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BITCOIN I DIVERSIFIKACIJA PORTFOLIJA: PERSPEKTIVA GLOBALNOG INVESTITORA

„Dobar portfolio je mnogo više od duge liste dobih akcija i obveznica. Portfolio je uravnotežena celina koja investitoru pruža zaštitu i mogućnosti u pogledu širokog spektra nepredviđenih situacija”, Harry Markowitz (1997).

Rezime

U ovom istraživanju se ispituje da li je poželjno u investicioni portfolio koji sadržava tradicionalne finansijske instrumente uključiti Bitcoin. Cilj rada jeste da se istraži da li Bitcoin može biti dobar izvor diversifikacije, ukoliko se posmatra investitor koji investira sredstva na globalnom finansijskom tržištu. Za tu svrhu kreirana su dva portfolija: portfolio kojem je cilj minimalizovanje rizika i portfolio koji je označen kao „agresivni”, a koji nudi veće stope povraćaja na dnevnom nivou, ali i veći rizik. Portfoliji su kreirani primenom Markowitz teorije optimizacije, a uključili su tradicionalne instrumente (akcije, obveznice, zlato) i Bitcoin. Korišćeni su podaci visoke frekvencije (dnevni podaci). Analizirani period je od kraja jula 2010. godine do kraja juna 2019. godine, što predstavlja period aktivnog trgovanja Bitcoin-om. Rezultati istraživanja pokazuju da Bitcoin može biti dobar izvor diversifikacije u portfoliju koji sadrži tradicionalne aktive (zlato, obveznice i akcije), kako za investitora koji nije sklon riziku, tako i za one investitore koji imaju veći apetit za rizik. Ipak, uzimajući u obzir visoki stepen volatilnosti, investitori treba da budu veoma pažljivi kada donose odluke o uključivanju Bitcoin-a u portfolio.

Ključne reči: diversifikacija, Bitcoin, portfolio teorija, optimizacija, rizik, povraćaj

JEL: G11, G12, G15, G24

BITCOIN IN PORTFOLIO DIVERSIFICATION: THE PERSPECTIVE OF A GLOBAL INVESTOR

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"A good portfolio is more than a long list of good stocks and bonds. It is a balanced whole, providing the investor with protections and opportunities with respect to a wide range of contingencies." Harry Markowitz (1997)

Summary

This paper examines whether it is advisable to include some portion of Bitcoin in a portfolio of traditional financial assets. The goal is to explore whether Bitcoin could be a good source of diversification from the perspective of a global investor. Two portfolios have been created for this purpose: a portfolio aimed at minimizing risk and a portfolio designated as "aggressive" that offers higher rates of daily return but also a higher risk. Portfolios were created using Markowitz's optimization theory and included traditional instruments (stocks, bonds, gold) and Bitcoin. In portfolio optimization, high-frequency data (daily data) were used. The analysed period is from the end of July 2010 to the end of June 2019, which is the period of active Bitcoin trading. The results show that Bitcoin could be a good source of diversification for a portfolio that consists of traditional financial instruments, for investors trading daily. It could be a good source of diversification for the risk-averse investor and those investors who have a higher risk appetite. Considering the high volatility of Bitcoin, the investors should be very careful when they decide to include Bitcoin in a portfolio.

Keywords: diversification, Bitcoin, portfolio theory, optimization, risk, return

JEL: G11, G12, G15, G24

1. Uvod

Teorija portfolija, u svojoj osnovi, sugerirše da nije mudra odluka „sva jaja držati u istoj korpi“. Jedan od načina da se dostigne dobar odnos između rizika i povraćaja jeste primena diversifikacije koja podrazumeva uključivanje onih instrumenata u portfolio koji nisu visoko korelisani. Na taj način se eliminiše nesistemska komponenta rizika.

