examined the strategy of economic ordering quantity and delivery for single-vendor multi-customer supply chains featuring multi-shipment. The unequal lot sizes were considered to cope with the variation of customers' demands. The authors used a direct search method to decide the decision variables' optimal values, justified their proposed lot-splitting solution versus no lot-splitting approach in terms of cost savings, and provided further analyses to demonstrate their effectiveness model. Additional studies [23-27] explored the influence of different characteristics of multiple shipments on various fabrication-transportation coordinated supply chains. As few prior works focused on examining the collective influence of expedited rate, unreliable machines, scrap, rework, and multi-shipment rule on the optimal production runtime decision in a vendor-buyer collaborative business environment, this work aims to bridge the gap.

The proposed coordinated vendor-buyer system

Problem description and formulation

The proposed coordinated vendor-buyer system's description with an expedited rate, unreliable machine, scrap, and rework is as follows. A batch fabrication plan is employed to meet a product with an annual demand λ and accelerate the manufacturing uptime at an expedited rate P_{1A} is used. Consequently, a higher setup KA and unit C_A costs are linked to this extra percentage of manufacturing rate α_1. The relationship of speedup-rate related parameters P_{1A}, K_A, and C_A and the standard-rate relevant variables are expressed as follows:

$$P_{1A}=(1+\alpha_1)P_1 \tag{1}$$

$$K_A=(1+\alpha_2)K \tag{2}$$

$$C_A=(1+\alpha_3)C \tag{3}$$

Due to various unforeseen factors, a random x portion of manufactured products is nonconforming. No shortages are permitted, so $P_{1A}–d_{1A}–\lambda>0$ is assumed (where $d_{1A}=xP_{1A}$). A θ_1 portion of nonconforming products is confirmed to be scrap (where $0\le\theta_1\le1$) and will be disposed of at extra cost C_S per unit. Right after each cycle's uptime completes, the rework of the other $(1–\theta_1)$ portion of nonconforming products starts at a rate P_{2A} and extra unit cost C_{RA}. The relationship of speedup-rate related parameters P_{2A} and C_{RA} and the standard-rate variables are expressed as follows:

$$P_{2A}=(1+\alpha_1)P_2 \tag{4}$$

$$C_{RA}=(1+\alpha_3)C_R \tag{5}$$

Moreover, A θ_2 portion of reworked stocks is identified as scrap (where $0\le\theta_2\le 1$, thus, $d_{2A}=\theta_2 P_{2A}$) and these scrap products will also be disposed. Therefore, the overall scrap rate of nonconforming products in a cycle, $\varphi=[\theta_1+(1-\theta_1)\theta_2]$. In addition to product quality issues, the manufacturing equipment is subject to the Poisson distributed failure with β as mean per year. An abort/resume stock controlling discipline is adopted whenever a failure happens. Under this discipline, the failed equipment is repaired immediately. The fabrication of the unfinished/interrupted lot is resumed as soon as the equipment is fixed/restored. A constant repair time of the equipment tr is assumed. However, if real repair time shall exceed tr, then a piece of rental equipment will be in place to avoid additional fabrication delay. Because of a failure to randomly happen in manufacturing uptime t_{1A}, two distinct situations are studied in subsections 2.3 and 2.4. Lastly, upon completion of the regular production and rework processes, n equal-size installments of the lot are transported at a fixed interval of time t'_{nA} to the buyer in t'_{3A}.

Situation one: A stochastic failure happens in uptime

Figure 1 illustrates the perfect stock level in situation one (i.e., when $t<t_{1A}$). It indicates that the stock level comes to H_0 when a stochastic failure happens; after the equipment is restored, the stock level climbs up to H_1 at the end of t_{1A}; and it arrives at H when t'_{2A} ends, before the distribution time t'_{3A} starts.

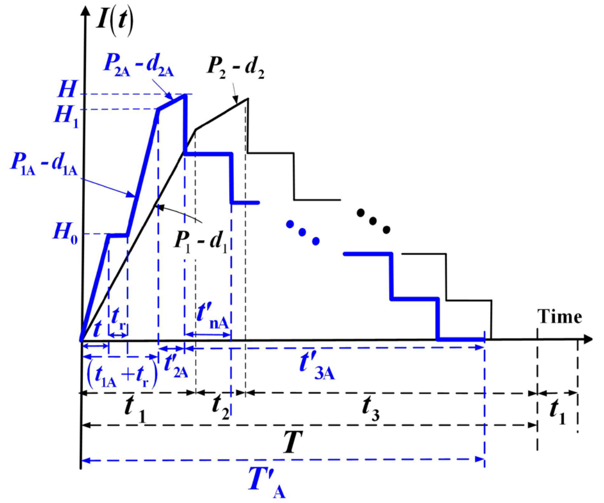

Figure 1: Level of perfect stocks in the proposed coordinated vendor-buyer system with an expedited rate, unreliable machine, scrap, and rework (in blue) as compared to that of the same system without expedited rate, nor machine failure (in black)

Figure 2 shows the safety stock level in situation one. It specifies that to meet the extra demand during t_r, the safety stocks λt_r will be added to the finished lot and distributed to the buyer in t'_{3A}.

Figure 3 illustrates the nonconforming and scrap items' level in situation one are depicted in Figures 3 and 4. From Figure 3, one notices that the maximal nonconforming stocks reach $d_{1A}t_{1A}$ when uptime t_{1A} ends; after disposal of scrap items, the level of nonconforming stocks drops to $d_{1A}t_{1A}(1-\theta_1)$; and at the end of t'_{2A}, it depletes to zero.

Figure 4 indicates that the level of scrap stocks comes to $(d_{1A}t)\theta_1$ when a stochastic failure happens; after the equipment is restored, the level scrap stocks climbs up to $(d_{1A}t_{1A})\theta_1$ at the end of t_{1A}; it reaches the maximal level $[(d_{1A}t_{1A})\theta_1+d_{2A}t'_{2A}]$ when t'_{2A} ends.

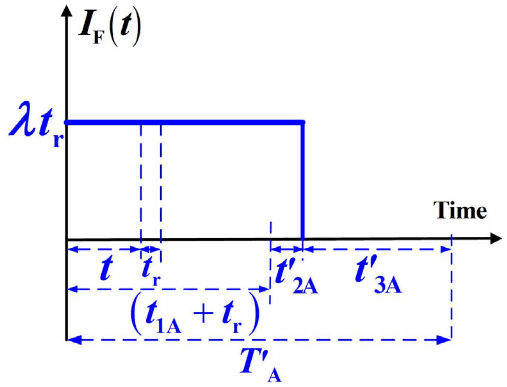

Figure 2: Level of safety stocks in situation one

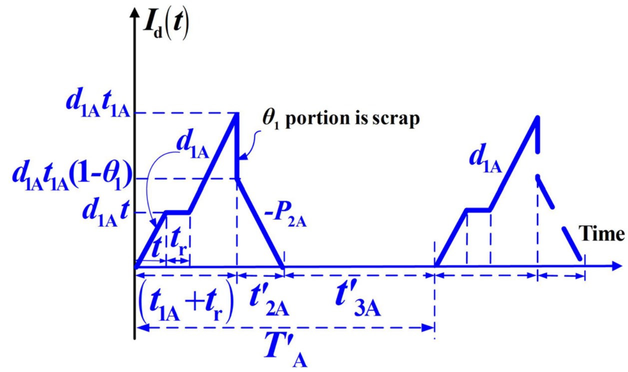

Figure 3: Level of nonconforming stocks in situation one

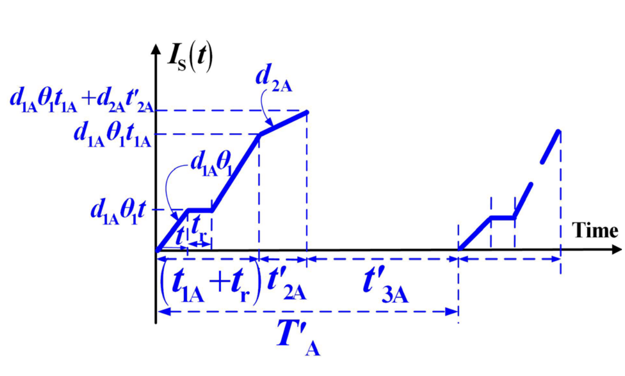

Figure 4: Level of scrap stocks in situation one

Based on problem description and Figures 1 to 4, the following equations are observed:

$$T'_A=t_{1A}+t_r+t'_{2A}+t'_{3A} \tag{6}$$

$$t_{1A}=\frac{Q}{P_{1A}}=\frac{H_1}{P_{1A}-d_{1A}} \tag{7}$$

$$t'_{2A}=\frac{Qx\left(1-\theta_1\right)}{P_{2A}} \tag{8}$$

$$t'_{3A}=T'_A-\left(t_{1A}+t'_{2A}+t_r\right) \tag{9}$$

$$T'_A=\frac{Q\left(1-x\varphi\right)}{\lambda}+t_r \tag{10}$$

$$t'_{3A}=\frac{Q}{\lambda}\left[\left(1-x\varphi\right)-\frac{\lambda}{P_{1A}}-\frac{\lambda x\left(1-\theta_1\right)}{P_{2A}}\right] \tag{11}$$

$$H_0=t\left(P_{1A}-d_{1A}\right) \tag{12}$$

$$H_1=\left(P_{1A}-d_{1A}\right)t_{1A} \tag{13}$$

$$H=H_1+\left(P_{2A}-d_{2A}\right)t'_{2A}+\lambda t_r=Q\left(1-x\varphi\right)+\lambda t_r \tag{14}$$

$$d_{1A}t_{1A}=xQ=\left(xP_{1A}\right)t_{1A} \tag{15}$$

$$\varphi\left(xQ\right)=\left[\theta_1+\left(1-\theta_1\right)\theta_2\right]\left(xQ\right) \tag{16}$$

Figure 5 exhibits the level of the producer's perfect stock in distribution time in situation one. Total stocks during t'_{3A} are expressed in Eq. (17) [5].

