Bankarstvo, 2017, vol. 46, br. 4 108
Primljen: 02.11.2017. Prihvaćen: 24.11.2017.

originalni naučni rad doi: 10.5937/bankarstvo1704108L

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MODELI OCENE PERFORMANSI PORTFOLIJA INVESTICIONIH FONDOVA - ŠARPOV, TREJNOROV I JENSENOV INDEKS

Prevod obezbedio autor

Rezime

Ocena performansi portfolija investicionih fondova je sastavni deo kontinuiranog procesa upravljanja portfoliom podređen unapređenju njegove efikasnosti. Važnost poznavanja modela ocene performansi portfolija investicionih fondova ogleda se u činjenici da se njihovom upotrebom pojedinačni i institucionalni investitori informišu o uspešnosti portfolio menadžmenta i performansnosti pojedinačnih investicionih fondova, a portfolio menadžeri fondova o prednostima i slabostima kreiranih portfolija. S tim u vezi, predmet rada su u teoriji najčešće korišćene i u praksi potpuno afirmisane mere performansi portfolija: Šarpov indeks, Trejnorov indeks i Jensenov ili alfa indeks. Budući da nijedna od navedenih mera performansi portfolija nije savršena, pažnja je posvećena i razumevanju problema i uvažavanju nedostataka i ograničenja sa kojima se suočavaju ovi modeli. Uvažavanjem nedostataka i ograničenja umanjuje se njihov značaj, a povećava značaj i validnost dobijenih rezultata.

Ključne reči: investicioni fondovi, performanse portfolija, Šarpov indeks, Trejnorov indeks, Jensenov indeks

JEL: G11, G23

Bankarstvo, 2017, Vol. 46, Issue 4
Received: 02.11.2017 Accepted: 24.11.2017

doi: 10.5937/bankarstvo1704108L

original scientific paper

MUTUAL FUNDS PORTFOLIO PERFORMANCE EVALUATION MODELS - SHARPE, TREYNOR AND JENSEN INDEX

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Summary

Portfolio performance evaluation is an integral part of a continuous portfolio management process aimed at improving its efficiency. The importance of being familiar with the evaluation models of mutual fund portfolio performance is reflected in the fact that such models inform individual and institutional investors about the success of portfolio management, as well as about the performance of individual mutual funds; moreover, the portfolio managers identify strengths and weaknesses of the created portfolios based on the mentioned models. In this respect, this paper looks at the methods that are most widely used, both in theory and in practice, to measure portfolio performance: Sharpe index, Treynor index and Jensen's alpha or index. Bearing in mind that none of these methods of portfolio performance measurement is perfect, the author focuses on identifying and understanding the disadvantages and limitations of these models. By identifying and understanding these disadvantages and limitations, their importance and effects are being reduced, while at the same time, the importance and accuracy of the results are being improved.

Keywords: mutual funds, portfolio performance, Sharpe index, Treynor index, Jensen index

JEL: G11, G23

Uvod

U nastojanju da privuku nove investitore investicioni fondovi međusobno konkurišu putem nižih provizija, boljih usluga, kvalitetnijeg izveštavanja i sl., ali se ipak osnovni aspekt konkurencije odnosi na ostvarene investicione performanse (Elton i saradnici, 2011). Budući da izbor konkretnog investicionog fonda dominantno zavisi od njegove uspešnosti, poznavanje modela ocene performansi portfolija investicionih fondova je od neprocenjivog značaja za pojedinačne i institucionalne investitore. Od uspešnosti investicionog fonda zavise i naredni potezi investitora u smislu zadržavanja sredstava u fondu, dodatnog ulaganja, ili pak povlačenja investiranih sredstava iz fonda.

Ocena performansi portfolija investicionih fondova je podjednako važna i za menadžere investicionih fondova, jer ih informiše o prednostima i slabostima kreiranog portfolija. Uočene prednosti nastoje da iskoriste, a slabosti da eliminišu promenom udela postojećih i uvođenjem novih hartija od vrednosti u portfolio. Na ovaj način ocena performansi portfolija dobija važnu ulogu u unapređenju procesa upravljanja portfoliom. Iako se u finansijskoj literaturi navodi kao poslednja faza portfolio menadžmenta, ocena performansi portfolija je podjednako važna koliko i portfolio analiza i selekcija, zbog čega se evaluaciji performansi portfolija mora pristupiti oprezno, stručno i objektivno. Prema Sharpe i saradnici (1995), ocena performansi portfolija je neka vrsta kontrolnog mehanizma koja portfolio menadžment može učiniti efikasnijim.

Ocenom portfolio performansi nastoji se utvrditi da li je kreirani portfolio ostvario prosečne, natprosečne ili ispodprosečne rezultate, kao i da li su ovi rezultati postignuti zahvaljujući sposobnostima portfolio menadžera ili sreći. Takođe, važno je ne samo da se oceni koliko dobro je investicioni fond poslovao u odnosu na druge fondove ili u odnosu na tržište, već i da se razume definisana investiciona politika fonda, kao i stepen u kojem su se portfolio menadžeri pridržavali ove politike. Uvidom u definisanu investicionu politiku i stepen poštovanja iste, ostvaruje se uvid u preuzeti rizik.

Razvoj modela ocene performansi portfolija

Razvoj modela ocene performansi portfolija odvijao se paralelno sa razvojem portfolio teorije. U vreme važenja tradicionalne portfolio teorije vladalo je uverenje da se ukupan rizik portfolija može eliminisati prostom (naivnom) diversifikacijom, pa su se performanse portfolija ocenjivale isključivo na osnovu ostvarene stope prinosa. Investicioni savetnici su u početku predlagali ulaganje sredstava u portfolio koji je u prethodnoj godini ostvario najvišu stopu prinosa. Međutim, ubrzo je primećeno da su performanse većine investicionih portfolija u periodu rastućeg tržišta bolje od performansi ostvarenih tokom opadajućeg tržišta. Stoga su najveštiji finansijski analitičari usmerili pažnju na portfolije koji su imali najvišu prosečnu stopu prinosa u reprezentativnom periodu, koji je obuhvatao kako periode rastućeg tržišta, tako i periode opadajućeg tržišta.

Opisani postupak ocene performansi i izbora portfolija bio je dominantan sve do objavljivanja Markowitz-evog članka „*Portfolio Selection*“ 1952. godine (Markowitz, 1952) i nastanka savremene portfolio teorije (*Modern Portfolio Theory - MPT*). Razvoj *MPT* značio je evoluciju procesa ocene performansi portfolija u smislu uključivanja u analizu kako prinosa, tako i pripadajućeg rizika. Jednostavno poređenje prinosa dva portfolija zamenjeno je poređenjem njihovih rizikom ponderisanih prinosa. Prinosi portfolija se moraju korigovati za rizik da bi njihovo poređenje imalo smisla.

Dalji razvoj modela ocene performansi portfolija odvijao se paralelno sa ravojem modela vrednovanja aktive. Primera radi, iz modela vrednovanja kapitalne aktive (*Capital Asset Pricing Model - CAPM*) izvedene su važne mere performansi, kao što su: Šarpov indeks, Trejnorov indeks i Jensenov ili alfa indeks, a sa razvojem teorije arbitražnog vrednovanja (*Arbitrage Pricing Theory - APT*) pojavile su se nove višeparametarske mere performansi portfolija. Zasnovanost modela ocene performansi portfolija na modelima vrednovanja aktive ukazuje na njihovu neraskidivu povezanost.

Konačno, sa razvojem post-savremene portfolio teorije (*Post-Modern Portfolio Theory -*

Introduction

In an effort to attract new investors, mutual funds fight competition by introducing lower fees, better service, improved reporting etc.; however, the main aspect of competition refers to the actual investment performance (Elton et al, 2011). Since the choice of a particular mutual fund predominantly depends on its performance, being familiar with the portfolio performance evaluation model of a mutual fund is of paramount importance for individual and institutional investors. The performance of the mutual fund determines the investor's course of action in terms of retaining assets in the fund, additional investing or making a withdrawal from the fund.

Portfolio performance evaluation of mutual funds is equally important for mutual fund managers, as it informs them on the strengths and weaknesses of a created portfolio. Best efforts are invested in trying to make the most of these perceived strengths, as well as in eliminating the weaknesses, normally by changing the proportion of the existing securities and introducing the new ones into the portfolio. In this way, the evaluation of the portfolio's performances gets an important role in improving the process of portfolio management. Although the financial literature considers portfolio performance evaluation as the last phase in portfolio management, it is just as important as portfolio analysis and selection, therefore, it must be approached carefully, professionally and objectively. According to Sharpe et al (1995), portfolio performance evaluation can be considered as a sort of a control mechanism which can contribute to the efficiency of portfolio management.

By evaluating portfolio performance, one seeks to determine whether the created portfolio has achieved an average, above average or below average results and whether these results have been achieved thanks to the skills of a portfolio manager, i.e. his/her selection capability, or pure luck. Furthermore, it is important not only to assess how successfully a mutual fund operated in relation to other funds or in relation to the market, but also to understand the defined investment policy of the fund, as well as the extent to which the portfolio managers complied with this policy. By examining the defined investment policy and the level of compliance, one can truly understand the taken risk.

Development of Portfolio Performance Evaluation Models

The development of portfolio performance evaluation models progressed in parallel with the development of the portfolio theory. At the time when the traditional portfolio theory was widely accepted, it was considered that the overall portfolio risk can be eliminated by the simple (naive) diversification; therefore, portfolio performance was evaluated solely on the basis of the actual rates of return. Initially, investment advisers suggested investing in a portfolio which had the highest rate of return in the previous year. However, soon after that, it was noticed that the performance of the most investment portfolios during the market growth periods had been better than the performance during the periods of market decline. Therefore, the most skilled financial analysts focused their attention on the portfolios that had the highest average rate of return in an established reference period, which included both the periods of growing and declining markets.

The above described method of portfolio performance evaluation and selection had been predominant until the publication of the Markowitz's article titled *"Portfolio Selection"* in 1952 (Markowitz, 1952) and the emergence of the *Modern Portfolio Theory (MPT)*. The development of the *MPT* revolutionized the process of portfolio performance evaluation by introducing the expected return and the associated risks in the analysis. A simple comparison of the returns of two portfolios was replaced by a comparison of their risk-weighted returns. However, these portfolio returns had to be risk-adjusted in order to achieve consistent comparison.

Further development of the portfolio performance evaluation model progressed in parallel with the development of asset pricing models. For example, the *Capital Asset Pricing Model (CAPM)* produced important performance measuring methods, such as: Sharpe index, Treynor index and Jensen or alpha index, while the development of

PMPT), kao proširene prinos-rizik paradigme, izvršena je zamena ustaljenih mera performansi portfolija. Novo shvatanje rizika i upotreba minimalno prihvatljivog prinosa (*Minimum Acceptable Return - MAR*), kao ličnog benčmarka prilikom ocene postignutih rezultata, uslovili su zamenu Jensenovog alfa indeksa sa omegom i Šarpovog racija sa Sortinovim raciom.

Nezavisno od dostignutog stepena razvoja portfolio teorije, prvi korak u oceni performansi portfolija je računanje stope prinosa portfolija ostvarene tokom analiziranog perioda, koji se u finansijskoj literaturi naziva period ocene ili evaluacioni period (*evaluation period*). Najjednostavniji slučaj računanja stope prinosa portfolija podrazumeva da investitor tokom analiziranog perioda niti ulaže dodatna sredstva u portfolio, niti povlači ranije investirana sredstva iz portfolija. U ovoj situaciji stopa prinosa portfolija (r_p) se računa tako što se tržišna vrednost portfolija sa kraja perioda (*ending value - V_e*) umanji za tržišnu vrednost portfolija sa početka perioda (*beginning value - V_b*) i dobijena razlika podeli sa tržišnom vrednošću portfolija sa početka perioda (Sharpe i saradnici, 1995):

$$r_p = \frac{V_e - V_b}{V_b} \quad (1)$$

S druge strane, ukoliko analizirani period karakterišu ulaganja (*deposits -D*) i povlačenja (*withdrawals - W*) sredstava iz portfolija, ključno pitanje koje se postavlja jeste kada su se ovi novčani tokovi dogodili: na početku perioda, na kraju ili u preostalom vremenu koje je obuhvaćeno analizom.