Bitcoin je po prvi put predstavio Nakamoto (2008) kao medijum razmene, kao oblik virtuelne valute. Interes za ovim instrumentom, ukoliko se u obzir uzme njegova cena, nije bio značajan do kraja 2015. godine. Tokom 2016. godine cena Bitcoin-a postepeno beleži rast, da bi maksimalni nivo cene bio dostignut krajem 2017. godine.

Od 2015. godine pojavljuje se obimna literatura koja nastoji da identifikuje prirodu Bitcoin-a, kao i njegovu ulogu u diversifikaciji portfolija. S obzirom na izraženu popularnost, kao i medijsko promovisanje Bitcoin-a, brojna istraživanja nastoje da ispituju i pokažu špekulativnu prirodu Bitcoin-a (Kristoufek, 2015; Yermack, 2015; Baek and Elbeck, 2015, Dyhrberg, 2016b). Kristoufek (2015) i Yermack (2015) ispituju prirodu Bitcoin-a i zaključuju da se ona ne može objasniti savremenim ekonomskim teorijama jer iskazuje više špekulativnih karakteristika u odnosu na druge instrumente i valute. Drugi autori pokušavaju da shvate da li je Bitcoin valuta ili dobro, roba (na primer Dyhrberg, 2016ab; Fry i Cheah, 2016; Katsiampa, 2017; Pieters i Vivanco, 2017). Pored toga, Dyhrberg (2016ab) pronalazi da Bitcoin ispoljava neke sličnosti sa ostalim klasama aktive poput kurseva valuta i zlata, te kao takav nudi široke mogućnosti diversifikacije portfolija. Sa druge strane Baur et al. (2018) pokazuju da su karakteristike Bitcoin-a drugačije u odnosu na ostale aktive poput zlata ili dolara. Za razliku od drugih klasa aktive Bitcoin nema neku aktivnu koja se nalazi u njegovoj osnovi, te su stoga investitori veoma oprezni prilikom ulaganja u ovaj instrument. S obzirom na slabu ili nikakvu povezanost Bitcoin-a sa ostalom aktivom, on se prepoznaje kao modus zaštite od rizika u menadžmentu portfolija. Koristeći mesečne podatke o povraćaju, Chan et al. (2019) demonstriraju da Bitcoin može efektivno poslužiti kao zaštita od rizika u odnosu na Euro

STOXX, Nikkei, Shanghai A-Share, S&P 500 kao i TSX Index. Ipak, ovi autori pronalaze da je zaštita od rizika u odnosu na S&P 500 i Euro STOXX atraktivna za podatke srednje frekvencije, dok je zaštita od rizika u odnosu na Shanghai A-Share najefektivnija kod niskofrekventnih podataka, kao što su mesečni povraćaji.

Ram (2019) sprovodi opsežnu analizu koristeći Sharpe ratio te pokazuje da Bitcoin obezbeđuje povraćaje ponderisane rizikom koji su veći u odnosu na druge klase aktive. Trautman i Dorman, (2018) sprovode korelacionu analizu među različitim klasama aktive poput akcija, zlata, nekretnina i obveznica te dokazuju da su koristi od diversifikacije povezane sa nižom korelacijom ovih aktiva sa drugim aktivama. Uprkos niskoj korelaciji sa drugim aktivama Bitcoin je veoma volatilna i nekada može postati nelikvidna, uzimajući u obzir da ne postoji neka aktiva koja se nalazi u njegovoj osnovi. Ovo može značiti da investitori nekada iznenada povuku investicije u ovaj instrument, u periodima krize, što najčešće rezultira značajnim padom cene, te nekada ta aktiva može postati bezvredna. Uzimajući u obzir navedeno, sprovedene su i analize koje su ispitale rizike povezane sa Bitcoin-om. Eisl et al. (2015) u analizi polaze od uslovnog VaR-a, odnosno okvira za njegovo izračunavanje, sa ciljem da ispituju da li Bitcoin optimizuje performanse portfolija. Njihovi nalazi sugerirše da, iako Bitcoin povećava VaR, dodatni rizik je kompenzovan visokim povraćajem koji Bitcoin nudi.