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1}i\right)H\left(t'_{3A}\right)=\left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right]H\left(t'_{3A}\right)=\left(\frac{n-1}{2n}\right)H\left(t'_{3A}\right) \tag{17}$$

The level of the customer's stock in situation one is displayed in Figure 6. Total stocks during the cycle time T'_A can be computed by Eq. (18) [5].

$$n\left(t'_{nA}\right)\left(D-\frac{\lambda\left(t'_{nA}\right)}{2}\right)+\frac{n(n-1)}{2}I\left(t'_{nA}\right)+\frac{nI}{2}\left(t_{1A}+t'_{2A}\right)=\frac{1}{2}\left[\frac{Ht'_{3A}}{n}+\left(H-\lambda t'_{3A}\right)T'_A\right] \tag{18}$$

where

$$D=\frac{H}{n} \tag{19}$$

$$I=D-\left(\lambda t'_{nA}\right) \tag{20}$$

In situation one, $TC(t_{1A})_1$ comprises the following variable and fixed expedited fabrication costs, the repair cost of failed equipment, safety stock relevant costs, rework and disposal costs, variable and fixed distribution costs, and total stock holding costs (comprising the reworked items, buyer's items, and the finished and nonconforming items in a cycle):

$$\begin{aligned}TC\left(t_{1A}\right)_1=&C_AQ+K_A+M+\left[h_3\left(\lambda t_r\right)\left(t_{1A}+t_r+t'_{2A}\right)\right]\\&+C_1\left(\lambda t_r\right)+C_{RA}xQ\left(1-\theta_1\right)+C_S\varphi xQ\\&+nK_1+C_T\left[Q\left(1-\varphi x\right)+\lambda t_r\right]+h_1\frac{P_{2A}t'_{2A}}{2}\left(t'_{2A}\right)\\&+\frac{h_2}{2}\left[\frac{Ht'_{3A}}{n}+T'_A\left(H-\lambda t'_{3A}\right)\right]\\&+h\left[\frac{H_1+d_{1A}t_{1A}}{2}\left(t_{1A}\right)+\left(H_0t_r\right)+\left(d_{1A}t\right)t_r+\frac{H_1+\left(H-\lambda t_r\right)}{2}\left(t'_{2A}\right)+\left(\frac{n-1}{2n}\right)Ht'_{3A}\right]\end{aligned} \tag{21}$$

To deal with the randomness of x, we apply the expected values and substitute Eqs. (1) to (20) in Eq. (21) to gain $E[TC(t_{1A})_1]$ as follows:

$$\begin{aligned}\left[TC\left(t_{1A}\right)_1\right]=&\left[\left(1+\alpha_2\right)K\right]+\left[\left(1+\alpha_3\right)C\right]\left[\left(1+\alpha_1\right)P_1t_{1A}\right]\\&+nK_1+C_T\left[\left[\left(1+\alpha_1\right)P_1t_{1A}\right]y_0+\lambda g\right]\\&+M+\left(1+\alpha_3\right)C_RE\left[x\right]\left[\left(1+\alpha_1\right)P_1t_{1A}\right]\left(1-\theta_1\right)\\&+C_S\varphi E\left[x\right]\left[\left(1+\alpha_1\right)P_1t_{1A}\right]+C_1\lambda g\\&+\frac{E\left[x\right]^2\left[\left(1+\alpha_1\right)P_1t_{1A}\right]^2\left(1-\theta_1\right)}{2\left[\left(1+\alpha_1\right)P_2\right]}\left[h_1\left(1-\theta_1\right)-h\right]\\&+\frac{\left[\left(1+\alpha_1\right)P_1t_{1A}\right]^2}{2n\lambda}\left(h_2-h\right)y_0\left(y_0-y_1\right)\\&+\frac{h_2\left[\left(1+\alpha_1\right)P_1t_{1A}\right]^2y_0}{2}\left(\frac{y_1}{\lambda}\right)+\frac{h\left[\left(1+\alpha_1\right)P_1t_{1A}\right]^2}{2\lambda}\left[{y_0}^2+\frac{\lambda E\left[x\right]\varphi}{\left(1+\alpha_1\right)P_1}+\frac{\lambda E\left[x\right]\left(1-\theta_1\right)}{\left(1+\alpha_1\right)P_2}\right]\\&+\frac{\left(h_2-h\right)g}{2n}\left[\left(1+\alpha_1\right)P_1t_{1A}\right]\left(y_0-y_1\right)\\&+\frac{h_2g}{2}\left[\left[\left(1+\alpha_1\right)P_1t_{1A}\right]\left(y_0+y_1\right)+\lambda g\right]\\&+hg\left[\left[\left(1+\alpha_1\right)P_1\right]t+\frac{\left(1+\alpha_1\right)P_1t_{1A}}{2}\left(y_0-y_1\right)\right]\\&+h_3g\left[\left[\left(1+\alpha_1\right)P_1t_{1A}\right]y_1+\lambda g\right]\end{aligned} \tag{22}$$

where

$$y_0=\left[1-E\left[x\right]\varphi\right]\quad y_1=\left[\frac{\lambda}{\left(1+\alpha_1\right)P_1}+\frac{E\left[x\right]\lambda\left(1-\theta_1\right)}{\left(1+\alpha_1\right)P_2}\right]$$

Situation two: No failures happen in uptime

Figure 5 exhibits the perfect stock level in situation two (i.e., when $t\geq t_{1A}$). It illustrates that the stocks climb up to H_1 when t_{1A} finishes, and it arrives at H when t_{2A} ends, before t_{3A} begins.

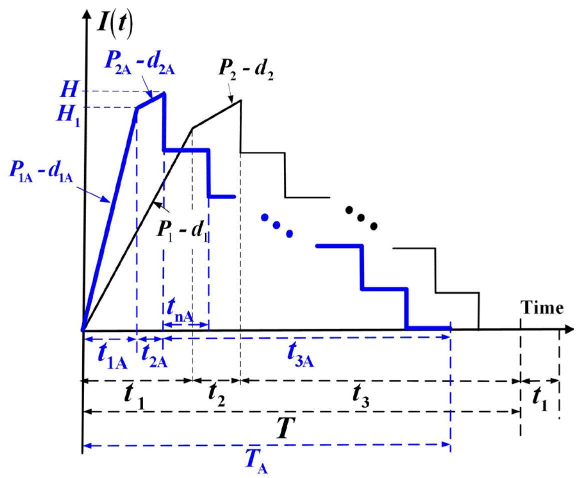

Figure 5: Level of perfect stocks in situation two of our coordinated vendor-buyer system with an expedited-rate, scrap, rework, but no equipment failure happening (in blue) as compared to the same system without expedited rate (in black)

Because of no failure happening in t_{1A}, the safe stocks are not used during T_A. The maximal level of nonconforming stocks reaches $d_{1A}t_{1A}$ when uptime t_{1A} ends; after disposal of scrap items, the level of nonconforming stocks drops to $d_{1A}t_{1A}(1–\theta_1)$; and it reduces to zero when t_{2A} ends (refer to Figure 3 excepts the following parameters are utilized: t_{2A}, t_{3A}, and T_A).

Similarly, the level of scrap stocks climbs up to $(d_{1A}t_{1A})\theta_1$ at the end of t_{1A}, and it reaches the maximal level $[(d_{1A}t_{1A})\theta_1+d_{2A}t_{2A}]$ at the end of rework time t_{2A} (see Figure 4 excepts the following parameters are used: t_{2A}, t_{3A}, and T_A).

Based on problem description for situation two and Figure 5, the following equations can be observed:

$$T_A=t_{1A}+t_{2A}+t_{3A} \tag{23}$$

$$t_{1A}=\frac{Q}{P_{1A}}=\frac{H_1}{P_{1A}-d_{1A}} \tag{24}$$

$$t_{2A}=\frac{Qx(1-\theta_1)}{P_{2A}} \tag{25}$$

$$t_{3A}=T_A-(t_{1A}+t_{2A}) \tag{26}$$

$$T_A=\frac{Q(1-x\varphi)}{\lambda} \tag{27}$$

$$t_{3A}=\frac{Q}{\lambda}\left[(1-x\varphi)-\frac{\lambda}{P_{1A}}-\frac{\lambda x(1-\theta_1)}{P_{2A}}\right] \tag{28}$$

$$H_1=(P_{1A}-d_{1A})t_{1A} \tag{29}$$

$$H=Q(1-x\varphi)=H_1+(P_{2A}-d_{2A})t_{2A} \tag{30}$$

Total stocks during t_{3A} [5] is expressed in Eq. (31).