Ako su se ulaganja i povlačenja sredstava dogodila na samom početku analiziranog perioda računanje stope prinosa portfolija se vrši prilagođavanjem početne tržišne vrednosti portfolija:

1. u slučaju ulaganja dodatnih sredstava u portfolio početna vrednost portfolija se uvećava za iznos uloženih sredstava:

$$r_p = \frac{V_e - (V_b + D)}{(V_b + D)} \quad (2)$$

2. u slučaju povlačenja sredstava iz portfolija početna vrednost portfolija se umanjuje za iznos povučenih sredstava:

$$r_p = \frac{V_e - (V_b - W)}{(V_b - W)} \quad (3)$$

Ako su se ulaganja i povlačenja sredstava dogodila na samom kraju analiziranog perioda stopa prinosa portfolija se računa prilagođavanjem krajnje tržišne vrednosti portfolija:

1. u slučaju ulaganja dodatnih sredstava u portfolio krajnja vrednost portfolija se umanjuje za iznos uloženih sredstava:

$$r_p = \frac{(V_e - D) - V_b}{V_b} \quad (4)$$

2. u slučaju povlačenja sredstava iz portfolija krajnja vrednost portfolija se uvećava za iznos povučenih sredstava:

$$r_p = \frac{(V_e + W) - V_b}{V_b} \quad (5)$$

Konačno, ako su se ulaganja i povlačenja sredstava dogodila u preostalom vremenu obuhvaćenom analizom, odnosno, negde između početnog i krajnjeg intervala vremena, računanje stope prinosa portfolija se vrši primenom vrednosno ponderisane (*value-weighted/money-weighted/dollar-weighted)* ili vremenski ponderisane (*time-weighted*) stope prinosa. Vrednosno ponderisana stopa prinosa se naziva još i interna stopa prinosa (*internal rate of return - IRR*), a predstavlja kamatnu stopu koja izjednačava sadašnju vrednost svih novčanih tokova (*cash flow - CF*) i krajnje tržišne vrednosti portfolija, sa početnom tržišnom vrednošću portfolija:

$$V_b = \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \cdots + \frac{CF_n + V_e}{(1+IRR)^n} \quad (6)$$

Novčani tok za bilo koji subperiod *k*, pri čemu je *k=1,2,...,n*, predstavlja razliku između ostvarenih priliva i odliva novčanih sredstava. Važno je primetiti da ovaj metod izračunavanja stope prinosa ne zahteva utvrđivanje tržišne vrednosti portfolija u pojedinačim subperiodima.

S druge strane, vremenski ponderisana stopa prinosa (r_p) se računa pomoću sledeće formule:

$$r_p = \prod_{k=1}^{n}(1 + r_k) - 1 = [(1 + r_1)(1 + r_2) \dots (1 + r_n)] - 1 \quad (7)$$

pri čemu je:

the *Arbitrage Pricing Theory (APT)* brought about the new multi-parameter portfolio performance measures. The development of portfolio performance evaluation models based on asset pricing models points to the close interrelatedness of these models.

Finally, with the development of the *Post-Modern Portfolio Theory (PMPT)*, as the extended return-risk paradigm, the long-established portfolio performance measuring methods were succeeded with the new ones. The new understanding of the risks and the use of the *minimum acceptable return (MAR)* as a personal benchmark in evaluating the achieved results, induced the replacement of the Jensen's alpha ratio by the omega ratio and the Sharpe ratio by the Sortino ratio.

Regardless of the achieved level of the portfolio theory development, the first step in evaluating portfolio performance is calculating the portfolio's rate of return achieved over the analyzed period of time, which is in financial literature referred to as the *evaluation period*. The simplest method of calculating the portfolio's rate of return implies that an investor should not invest additional funds in the portfolio, nor withdraw the previously invested funds, during the evaluation period. In this case, the portfolio's return (r_p) is calculated as the *ending value* (V_e) minus the *beginning value* (V_b) and then the resulting difference is divided by the *beginning value* (Sharpe et al, 1995):

$$r_p = \frac{V_e - V_b}{V_b} \quad (1)$$

On the other hand, if the evaluation period is characterized by the *deposits (D)* and *withdrawals* (W), the key thing that one should look into is their timeline, i.e. when these cash flows took place: at the beginning of the evaluation period, at the end of the evaluation period or at some other point during the evaluation period.

If deposits and withdrawals took place at the beginning of the evaluation period, the portfolio's return is calculated by adjusting the beginning value of the portfolio:

1. If some additional deposits are made, the amount of these deposits is added to the beginning value:

$$r_p = \frac{V_e - (V_b + D)}{(V_b + D)} \quad (2)$$

2. If some withdrawals are made, the beginning value is reduced by the amount of the assets withdrawn:

$$r_p = \frac{V_e - (V_b - W)}{(V_b - W)} \quad (3)$$

If the assets are deposited or withdrawn at the end of the evaluation period, the portfolio's return is calculated by adjusting the ending (terminal market) value:

1. If additional portfolio investments are made, the ending value is reduced by the amount of the assets invested:

$$r_p = \frac{(V_e - D) - V_b}{V_b} \quad (4)$$

2. If withdrawals from the portfolio are made, the overall amount of these withdrawals is added to the ending value:

$$r_p = \frac{(V_e + W) - V_b}{V_b} \quad (5)$$

And, finally, if the assets are deposited and withdrawn from the portfolio at some other point during the evaluation period, i.e. somewhere between the beginning and the end of the defined time interval, the portfolio's return is calculated by using the *value-weighted/ money-weighted/dollar-weighted* or *time-weighted* rate of return. The value-weighted rate of return is also called the *internal rate of return (IRR)* and it actually represents the interest rate which makes the net present value of all the *cash flows (CF)* plus the portfolio's terminal market value equal to the portfolio's initial market value:

$$V_b = \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \cdots + \frac{CF_n + V_e}{(1+IRR)^n} \quad (6)$$

Cash flow for any subperiod *k*, where $k = 1, 2, \ldots, n$, represents the difference between the cash inflows and the cash outflows. Please notice that when using this method of calculating the rate of return, it is not necessary to know the portfolio's market value for each subperiod.

On the other hand, the time-weighted rate of return (r_p) is calculated by using the following formula:

$$r_p = \prod_{k=1}^{n}(1 + r_k) - 1 = [(1 + r_1)(1 + r_2) \ldots (1 + r_n)] - 1 \quad (7)$$

where:

$$r_k = \frac{V_{e,k} - V_{b,k}}{V_{b,k}} \quad (8)$$

gde su:

r_k - stopa prinosa portfolija ostvarena u subperiodu k, za $k=1,2,...,n$,

$V_{e,k}$ - tržišna vrednost portfolija na kraju subperioda k, odnosno, neposredno pre novčanog toka,

$V_{b,k}$ - tržišna vrednost portfolija na početku subperioda k, odnosno, neposredno nakon novčanog toka.

Nije teško zaključiti da svaki novčani tok označava kraj jednog, a početak narednog subperioda. Glavni nedostatak vremenski ponderisane stope prinosa se ogleda u činjenici da ovaj metod zahteva utvrđivanje tržišne vrednosti portfolija svaki put kada se novčani tok dogodi. Samim tim, vremenski ponderisana stopa prinosa je precizniji, ali i složeniji metod od alternativne vrednosno ponderisane stope prinosa. Ključni argument u korist izbora vremenski ponderisane stope prinosa je da je alternativna vrednosno ponderisana stopa prinosa pod snažnim uticajem veličine novčanih tokova nad kojima portfolio menadžeri nemaju kontrolu.

Početni modeli ocene performansi portfolija podrazumevali su poređenje ovako izračunate stope prinosa portfolija investicionog fonda sa stopama prinosa portfolija drugih investicionih fondova, ili sa stopom prinosa odabranog benčmarka. Međutim, jednostavno poređenje bruto prinosa portfolija investicionih fondova može da dovede do pogrešnih zaključaka o njihovoj performansnosti iz dva razloga. Prvi razlog se ogleda u činjenici da su različiti investicioni fondovi opterećeni različitim troškovima. Stoga je bolja mera performansi stopa neto prinosa portfolija fonda, koja se dobija kao razlika između stope bruto prinosa i stope ukupnih troškova. Investicioni fond može da ostvari viši prinos od benčmarka pre oduzimanja troškova, ali da nakon uključivanja troškova u analizu prinos fonda bude niži od benčmarka. Prema Aragon i Ferson (2006), ukoliko prinos investicionog fonda uspeva da nadmaši benčmark nakon oduzimanja troškova (*on an after-cost basis*), onda se kaže da fond dodaje vrednost investitorima (*adds value for investors*). S druge strane, ukoliko prinos investicionog fonda uspeva da nadmaši benčmark pre oduzimanja troškova (*on a before-cost basis*), ali ne i nakon uključivanja troškova u analizu, smatra se da portfolio menadžer ima investicionu sposobnost (*investment ability*), ali da naplaćuje visoke provizije i/ili postignute rezultate troši na visoke transakcione troškove.

Drugi razlog zbog kojeg poređenje prinosa portfolija različitih investicionih fondova može da dovede do pogrešnih zaključaka o njihovoj performansnosti jesu različiti nivoi rizika kojima su ovi fondovi izloženi. Stoga celovita ocena performansi portfolija mora da uključi ne samo merenje prinosa portfolija, već i merenje pripadajućeg rizika. Drugim rečima, da bi se rezultati poređenja performansi portfolija smatrali validnim, izračunata stopa prinosa portfolija mora biti korigovana za rizik.

Prema Bodie i saradnici (2009), najjednostavniji način korigovanja prinosa za rizik portfolija jeste da se stope prinosa analiziranih investicionih fondova uporede sa stopama prinosa koje ostvaruju drugi investicioni fondovi sa sličnim karakteristikama rizika. Investicioni fondovi sa sličnim karakteristikama rizika se svrstavaju u grupe (fondovi koji ulažu u akcije, fondovi koji ulažu u obveznice, fondovi koji ulažu u instrumente novčanog tržišta i sl.), a zatim se porede prinosi fondova iste grupe. Međutim, u okviru svake grupe mogu se formirati podgrupe koje karakteriše različit nivo rizika, tako da poređenje prinosa investicionih fondova iste grupe može rezultirati pogrešnim zaključcima. Na osnovu navedenog zaključuje se da su potrebne preciznije metode korekcije prinosa za rizik, odnosno, precizniji modeli ocene performansi portfolija investicionih fondova.

Kao važno nameće se i pitanje određivanja adekvatne mere rizika. Da li rizik meriti standardnom devijacijom ili beta koeficijentom? Odgovor na ovo pitanje zavisi od toga da li klijent investicionog fonda poseduje drugu značajnu aktivu osim njegovog udela u fondu. Ukoliko je udeo u investicionom fondu jedina aktiva u vlasništvu investitora, onda je standardna devijacija relevantna mera rizika. Međutim, ukoliko klijent poseduje i brojnu drugu aktivu, beta koeficijent se smatra relevantom merom rizika, jer će se, zahvaljujući sprovedenoj diversifikaciji ulaganja, nesistemski rizik klijentovog portfolija eliminisati. Treba istaći i da je zahvaljujući razvoju *PMPT* sve više zagovornika upotrebe negativne

$$r_k = \frac{V_{e,k} - V_{b,k}}{V_{b,k}} \quad (8)$$

with:

r_k - portfolio's rate of return achieved in the subperiod k, for $k=1,2,...,n$,

$V_{e,k}$ - portfolio's market value at the end of the subperiod k, i.e. immediately before the cash flow occurrence,

$V_{b,k}$ - portfolio's market value at the beginning of the subperiod k, i.e., immediately after the cash flow occurrence.

Therefore, it is understandable that each cash flow marks the end of a subperiod and the beginning of the next one. The main weakness of the time-weighted rate of return is reflected in the fact that it is necessary to determine market value of the portfolio every time the cash flow occurs. Therefore, the time-weighted rate of return is a more accurate, but also a more complex method than the alternative value-weighted rate of return. The key argument in favor of the time-weighted rate of return is that the alternative value-weighted rate of return is strongly influenced by the size of the cash flows that cannot be controlled by the portfolio managers.

The early portfolio performance evaluation models included the comparison of the thus calculated portfolio returns of a particular mutual fund with the portfolio returns of other mutual funds, or the rate of return of a chosen benchmark. However, a simple comparison of the gross rates of return of the different mutual funds' portfolios may lead to the wrong conclusions concerning the performance of these funds for two reasons. The first reason is reflected in the fact that different mutual funds are burdened by different costs. Therefore, it is better to measure the performance via the net rate of return, which is calculated as the difference between the gross rate of return minus the total costs. A mutual fund can achieve a higher rate of return than the benchmark on *a before-cost basis*; however, after introducing costs into the analysis, the fund's rate of return is lower than the benchmark. According to Aragon and Ferson (2006), if a mutual fund manages to outperform the benchmark on *an after-cost basis*, it is considered that this fund adds value for investors. On the other hand, if the rate of return of a mutual fund manages to outperform the benchmark on *a before-cost basis*, but not after the inclusion of the costs into the analysis, it is considered that the portfolio manager has investment *ability*, but that he/she also charges high commissions and/or spends the achieved results on high transaction costs.

Another reason why comparing the portfolios of different mutual funds may be misleading in terms of their performance is the fact that different portfolios are exposed to different risk levels. Therefore, a thorough portfolio performance evaluation must include not only the measurement of the portfolio's rate of return, but the measurement of the associated risks as well. In other words, in order to get valid results in terms of portfolio performance, the calculated portfolio's rate of return must be risk-adjusted.