Koristeći multivarijantnu dinamičko uslovnu korelaciju (DCC model) Chuen et al. (2018) ispituju da li uključivanje deset kriptovaluta u tradicionalni portfolio donosi dodatne koristi povraćaju prilagođenom za rizik.

Rezultati istraživanja ovih autora sugerirše da uključivanje indeksa kriptovaluta obezbeđuje značajna poboljšanja u sveukupnom portfoliju i njegovim performansama, što je potvrđeno i spaninig testom. Slične nalaze iznose i Ehlers i Gauer (2019) koji primenjuju Kolmogorov-Smirnov test kao i test racija varijanse pod pretpostavkom heteroskedastičnosti kako bi pokazali ulogu pet vodećih kriptovaluta kao što su Bitcoin, Ethereum, Ripple, Litecoin i Dash u portfoliju. Ovi autori dolaze do saznanja da jedino Bitcoin i Ripple pružaju minimalne pogodnosti u portfoliju i volatilnosti koji su u

1. Introduction

The very basics of portfolio theory suggest that it is not a wise decision to put all your eggs in one basket. One way to achieve a good risk-return trade-off is to spread the risk and reward associated with investment portfolio by diversifying the individual assets with a variety of non-correlated assets, which nearly eliminates the unsystematic component of risk.

Bitcoin was first introduced by Nakamoto (2008) as a medium of exchange and there was no significant interest shown by investors until 2015, after which its price began to increase gradually. Interest in this instrument, given its price, was not significant until the end of 2015. During 2016, the price of Bitcoin gradually increased, reaching its maximum level at the end of 2017.

Since 2015, voluminous literature started dealing with identifying the nature and diversification benefits of Bitcoin. Given the popularity and excessive media coverage promoting Bitcoin, several scholars attempted to examine the speculative nature of Bitcoin (See e.g. Kristoufek, 2015; Yermack, 2015; Baek and Elbeck, 2015; Dyhrberg, 2016b). In particular, Kristoufek (2015) and Yermack (2015) studied the nature of Bitcoin and concluded that it cannot be explained by contemporary theories of economics, as it exhibits more speculative characteristics than a medium of exchange. Another section of scholars attempted to understand whether Bitcoin is a currency or a commodity (See for example Dyhrberg, 2016ab; Fry and Cheah, 2016; Katsiampa, 2017; Pieters and Vivanco, 2017). Also, Dyhrberg (2016ab) finds that Bitcoin exhibits some similarities to other classes of assets such as exchange rate and gold and, as such, it offers a wide array of opportunities for portfolio diversification. Arguing to the contrary, Baur et al. (2018) demonstrated that the characteristics of Bitcoin are different from that of other assets such as gold and the US dollar. Unlike other classes of assets, Bitcoin does not have an underlying asset and, as such, investors must invest with caution. Given the little or no correlations with other classes of assets, Bitcoin has now been recognized as a mode of hedging risk in portfolio management. Using monthly

return data, Chan et al. (2019) demonstrated that Bitcoin can effectively be hedged against Euro STOXX, Nikkei, Shanghai A-Share, S&P 500, and the TSX Index. However, they found that hedging against the S&P 500 and Euro STOXX is attractive for medium-frequency data and hedging against Shanghai A-Share is most effective for low-frequency data, such as monthly returns.

Ram (2019) carried out an extensive analysis using the Sharpe ratio and showed that the Bitcoin provides risk-adjusted returns over and above the return on other classes of assets. Trautman and Dorman, (2018) carried out a correlation analysis among different asset classes such as stocks, gold, real estates and bonds and concluded that the diversification benefits are associated with its low correlation with other classes of assets. Despite its low correlation with other assets, Bitcoin has high volatility and may sometimes become illiquid given the fact that there is no underlying asset. This might cause the investors' sudden withdrawal of investments in the case of a crisis, which usually results in a significant drop in price; it may sometimes become valueless. As such, studies must include adequate controls in the regression analyses for this illiquidity risk. Eisl et al. (2015) use conditional value-at-risk a framework to examine whether the Bitcoin optimizes the portfolio performance. Their findings suggest that—although the Bitcoin increases the value-at-risk—the additional risk is compensated for by the excessive return on Bitcoin investments.