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1}i\right)H(t_{3A})=\left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right]H(t_{3A})=\left(\frac{n-1}{2n}\right)H(t_{3A}) \tag{31}$$

Total stocks during the cycle time TA can be calculated [5] by Eq. (32).

$$\frac{1}{2}\left[\frac{Ht_{3A}}{n}+T_A(H-\lambda t_{3A})\right] \tag{32}$$

In situation two, $TC(t_{1A})_2$ includes the following variable and fixed expedited production costs, safety stock holding costs, rework and disposal costs, fixed and variable distribution costs, and total holding costs (comprising reworked items, buyer's items, and the finished and nonconforming items in a cycle):

$$\begin{aligned}TC(t_{1A})_2=&C_AQ+K_A+h_3(\lambda t_r)T_A+C_{RA}xQ(1-\theta_1)+C_S\varphi xQ+nK_1\\&+C_T[Q(1-\varphi x)]+h_1\frac{P_{2A}t_{2A}}{2}(t_{2A})+\frac{h_2}{2}\left[\frac{Ht_{3A}}{n}+T_A(H-\lambda t_{3A})\right]\\&+h\left[\frac{H_1+d_{1A}t_{1A}}{2}(t_{1A})+\frac{H_1+H}{2}(t_{2A})+\left(\frac{n-1}{2n}\right)Ht_{3A}\right]\end{aligned} \tag{33}$$

To deal with the randomness of x, we apply the expected values and substitute Eqs. (15) to (16) and (23) to (32) in Eq. (33) to gain the following $E[TC(t_{1A})_2]$:

$$\begin{aligned}E[TC(t_{1A})_2]=&[(1+\alpha_2)K]+[(1+\alpha_3)C][(1+\alpha_1)P_1t_{1A}]\\&+nK_1+C_T[(1+\alpha_1)P_1t_{1A}]y_0\\&+(1+\alpha_3)C_RE[x][(1+\alpha_1)P_1t_{1A}](1-\theta_1)\\&+C_S\varphi E[x][(1+\alpha_1)P_1t_{1A}]+h_3g[(1+\alpha_1)P_1t_{1A}]y_0\\&+\frac{E[x]^2[(1+\alpha_1)P_1t_{1A}]^2(1-\theta_1)}{2[(1+\alpha_1)P_2]}[h_1(1-\theta_1)-h]\\&+\frac{[(1+\alpha_1)P_1t_{1A}]^2}{2n\lambda}(h_2-h)y_0(y_0-y_1)\\&+\frac{h_2[(1+\alpha_1)P_1t_{1A}]^2y_0}{2}\left(\frac{y_1}{\lambda}\right)\\&+\frac{h[(1+\alpha_1)P_1t_{1A}]^2}{2\lambda}\left[y_0^2+\frac{\lambda E[x]\varphi}{(1+\alpha_1)P_1}+\frac{\lambda E[x](1-\theta_1)}{(1+\alpha_1)P_2}\right]\end{aligned} \tag{34}$$

where

$$y_0=[1-E[x]\varphi]\quad y_1=\left[\frac{\lambda}{(1+\alpha_1)P_1}+\frac{E[x]\lambda(1-\theta_1)}{(1+\alpha_1)P_2}\right]$$

Solving the proposed vendor-buyer coordinated system

Since the mean of the Poisson distributed failure rate is β, so the time to failure has the Exponentially distributed density function $\beta e^{-\beta t1A}$ (i.e., $f(t)$) and cumulative density function $(1–e^{-\beta t1A})$ (i.e., $F(t)$). Besides, the cycle time is variable due to random scrap rate φ. The renewal reward theorem is employed to deal with the variable cycle time. Thus, $E[TCU(t_{1A})]$ is computed as follows:

$$E[TCU(t_{1A})]=\frac{\left\{\int_0^{t_{1A}}E[TC(t_{1A})_1]\times f(t)dt+\int_{t_{1A}}^{\infty}E[TC(t_{1A})_2]\times f(t)dt\right\}}{E[T_A]} \tag{35}$$

where $E[T_A]$, $E[T'_A]$, and $E[T_A]$ represent the following:

$$E[T_A]=\int_0^{t_{1A}}E[T'_A]\cdot f(t)dt+\int_{t_{1A}}^{\infty}E[T_A]\cdot f(t)dt \tag{36}$$

$$E[T'_A]=\frac{Q[1-\varphi\cdot E[x]]+\lambda t_r}{\lambda}=\frac{t_{1A}P_{1A}[1-\varphi\cdot E[x]]+\lambda t_r}{\lambda} \tag{37}$$

$$E[T_A]=\frac{Q[1-\varphi\cdot E[x]]}{\lambda}=\frac{t_{1A}P_{1A}[1-\varphi\cdot E[x]]}{\lambda} \tag{38}$$

Substitute equations (22), (34), and (36) in Eq. (35), along with extra derivation efforts one gains the following $E[TCU(t_{1A})]$ (for details, see Appendix A):

$$E[TCU(t_{1A})]=\left[\frac{\lambda}{y_0+\frac{\lambda g(1-e^{-\beta t_{1A}})}{(t_{1A})[(1+\alpha_1)P_1]}}\right]\left[\frac{z_0}{t_{1A}}+\frac{z_1}{t_{1A}}+z_2e^{-\beta t_{1A}}+\frac{z_3}{t_{1A}}e^{-\beta t_{1A}}-z_4e^{-\beta t_{1A}}+z_4+z_5t_{1A}+z_6\right] \tag{39}$$

The first- and second-derivatives of $E[TCU(t_{1A})]$ are derived and exhibited in Eqs. (B-1) and (B-2) (see Appendix B). If Eq. (B-3) is verified to be accurate, then we can solve the optimal t_{1A}* by setting the first-derivative of $E[TCU(t_{1A})]$=0 (refer to Eq. (B-1)). Since the first term on the RHS of Eq. (B-1) is positive, we have the following:

$$\begin{bmatrix} -(z_0+z_1)\left[y_0\left[(1+\alpha_1)P_1\right]+e^{-\beta t_{1A}}\lambda g\beta\right]-(z_4+z_6)(\lambda g)\left(e^{-\beta t_{1A}}\beta(t_{1A})+e^{-\beta t_{1A}}-1\right) \\ +(z_2-z_4)\left[-e^{-\beta t_{1A}}\beta(t_{1A})^2 y_0\left[(1+\alpha_1)P_1\right]-e^{-\beta t_{1A}}\beta(t_{1A})\lambda g-e^{-2\beta t_{1A}}\lambda g+e^{-\beta t_{1A}}\lambda g\right] \\ +z_3\left[-e^{-\beta t_{1A}}\beta(t_{1A})y_0\left[(1+\alpha_1)P_1\right]-e^{-\beta t_{1A}}\beta\lambda g-e^{-\beta t_{1A}}y_0\left[(1+\alpha_1)P_1\right]\right] \\ +z_5(t_{1A})\left[(t_{1A})\left[(1+\alpha_1)P_1\right]y_0+2\lambda g\left(1-e^{-\beta t_{1A}}\right)-e^{-\beta t_{1A}}(t_{1A})\lambda g\beta\right] \end{bmatrix}=0 \quad (40)$$

Let v_0, v_1, and v_2 represent the following:

$$v_0=\left[(z_2-z_4)\left[-e^{-\beta t_{1A}}\beta y_0(1+\alpha_1)P_1\right]+z_5\left[(1+\alpha_1)P_1 y_{+0}-e^{-\beta t_{1A}}\lambda g\beta\right]\right]$$

$$v_1=\left[-\left(e^{-\beta t_{1A}}\beta\right)\left[(z_4+z_6)(\lambda g)+(z_2-z_4)(\lambda g)+z_3 y_0(1+\alpha_1)P_1\right]+z_5\left[2\lambda g\left(1-e^{-\beta t_{1A}}\right)\right]\right]$$

$$v_2=\begin{bmatrix} -(z_0+z_1)\left[y_0(1+\alpha_1)P_1+e^{-\beta t_{1A}}\lambda g\beta\right]-(z_4+z_6)(\lambda g)\left(e^{-\beta t_{1A}}-1\right) \\ +(z_2-z_+)\lambda g\left(-e^{-2\beta t_{1A}}+e^{-\beta t_{1A}}\right)-z_3 e^{-\beta t_{1A}}\left[\beta\lambda g+y_0(1+\alpha_1)P_1\right] \end{bmatrix}$$