According to Bodie et al. (2009), the simplest way to risk-adjust portfolio returns is to compare the returns achieved by the evaluated mutual funds with the returns achieved by other mutual funds with similar risk characteristics. Mutual funds with similar risk characteristics are first classified into groups (funds that invest in stocks, funds that invest in bonds, funds that invest in money market instruments etc.), and only then the returns of the funds that are in the same group can be compared. However, different subgroups which are characterized by different risk levels may be formed within each group; therefore, even the comparison of the returns of the funds that are classified into the same group may produce incorrect results. Hence, it can be concluded that the more accurate methods of calculating risk-adjusted returns are needed, i.e. the more accurate portfolio performance evaluation methods.

Selecting an adequate method of risk measurement is also an important issue. Should the risk be measured by the standard deviation or by the beta coefficient? The answer to this question depends on whether the client of a certain mutual fund has other significant assets other than his/her share in the fund. If the share in the investment fund is the only asset owned by an investor, then the standard deviation is the suitable method for risk measurement. However, if a client has a number of other assets, the beta coefficient is considered a more relevant measure of risk, since based on the realized investment diversification,

devijacije (*downside deviation*) kao mere rizika. Negativna devijacija ili polu-devijacija je mera negativnog rizika, a računa se kao standardna devijacija negativnih prinosa, odnosno prinosa ispod *MAR*.

Konačno, nakon korekcije prinosa za rizik vrši se poređenje rizikom ponderisanih prinosa portfolija investicionih fondova sa rizikom ponderisanim prinosom odabranog benčmarka. Na ovaj način investitori u investicione fondove i portfolio menadžeri fondova saznaju da li ostvarena stopa prinosa predstavlja prosečan, superioran, ili inferioran rezultat.

U finansijskoj teoriji najčešće korišćene i u investicionoj praksi potpuno afirmisane mere performansi portfolija koje su na adekvatan način korigovale prinos za rizik su: Šarpov indeks, Trejnorov indeks i Jensenov ili alfa indeks.

Šarpov indeks

William Sharpe (1966) je konstruisao indeks za merenje performansi portfolija koji se zasniva na ukupnom riziku iskazanom standardnom devijacijom. Šarpov indeks se računa deljenjem riziko premije, tj. viška prinosa portfolija iznad bezrizične stope prinosa, sa standardnom devijacijom prinosa portfolija kao merom ukupnog rizika:

$$S_i = \frac{r_p - r_f}{\sigma_p} \quad (9)$$

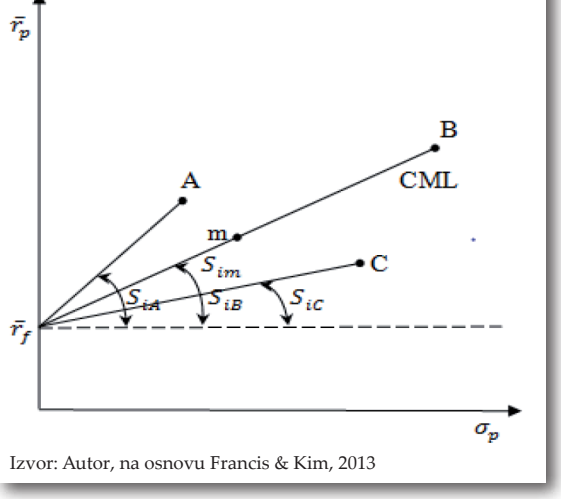
gde su:
S - Šarpov indeks,
$\bar{r}_p$ - prosečan prinos portfolija,
$\bar{r}_f$ - prosečna bezrizična stopa prinosa,
σ_p - standardna devijacija prinosa portfolija.

Brojilac u matematičkoj interpretaciji Šarpovog indeksa pokazuje inkrementalni prinos portfolija u odnosu na alternativnu investiciju u bezrizičnu aktivu, dok imenilac ukazuje na povećanje volatilnosti portfolija u odnosu na bezrizičnu alternativu (Bodie i saradnici, 2009). Ovaj indeks sažima koristi i troškove investiranja - prosečan prinos i standardnu devijaciju, u jednu meru performansi i pokazuje da li su prinosi investicionih fondova rezultat pametnih investicionih odluka ili neumerenog rizika. Prednost upotrebe Šarpovog racija prilikom ocene performansi portfolija je u tome što se prilikom njegovog računanja ne mora odrediti benčmark kao zamena za tržište, tako da izbor benčmarka ne utiče na rangiranje investicionih fondova prema ovom indeksu. Sa druge strane, veliki nedostatak Šarpovog racija leži u činjenici da je on pouzdan pokazatelj performansi jedino nediversifikovanog ili slabo diversifikovanog portfolija. Takođe, bez obzira na činjenicu da računanje ovog indeksa ne zahteva određivanje benčmarka, uporedna analiza performansi portfolija investicionih fondova i prosečnih tržišnih performansi podrazumeva poređenje Šarpovog racija za portfolio sa Šarpovim raciom za benčmark. Samim tim, jasno je da je određivanje benčmarka neophodan uslov za dobijanje odgovora na pitanje da li analizirani investicioni fond ostvaruje superiorne, prosečne, ili inferiorne performanse u odnosu na tržište.

Šarpov indeks je kreiran sa ciljem poređenja i rangiranja investicionih portfolija koji pripadaju različitim klasama rizika i koji imaju različite prosečne stope prinosa. Opisani indeks najčešće uzima vrednosti između 0,5 i 3, s tim da veći Šarpov indeks znači bolje performanse portfolija. Ako je godišnji Šarpov indeks veći od 1,0 performanse investicionog fonda su dobre, dok izvanredni fondovi imaju Šarpov indeks veći od 2,0 (Jagrič i saradnici, 2007). U periodima jake krize, kada je cilj aktivnog menadžmenta ne da dobije više već da izgubi manje, Šarpov indeks može biti i negativan. Interpretacija negativnog Šarpovog indeksa je ista kao i interpretacija pozitivnog. Drugim rečima, pravilo da su, što je ovaj indeks veći, bolje performanse portfolija, i dalje važi (Sajter, 2011).

Grafički prikazano Šarpov indeks predstavlja ugao prave linije koja povezuje bezrizičnu stopu prinosa i analizirani portfolio u dvodimenzionalnom prostoru ($\bar{r}_p, \sigma_p$) (Slika 1).

Slika 1: Grafički prikaz Šarpovog indeksa



Izvor: Autor, na osnovu Francis & Kim, 2013

unsystematic risks are eliminated from the client's portfolio. It should be noted that thanks to the development of the *PMPT*, the application of the *downside deviation* as a risk measurement method really caught on. Downside deviation, or semi-standard deviation, is a negative risk measure which is calculated as the standard deviation of the returns that fall below a *MAR*.

Finally, after the rate of return has been risk adjusted, the comparison is made between the risk-weighted returns of the mutual funds' portfolios and the risk-weighted return of the selected benchmark. In this way, the mutual fund investors and portfolio managers discover whether the actual rate of return can be considered as an average, superior or inferior result.

The portfolio performance measuring methods that are most commonly used in finance theory, fully affirmed in practice and adequately risk-adjusted are: Sharpe index, Treynor index and Jensen or alpha index.

Sharpe Index

William Sharpe (1966) developed an index to measure portfolio performance based on the overall risk determined by the standard deviation. Sharpe index, also called Sharpe ratio, is calculated by dividing the risk premium, i.e. the excess return of the portfolio over the risk-free return, by the standard return deviation as a measure of the overall risk:

$$S_i = \frac{\bar{r}_p - \bar{r}_f}{\sigma_p} \quad (9)$$

where:

S - Sharpe index,

$\bar{r}_p$ - average return of the portfolio,

$\bar{r}_f$ - average risk-free rate of return,

σ_p - standard deviation of portfolio return.

The numerator in the mathematical interpretation of Sharpe index shows the incremental return of the portfolio in relation to an alternative investment in the risk-free asset, while the denominator indicates an increase in the volatility of the portfolio in relation to the risk-free alternative (Bodie et al, 2009). This index fuses the benefits and costs of investing - the average return and the standard deviation - into a single measure of performance and indicates whether the returns of mutual funds are the result of smart investment decisions or excessive risk-taking. The advantage of using the Sharpe ratio in portfolio performance evaluation is reflected in the fact that it is not necessary to set a benchmark as a market proxy in order to perform the calculation, therefore, based on this index, the choice of benchmarks does not affect the ranking of mutual funds. On the other hand, a major disadvantage of using the Sharpe ratio is the fact that this ratio is reliable only when evaluating portfolio performance of undiversified or poorly diversified portfolios. Furthermore, regardless of the fact that the calculation of this ratio does not require a benchmark, the comparative analysis of the portfolio performance of mutual funds and the average performance of markets, implies the comparison of the Sharpe ratio relative to the portfolio with the Sharpe ratio relative to the benchmark. Therefore, it is clear that the determination of benchmarks is a necessary requirement in order to answer the question whether the evaluated mutual fund achieves superior, average or inferior performance compared to the market.

Sharpe index was developed in order to enable the comparison and ranking of investment portfolios belonging to different risk classes and with different average rates of return. This index usually takes values between 0.5 and 3, where, the higher the Sharpe index the better the portfolio performance. If the annualized Sharpe index is greater than 1.0, the performance of the mutual fund is considered satisfactory, while the best funds are characterized with the value of the Sharpe index greater than 2.0 (Jagrič et al, 2007). In the times of a severe crisis, when the goal of the active management is not to earn more, but to lose less, the value of the Sharpe index may be negative. The interpretation of the negative Sharpe index is the same as for the positive index. In other words, the rule: the higher the Sharpe index, the better the portfolio performance, still applies (Sajter, 2011).

In graphical terms, the Sharpe index is an angle of a straight line which connects the risk-free rate of return and the evaluated portfolio in the two-dimensional space ($\bar{r}_p, \sigma_p$) (Figure 1).

Na prethodnoj slici grafički je prikazan Šarpov indeks za tri različita portfolija (A, B i C), kao i Šarpov indeks za tržišni portfolio *m*. Na ovaj način obezbeđeno je poređenje performansi različitih portfolija, ali i poređenje njihovih performansi sa prosečnim tržišnim performansama. Među navedenim portfolijima, u kontekstu Šarpovog indeksa, najpoželjniji je portfolio A, budući da prava linija koja povezuje bezrizičnu stopu prinosa i analizirani portfolio ima najveći ugao, tj. najveću vrednost Šarpovog indeksa ($S_{iA}>S_{iB}>S_{iC}$). To znači da bilo koja kombinacija portfolija A i aktive oslobođene rizika donosi viši prinos za isti nivo rizika u odnosu na moguće kombinacije preostala dva portfolija (B i C) i bezrizične aktive.

S druge strane, kao reper za poređenje performansi posmatranih portfolija sa prosečnim tržišnim performansama služi linija tržišta kapitala (*Capital Market Line - CML*), koja povezuje bezrizičnu stopu prinosa i tržišni portfolio *m* u dvodimenzionalnom prostoru ($\bar{r}_p, \sigma_p$). Budući da je Šarpov indeks za tržišni portfolio (S_{im}) jednak uglu *CML* linije nije teško zaključiti sledeće:

- Portfoliji koji se nalaze iznad *CML* linije imaju veći Šarpov indeks od Šarpovog indeksa za tržišni portfolio i ostvaruju superiorne performanse u odnosu na tržište (portfolio A);
- Portfoliji koji leže na *CML* liniji imaju Šarpov indeks jednak Šarpovom indeksu za tržišni portfolio i ostvaruju prosečne tržišne performanse (portfolio B);
- Portfoliji koji se nalaze ispod *CML* linije imaju manji Šarpov indeks od Šarpovog indeksa za tržišni portfolio i ostvaruju inferiorne performanse u odnosu na tržište (portfolio C).

O značaju predstavljenog Šarpovog indeksa i Šarpovog rada uopšte najbolje govori činjenica da je 1990. godine njegov model portfolio performansi nagrađen Nobelovom nagradom za ekonomiju.