Using the multivariate dynamic conditional correlation (DCC) model, Chuen et al. (2018) examined whether the inclusion of ten cryptocurrencies in a traditional portfolio of nine assets will bring additional benefits on risk-adjusted returns.

Their findings suggest that the inclusion of the cryptocurrency index provides large improvements in the overall portfolio performance based on mean-variance, which was confirmed by the spanning test employed. Similarly, Ehlers and Gauer (2019) employed the Kolmogorov–Smirnov test and variance ratio test (VRT) with heteroscedasticity adjustment to understand the role of five leading cryptocurrencies such as Bitcoin, Ethereum,

skladu sa idejom optimizacije portfolija prema Markowitz teoriji.

Briere et al. (2015) su u analizi koristili nedeljne podatke u periodu od 2010 - 2013. godine i analizirali su investiranje u Bitcoin sa aspekta SAD investitora koji investira u tradicionalne instrumente (akcije, obveznice i ključne valute) kao i alternativne investicije (dobra, hedž fondove i nekretnine). Ovi autori zaključuju da, tokom analiziranog perioda, ulaganja u Bitcoin imaju vrlo prepoznatljive karakteristike uključujući veoma visoki nivo povraćaja i rizika. Takođe, ovi autori zaključuju da Bitcoin investicije nude značajne koristi diversifikacije.

Carpenter (2016) koristi modifikovani pristup koji je zasnovan na varijansi i očekivanoj vrednosti i pokazuje da Bitcoin može biti održiv diversifikacijski alat.

Bouri et al. (2017) koristi dinamički korelacioni model i ispituje da li Bitcoin može da deluje kao zaštita od rizika i kao sigurna aktiva u portfoliju koji uključuje glavne akcijske indekse, obveznice, naftu, zlato, te generalne indekse dobara kao što je US dolar indeks. Autori u istraživanju koriste dnevne i nedeljne podatke u periodu od jula 2011. godine do decembra 2015. godine. Empirijski rezultati su pokazali da Bitcoin nudi siromašnu zaštitu od rizika i može da posluži samo kao diversifikacijski alat.

Cilj ovog istraživanja jeste da ispita da li je preporučljivo uključiti određeni udeo Bitcoin-a u portfolio koji sadrži tradicionalne finansijske instrumente. Polazi se od hipoteze da Bitcoin predstavlja osnovu za diversifikaciju portfolija. Za potrebe ove analize korišćena je tehnologija optimizacije portfolija kojom je procenjen optimalan udeo tradicionalnih finansijskih instrumenata i Bitcoin-a u portfoliju. Za potrebe optimizacije korišćena je Markowitz portfolio teorija.

Istraživanje je segmentirano kroz nekoliko delova. Prvi deo jeste uvodni deo, dok drugi deo nudi metodološki pristup, a treći prikazuje podatke koji su korišćeni, kao i deskriptivnu statistiku. Takođe, u ovom delu je ukazano na ključne nalaze istraživanja, te je data diskusija. Četvrti deo pokazuje testiranje diversifikacije primenom Wald testa, dok peti deo predstavlja zaključak.