Eq. (40) becomes as follows:

$$v_0(t_{1A})^2+v_1(t_{1A})+v_2=0 \quad (41)$$

Apply the square root solution, t_A* can be determined as follows:

$$t_{1A}^*=\frac{-v_1\pm\sqrt{v_1^2-4v_0v_2}}{2v_0} \quad (42)$$

Since $F(t_{1A})=(1–e^{–\beta t1A})$ is over the interval of [0, 1], so does its complement $e^{–\beta t1A}$. Furthermore, Eq. (40) can be rearranged as follows:

$$e^{-\beta t_{1A}}=\frac{-(z_0+z_1)y_0\left[(1+\alpha_1)P_1\right]+(z_4+z_6)(\lambda g)+z_5 t_{1A}\left[t_{1A}(1+\alpha_1)P_1 y_0+2\lambda g\right]}{\left\{\begin{matrix}(z_0+z_1)\lambda g\beta+(z_4+z_6)(\lambda g)(\beta t_{1A}+1)+z_3\left[\beta\lambda g+y_0(1+\alpha_1)P_1(1+\beta t_{1A})\right] \\ +(z_2-z_4)\left[\beta t_{1A}^2 y_0(1+\alpha_1)P_1+\lambda g\left(\beta t_{1A}+e^{-\beta t_{1A}}-1\right)\right]+z_5\lambda g t_{1A}(2+t_{1A}\beta)\end{matrix}\right\}} \quad (43)$$

To find t_{1A}*, we set $e^{–\beta t1A}$=1 and $e^{–\beta t1A}$=0 initially, then compute Eq. (42) to gain the initial bounds for t_{1A} (i.e., t_{1AU} and t_{1AL}). The next step is to use the present t_{1AU} and t_{1AL} to calculate and obtain the updated values of $e^{–\beta t1AU}$ and $e^{–\beta t1AL}$. Then, re-calculate Eq. (42) with the present $e^{–\beta t1AL}$ and $e^{–\beta t1AU}$ to gain a new set of bounds t_{1AL} and t_{1AU}. If (t_{1AL}=t_{1AU}) is true, then t_{1A}* is found (i.e., t_{1A}*=t_{1AU}=t_{1AL}); otherwise, repeat the iterations above until t_{1AL}=t_{1AU}.

Numerical example with discussion

The following values of parameters are considered in the proposed vendor-buyer coordinated system with expedited rate, unreliable machine, and scrap/rework of defective items (see Table 1).

We start with the convexity test (refer to Eq. (B-3)) for $E[TCU(t_{1A})]$ by setting $e^{–\beta t1A}$=0 and $e^{–\beta t1A}$=1 (where β=1.0). Apply Eq. (42) we obtain t_{1AU}=0.3106 and t_{1AL}=0.0914. Then, applying t_{1AU} and t_{1AL} to compute $e^{–\beta t1AU}$=0.7330 and $e^{–\beta t1AL}$=0.9127. Finally, apply Eq. (B-3) with the obtained values of $e^{–\beta t1AU}$ and $e^{–\beta t1AL}$, we confirm that $\gamma(t_{1AU})$=0.5388>t_{1AU}=0.3106>0 and $\gamma(t_{1AL})$=0.2931>t_{1AL}=0.0914>0. Therefore, for β=1 the convexity of $E[TCU(t_{1A})]$ is confirmed. Thus, the optimal t_{1A}* exists. To demonstrate the broader applicability of our study, a wider range of β values were used for the convexity tests and Table C-1 (Appendix C) shows the results.

Upon verification of the convexity of $E[TCU(t_{1A})]$, we start to find t_{1A}*. We set $e^{–\beta t1A}$=0 and $e^{–\beta t1A}$=1, then apply Eq. (42), for β=1.0 we obtain t_{1AU}=0.3106 and t_{1AL}=0.0914. Then, using the current values of t_{1AU} and t1AL to compute $e^{–\beta t1AU}$=0.7330 and $e^{–\beta t1AL}$=0.9127. Repeatedly apply Eq. (42) to update t_{1AU} and t_{1AL}, and recalculate updated values of $e^{–\beta t1AU}$ and $e^{–\beta t1AL}$ until t_{1AU}=t_{1AL}. Accordingly, we obtain t_{1A}*=0.1280 and by applying Eq. (39) $E[TCU(t_{1A}*)]$=$13,792.94 is obtained. Table C-2 (in Appendix C) exhibits the iterative outcomes for deriving t_{1A}*. The influence of changes in t_{1A} on $E[TCU(t_{1A})]$ is illustrated in Figure 6. It specifies $E[TCU(t_{1A})]$ increases significantly in both directions, as t_{1A} deviates from t_{1A}*.

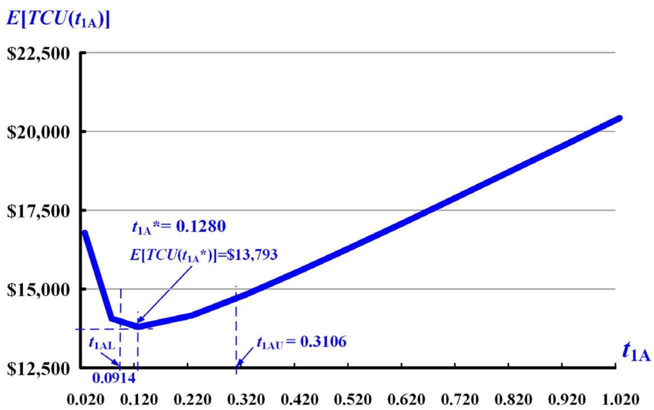

Figure 6: The impact of variations in t_{1A} on $E[TCU(t_{1A})]$

Figure 7 depicts the influence of differences in mean-time-to-breakdown 1/β with different scrap rates on the optimal cost $E[TCU(t_{1A}*)]$. It discloses that as 1/β increases, $E[TCU(t_{1A}*)]$ declines, and $E[TCU(t_{1A}*)]$ decreases radically starting from 1/β≥0.20 (i.e., β≤5) and it continues to drop and reach $13,149 when 1/$\beta$ approaches infinite (i.e., when a system has no breakdown instance). Figure 7 also points out that as φ goes up, the system cost increases slightly.

The impact of variations in the delivery's frequency n on the system related costs is demonstrated in Figure 8. It exposes that when n=1, there is a significantly higher holding cost at the customer end; and as n increases,

Table 1: The values of parameters used in the example

Parameters	C_A	C	C_{RA}	C_R	C_S	C_T	K_A	P_{1A}	P_{2A}	α_1
Values	2.5	2.0	1.25	1.0	0.1	0.01	220	15000	7500	0.5
	β	θ_1	θ_2	φ	h	h_1	K	P_1	P_2	α_2
	1	0.3	0.3	0.51	0.4	0.4	200	10000	5000	0.1
	C_1	n	x	g	h_2	h_3	K_1	λ	M	α_3
	2.0	3	20%	0.018	1.6	0.4	90	4000	2500	0.25

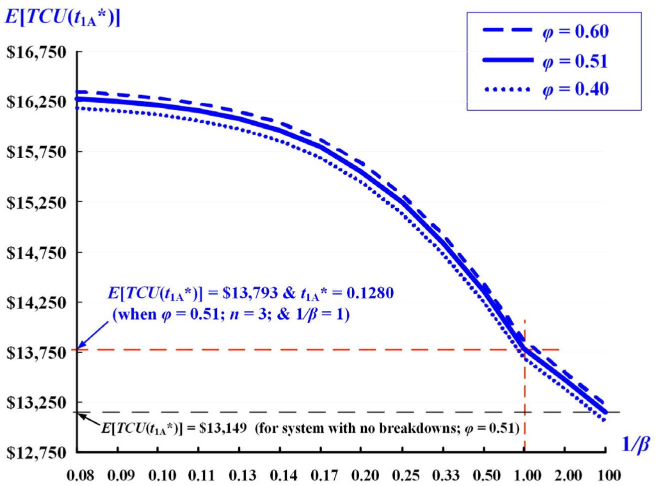

Figure 7: The influence of differences in 1/β with different φ on E[TCU(t_{1A})]*

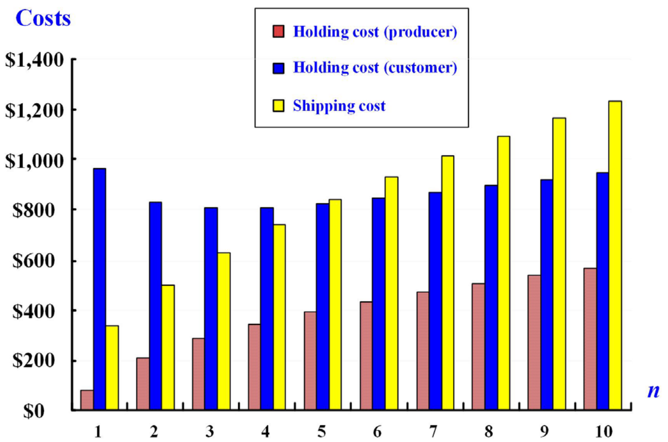

Figure 8: The impact of variations in n on the system related costs

both the shipping cost and producer's holding cost increase drastically (the former is due to more frequent deliveries and the latter is owing to a slow stock movement from the producer to the customer).