Trejnorov indeks

Za razliku od Šarpovog indeksa koji meri riziko premiju portfolija po jedinici ukupnog rizika, Trejnorov indeks (Treynor, 1965) je pokazatelj viška prinosa portfolija iznad bezrizične stope prinosa po jedinici sistemskog rizika. Kod Trejnorovog indeksa prinos portfolija je korigovan samo za sistemski rizik, jer se polazi od pretpostavke o izvršenoj efikasnoj diversifikaciji ulaganja i odsustvu nesistemskog rizika. Dakle, Trejnorov indeks je sličan Šarpovom indeksu, osim što se umesto standardne devijacije kao pokazatelja volatilnosti prinosa oko njihove srednje vrednosti koristi beta koeficijent portfolija:

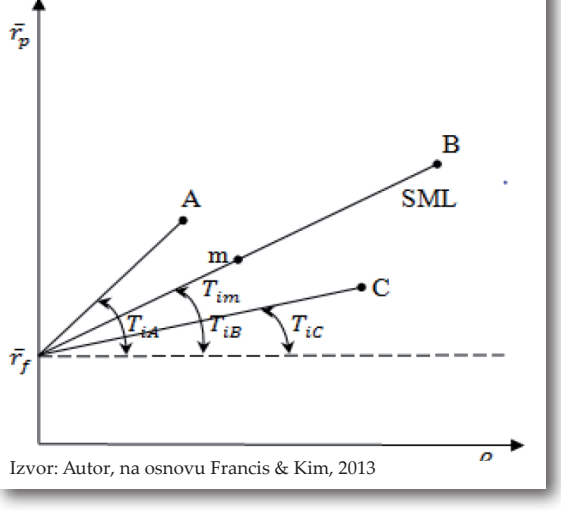
$$T_i = \frac{\bar{r}_p - \bar{r}_f}{\beta_p} \quad (10)$$

gde su:
T_i - Trejnorov indeks,
β_p - beta koeficijent portfolija.

Trejnorov indeks se računa deljenjem stope prinosa portfolija iznad bezrizične stope prinosa sa beta koeficijentom portfolija kao merom sistemskog rizika. Veća vrednost indeksa znači bolje performanse portfolija. Ako je portfolio savršeno diversifikovan obe mere performansi, i Šarpov i Trejnorov indeks, daće isti rezultat, jer je tada ukupan rizik jednak sistemskom, dok veći Trejnorov indeks od Šarpovog govori o nedovoljnoj diversifikaciji portfolija i prisustvu nesistemskog rizika.

Grafički predstavljeno Trejnorov indeks je ugao prave linije koja povezuje bezrizičnu stopu prinosa i analizirani portfolio u dvodimenzionalnom prostoru ($\bar{r}_p, \beta_p$) (Slika 2).

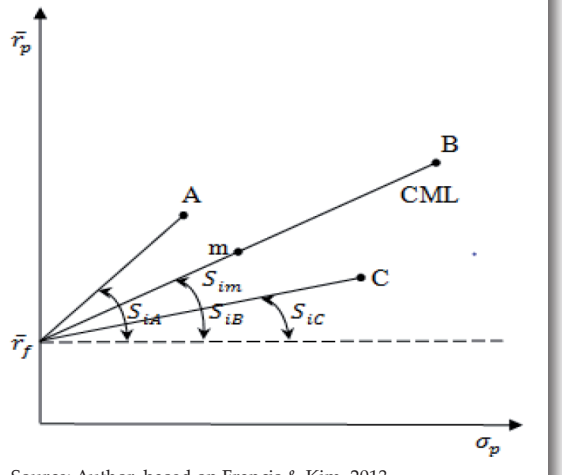
Slika 2: Grafički prikaz Trejnorovog indeksa



Izvor: Autor, na osnovu Francis & Kim, 2013

Među prikazanim linijama koje povezuju bezrizičnu stopu prinosa i analizirane portfolije, najveći nagib ima linija $\bar{r}_f A$, što upućuje na zaključak da je od navedenih portfolija (A,

Figure 1: Graphical representation of the Sharpe index



Source: Author, based on Francis & Kim, 2013

The figure above graphically represents the Sharpe index for three different portfolios (A, B and C), as well as the Sharpe index for the market portfolio *m*. Not only does this allow the comparison of the performance of different portfolios, but also the comparison of their performance with the average performance of the market. Among the mentioned portfolios, in terms of the Sharpe index, the most preferred portfolio is the portfolio A, since the straight line connecting the risk-free rate of return and the evaluated portfolio shows the largest angle, i.e. the maximum value of the Sharpe index (S_{iA}>S_{iB}>S_{iC}). This means that any combination of the portfolio A and risk-free assets has higher returns for the same level of risk, than any other combination of other two portfolios (B and C) and the risk-free assets.

On the other hand, the *capital market line (CML)* represents a benchmark for the comparison of the evaluated performance of portfolios with the average market performance, and this line actually connects the risk-free rate of return and the market portfolio *m* in the two-dimensional space ($\bar{r}_p, \sigma_p$). Since the Sharpe index of the market portfolio (S_{im}) is the same as the angle of the *CML*, the following can be concluded:

Portfolios above the *CML* are characterized by a higher value of the Sharpe index compared to the value of the Sharpe index of the market portfolio and are outperforming the market (portfolio A);

Portfolios that are located on the *CML* have the same value of the Sharpe index as the Sharpe index of the market portfolio, thus achieving the average market performance (portfolio B);

Portfolios that are under the *CML* have a lower value of the Sharpe index than the value of the Sharpe index for the market making their performance inferior compared to the market (portfolio C).

The significance of the discussed Sharpe index and Sharpe's work is best illustrated by the fact that in 1990 William Sharpe won the Nobel Prize in Economic Sciences for his contribution to the portfolio performance evaluation model.

Treynor Index

While the Sharpe index measures the portfolio's risk premium per unit of the overall risk, the Treynor index (Treynor, 1965) is an indicator of the excess return of a portfolio over the risk-free rate per unit of systemic risk. In terms of the Treynor index, the portfolio's return is adjusted only for systemic risk, because it is assumed that investments are adequately diversified and that, thus, non-systemic risk is eliminated. Therefore, the Treynor index is similar to the Sharpe index, except that instead of the standard deviation it uses the portfolio's beta coefficient as a measure of volatility of returns around their mean value:

$$T_i = \frac{\bar{r}_p - \bar{r}_f}{\beta_p} \quad (10)$$

where:
T_i - Treynor index,
β_p - portfolio's beta coefficient.

The Treynor index is calculated by dividing the portfolio's return over the risk-free rate with the portfolio's beta coefficient as a measure of systemic risk. The higher the Traynor index value, the better the portfolio performance. If a portfolio is perfectly diversified, both performance measures - the Sharpe index and the Treynor index will produce the same result, because in this case, the overall risk equals systemic risk, while the value of the Treynor index higher than the Sharpe index indicates insufficient diversification and the presence of non-systemic risk.

Graphically presented, the Treynor index is an angle of a straight line that connects the risk-free rate of return and the evaluated portfolio in the two-dimensional space ($\bar{r}_p, \beta_p$) (Figure 2).

B i C), u kontekstu Trejnorovog indeksa, najpoželjniji portfolio A. Na drugom mestu prema ostvarenim performansama je portfolio B, dok je portfolio C poslednje rangiran, jer ima najmanju vrednost Trejnorovog indeksa ($T_{iA}>T_{iB}>T_{iC}$).

Na prethodnoj slici, pored Trejnorovog indeksa za tri različita portfolija (A, B i C), grafički je prikazan i Trejnorov indeks za tržišni portfolio *m*. Trejnorov indeks za tržišni portfolio (T_{im}) jednak je riziko premiji, tj. višku prinosa tržišnog portfolija iznad prinosa na bezrizičnu aktivu $(T_{im} = \bar{r}_m - \bar{r}_f)$. Imenilac u formuli za Trejnorov indeks se gubi, jer je beta koeficijent tržišnog portfolija jednak jedinici ($\beta_m=1$). Grafičkim prikazom Trejnorovog indeksa za tržišni portfolio omogućeno je poređenje performansi posmatranih portfolija sa prosečnim performansama tržišta. Kao reper za poređenje služi tržišna linija hartija od vrednosti (*Security Market Line - SML*), jer je njen nagib jednak Trejnorovom raciju za tržišni portfolio. Imajući u vidu navedeno izvode se sledeći zaključci:

- Portfoliji koji se nalaze iznad *SML* linije imaju veći Trejnorov indeks od Trejnorovog indeksa za tržišni portfolio i ostvaruju superiorne performanse u odnosu na tržište (portfolio A);
- Portfoliji koji leže na *SML* liniji imaju Trejnorov indeks jednak Trejnorovom indeksu za tržišni portfolio i ostvaruju prosečne tržišne performanse (portfolio B);
- Portfoliji koji se nalaze ispod *SML* linije imaju manji Trejnorov indeks od Trejnorovog indeksa za tržišni portfolio i ostvaruju inferiorne performanse u odnosu na tržište (portfolio C).

Na kraju treba istaći da predstavljeni Šarpov i Trejnorov indeks mogu dati oprečne rezultate kada je poređenje performansi portfolija različitih investicionih fondova u pitanju, ali i poređenje njihovih performansi sa prosečnim tržišnim performansama. Šarpov indeks može da ukaže na inferiorne, a Trejnorov indeks na superiorne performanse analiziranog investicionog fonda u odnosu na konkurentski fond ili u odnosu na tržište. Uzrok oprečnih rezultata su različite mere rizika, odnosno, zasnovanost Šarpovog indeksa na standardnoj devijaciji, a Trejnorovog indeksa na beta koeficijentu kao meri rizika. Proizvoljni portfolio može istovremeno imati nizak nivo sistemskog rizika i visok nivo ukupnog rizika, što se objašnjava nedovoljnom diversifikacijom ulaganja. Navedeno za posledicu ima relativno visok Trejnorov indeks i relativno nizak Šarpov indeks.

U opisanoj situaciji kao logično nameće se pitanje izbora adekvatne mere portfolio performansi: da li performanse portfolija meriti Šarpovim ili Trejnorovim indeksom? Izbor između Šarpovog i Trejnorovog indeksa određen je odabirom odgovarajuće mere rizika, posmatrano iz ugla klijenta investicionog fonda. Ukoliko klijent investicionog fonda ne poseduje drugu značajnu aktivu osim njegovog udela u fondu, standardna devijacija prinosa je relevantna mera rizika, a Šarpov indeks odgovarajuća mera performansi portfolija. S druge strane, ukoliko je pored udela u fondu i brojna druga aktiva u vlasništvu investitora, beta koeficijent se smatra relevantnom merom rizika, a Trejnorov indeks merodavnim parametrom performansi portfolija.

Dodatni razlog eventualnog oprečnog rangiranja investicionih fondova po osnovu ostvarenih performansi, u kontekstu Šarpovog i Trejnorovog indeksa, ogleda se u činjenici da, za razliku od Šarpovog indeksa, računanje Trejnorovog indeksa zahteva određivanje benčmarka kao zamene za tržište. Prema tome, izbor benčmarka ne utiče na rangiranje investicionih fondova prema Šarpovom raciju, dok u slučaju Trejnorovog racija kao mere performansi, izbor benčmarka može presudno uticati na poziciju investicionih fondova.

Jensenov indeks

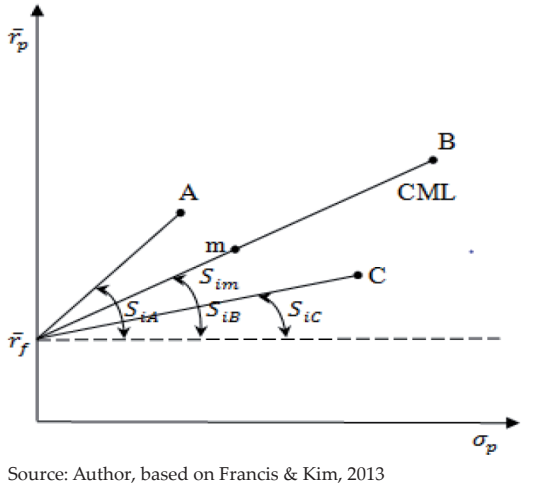
Iako su korisni instrumenti merenja performansi portfolija investicionih fondova, niti Šarpov niti Trejnorov indeks ne pokazuju procentualni ekstra prinos portfolija u odnosu na tržište, postignut zahvaljujući aktivnom portfolio menadžmentu. Kako bi eliminisao navedeni nedostatak Jensen (1968) je iz regresione jednačine *CAMP* modela izveo alfa indeks:

$$\alpha_i = \bar{r}_p - [\bar{r}_f + \beta_p (\bar{r}_m - \bar{r}_f)] \qquad (11)$$

gde su:

α_i - Jensenov ili alfa indeks,

Figure 2: Graphical representation of the Treynor index



Source: Author, based on Francis & Kim, 2013

If we look at the lines represented in the picture above which connect the risk-free rate of return and the evaluated portfolios, it is clear that line $\bar{r}_f A$, has the greatest inclination; this means that out of all evaluated portfolios (A, B and C), based on the Treynor index, the most preferred portfolio is portfolio A. Regarding the achieved performance, portfolio B comes second, while portfolio C has the lowest ranking due to the lowest Treynor index value ($T_{iA}>T_{iB}>T_{iC}$).