2. Metodološki pristup

Sa ciljem da se analizira uloga Bitcoin-a u diversifikaciji portfolija, neophodno je izračunati povraćaje svake aktive koja se uključuje u portfolio. Dnevni povraćaj za svaki instrument je izračunat prema formuli koju su koristili Bodie, et al. (2014) kao i Šoja (2019), na sledeći način:

$$r_{it} = \ln\left(\frac{p_{it}}{p_{it-1}}\right)$$

Pri čemu r_{it} predstavlja povraćaj aktive i u portfoliju, a \ln predstavlja prirodni logaritam. Dalje, p_{it} predstavlja cenu (odnosno vrednost indeksa) u periodu t , a p_{it-1} je cena, odnosno vrednost aktive u prethodnom periodu (i.e. $t - 1$).

Prosečan povraćaj može biti izračunat na sledeći način:

$$r_i = \frac{r_{it} + \dots + r_{in}}{N}$$

Pri čemu N predstavlja broj opservacija.

Standardna devijacija je korišćena sa ciljem da se pokaže i shvati disperzija povraćaja serije podataka u odnosu na srednju vrednost, a izračunata je kao kvadratni koren varijanse. Novija literatura pokazuje da korišćenje modela volatilnosti, poput GARCH modela, rezultira u inkorektnim VaR procenama (Caporale and Zekokh 2019) zbog prisustva podataka koji mogu značajno da utiču na model i mere rizika (Trucíos 2019). Polazeći od toga, u ovom istraživanju korišćena je metodologija predložena od strane Bodie, et al. (2014).

Aktiva koja je volatilna ima visoku standardnu devijaciju, dok je standardna devijacija kod relativno stabilne aktive najčešće niska. Sa druge strane, ona meri neizvesnost kao rizik, čak i u slučaju kada postoje natprosečni povraćaji koji su poželjni za investitora. Standardna devijacija je izračunata na sledeći način:

$$\text{Standardna devijacija} = \sqrt{\frac{\sum_{j=1}^n (r_j - \bar{r})^2}{N}}$$

Pri čemu:

r_j predstavlja vrednost j th u datom setu podataka.

\bar{r} predstavlja prosečan povraćaj posmatrane

Ripple, Litecoin, and Dash in a portfolio. They unearthed interesting findings that only Bitcoin and Ripple were shown to provide minimum variance portfolio benefits and volatility exposure, in line with Harry Markowitz's idea on the mean-variance portfolio.

Briere et al. (2015) used weekly data over the period from 2010-2013, and they analyzed a Bitcoin investment from the standpoint of a U.S. investor with a diversified portfolio including both traditional assets (worldwide stocks, bonds, hard currencies) and alternative investments (commodities, hedge funds, real estate). They concluded that, during the observed period, Bitcoin investment had highly distinctive features, including the exceptionally high average return and volatility. Also, they concluded that Bitcoin investments offer significant diversification benefits.

Carpenter (2016) used a modified mean-variance framework and showed that Bitcoin can be a viable diversification tool. His research showed that Bitcoin investment could be skewed by return activity that occurred during a speculative bubble in 2013.

Bouri et al. (2017) used a dynamic conditional correlation model and examined whether Bitcoin can act as a hedge and a haven in a portfolio which included major world stock indices, bonds, oil, gold, the general commodity index, and the US dollar index. The authors used daily and weekly data from July 2011 to December 2015. The empirical results showed that Bitcoin offers poor hedge and only can serve as a diversification tool.

The objective of this research is to examine whether it is advisable to include some portion of Bitcoin in a portfolio of traditional financial instruments. The portfolio optimization technique is used to estimate the optimal portfolio level using Markowitz's portfolio theory.

This paper is organized as follows: section one is the introduction, while section two provides the methodological approach and section three outlines the data set used and provides descriptive statistics. It also recapitulates the findings and discusses the practical implications. Section five shows the diversification testing via the Wald test, and section four concludes the paper.

2. Methodological Approach

In order to analyze the role of Bitcoin in portfolio diversification, the return of each asset in the portfolio of assets is computed in the following manner for the three regimes. The daily return of each instrument is computed in the sense of Bodie, et al. (2014) and Šoja (2019) as follows.

$$r_{it} = \ln\left(\frac{p_{it}}{p_{it-1}}\right)$$

Where r_{it} is the return of asset i in the portfolio and \ln denotes the natural logarithm. p_{it} is the price (or the index value) at the time period t and p_{it-1} is the price or value of assets in the prior period (i.e. $t - 1$).