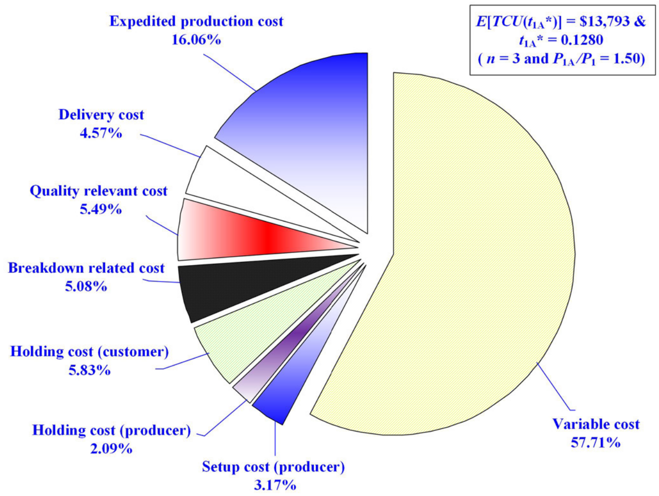

Figure 9: The breakup of E[TCU(t_{1A})]*

Figure 9 exhibits the breakup of E[TCU(t1A*)] where the cost contributors to E[TCU(t1A*)] are exposed. For instance, 16.06% of the system cost is related to the price for expedited rate, 5.08% is the machine breakdown relevant cost, and a 5.49% cost relates to the quality assurances issues, etc.

Detailed analysis of the quality cost contributors is performed, and the results are displayed in Figure 10. It shows that the extra production cost contributes 56.79% of the quality cost for making up scrapped items.

Figure 11 depicted the combined impact of differences in mean-time-to-breakdown 1/*β* and the expedited-rate α_1 on t_{1A}*. It discloses that t_{1A}* decreases radically, as both α_1 and 1/*β* goes up.

Figure 12 reveals the combined influence of changes in the expedited-rate α_1 and overall scrap rate *φ* on the optimal *E*[*TCU*(t_{1A}*)]. It indicates that *E*[*TCU*(t_{1A}*)] goes up drastically as α1 increases, and it only goes up slightly as *φ* rises. Hence, we know that α_1 has more influence on *E*[*TCU*(t_{1A}*)] than that of *φ*.

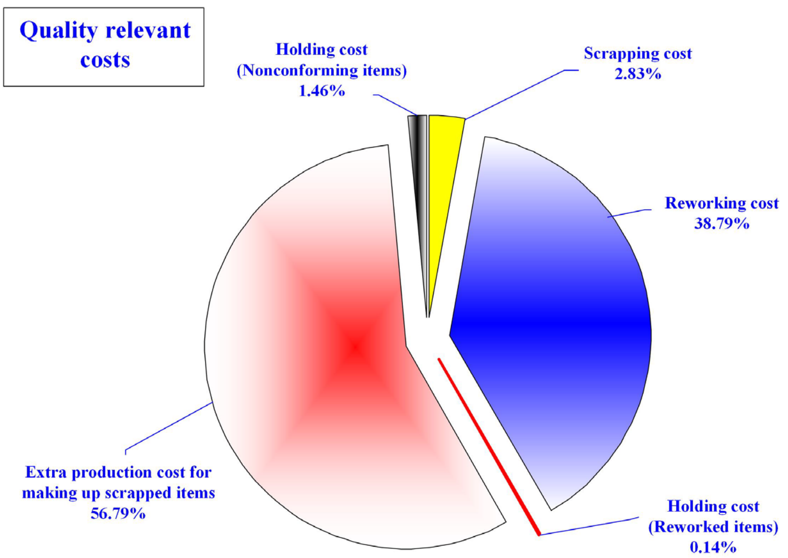

Figure 10: A further breakup of the product quality relevant costs

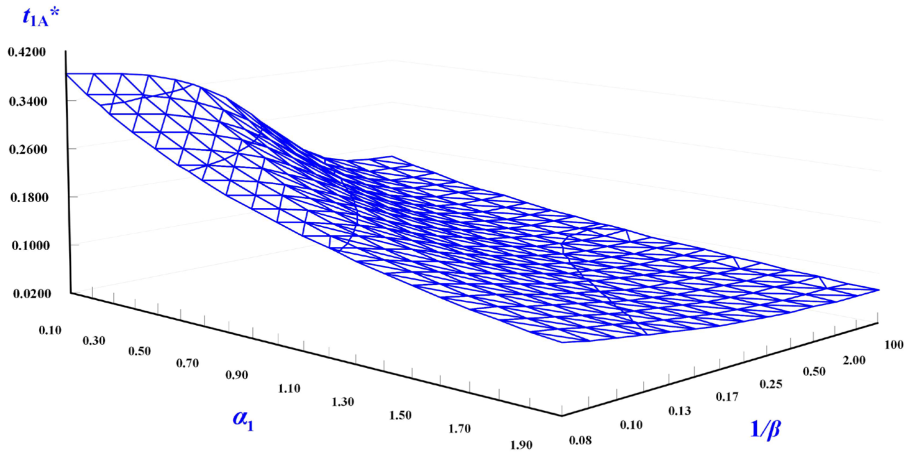

Figure 11: The combined impact of differences in α_1 and 1/β on t_{1A}*

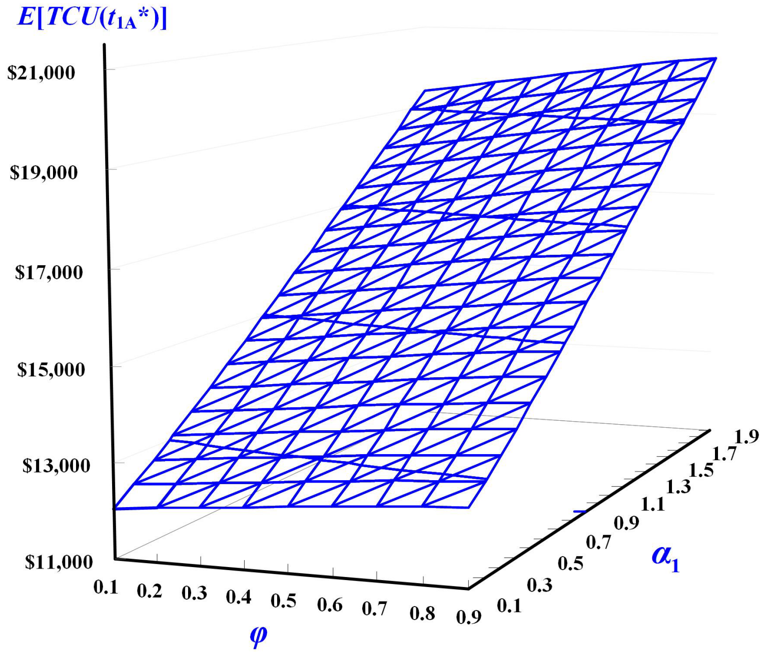

Figure 12: The collective influence of variations in α_1 and φ on $E[TCU(t_{1A}^*)]$

Figure 13 depicts the impact of changes in the expedited-rate α_1 on different system cost contributors. It shows that as α_1 increases, the expedited cost goes up radically, but other system costs change insignificantly. Besides, it reconfirms that $E[TCU(t_{1A}^*)]$=$13,793, at α_1=0.5.