In the Figure 2, in addition to the Treynor index for three different portfolios (A, B and C), the Treynor index for market portfolio *m* is also graphically represented. Treynor index for market portfolio *m* (T_{im}) is equal to the risk premium, i.e. the excess return of the market portfolio over the returns on risk-free assets ($T_{im} = \bar{r}_m - \bar{r}_f$). The denominator in the formula for the Treynor index is cleared, because the beta coefficient of the market portfolio is equal to 1 ($\beta_m=1$). The graphical representation of the Treynor index for market portfolio *m* allows the comparison between the performance of the evaluated portfolio and the average performance of the market. The *security market line (SML)* is used as a benchmark for comparison, since its inclination is the same as the Treynor index for market portfolio *m*. Therefore, the following can be concluded:

- Portfolios above the *SML* have a higher Treynor index value than the Treynor index for market portfolio *m*, therefore outperforming the market (portfolio A);
- -Portfolios that are located on the *SML* have the same values of the Treynor index and the Treynor index for market portfolio *m* and thus have the average market performance (portfolio B);
- Portfolios below the *SML* have a lower value of the Treynor index than the Treynor index for market portfolio *m* and achieve inferior performance compared to the market (portfolio C).

Finally, it should be noted that the Sharpe index and the Treynor index can show conflicting results when comparing the portfolio performance of different mutual funds, as well as when comparing their performance with the average performance of the market. The Sharpe index may indicate inferior performance, while the Treynor index might identify superior performance of the evaluated investment fund compared to the other funds or the market. The reason for such conflicting results is explained by the application of different risk measures, i.e. the Sharpe index measures risk based on the standard deviation, while the Treynor index uses the beta coefficient. Some portfolios can at the same time be characterized by a low level of the systemic risk and a high level of the overall risk, which is explained by the insufficient diversification of investments; this results in a relatively high Treynor index value and a relatively low Sharpe index value.

With this in mind, the logical question is whether to measure the portfolio performance by applying the Sharpe or the Treynor index. The choice between the Sharpe and the Treynor index is determined by selecting the appropriate risk measure, having in mind the best interest of the mutual fund client. If the client does not hold any significant assets other than his/her share in the fund, the standard deviation of the returns is the more suitable measure of risk, while the Sharpe index is the adequate portfolio performance measure. On the other hand, if an investor owns a number of other assets in addition to his/her shares in the fund, the beta coefficient is considered as a more suitable measure of risk and the Treynor index as the more appropriate portfolio performance measure.

Another reason for the possibly different rankings of mutual funds based on their actual performance, in terms of the Sharpe and the Treynor index, is the fact that the calculation of

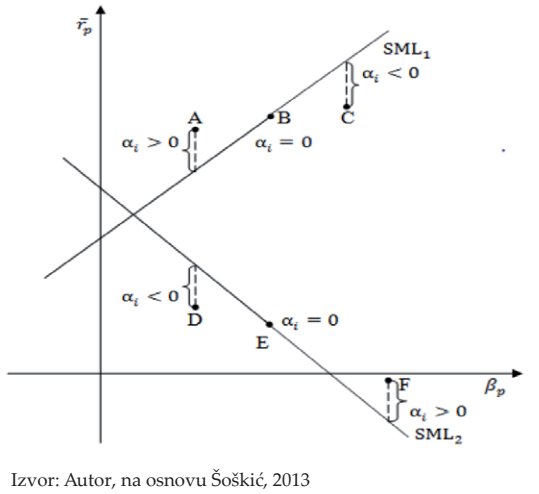
$\bar{r}_m$ - prosečan tržišni prinos.

Alfa indeks pokazuje razliku između stvarnog prinosa portfolija investicionog fonda i očekivanog prinosa za dati nivo rizika. Ako je stvarni prinos portfolija veći od očekivanog prinosa alfa indeks je pozitivan, performanse fonda superiorne, a portfolio menadžer investicionog fonda je uspeo da ostvari ekstra prinos i „pobedi" tržište pokazujući zavidnu selekcionu sposobnost. Ako je pak ostvarena stopa prinosa portfolija niža od očekivane stope za dati nivo rizika portfolija, alfa indeks je negativan, performanse fonda su inferiorne, a portfolio menadžeru investicionog fonda nedostaje potrebna veština izbora akcija. Konačno, jednakost stvarnog i očekivanog prinosa ukazuje na prosečne performanse investicionog fonda koji se smatra tržišnim, a alfa indeks je u ovom slučaju jednak nuli (Jakšić i saradnici, 2015).

Pozitivna Jensenova alfa iz regresione jednačine postiže se zahvaljujući selekcionoj sposobnosti portfolio menadžera i predstavlja čisto dodatu vrednost koju donosi aktivno upravljanje portfoliom. Jensenova alfa meri veštinu izbora hartija od vrednosti od strane portfolio menadžera investicionih fondova, ukazujući na njihovu inferiornost ili superiornost. Inferiorni menadžer ima Jensenovu alfu koja je statistički značajno negativna (α_i<0), dok superiorni menadžer ima pozitivnu i statistički značajnu vrednost alfa indeksa (α_i>0) (Jagrič i saradnici, 2007).

Grafički prikazano Jensenov indeks predstavlja vertikalno odstojanje pozicije portfolija od *SML* linije (Slika 3).

Slika 3: Grafički prikaz Jensenovog indeksa



Izvor: Autor, na osnovu Šoškić, 2013

Na prethodnoj slici prikazane su dve *SML* linije: *SML* linija u uslovima rastućeg tržišta (SML_1) i *SML* linija u uslovima opadajućeg tržišta (SML_2). U uslovima rastućeg tržišta realno je očekivati više stope prinosa kod investicionih fondova sa višim beta koeficijentom u odnosu na fondove sa nižim beta koeficijentom. Međutim, navedeno ne znači da će viši beta koeficijent u uslovima rastućeg tržišta rezultirati nužno pozitivnom alfom. Slično, u uslovima opadajućeg tržišta realno je očekivati niže stope prinosa kod investicionih fondova sa višim beta koeficijentom u odnosu na fondove sa nižim beta koeficijentom. Ipak, ovo ne znači da će viši beta koeficijent u uslovima opadajućeg tržišta rezultirati nužno negativnom alfom. Na Slici 3 investicioni fondovi sa višim beta koeficijentom su u uslovima rastućeg tržišta ostvarili inferiorne, a u uslovima opadajućeg tržišta superiorne performanse. Navedeno ukazuje da rangiranje investicionih fondova po osnovu Jensenovog indeksa nije zavisno od rizika fonda i opšteg kretanja tržišta (Šoškić, 2013).

Grafičko tumačenje ostvarenih performansi portfolija prikazanih na Slici 3 je jednostavno:

- Portfoliji koji se nalaze iznad *SML* linije imaju pozitivan alfa indeks (α_i>0) i ostvaruju superiorne performanse u odnosu na tržište (portfoliji A i F);
- Portfoliji koji leže na *SML* liniji imaju alfa indeks jednak nuli (α_i=0) i ostvaruju prosečne tržišne performanse (portfoliji B i E);
- Portfoliji koji se nalaze ispod *SML* linije imaju negativan alfa indeks (α_i<0) i ostvaruju inferiorne performanse u odnosu na tržište (portfoliji C i D).

Ovde je važno ukazati na razliku između *ex post* Jensenove alfe i *ex ante* Jensenove alfe. Alfa indeks prikazan jednačinom (11) predstavlja *ex post* verziju Jensenove alfe koja meri prošle performanse posmatranog portfolija. Zamenom istorijskih vrednosti sa očekivanim vrednostima, tj. prosečnih prinosa portfolija ($\bar{r}$) sa očekivanim prinosima ($E(r)$), *ex post* Jensenova alfa se transformiše u *ex ante* Jensenovu alfu koja meri buduće performanse portfolija (Francis & Kim, 2013):

$$\alpha_i = E(r_p) - [r_f + \beta_p (E(r_m) - r_f)] \qquad (12)$$

U slučaju primene *ex ante* Jensenove

the Treynor index requires a benchmark to be set as a market proxy. Therefore, the selection of benchmarks does not affect the ranking of mutual funds in terms of the Sharpe ratio, while in case of the Treynor ratio as a performance measure, the selection of benchmarks may have a decisive role in the ranking of mutual funds.

Jensen Index

Despite being useful portfolio performance measures, neither Sharpe or Treynor index can show the percentage of the portfolio's excess returns in relation to the market achieved owing to the active portfolio management. In order to bridge the mentioned gap, Jensen (1968) derived the alpha index from the *CAPM* regression equation:

$$\alpha_i = \bar{r}_p - [\bar{r}_f + \beta_p(\bar{r}_m - \bar{r}_f)] \qquad (11)$$

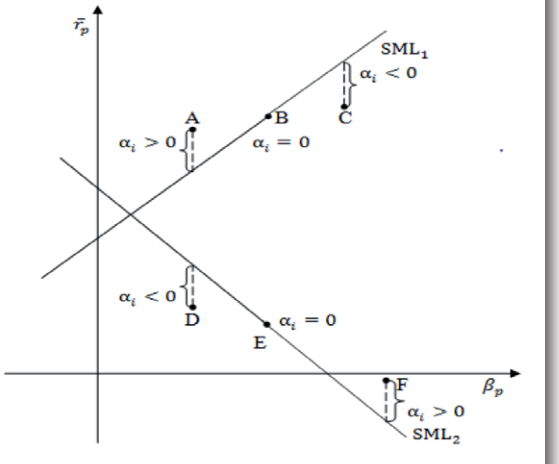
where:
α_i - Jensen or alpha index,
$\bar{r}_m$ - average market return.

The alpha index denotes the difference between the actual mutual fund portfolio return and the expected return at a given level of risk. If the actual portfolio return exceeds the expected return, the alpha index is positive, the fund performance superior, while the mutual fund manager is capable of achieving the extra return and "beating" the market, thus showing the significant selection capability. However, if the actual portfolio return is lower than the expected at a given level of risk, the alpha index is negative, the fund performance inferior, while the mutual fund manager does not have the necessary selection capability. Finally, the equal values of the actual and expected returns indicate the average mutual fund performance, which means that the fund performs in line with the market, while the alpha index in this case equals zero (Jakšić et al, 2015).

A positive Jensen's alpha derived from the regression equation is achieved thanks to a portfolio manager's selection capability and is considered added-value provided by the active portfolio management. The Jensen's alpha also measures the mutual fund managers' selection capability of securities, thus indicating their inferiority or superiority. An inferior manager has the Jensen's alpha that is statistically significant and negative (α_i<0), while a superior manager has a positive and statistically significant alpha index value (α_i>0) (Jagrič et al, 2007).

Illustrated in graphical terms, the Jensen index represents the vertical distance between the position of the portfolio and the *SML* (Figure 3).

Figure 3: Graphical representation of the Jensen index



Source: Author, based on Šoškić, 2013

Figure 3 shows two *SML*s: *SML* in the conditions of the growing market (SML_1) and *SML* in the conditions of the declining market (SML_2). In the conditions of the growing market, it is reasonable to expect higher returns from mutual funds that have higher beta coefficients than the funds with lower beta coefficients. However, this does not mean that a higher beta coefficient, in terms of the growing market, necessarily results in a positive alpha. Accordingly, in the conditions of the declining market, it is realistic to expect lower returns from the mutual funds with higher beta coefficients compared to the funds with lower beta coefficients. However, this does not mean that a higher beta coefficient in the conditions of a declining market will always result in a negative alpha. In Figure 3, mutual funds with higher beta coefficients achieved inferior performance in the conditions of the growing market, while in the conditions of the declining market these funds achieved superior performance. This indicates that the ranking of investment funds based on the Jensen index does not depend either on the fund's level of risk or the general market movements (Šoškić, 2013).

Interpretation of the achieved performance of portfolios illustrated in Figure 3 is quite simple:

alfe izrazi superiorni, prosečni i inferiorni portfolio zamenjuju se izrazima potcenjeni, pravilno vrednovani i precenjeni portfolio. Izrazi superiorni, prosečni i inferiorni portfolio označavaju portfolije koji su ostvarili natprosečne, prosečne i ispodprosečne rezultate. S druge strane, budući da *ex ante* Jensenova alfa meri buduće performanse portfolija, izrazi potcenjeni, pravilno vrednovani i precenjeni portfolio označavaju portfolije od kojih se očekuje ostvarivanje natprosečnih, prosečnih i ispodprosečnih rezultata. Imajući u vidu navedeno zaključuje se da na Slici 3:

- Portfoliji A i F predstavljaju superiorne portfolije u kontekstu *ex post* Jensenove alfe, odnosno, potcenjene portfolije u kontekstu *ex ante* Jensenove alfe;
- Portfoliji B i E predstavljaju portfolije sa prosečnim performansama u kontekstu *ex post* Jensenove alfe, odnosno, pravilno vrednovane portfolije u kontekstu *ex ante* Jensenove alfe;
- Portfoliji C i D predstavljaju inferiorne portfolije u kontekstu *ex post* Jensenove alfe, odnosno, precenjene portfolije u kontekstu *ex ante* Jensenove alfe.