The average of return can be computed in the following manner:

$$r_i = \frac{r_{it} + \dots + r_{in}}{N}$$

Where N is the number of observations.

The standard deviation is used to understand the dispersion of return series relative to its mean and is calculated as the square root of the variance. Recent literature shows that the use of *volatility models such as GARCH results in incorrect VaR estimates* (Caporale and Zekokh 2019) because the presence of outliers has a critical impact on the model's performance to produce more accurate estimates of risk measures (Trucíos 2019). As such, this paper utilizes the methodology proposed by Bodie, et al. (2014).

A volatile asset has a high standard deviation, while the standard deviation of a relatively stable asset is usually rather low. On the downside, it measures the uncertainty as risk even at times when above average returns favour the investors. The standard deviation is computed as follows.

$$\text{Standard Deviation} = \sqrt{\frac{\sum_{j=1}^n (r_j - \bar{r})^2}{N}}$$

Where:

r_j is the value of j th point in the data set.

\bar{r} is the mean return in the return series.

N is the number of observations.

Value-at-risk (VaR) is used to model the

serije povraćaja.

N je broj opservacija.

Value-at-risk (VaR) predstavlja metodologiju koja se često koristi za izračunavanje potencijalnih gubitaka na investiciji kao i verovatnoće nastanka takvih gubitaka. Putem VaR modela se procenjuje najlošiji scenario gubitaka sa kojim se investitor može suočiti po pitanju ulaganja u konkretnu aktivu. Tri komponente se uključuju u procenu VaR-a: vremenski period, stepen poverenja i suma gubitka izražena procentualno. VaR pokazatelj, prema Bodie, et al. (2014) izračunava se na sledeći način:

$$VaR_{x,\alpha} = -z_\alpha \sigma S$$

Pri čemu z_α predstavlja kvantilni red dok je α standardizovana slučajna varijabla a S je vrednost pozicije. Sa ciljem da se izračuna VaR portfolija primenom Makrowitz metodologije, neophodno je izračunati kovarijansu i korelaciju među aktivama, očekivani povraćaj i rizik pojedinačne aktive koja se uključuje u portfolio. Prema Levišauskait (2010), očekivani povraćaj pojedinačne aktive u portfoliju može biti izračunat korišćenjem sledećeg izraza:

$$E_{r(p)} = \sum_{i=1}^n W_i E_{i(r)} = W_1 E_{1(r)} + W_2 E_{2(r)} + \dots + W_N E_{N(r)}$$

U ovom slučaju $E_{r(p)}$ predstavlja očekivani povraćaj r portfolija, p i W predstavljaju udele aktive i u portfoliju p . Sa ciljem da se shvati kako se kreću aktive koje se uključuju u portfolio, neophodno je izračunati korelaciju među njima. Uprkos tome što neki autori jednostavno zaključuju da Bitcoin nudi mogućnosti diversifikacije, odnosno nosi koristi kroz diversifikaciju na osnovu korelacione analize, korisno je uključiti i VaR u analizu, uzimajući u obzir visoki stepen volatilnosti Bitcoin-a, s obzirom na to da on u sebi ne sadrži neku aktivu koja bi činila njegovu osnovu.