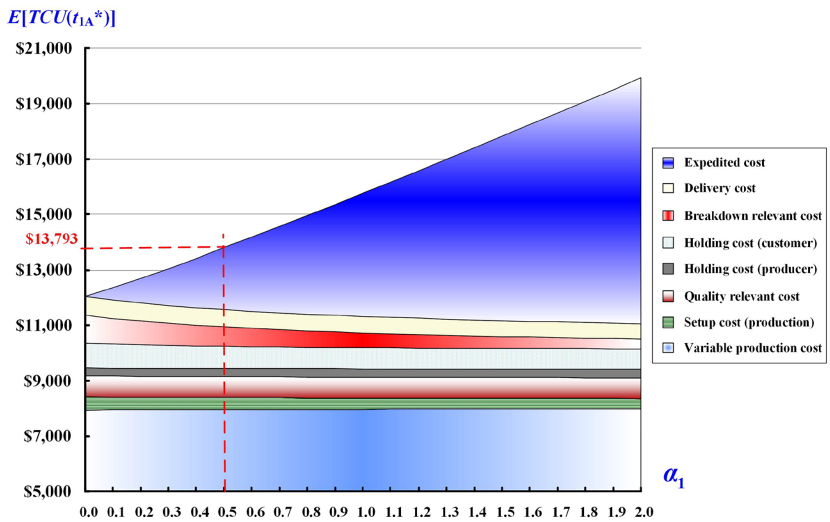

Figure 13: The impact of changes in α_1 on distinct system cost contributors

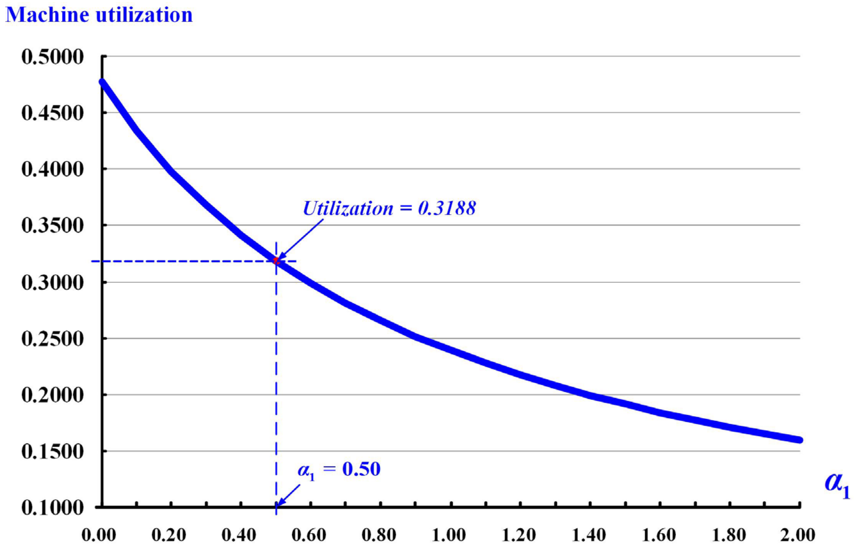

Figure 14: The behavior of machine utilization with reference to α_1

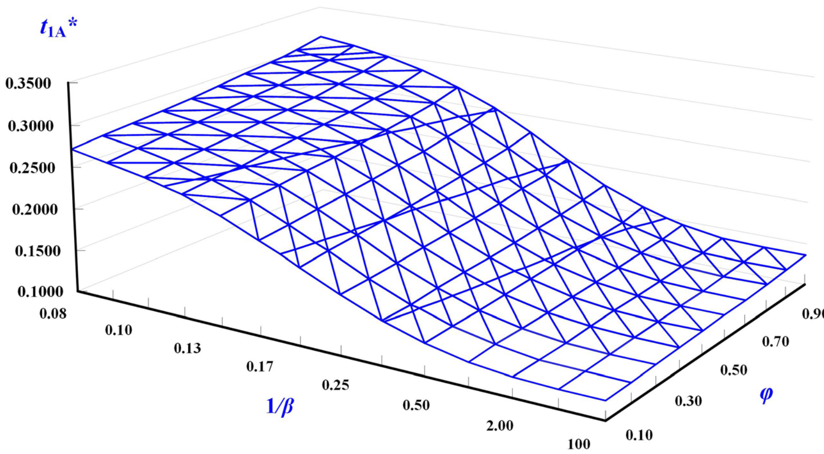

Figure 15: The joint impact of differences in φ and 1/β on the optimal runtime t_{1A}^*

The behavior of machine utilization concerning α_1 is exhibited in Figure 14. It indicates that utilization declines from 0.4772 to 0.3188 (i.e., a 33.19% decreases), as we expedite the fabrication rate by 50% (i.e., α_1=0.5).

Figure 15 exhibits the joint impact of differences in overall scrap rate φ and mean-time-to- breakdown $1/\beta$ on the optimal runtime t_{1A}^*. It exposes that t_{1A}^* rises slightly as φ goes up. Conversely, t_{1A}^* declines significantly as $1/\beta$ increases. Thus, we find that $1/\beta$ has more impact on t_{1A}^* than that of φ.

Discussion and limitation

This study develops the replenishment runtime model based on a case where only one or no breakdowns occur during a production cycle. Table D-1 (see Appendix D) provides the probabilities of different Poisson-distributed breakdown rates. It shows that for production equipment in “good” condition or with an average of less than one random breakdown per year, the proposed model is appropriate, as there is over 99.25% chance that only one or no breakdowns will occur (see Table D-1).

Further, for equipment in “fair” condition, or with an average of less than or equal to three breakdowns per year, our model is suitable with an over 92.94% chance of one or no breakdown occurring (see Table D-1). However, if production equipment in “worse” condition (or having over four random breakdowns per year), our proposed model’s suitability will decline to less than 80%. In this case, we suggest production researchers/managers should develop a multiple failures model for this specific condition.

CONCLUSIONS

Seeking to help transnational manufacturing firms achieve operating goals of reducing order response time/fabrication cycle time, keeping the desired product quality, managing the unanticipated machine failures, and providing timely delivery, a specific fabrication-shipment model is built. It captures all of the problem’s particular characteristics, such as the expedited rate, Poisson distributed breakdowns, scrap/rework of defective items, and multiple shipments policy (refer to Figures 1 to 8). Mathematical and optimization approaches and a proposed algorithm enable us to decide the optimal fabrication runtime policy (see Subsections 3 and 4, Tables C and D, and Figure 6). The applicability of our model and the expose of crucial system information are provided (see Figures 7 to 15).

This work's contribution includes the following:

a. Developing a decision support model that assists in exploring the problem (see Subsections 2 and 3);
b. Deciding the optimal runtime of the proposed problem (refer to Subsections 3 and 4, and Figure 6); and
c. Exposing a diverse set of crucial information regarding the collective/individual impact of differences in the expedite rates, mean-time-to-breakdown, frequency of shipment, and rework/disposal rates of nonconforming items on the optimal runtime policy, utilization, total operating cost, and various cost contributors (refer to Figures 7 to 15).

This information facilitates better decision making. Future studies can extend the problem by exploring the influence of stochastic demand.

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NOMENCLATURE

λ=product demand rate per year,

P_{1A}=expedited manufacturing rate per year,

t_{1A}=the uptime in the failure happening case – the decision variable,

t'_{2A}=the rework time in the failure happening case,

t'_{3A}=the finished stock distribution time in the failure happening case,

$T_{'A}$=the cycle time in the failure happening case,

Q=the lot size,

β=mean machine breakdown rate per year – follows Poisson distribution,

t=time to a random machine failure,

M=the repair cost per failure instance,

t_r=time required to repair the failure,

x=nonconforming rate – which obeys the uniform distribution,

d_{1A}=fabrication rate of nonconforming items in t_{1A} when P_{1A} is used,

H_0=level of finished stock when a stochastic failure happens,

H_1=level of finished stock when fabrication uptime completes,

P_{2A}=the expedited reworking rate per year,

d_{2A}=fabrication rate of scrap items during t'_{2A} when P_{2A} is used,

H=level of finished stock when the rework time completes,

$t'nA$=time interval between any two deliveries in the failure happening case,

P_1=standard annual fabrication rate,

P_2=standard annual reworking rate,

K_A=the setup cost when P_{1A} is used,

C_A=the unit cost if P_{1A} is used,

C_{RA}=the reworking cost if P_{2A} is used,

K=the setup cost if P_1 is used,

C=the unit cost when P_1 is used,

C_R=the reworking cost when P_2 is used,

α_1=the connecting factor between P_{1A} and P_1,

α_2=the connecting factor between K_A and K,

α_3=the connecting factor between C_A and C, and C_{RA} and C_R,

t_{2A}=the reworking time in the no failure happening case,

t_{3A}=the finished stock distribution time in the no failure happening case,

T_A=the cycle time in the no failure happening case,

t_{nA}=time interval between any two deliveries in the no failure happening case,

d_1=fabrication rate of nonconforming items in t_{1A} when P1 is used,

d_2=fabrication rate of scrap items during t_2 when P_2 is used,

t_1=the uptime for a system without expedited rate, nor machine failure,

t_2=the reworking time for a system without expedited rate, nor machine failure,

t_3=the distribution time for a system without expedited rate, nor machine failure,

T=the cycle time for a system without expedited rate, nor machine failure,

h=unit holding cost,

C_S=unit disposal cost per scrap stock,

C_1=safety stock's unit cost,

C_T=unit distribution cost,

h_1=reworked stock's unit holding cost,

h_2=buyer stock's unit holding cost,

h_3=safety stock's unit holding cost,

K_1=fixed distribution cost,

g=tr, fixed failure repair time,

$I(t)$=level of perfect stocks at time t,

$I_F(t)$=level of safety stocks at time t,

$I_d(t)$=level of nonconforming stocks at time t,

θ_1=the scrap portion of the nonconforming products,

θ_2=the scrap portion of the reworked products,

φ=the overall scrap rate of the nonconforming products,

$Is(t)$=level of scraps at time t,

D=the quantity per shipment,

I=the leftover stocks after each distribution time interval,

$I_c(t)$=the buyer's stock level at time t,

$TC(t_{1A})_1$=total cost per cycle in the failure happening case,

$E[TC(t_{1A})_1]$=the expected total cost per cycle in the failure happening case,

$E[T'_A]$=the expected cycle time in the failure happening case,

$TC(t_{1A})_2$=total cost per cycle in the no failure happening case,

$E[TC(t_{1A})_2]$=the expected total cost per cycle in the no failure happening case,

$E[T_A]$=the expected cycle time in the no failure happening case,

T_A=replenishment cycle time for the proposed system with or without a failure happening,

$E[TCU(t_{1A})]$=the expected system cost per unit time for the proposed system with or without a failure happening.

APPENDIX – A

Detailed derivations for Eq. (39) are given below.