Na osnovu Slike 3 dolazi se i do važnog zaključka o stepenu kompatibilnosti Jensenovog i Trejnorovog indeksa, kao mera performansi portfolija zasnovanih na sistemskom riziku. U uslovima rastućeg tržišta i rastuće *SML* linije Jensenov i Trejnorov indeks daju istovetne zaključke kada je poređenje performansi portfolija različitih investicionih fondova sa prosečnim tržišnim performansama u pitanju. Portfoliji koji se nalaze iznad rastuće *SML* linije imaju pozitivan alfa indeks i istovremeno imaju Trejnorov indeks veći od Trejnorovog indeksa za tržišni portfolio, jer su nagibi linija koje povezuju bezrizičnu stopu prinosa i analizirane portfolije veći od nagiba *SML* linije. Slično, portfoliji koji se nalaze ispod rastuće *SML* linije imaju negativan alfa indeks i istovremeno imaju Trejnorov indeks manji od Trejnorovog indeksa za tržišni portfolio, jer su nagibi linija koje povezuju bezrizičnu stopu prinosa i analizirane portfolije manji od nagiba *SML* linije. U prvom slučaju obe mere performansi ukazuju na superiorne rezultate posmatranih portfolija u odnosu na tržište, dok u drugom slučaju obe mere performansi ukazuju na inferiorne rezultate posmatranih portfolija.

S druge strane, u uslovima opadajućeg tržišta i opadajuće *SML* linije Jensenov i Trejnorov indeks daju oprečne zaključke o performansnosti portfolija investicionih fondova u odnosu na tržište. Portfoliji koji se nalaze iznad opadajuće *SML* linije imaju pozitivan alfa indeks, ali istovremeno imaju Trejnorov indeks manji od Trejnorovog indeksa za tržišni portfolio, jer su nagibi linija koje povezuju bezrizičnu stopu prinosa i analizirane portfolije manji od nagiba *SML* linije. Slično, portfoliji koji se nalaze ispod opadajuće *SML* linije imaju negativan alfa indeks i istovremeno veći Trejnorov indeks od Trejnorovog indeksa za tržišni portfolio, jer su nagibi linija koje povezuju bezrizičnu stopu prinosa i analizirane portfolije veći od nagiba *SML* linije. U prvom slučaju Jensenov indeks ukazuje na superiorne, a Trejnorov indeks na inferiorne performanse posmatranih portfolija u odnosu na tržište, dok u drugom opisanom slučaju Jensenov indeks ukazuje na inferiorne, a Trejnorov indeks na superiorne performanse portfolija u odnosu na tržište.

Na kraju treba istaći da se sve tri do sada predstavljene mere performansi portfolija (Šarpov, Trejnorov i Jensenov indeks) mogu prikazati kao linearna transformacija jedne mere u drugu. Jensen (1968) je u jednačinu standardnog *CAMP* modela dodao indeks alfa i zapisao je na sledeći način (Francis & Kim, 2013):

$$\bar{r}_p - \bar{r}_f = \alpha_i + (\bar{r}_m - \bar{r}_f)\,\beta_p \qquad (13)$$

Deljenjem obe strane jednačine sa beta koeficijentom (β_p), dobija se Trejnorova mera performansi portfolija (T_i):

$$T_i = \frac{r_p - r_f}{\beta_p} = \frac{\alpha_i}{\beta_p} + (\bar{r}_m - \bar{r}_f) \qquad (14)$$

Prethodna jednačina ukazuje da Trejnorov indeks (T_i) predstavlja linearnu transformaciju Jensenovog indeksa alfa (α_i), jer je izraz $(\bar{r}_m - \bar{r}_f)$ konstantan:

$$T_i = \frac{\alpha_i}{\beta_p} + (constant) \qquad (15)$$

Pored Trejnorovog indeksa (T_i), i Šarpov indeks (S_i) se može izvesti iz Jensenove jednačine (13) uvođenjem sledećeg izraza

- Portfolios above the *SML* have a positive alpha index (α_i>0) and achieve superior performance compared to the market (portfolios A and F);
- Portfolios on the *SML* have an alpha index that is equal to zero (α_i=0) and achieve the average market performance (portfolios B and E);
- Portfolios below the *SML* have a negative alpha index (α_i<0) and achieve inferior performance compared to the market (portfolios C and D).

Here, it is important to notice the difference between the *ex post* Jensen's alpha and the *ex ante* Jensen's alpha. The alpha index described by the equation (11) is the *ex post* version of the Jensen's alpha, which measures the past performance of the evaluated portfolio. By substituting the historical values with the expected ones, i.e. the average portfolio returns ($\bar{r}$) with the expected returns ($E(r)$), the *ex post* Jensen's alpha is transformed to the *ex ante* Jensen's alpha which measures the future portfolio performance (Francis & Kim, 2013):

$$\alpha_i = E(r_p) - [r_f + \beta_p(E(r_m) - r_f)] \qquad (12)$$

When applying the Jensen's *ex ante* alpha, the terms: superior, average and inferior portfolio are replaced by the terms: undervalued, properly valued and overvalued portfolio. The mentioned terms: superior, average and inferior portfolio imply the portfolios that have achieved the above-average, average and below-average results. On the other hand, since the Jensen's *ex ante* alpha measures the future performance of the portfolio, the terms: undervalued, properly valued and overvalued portfolio indicate the portfolios which are expected to provide the above-average, average and below-average results. In this respect, the following can be concluded regarding Figure 3:

- Portfolios A and F are superior portfolios in terms of the Jensen's *ex post* alpha, i.e. the undervalued portfolios in terms of the *ex ante* alpha;
- Portfolios B and E are average portfolios in terms of the Jensen's *ex post* alpha, i.e. the properly valued portfolios in terms of the *ex ante* alpha;
- Portfolios C and D are inferior portfolios in terms of the Jensen's *ex post* alpha, i.e. the overvalued portfolios in terms of the *ex ante* alpha.

Based on Figure 3, an important conclusion can be drawn about the degree of compatibility between the Jensen and the Treynor index as measures of portfolio performance based on systemic risk. In terms of the growing market and the increasing *SML*, the Jensen and the Treynor index produce the identical findings in the comparison of portfolio performance of different mutual funds with an average market performance. Portfolios above the increasing *SML* line have a positive alpha index and at the same time their Treynor index is greater than the Treynor index for market portfolio, because the inclinations of the lines connecting the risk-free returns and the evaluated portfolios are greater than the *SML*'s inclination. Similarly, the portfolios which are below the increasing *SML* have a negative alpha index and at the same time their Treynor index is lower than the Treynor index for market portfolio, because the inclinations of the lines connecting the risk-free returns and the evaluated portfolios are smaller than the *SML*'s inclination. In the first case, both performance measures indicate superior results of the evaluated portfolios compared to the market, while in the second both performance measures indicate inferior performance of the evaluated portfolios.

On the other hand, in terms of the declining market and the decreasing *SML*, the Jensen and the Treynor index produce conflicting findings in respect of the portfolio performance of mutual funds in relation to the market. Portfolios above the decreasing *SML* have a positive alpha index, but at the same time their Treynor index is lower than the Treynor index for market portfolio, because the inclinations of the lines that connect the risk-free returns and the evaluated portfolios are smaller than the inclination of the *SML*. Similarly, the portfolios below the decreasing *SML* have a negative alpha index and at the same time their Treynor index is greater than the Treynor index for market portfolio, because the inclinations of the lines that connect the risk-free returns and the evaluated portfolios are greater than the *SML* inclination. In the first case, the Jensen index indicates superior performance of the evaluated

za beta koeficijent $\beta_p = \frac{\rho_{pm}\sigma_p\sigma_m}{\sigma_m^2}$, gde su p_{pm} - koeficijent korelacije portfolija p i tržišnog portfolija, σ_m - standardna devijacija prinosa tržišnog portfolija, σ_m^2 - varijansa prinosa tržišnog portfolija:

$$\bar{r}_p - \bar{r}_f = \alpha_i + (\bar{r}_m - \bar{r}_f)\left(\frac{\rho_{pm}\sigma_p\sigma_m}{\sigma_m^2}\right) \quad (16)$$

Izostavljanjem koeficijenta korelacije posmatranog i tržišnog portfolija, budući da je njegova vrednost za efikasno diversifikovane portfolije približno jednaka jedinici ($\rho_{pm} \cong 1$), i deljenjem prethodne jednačine sa σ_p dobija se Šarpova mera performansi portfolija (S_i):

$$S_i = \frac{\bar{r}_p - \bar{r}_f}{\sigma_p} \cong \frac{\alpha_i}{\sigma_p} + \left(\frac{\bar{r}_m - \bar{r}_f}{\sigma_m}\right) \quad (17)$$

Prethodna jednačina ukazuje da, pored Trejnorovog indeksa (T_i), i Šarpov indeks (S_i) predstavlja linearnu transformaciju Jensenovog indeksa alfa (α_i), jer je izraz $\left(\frac{\bar{r}_m - \bar{r}_f}{\sigma_m}\right)$ konstantan:

$$S_i = \frac{\alpha_i}{\beta_p} + (constant) \quad (18)$$

Konačno, ukoliko se u jednačini Trejnorovog indeksa $T_i = \frac{\bar{r}_p - \bar{r}_f}{\beta_p}$, beta koeficijent ($\beta_p$) zameni izrazom $\beta_p = \frac{\rho_{pm}\sigma_p\sigma_m}{\sigma_m^2}$, i potom izostavi koeficijent korelacije posmatranog i tržišnog portfolija, budući da je njegova vrednost za efikasno diversifikovane portfolije približno jednaka jedinici ($\rho_{pm} \cong 1$), dobija se izraz:

$$T_i = S_i\sigma_m \quad (19)$$

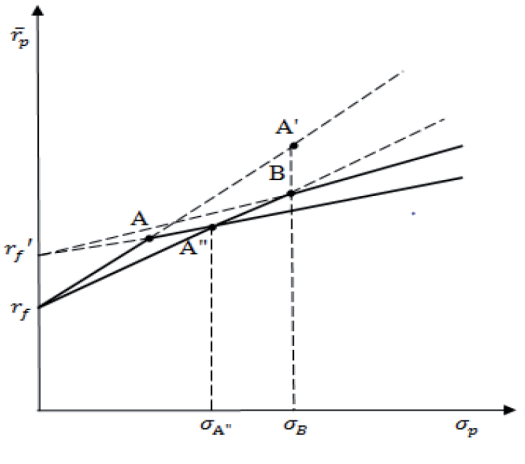
Navedeni izraz ukazuje da Trejnorov indeks (T_i) i Šarpov indeks (S_i) predstavljaju ne samo linearnu transformaciju Jensenovog indeksa alfa (α_i), već i linearnu transformaciju jedne mere u drugu.

Kritika modela ocene performansi portfolija

Imajući u vidu zasnovanost do sada predstavljenih modela ocene performansi portfolija na *CAPM* modelu, jasno je da odsustvo validnosti pretpostavki *CAPM* modela istovremeno znači i odsustvo validnosti predstavljenih modela ocene performansi portfolija.

Jedna od polaznih pretpostavki *CAPM* modela koja pojednostavljuje tržišne uslove privređivanja jeste jednakost kamatne stope na uzimanje novca na zajam i kamatne stope na davanje novca na zajam. Na Slici 4 portfolio A, u kontekstu Šarpovog indeksa, ostvaruje bolje performanse od portfolija B, pod uslovom važenja navedene pretpostavke. Portfolio A dominira nad portfoliom B, jer bilo koja kombinacija portfolija A i bezrizične aktive donosi višu stopu prinosa za isti nivo rizika, u poređenju sa kombinacijama portfolija B i bezrizične aktive. Primera radi, pozajmljivanjem novca po bezrizičnoj kamatnoj stopi r_f i njegovim investiranjem u portfolio A dobija se kombinovani portfolio A', koji pri istom nivou rizika (σ_B) donosi viši prinos od portfolija B. Međutim, ukoliko je kamatna stopa na uzimanje novca na zajam (r_f') viša od kamatne stope na davanje novca na zajam (r_f), dominantnost performansi portfolija A nad performansama portfolija B se dovodi u pitanje. U situaciji predstavljenoj na Slici 4 portfolio A ostvaruje bolje performanse od portfolija B sve do nivoa rizika koji odgovara kombinovanom portfoliju A" ($\sigma_{A"}$), dok su pri višim nivoima rizika performanse portfolija B superiornije od performansi portfolija A. Na osnovu navedenog zaključuje se da narušenost samo jedne od polaznih pretpostavki *CAPM* modela dovodi u pitanje validnost Šarpovog indeksa, kao i validnost svih ostalih mera performansi portfolija zasnovanih na ovom modelu.

Slika 4: Nedostaci Šarpovog indeksa



Izvor: Autor, na osnovu Elton i saradnici, 2011

portfolios compared to the market, while the Treynor index indicates inferior performance. In the second case, the Jensen index indicates inferior performance, while the Treynor index points to the superior performance of the evaluated portfolio in terms of the market.