Kovarijansa između dve aktive u portfoliju se izračunava na sledeći način:

$$\text{Kovarijansa (A, B)} = \frac{\sum (r_A - \bar{r}_A)(r_B - \bar{r}_B)}{N}$$

Pri čemu r_A predstavlja povraćaj aktive A, a r_B predstavlja povraćaj aktive B koja se nalazi u portfoliju, dok \bar{r}_A predstavlja prosečan povraćaj

aktive A, a \bar{r}_B prosečan povraćaj aktive B. N i u ovom slučaju predstavlja broj opservacija. Korelacija između aktiva A i B može biti izračunata na sledeći način:

$$\text{Korelacija (p)} = \frac{Cov(A,B)}{\sigma_A \sigma_B}$$

Pri čemu $Cov(A,B)$ predstavlja kovarijansu između aktive A i B, dok σ_A i σ_B predstavljaju standardnu devijaciju aktive A i aktive B.

Za testiranje diversifikacije portfolija u koji se uključuje Bitcoin korišćen je Wald test. U prvom koraku korišćen je test koji su razvili Huberman i Kendel (1987) prema kojem se koristi OLS regresija u koju je uključen povraćaj Bitcoin-a R_E koji je stavljen u regresijski odnos sa povraćajem K aktive koja čini portfolio, odnosno koji čini efikasan portfolio, bez Bitcoin-a a koji je označen kao R^k_B , $k=1, \dots, K$. Ovaj model je sledeći:

$$R_{e,t} = \alpha + \sum_{k=1}^K \beta_k R^k_{B,t} + \varepsilon_t; \quad t=1,2,\dots,T$$

Neophodan uslov za ovaj test jeste sledeći:

$$H_0: \alpha = 0 \text{ and } \sum_{k=1}^K \beta_k = 1$$

Nulta hipoteza koja se testira jeste da portfolio koji sadrži K aktivu ima isti očekivani povraćaj ali nižu varijansu nego aktiva koja se testira, s obzirom na to da su instrumenti uključeni u portfolio K nekorelisani sa ε_t . Varijable su testirane primenom Wald testa kojim se ispituje značaj varijabli u modelu. Formula za Wald test je sledeća:

$$W_T = \frac{[\hat{\theta} - \theta_0]^2}{1/I_n(\hat{\theta})} = I_n(\hat{\theta})[\hat{\theta} - \theta_0]^2$$

Putem Wald testa se testira hipoteza da je alfa u modelu jednaka nuli, a da je beta jednaka jedinici. Isto tako, Wald test ispituje da li je posmatrana varijabla statistički značajna u modelu. Ukoliko je značajna, ona povećava validnost modela i kao takva treba biti uključena u isti.

3. Podaci i empirijski nalazi

Prilikom određivanja optimalnog portfolija koji uključuje Bitcoin, posmatrano sa aspekta investitora koji ulaže na globalnom nivou, izabrano je nekoliko instrumenata koji aproksimiraju globalne investicijske

potential for any loss in the investment asset being considered and the probability of occurrence of such loss. It assesses the worst-case scenario that an investor could face with regard to investment in assets. Three components are involved in the estimates of VaR statistics: time period, the confidence level and the amount of loss in percentages. The VaR in the sense of Bodie, et al. (2014) can be computed as follows:

$$VaR_{x,\alpha} = -z_{\alpha} \sigma S$$

Where z_{α} is the quantile order α of the standardized random variable and S is the value of the position. In order to compute the VaR using Markowitz's method, one must compute the correlation and covariance among assets, expected returns and the risk of individual assets in the portfolio. In the sense of Levišauskait, (2010), the expected return of individual assets in the portfolio could be computed by the following equation.

$$E_{r(p)} = \sum_{i=1}^n W_i E_{i(r)} = W_1 E_{1(r)} + W_2 E_{2(r)} + \dots + W_N E_{N(r)}$$

Where $E_{r(p)}$ is the expected return r on the portfolio p and W is the weight of asset i in the portfolio p . In order to understand to what extend the assets in the portfolio move together, one must need to compute the correlation among individual assets. Although some authors simply conclude that the Bitcoin offers diversification benefits based on the correlation analysis, it would be more appropriate to consider VaR given the high volatility and the fact that there are no underlying assets.