By substituting equations (22), (34), and (36) in Eq. (35), along with extra derivation efforts, we first gain $E[TCU(t_{1A})]$ as exhibited in Eq. (A-1):

$$E\left[TCU\left(t_{1A}\right)\right]=\left[\frac{\lambda}{y_0+\frac{\lambda g\left(1-e^{-\beta t_{1A}}\right)}{\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]}}\right]\cdot \tag{A-1}$$

$$\left[\begin{aligned}&\frac{\left[\left(1+\alpha_2\right)K\right]}{\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]}+\frac{nK_1}{\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]}+\left[\left(1+\alpha_3\right)C\right]+\left[\left(1+\alpha_3\right)C_R\right]E[x]\left(1-\theta_1\right)+C_S\varphi E[x]\\&+C_Ty_0+h_3gy_0\left(e^{-\beta t_{1A}}\right)+\frac{E[x]^2\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]\left(1-\theta_1\right)}{2\left[\left(1+\alpha_1\right)P_2\right]}\left[h_1\left(1-\theta_1\right)-h\right]\\&+\frac{\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]}{2n\lambda}\left(h_2-h\right)y_0\left(y_0-y_1\right)+\frac{h_2\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]y_0}{2}\left(\frac{y_1}{\lambda}\right)\\&+\frac{h\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]}{2\lambda}\left[{y_0}^2+\frac{\lambda E[x]\varphi}{\left(1+\alpha_1\right)P_1}+\frac{\lambda E[x]\left(1-\theta_1\right)}{\left(1+\alpha_1\right)P_2}\right]+\frac{M}{\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]}\left(1-e^{-\beta t_{1A}}\right)\\&+\frac{C_T\lambda g}{t_{1A}\left[\left(1+\alpha_1\right)P_1\right]}\left(1-e^{-\beta t_{1A}}\right)+\frac{C_1\lambda g}{\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]}\left(1-e^{-\beta t_{1A}}\right)+\frac{hg}{t_{1A}}\left[-t_{1A}e^{-\beta t_{1A}}-\frac{1}{\beta}e^{-\beta t_{1A}}+\frac{1}{\beta}\right]\\&+\frac{h_2\lambda g^2}{2\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]}\left(1-e^{-\beta t_{1A}}\right)+\frac{g}{2n}\left(h_2-h\right)\left(y_0-y_1\right)\left(1-e^{-\beta t_{1A}}\right)+\frac{g}{2}h_2\left(y_0+y_1\right)\left(1-e^{-\beta t_{1A}}\right)\\&+\frac{g}{2}h\left(y_0-y_1\right)\left(1-e^{-\beta t_{1A}}\right)+h_3g\left[y_1+\frac{\lambda g}{\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]}\right]\left(1-e^{-\beta t_{1A}}\right)\end{aligned}\right]$$

Let z_0, z_1, z_2, z_3, z_4, z_5, and z_6 represent the following:

$$z_0=\frac{\left(1+\alpha_2\right)K}{\left(1+\alpha_1\right)P_1}+\frac{nK_1}{\left(1+\alpha_1\right)P_1}$$

$$z_1=\frac{M}{\left(1+\alpha_1\right)P_1}+\frac{C_T\lambda g}{\left(1+\alpha_1\right)P_1}+\frac{C_1\lambda g}{\left(1+\alpha_1\right)P_1}+\frac{h_3\lambda g^2}{\left(1+\alpha_1\right)P_1}+\frac{h_2\lambda g^2}{2\left(1+\alpha_1\right)P_1}+\frac{hg}{\beta}$$

$$z_2=-hg$$

$$z_3=-\frac{M}{\left(1+\alpha_1\right)P_1}-\frac{C_T\lambda g}{\left(1+\alpha_1\right)P_1}-\frac{C_1\lambda g}{\left(1+\alpha_1\right)P_1}-\frac{h_3\lambda g^2}{\left(1+\alpha_1\right)P_1}-\frac{h_2\lambda g^2}{2\left(1+\alpha_1\right)P_1}-\frac{hg}{\beta}$$

$$z_4=\frac{g}{2}\left[h\left(y_0-y_1\right)+\frac{\left(h_2-h\right)\left(y_0-y_1\right)}{n}+\left(h_2+2h_3\right)\left(y_0+y_1\right)\right]$$

$$\begin{aligned}z_5=&\frac{E[x]^2\left[\left(1+\alpha_1\right)P_1\right]\left(1-\theta_1\right)}{2\left[\left(1+\alpha_1\right)P_2\right]}\left[h_1\left(1-\theta_1\right)-h\right]+\frac{\left[\left(1+\alpha_1\right)P_1\right]\left(h_2-h\right)y_0}{2n\lambda}\left(y_0-y_1\right)\\&+\frac{h_2\left[\left(1+\alpha_1\right)P_1\right]y_0}{2}\left(\frac{y_1}{\lambda}\right)+\frac{h\left[\left(1+\alpha_1\right)P_1\right]}{2\lambda}\left[{y_0}^2+\frac{\lambda E[x]\varphi}{\left(1+\alpha_1\right)P_1}+\frac{\lambda E[x]\left(1-\theta_1\right)}{\left(1+\alpha_1\right)P_2}\right]\end{aligned}$$

$$z_6=\left[\left(1+\alpha_3\right)C\right]+\left[\left(1+\alpha_3\right)C_R\right]E[x]\left(1-\theta_1\right)+C_S\varphi E[x]+C_Ty_0$$

and

$$y_0=\left[1-E[x]\varphi\right]\quad y_1=\left[\frac{\lambda}{\left(1+\alpha_1\right)P_1}+\frac{E[x]\lambda\left(1-\theta_1\right)}{\left(1+\alpha_1\right)P_2}\right]$$

Then, Eq. (A-1) becomes the following:

$$E\left[TCU\left(t_{1A}\right)\right]=\left[\frac{\lambda}{y_0+\frac{\lambda g\left(1-e^{-\beta t_{1A}}\right)}{\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]}}\right] \tag{39}$$

$$\left[\frac{z_0}{t_{1A}}+\frac{z_1}{t_{1A}}+z_2e^{-\beta t_{1A}}+\frac{z_3}{t_{1A}}e^{-\beta t_{1A}}-z_4e^{-\beta t_{1A}}+z_4+z_5t_{1A}+z_6\right] \tag{39}$$

APPENDIX – B

The first- and second-derivatives of $E[TCU(t_{1A})]$ are derived and exhibited as follows:

$$\frac{dE\left[TCU\left(t_{1A}\right)\right]}{d\left(t_{1A}\right)}=\frac{\lambda\left[\left(1+\alpha_1\right)P_1\right]}{\left[\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]y_0+\lambda g\left(1-e^{-\beta t_{1A}}\right)\right]^2}\cdot \tag{B-1}$$

$$\left[\begin{array}{l}-\left(z_0+z_1\right)\left[y_0\left[\left(1+\alpha_1\right)P_1\right]+e^{-\beta t_{1A}}\lambda g\beta\right]-\left(z_4+z_6\right)\left(\lambda g\right)\left(e^{-\beta t_{1A}}\beta\left(t_{1A}\right)+e^{-\beta t_{1A}}-1\right)\\ +\left(z_2-z_4\right)\left[-e^{-\beta t_{1A}}\beta\left(t_{1A}\right)^2y_0\left[\left(1+\alpha_1\right)P_1\right]-e^{-\beta t_{1A}}\beta\left(t_{1A}\right)\lambda g-e^{-2\beta t_{1A}}\lambda g+e^{-\beta t_{1A}}\lambda g\right]\\ +z_3\left[-e^{-\beta t_{1A}}\beta\left(t_{1A}\right)y_0\left[\left(1+\alpha_1\right)P_1\right]-e^{-\beta t_{1A}}\beta\lambda g-e^{-\beta t_{1A}}y_0\left[\left(1+\alpha_1\right)P_1\right]\right]\\ +z_5\left(t_{1A}\right)\left[\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]y_0+2\lambda g\left(1-e^{-\beta t_{1A}}\right)-e^{-\beta t_{1A}}\left(t_{1A}\right)\lambda g\beta\right]\end{array}\right]$$

and

$$\frac{d^2E\left[TCU\left(t_{1A}\right)\right]}{d\left(t_{1A}\right)^2}=\frac{\lambda\left[\left(1+\alpha_1\right)P_1\right]}{\left[\left(t_{1A}\right)\left[\left(1+\alpha_1\right)P_1\right]y_0+\lambda g\left(1-e^{-\beta t_{1A}}\right)\right]^3}\cdot \tag{B-2}$$