Finally, it should be highlighted that all the three so far presented portfolio performance measures (Sharpe, Treynor and Jensen index) can be represented as a linear transformation of one measure to another. Jensen (1968) introduced the alpha index into the standard *CAMP* equation and wrote as follows (Francis & Kim, 2013):

$$\bar{r}_p - \bar{r}_f = \alpha_i + (\bar{r}_m - \bar{r}_f)\beta_p \qquad (13)$$

By dividing both sides of the equation by the beta coefficient (β_p), we obtain the Treynor portfolio performance measure (T_i):

$$T_i = \frac{\bar{r}_p - \bar{r}_f}{\beta_p} = \frac{\alpha_i}{\beta_p} + (\bar{r}_m - \bar{r}_f) \qquad (14)$$

From the above equation follows that the Treynor index (T_i) represents a linear transformation of the Jensen's alpha (α_i), since the expression $(\bar{r}_m - \bar{r}_f)$ is a constant:

$$T_i = \frac{\alpha_i}{\beta_p} + (constant) \qquad (15)$$

In addition to the Treynor index (T_i), the Sharpe index (S_i) can be derived from the Jensen's equation as well (13) by introducing the following expression for the beta coefficient $\beta_p = \frac{\rho_{pm}\sigma_p\sigma_m}{\sigma_m^2}$, with p_{pm} - correlation coefficient of portfolio p and market portfolio, σ_m - standard deviation of market portfolio returns, σ_m^2 - variance of returns on the market portfolio:

$$\bar{r}_p - \bar{r}_f = \alpha_i + (\bar{r}_m - \bar{r}_f)\left(\frac{\rho_{pm}\sigma_p\sigma_m}{\sigma_m^2}\right) \qquad (16)$$

By omitting the coefficient of correlation of the evaluated and market portfolio, since its value for efficiently diversified portfolios approximately equals 1 ($\rho_{pm} \cong 1$), and by dividing the previous equation by σ_p we get the Sharpe portfolio performance measure (S_i):

$$S_i = \frac{\bar{r}_p - \bar{r}_f}{\sigma_p} \cong \frac{\alpha_i}{\sigma_p} + \left(\frac{\bar{r}_m - \bar{r}_f}{\sigma_m}\right) \qquad (17)$$

This equation confirms that, in addition to the Treynor index (T_i), the Sharpe index (S_i) also represents a linear transformation of the Jensen's alpha (α_i), since the expression $\left(\frac{\bar{r}_m - \bar{r}_f}{\sigma_m}\right)$ is a constant:

$$S_i = \frac{\alpha_i}{\beta_p} + (constant) \qquad (18)$$

Finally, if we replace the beta coefficient (β_p) in the Treynor equation $T_i = \frac{\bar{r}_p - \bar{r}_f}{\beta_p}$, with the expression $\beta_p = \frac{\rho_{pm}\sigma_p\sigma_m}{\sigma_m^2}$, and then omit the coefficient of correlation of the evaluated and market portfolio, since its value for efficiently diversified portfolios approximately equals 1 ($\rho_{pm} \cong 1$), we get:

$$T_i = S_i\sigma_m \qquad (19)$$

This expression confirms that not only that Treynor index (T_i) and the Sharpe index (S_i) represent a linear transformation of the Jensen's alpha (α_i), but also a linear transformation of one measure into another.

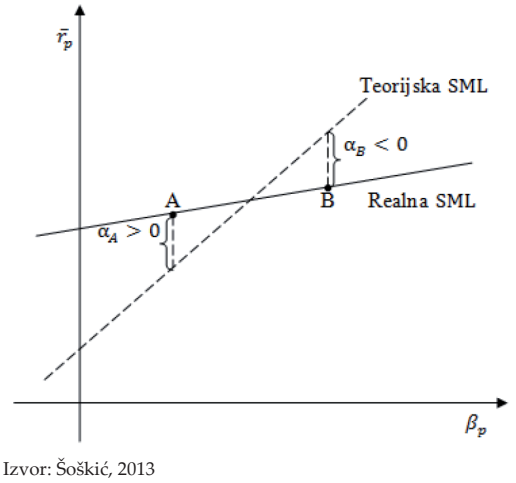
Criticism of Portfolio Performance Evaluation Models

Bearing in mind that all the discussed portfolio performance evaluation models are based on the *CAPM* model, it is only logical to conclude that if the *CAPM* model is challenged, the discussed portfolio performance evaluation models will also be challenged.

One of the essential assumptions of the *CAPM* model, which simplifies the market economy, is the equality of the borrowing and the lending interest rates. In Figure 4, in terms of the Sharpe index, portfolio A achieves better performance than portfolio B, under the condition that the aforementioned assumption is valid. Portfolio A outperforms portfolio B, because any combination of portfolio A and the risk-free assets delivers higher returns at the same level of risk compared with all the possible combinations of portfolio B and the risk-free assets. For example, by lending money at a risk-free interest rate r_f to be invested in portfolio A, we get the combined portfolio A', which at the same level of risk (σ_B) delivers higher returns than portfolio B. However, if the borrowing interest rate (r_f') is higher than the lending interest rate (r_f), the superiority of

Takođe, eventualna nepreciznost *CAPM* modela u smislu odstupanja realne od teorijske *SML* linije ima za posledicu odstupanje realne u odnosu na teorijsku ocenu performansi portfolija (Slika 5). Posmatranjem teorijske *SML* linije zaključuje se da portfolio A ima pozitivan alfa indeks (α_A>0) i ostvaruje superiorne performanse u odnosu na tržište, dok portfolio B ima negativan alfa indeks (α_B<0) i ostvaruje inferiorne performanse u odnosu na tržište. Međutim, realna *SML* linija navodi analitičare na drugačiji zaključak. Budući da se i portfolio A i portfolio B nalaze na realnoj *SML* liniji, njihovi alfa indeksi su jednaki nuli (α_A=α_B=0), a performanse jednake prosečnim tržišnim performansama. Dakle, teorijska *SML* linija sugeriše da su performanse portfolija A superiornije od performansi portfolija B, dok realna *SML* linija ukazuje na jednakost njihovih performansi (Slika 5).

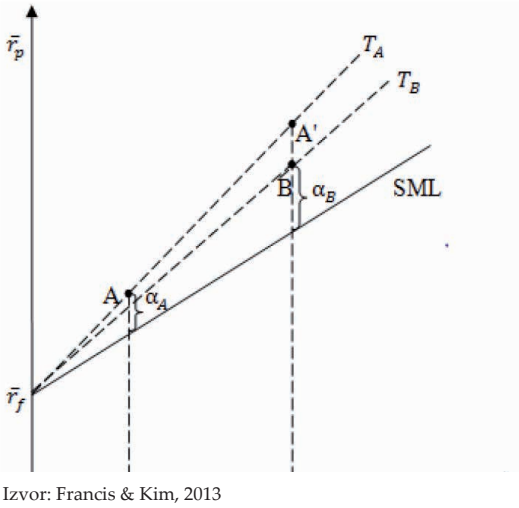
Slika 5: Odstupanje realne u odnosu na teorijsku ocenu performansi portfolija



Izvor: Šoškić, 2013

Pored navedenog, treba ukazati i na važno ograničenje Jensenovog indeksa koji se može koristiti za upoređivanje performansi investicionih fondova sa prosečnim tržišnim performansama, ali se za razliku od Šarpovog i Trejnorovog indeksa ne može samostalno koristiti za upoređivanje performansi investicionih fondova i njihovo rangiranje. Iako je na Slici 6 Jensenova alfa za portfolio B veća od Jensenove alfe za portfolio A (α_B>α_A), pogrešno je zaključiti da su performanse portfolija B superiornije od performansi portfolija A. Detaljnija analiza pokazuje da su čak performanse portfolija A bolje od performansi portfolija B, jer bilo koja kombinacija portfolija A i bezrizične aktive donosi viši Jensenov indeks pri istom nivo rizika, u poređenju sa kombinacijama portfolija B i bezrizične aktive (Francis & Kim, 2013). Primera radi, pozajmljivanjem novca po bezrizičnoj kamatnoj stopi r_f i njegovim investiranjem u portfolio A dobija se kombinovani portfolio A', koji pri istom nivou rizika (β_B) ima veću Jensenovu alfu od portfolija B (α_A>α_B). Superiornost performansi portfolija A nad performansama portfolija B potvrđuje i Trejnorova mera performansi portfolija. Uprkos činjenici da je Jensenov indeks za portfolio A niži od Jensenovog indeksa za portfolio B (α_A<α_B), Trejnorov indeks za portfolio A je viši od Trejnorovog indeksa za portfolio (T_A>T_B). Potvrda navednom je i veći nagib linije T_A u odnosu na liniju T_B (Slika 6).

Slika 6: Nedostaci Jensenovog indeksa kao mere performansi portfolija

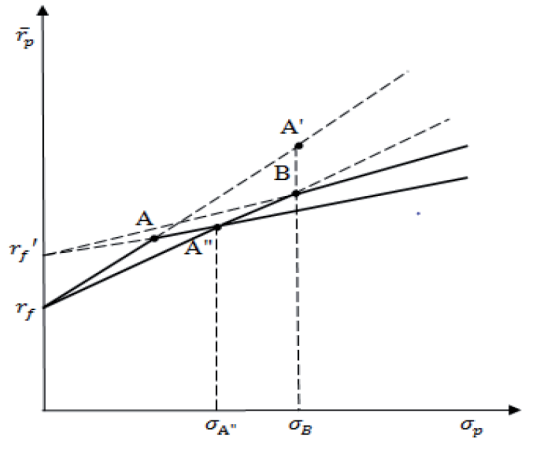


Izvor: Francis & Kim, 2013

Dodatnu relativizaciju modela ocene performansi portfolija predstavlja Roll-ova kritika (Roll, 1977), koja tvrdi da je nemoguće pronaći dovoljno preciznu zamenu za tržišni portfolio, zbog čega je nemoguće precizno meriti performanse portfolija na bazi Jensenovog i Trejnorovog indeksa. Prema Roll-ovoj kritici, analitičari nikada neće biti sigurni da li su izmerene performanse portfolija rezultat relativne sposobnosti portfolio menadžera, ili činjenice da je izabrani tržišni indeks ipak slaba aproksimacija stvarnog tržišnog portfolija i njegovog položaja u odnosu na efikasni set (Šoškić, 2013). Takođe, učestale su tvrdnje da izbor tržišnog indeksa kao benčmarka u

portfolio A's performance over the performance of portfolio B is brought into question. In the situation represented in Figure 4, portfolio A achieves better performance than portfolio B until reaching the level of risk corresponding to the combined portfolio A" ($\sigma_{A"}$), while at higher risk levels, the performance of portfolio B is superior to the performance of portfolio A. Given the above facts, it can be concluded that, even if only one of the essential assumptions of the *CAPM* model gets undermined, the validity of the Sharpe index will also be brought into question, as well as the validity of all other portfolio performance measures based on this model.

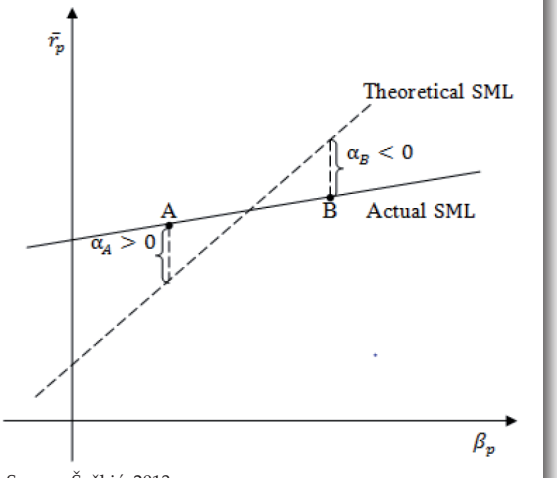
Figure 4: Disadvantages of the Sharpe index



Source: Author, based on Elton et al, 2011

In addition, the possible imprecision of the *CAPM* model in terms of the divergence of the actual *SML* from the theoretical one, results in a divergence of the actual evaluation from the theoretical evaluation of portfolio performance (Figure 5). By observing the theoretical *SML*, it is concluded that portfolio A has a positive alpha index (α_A>0) and achieves superior performance in relation to the market, while portfolio B has a negative alpha index (α_B>0) and achieves inferior performance compared to the market. However, the actual *SML* leads the analysts to a different conclusion. Since both portfolio A and portfolio B are located on the actual *SML*, their alpha indices equal zero (α_A=α_B=0), while their performance is the same as the average market performance. Therefore, the theoretical *SML* suggests that the performance of portfolio A is superior to the performance of portfolio B, while the actual *SML* indicates their equal performance (Figure 5).