$$\left[\begin{array}{l}\left(z_0+z_1\right)\left[\begin{array}{l}\left(\lambda g\beta\right)^2\left(e^{-2\beta t_{1A}}+e^{-\beta t_{1A}}\right)+e^{-\beta t_{1A}}\left(t_{1A}\right)y_0\left(\lambda g\right)\beta^2\left[\left(1+\alpha_1\right)P_1\right]\\ +4e^{-\beta t_{1A}}y_0\left(\lambda g\right)\beta\left[\left(1+\alpha_1\right)P_1\right]+2y_0^{\,2}\left[\left(1+\alpha_1\right)P_1\right]^2\end{array}\right]\\ +\left(z_2-z_4\right)e^{-2\beta t_{1A}}\left[\begin{array}{l}\left(\lambda g\beta\right)^2t_{1A}\left(1+e^{\beta t_{1A}}\right)+2\beta\left(\lambda g\right)^2\left(1-e^{\beta t_{1A}}\right)+e^{\beta t_{1A}}\beta^2\left(t_{1A}\right)^3y_0^{\,2}\left[\left(1+\alpha_1\right)P_1\right]^2\\ +y_0\lambda g\left(1+\alpha_1\right)P_1\left[\beta^2\left(t_{1A}\right)^2\left(2e^{\beta t_{1A}}+1\right)+2\beta\left(t_{1A}\right)\left(2-e^{\beta t_{1A}}\right)+2\left(1-e^{\beta t_{1A}}\right)\right]\end{array}\right]\\ +\left(z_4+z_6\right)\left(\lambda g\right)\left[\begin{array}{l}\left(\lambda g\right)\beta^2\left(t_{1A}\right)\left(e^{-2\beta t_{1A}}+e^{-\beta t_{1A}}\right)+2\left(\lambda g\beta\right)\left(e^{-2\beta t_{1A}}-e^{-\beta t_{1A}}\right)\\ +e^{-\beta t_{1A}}y_0\left[\left(1+\alpha_1\right)P_1\right]\left[\beta^2\left(t_{1A}\right)^2+2\beta\left(t_{1A}\right)+2\right]-2y_0\left[\left(1+\alpha_1\right)P_1\right]\end{array}\right]\\ +z_3\left(e^{-\beta t_{1A}}\right)\left[\begin{array}{l}\left(\lambda g\beta\right)^2\left(e^{-\beta t_{1A}}+1\right)+\beta^2\left(t_{1A}\right)y_0\left(\lambda g\right)\left[\left(1+\alpha_1\right)P_1\right]\left(2+e^{-\beta t_{1A}}\right)\\ +y_0^{\,2}\left[\left(1+\alpha_1\right)P_1\right]^2\left[\beta^2\left(t_{1A}\right)^2+2\beta\left(t_{1A}\right)+2\right]+2\beta y_0\left(\lambda g\right)\left[\left(1+\alpha_1\right)P_1\right]\left(1+e^{-\beta t_{1A}}\right)\end{array}\right]\\ +z_5\left(\lambda g\right)\left[\begin{array}{l}2e^{-2\beta t_{1A}}\left(\lambda g\right)+e^{-2\beta t_{1A}}\left(t_{1A}\right)^2\left(\lambda g\right)\beta^2+4\left(t_{1A}\right)\left(\lambda g\right)\beta\left(e^{-2\beta t_{1A}}-e^{-\beta t_{1A}}\right)\\ +e^{-\beta t_{1A}}\left(t_{1A}\right)^2\left(\lambda g\right)\beta^2+e^{-\beta t_{1A}}\left(t_{1A}\right)^3y_0\beta^2\left[\left(1+\alpha_1\right)P_1\right]-4e^{-\beta t_{1A}}\left(\lambda g\right)+2\left(\lambda g\right)\end{array}\right]\end{array}\right]$$

Since the first term on the right-hand side (RHS) of Eq. (B-2) is positive, we know that if the second term on the RHS of Eq. (B-2) is also positive (i.e., if $\gamma(t_{1A})>t_{1A}>0$ holds), then $E[TCU(t_{1A})]$ is convex.

$$\gamma\left(t_{1A}\right)=\frac{\begin{array}{l}-\left(z_0+z_1\right)\left[\left(\lambda g\beta\right)^2\left(e^{-2\beta t_{1A}}+e^{-\beta t_{1A}}\right)+y_0\left(1+\alpha_1\right)P_1\left[4e^{-\beta t_{1A}}\left(\lambda g\beta\right)+2y_0\left[\left(1+\alpha_1\right)P_1\right]\right]\right]\\ -\left(z_2-z_4\right)\left(e^{-2\beta t_{1A}}\right)\left(1-e^{\beta t_{1A}}\right)\left[2\beta\left(\lambda g\right)^2+2y_0\left(\lambda g\right)\left[\left(1+\alpha_1\right)P_1\right]\right]\\ -\left(z_4+z_6\right)\left(\lambda g\right)\left[2\left(\lambda g\right)\beta\left(e^{-2\beta t_{1A}}-e^{-\beta t_{1A}}\right)-2y_0\left[\left(1+\alpha_1\right)P_1\right]\left(1-e^{-\beta t_{1A}}\right)\right]\\ -z_3\left(e^{-\beta t_{1A}}\right)\left[\left(\lambda g\beta\right)^2\left(1+e^{-\beta t_{1A}}\right)+2y_0\left[\left(1+\alpha_1\right)P_1\right]\beta\left(\lambda g\right)\left(1+e^{-\beta t_{1A}}\right)+2y_0^{\,2}\left[\left(1+\alpha_1\right)P_1\right]^2\right]\\ -z_5\left(\lambda g\right)^2\left(2e^{-2\beta t_{1A}}-4e^{-\beta t_{1A}}+2\right)\end{array}}{\begin{array}{l}\left(z_0+z_1\right)\left[e^{-\beta t_{1A}}y_0\left(\lambda g\right)\beta^2\left[\left(1+\alpha_1\right)P_1\right]\right]\\ +\left(z_2-z_4\right)\left(e^{-2\beta t_{1A}}\right)\left[\begin{array}{l}\left(\lambda g\beta\right)^2\left(1+e^{\beta t_{1A}}\right)+e^{\beta t_{1A}}\beta^2t_{1A}^2y_0^{\,2}\left[\left(1+\alpha_1\right)P_1\right]^2\\ +\left(\lambda g\right)y_0\left[\left(1+\alpha_1\right)P_1\right]\left[2e^{\beta t_{1A}}\beta^2t_{1A}+\beta^2t_{1A}+4\beta-2e^{\beta t_{1A}}\beta\right]\end{array}\right]\\ +\left(z_4+z_6\right)\left(\lambda g\beta\right)\left[\left(\lambda g\beta\right)\left[e^{-2\beta t_{1A}}+e^{-\beta t_{1A}}\right]+y_0\left[\left(1+\alpha_1\right)P_1\right]\left[e^{-\beta t_{1A}}\beta t_{1A}+2e^{-\beta t_{1A}}\right]\right]\\ +z_3\left(e^{-\beta t_{1A}}\right)\left(1+\alpha_1\right)P_1y_0\left[2\beta^2\left(\lambda g\right)+e^{-\beta t_{1A}}\beta^2\left(\lambda g\right)+\beta^2t_{1A}y_0\left(1+\alpha_1\right)P_1+2\beta y_0\left(1+\alpha_1\right)P_1\right]\\ +z_5\left(\lambda g\right)\left[\left(\lambda g\right)\beta\left(e^{-2\beta t_{1A}}t_{1A}\beta+4e^{-2\beta t_{1A}}+e^{-\beta t_{1A}}t_{1A}\beta-4e^{-\beta t_{1A}}\right)+e^{-\beta t_{1A}}t_{1A}^2y_0\beta^2\left[\left(1+\alpha_1\right)P_1\right]\right]\end{array}}>t_{1A}>0 \tag{B-3}$$

APPENDIX – C

Table C-1: Additional convexity tests of $E[TCU(t_{1A})]$ using a wider choice of β values

β	$\gamma(t_{1AL})$	t_{1AL}	$\gamma(t_{1AU})$	t_{1AU}
10	0.0437	0.0203	0.9317	0.3055
8	0.0536	0.0248	0.6981	0.3057
6	0.0697	0.0318	0.5509	0.3059
4	0.0996	0.0441	0.4682	0.3064
3	0.1269	0.0542	0.4509	0.3069
2	0.1753	0.0690	0.4590	0.3078
1	0.2931	0.0914	0.5388	0.3106
0.5	0.4851	0.1065	0.7149	0.3162
0.01	3.8252	0.1245	4.3759	0.6654

Table C-2: Iterative outcomes from a recursive algorithm for deriving t_{1A}^*

Iteration#	t_{1AU}	$e^{-\beta t1AU}$	$E[TCU(t_{1AU})]$	t_{1AL}	$e^{-\beta t1AL}$	$E[TCU(t_{1AL})]$
-	-	0	-	-	1	-
1	0.3106	0.7330	$14,693.18	0.0914	0.9127	$13,915.55
2	0.1659	0.8471	$13,865.57	0.1186	0.8881	$13,799.10
3	0.1369	0.8720	$13,797.85	0.1256	0.8819	$13,793.29
4	0.1301	0.8780	$13,793.24	0.1274	0.8804	$13,792.96
5	0.1285	0.8794	$13,792.95	0.1278	0.8800	$13,792.94
6	0.1281	0.8798	$13,792.94	0.1279	0.8799	$13,792.94
7	0.1280	0.8799	$13,792.94	0.1280	0.8799	$13,792.94

APPENDIX – D

Table D-1: Probabilities of different Poisson-distributed breakdown rates

β	T_{1A}^*	P(x=0)	P(x=1)	P(x≤1)	P(x>1)
6.0	0.2039	29.43%	36.00%	65.43%	34.57%
5.0	0.1807	40.52%	36.60%	77.12%	22.88%
4.0	0.1601	52.71%	33.75%	86.46%	13.54%
3.0	0.1444	64.85%	28.09%	92.94%	7.06%
2.0	0.1338	76.52%	20.48%	97.00%	3.00%
1.0	0.1280	87.99%	11.26%	99.25%	0.75%
0.5	0.1267	93.86%	5.95%	99.81%	0.19%
0.01	0.1266	99.87%	0.13%	100.00%	0.00%

$$\frac{e^{-\beta t_{1A}^*}\left(\beta t_{1A}^*\right)^x}{x!} \tag{D-1}$$

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