Figure 5: Divergence of the actual from the theoretical portfolio performance evaluation



Source: Šoškić, 2013

In addition to the aforesaid, an important limitation of the Jensen index should be noted: the Jensen index can be used to compare the performance of mutual funds with the average market performance, however, this index alone cannot be used to compare the performance of mutual funds nor can it be used for their ranking, while the Sharpe index and the Treynor index can be used on their own for the mentioned purposes. Although in Figure 6, the Jensen's alpha for portfolio B is greater than the Jensen's alpha for portfolio A (α_B>α_A), it would be a mistake to conclude that the performance of portfolio B is superior to the performance of portfolio A. A more detailed analysis shows that the performance of portfolio A is even better than the performance of portfolio B, because any combination of portfolio A and risk-free assets has a higher Jensen index at the same level of risk, compared to the any possible combination of portfolio B and risk-free assets (Francis & Kim, 2013). For example, by borrowing money at a risk-free interest rate r_f to be invested in portfolio A, we get the combined portfolio A', which, at the same level of risk (β_B) has a higher Jensen's alpha than portfolio B (α_A>α_B). Superior performance of portfolio A over portfolio B is confirmed by the Treynor portfolio performance measure. Despite the fact that the Jensen index of portfolio A is lower than the Jensen index of portfolio B ($\alpha_{A'}$<α_B), the

značajnoj meri utiče na rezultate evaluacije performansi portfolija, kao i da se sa promenom tržišnog indeksa drastično menja rangiranje portfolija po osnovu ostvarenih performansi.

Kontraargumente navedenim kritikama pružili su Peterson i Rice (1980). Autori su istražujući Roll-ovu tvrdnju da modeli ocene performansi portfolija zasnovani na *CAPM* modelu rezultiraju dvosmislenim i nesigurnim ocenama performansi, zaključili da izbor tržišnog indeksa kao benčmarka i izbor mere performansi imaju relativno beznačajan uticaj na rangiranje portfolija po osnovu ostvarenih performansi. U njihovom istraživanju tri različite mere performansi (Šarpov, Trejnorov i Jensenov indeks) su rezultirale sličnim rangiranjem portfolija po osnovu ostvarenih performansi. Takođe, upotreba četiri različita tržišna indeksa: *Dow-Jones Industrial Average (DJIA), Standard and Poor's Industrials (S&P 500)* i dva indeksa koja obuhvataju sve akcije listirane na Njujorškoj berzi (*New York Stock Exchange - NYSE*), nije imala značajniji uticaj na rezultate evaluacije.

Važna kritika upućena modelima ocene performansi portfolija odnosi se i na činjenicu da se većina empirijskih istraživanja performansi investicionih fondova zasniva na istorijskim podacima koji se odnose samo na one investicione fondove koji su opstali u posmatranom (uzorkovanom) periodu. Nepotpunost podataka o investicionim fondovima koji nisu više aktivni rezultira njihovim izostavljanjem iz analize. Kao posledica navedenog uzorak sastavljen samo od aktivnih investicionih fondova precenjuje ostvarene prinose, a potcenjuje podnete rizike u poređenju sa ukupnim brojem fondova koji su postojali u analiziranom periodu (Francis & Kim, 2013).

Predstavljeni nedostaci jednoparametarskih modela ocene performansi portfolija zasnovanih na *CAPM* modelu uslovili su pojavu višeparametarskih modela merenja performansi portfolija baziranih na *APT* modelu. Međutim, i višeparametarski modeli se suočavaju sa brojnim problemima, od kojih je najvažniji mogućnost proizvoljnog izbora faktora sistemskog rizika i posledičnog dobijanja željenih rezultata. Različiti faktori sistemskog rizika upotrebljeni u modelima ocene performansi portfolija često rezultiraju i različitim ocenama performansi. Stoga je izborom odgovarajućih faktora moguće dobiti željenu ocenu performansi portfolija i željenu ocenu efikasnosti portfolio menadžmenta. Dakle, zamenom jednoparametarskih modela ocene performansi portfolija zasnovanih na *CAPM* modelu sa višeparametarskim modelima merenja performansi portfolija baziranim na *APT* modelu izvršena je zamena jednih problema u merenju performansi drugim problemima. Stoga, prilikom evaluacije performansi portfolija prioritet ne treba dati izboru modela ocene performansi, već razumevanju problema i uvažavanju nedostataka i ograničenja sa kojima se suočavaju ovi modeli. Na ovaj način umanjiće se značaj problema, a povećati validnost dobijenih rezultata.

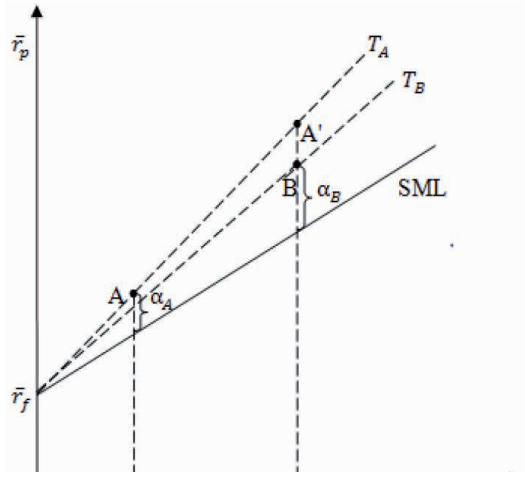
Zaključak

U cilju potpunijeg zadovoljenja različitih preferencija investitora i ostvarenja konkurentske prednosti razvile su se brojne vrste i podvrste investicionih fondova koji međusobno konkurišu smanjenjem provizija, poreskim olakšicama, kvalitetnijim izveštavanjem, investiranjem u posebne vrste imovine i sl. Svaki od navedenih aspekata konkurencije je važan, ali investitori prilikom izbora investicionog fonda najveći značaj pridaju ostvarenim investicionim performansama. U vreme važenja tradicionalne portfolio teorije rizik nije bio predmet analize, a ocena performansi portfolija je vršena isključivo na osnovu ostvarenih stopa prinosa. Međutim, da bi se ocena investicionih performansi smatrala merodavnom, ostvarena stopa prinosa portfolija mora biti korigovana za rizik, a prve mere performansi portfolija koje su na adekvatan način korigovale prinos za rizik su Šarpov, Trejnorov i Jensenov ili alfa indeks.

Iako su korisni instrumenti merenja performansi portfolija investicionih fondova, Šarpov i Trejnorov indeks se smatraju inferiornijim merama performansi u poređenju sa Jensenovim indeksom. Ključni razlozi zbog kojih se Jensenov indeks smatra superiornijom merom performansi su: 1) Jensenov indeks je izražen u procentima i stoga pogodniji za interpretaciju, za razliku od Šarpovog

Treynor index for portfolio A is higher than the Treynor index for portfolio B (T_A>T_B). This is supported by the greater inclination of the line T_A compared to the line T_B (Figure 6).

Figure 6: Disadvantages of the Jensen index as a portfolio performance measure



Source: Francis & Kim, 2013

Roll's critique (Roll, 1977) is another economic idea that challenges portfolio performance evaluation models, since Roll argues that it is impossible to find an accurate proxy for the market portfolio, therefore, it impossible to accurately measure the performance of the portfolio based on the Jensen and the Treynor index. According to the Roll's critique, analysts can never be sure whether the measured portfolio performance is a result of the selection capability of portfolio managers, or the fact that the chosen market index is still a poor approximation of the actual market portfolio and its position in relation to the effective set (Šoškić, 2013). Also, the opinion that the choice of a market index as a benchmark significantly affects the results of portfolio performance evaluation is becoming more widely accepted, as well as the viewpoint that any change in the market index drastically changes the ranking of the portfolio based on the actual performance.

Counterarguments to the said criticism were put forward by Peterson and Rice (1980). While analyzing the Roll's claim that portfolio performance evaluation models based on the *CAPM* model result in ambiguous and unreliable performance evaluations, these authors concluded that both the choice of a market index as a benchmark and the choice of the performance measure have a relatively insignificant impact on the portfolio ranking based on the actual performance. In their study, three different performance measures were applied (Sharpe, Treynor and Jensen index) and all three produced similar rankings of portfolios based on the actual performance. Furthermore, the application of four different market indices: *Dow-Jones Industrial Average (DJIA)*, *Standard and Poor's Industrials (S & P 500)* and two indices which include all shares listed on the *New York Stock Exchange (NYSE)*, did not have any significant influence on the evaluation results.

An important criticism of portfolio performance evaluation models refers to the fact that the majority of empirical studies in the field of mutual fund performance are based on historical data relating to only those mutual funds that survived the evaluated (sampled) period. The incomplete data on mutual funds that are no longer active result in their omission from the analysis. Accordingly, a sample being composed of only active mutual funds overestimates the returns and underestimates the risks compared to the total number of funds that were operating in the analyzed period (Francis & Kim, 2013).

The illustrated disadvantages of single-parameter models for portfolio performance evaluation based on the *CAPM* model enabled the development of multi-parameter models for measuring portfolio performance based on the *APT*. However, multi-parameter models are also facing numerous problems, the most important being the possibility of an arbitrary selection of systemic risk factors and the consequent achievement of the desired results. The different factors of systemic risk applied in the portfolio performance evaluation models often result in the different performance estimates. Therefore, by choosing the appropriate factors, one can get the desired evaluation of both portfolio performance and efficiency of the portfolio management. Therefore, the replacement of single-parameter portfolio performance evaluation models based on the *CAPM* model by multi-parameter models of measuring portfolio performance based on the *APT* model, implies the replacement of one type of problems related to performance measurement by another type of

i Trejnorovog indeksa koji predstavljaju racio brojeve, 2) Jensenov indeks se računa pomoću linearne regresione jednačine, što pruža mogućnost utvrđivanja statističke značajnosti dobijenih rezultata i 3) Jensenov indeks podrazumeva upotrebu vremenski varirajuće bezrizične stope prinosa, dok se u slučaju Šarpovog i Trejnorovog indeksa koristi konstantna prosečna bezrizična stopa prinosa.

Pored navedenog, bitno je razumeti i uvažiti nedostatke i ograničenja sa kojima se suočavaju opisani modeli ocene performansi portfolija, poput nemogućnosti adekvatne aproksimacije tržišnog portfolija, ili pak njihove zasnovanosti na *CAPM* modelu čije su nerealne polazne pretpostavke predmet brojnih kritika. Nemogućnost adekvatne aproksimacije tržišnog portfolija rezultira odsustvom preciznog merenja performansi portfolija, dok narušenost polaznih pretpostavki *CAPM* modela implicira odsustvo validnosti mera performansi portfolija zasnovanih na ovom modelu. Razumevanje navedenih nedostataka je svojevrsni preduslov povećanja validnosti dobijenih rezultata i iz njih izvedenih zaključaka.

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problems. With this in mind, when evaluating portfolio performance, the priority should not be given to the choice of the performance measurement model, but to the understanding and appreciation of the problem in terms of the disadvantages and limitations encountered by these models. In this way, the significance of the problem will be reduced, while the validity of the results will be improved.

Conclusion

In order to better meet the different requirements of investors and achieve competitive advantage, numerous types and subtypes of mutual funds have developed over time. These funds compete with each other by lowering their fees, introducing tax incentives, producing quality reporting, offering investments in specific types of property and the like. Each of these aspects of competition is important, yet, when choosing a suitable mutual fund, investors consider investment performance to be the most significant feature. At the times when the traditional portfolio theory was applied, risk was not included in the portfolio performance evaluation, which was carried out solely on the basis of the actual returns. However, an investment performance evaluation, in order to be considered reliable, needs to be risk-adjusted and the first portfolio performance measures which have adequately applied risk-adjustment to returns are the Sharpe index, the Treynor index and the Jensen or alpha index.

Despite being useful tools for measuring the portfolio performance of mutual funds, the Sharpe and the Treynor index are considered inferior performance measures compared to the Jensen index. The key reasons for the Jensen index to be considered a superior performance measure are as follows: 1) the Jensen index is expressed as a percentage, and is, therefore, more suitable for interpretation, unlike the Sharpe and the Treynor index which are expressed by ratio numbers, 2) the Jensen index is calculated by using a linear regression equation, which provides the possibility of determining the statistical significance of the obtained results, and 3) the Jensen index includes the use of the time-varying risk-free rates of return, whereas the Sharpe and the Treynor index use a constant average risk-free rate of return.

In addition, it is important to understand and know the disadvantages and limitations faced by the described portfolio performance evaluation models, such as the impossibility of determining an adequate proxy of the market portfolio, or their foundation in the *CAPM* model whose unrealistic initial assumptions are the subject of much criticism. The impossibility to adequately set a market portfolio proxy results in the inability to precisely measure the portfolio performance, while the challenging of the initial *CAPM* assumptions implies a questionable validity of portfolio performance measures based on this model. Understanding the above described disadvantages and deficiencies is a precondition for improving the validity of the obtained results and the conclusions derived thereof.