The covariance between any two assets in the portfolio is computed as follows.

$$\text{Covariance (A, B)} = \frac{\sum(r_A - \bar{r}_A)(r_B - \bar{r}_B)}{N}$$

Where r_A is the return of asset A and r_B is the return of asset B in the portfolio and \bar{r}_A denotes the average return of asset A and \bar{r}_B denotes the average return of asset B. N as usual is the number of observations. The correlation between assets A and B could be computed in the following manner:

$$\text{Correlation (p)} = \frac{\text{Cov (A,B)}}{\sigma_A \sigma_B}$$

Where $\text{Cov}(A,B)$ is the covariance between asset A and asset B. σ_A and σ_B are the standard deviation of asset A and asset B.

We test the diversification of portfolios that include Bitcoin via the Wald test. At the first stage we used the test developed by Huberman and Kendel (1987) which includes running OLS regression of Bitcoin return R_E on the returns of K assets that represents an efficient frontier without Bitcoin, R^k_B , $k = 1, \dots, K$. That model is the following:

$$R_{e,t} = \alpha + \sum_{k=1}^K \beta_k R_{B,t}^k + \varepsilon_t; \quad t=1,2,\dots, T$$

The necessary condition for this test is:

$$H_0: \alpha = 0 \text{ and } \sum_{k=1}^K \beta_k = 1$$

The null hypothesis in this test is that portfolio comprising K assets has the same expected return but a lower variance than the tested assets since the K benchmark assets are uncorrelated with ε_t . After this condition we run a Wald test to assume if the explanatory variables in a model are significant. The formula for Wald test is:

$$W_T = \frac{[\hat{\theta} - \theta_0]^2}{1/I_n(\hat{\theta})} = I_n(\hat{\theta})[\hat{\theta} - \theta_0]^2$$

Through the Wald test we test the hypothesis that the alpha in the model is equal to zero and beta is equal to 1. Also, the Wald test examines whether the observed variable is statistically significant in the model. If so, it increases the validity of the model and, as such, should be included in it.

3. Data and Empirical Findings

In order to explore the optimal portfolio diversification using Bitcoin for a global investment portfolio, a few investment instruments were chosen, because they provide a good approximation for the global investment opportunities. Financial instruments considered for the optimization process are as follows:

- Monetary gold denominated in USD;
- S&P 500, which represents stock market

mogućnosti. Korišćeni su podaci visoke frekvencije i dnevni podaci na zatvaranju tržišta. Finansijski instrumenti koji su uključeni u proces optimizacije su:

- Monetarno zlato denominirano u dolarima;
- Akcijski indeks S&P koji predstavlja globalno tržište akcija;
- Globalni indeks obveznica dospeća 1-10 godina denominovan u dolarima, a koji predstavlja globalno tržište obveznica;
- Bitcoin denominovan u dolarima.

Svi instrumenti su posmatrani na dnevnom nivou u periodu od 30.7.2010. do 30.6.2019. U

deskriptivne statistike za svaki analizirani instrument. U drugom koraku izračunata je korelacija među instrumentima, a nakon toga su određeni optimalni portfoliji.

3.1. Deskriptivna statistika

Deskriptivna statistika za analizirani period prikazana je tabelom broj 1 i obuhvata podatke o prosečnom povraćaju, standardnoj devijaciji, učestalosti negativnog povraćaja i VaR-u.

Dnevni povraćaji za analizirane instrumente pokazuju da je povraćaj kod tradicionalnih instrumenata veoma skroman i kreće se nešto

Tabela 1. Deskriptivna statistika: dnevni podaci 19.7.2010. do 30.6.2019.

	Bitcoin	S&P Index	Zlato	1-10 g globalni indeks obveznica
Prosek	0,72%	0,04%	0,01%	0,00%
StDev	6,67%	0,94%	0,95%	0,29%
Freq<0	43,03%	45,62%	48,42%	48,68%
Parametric VaR	-10,25%	-1,50%	-1,55%	-0,48%
Empirical VaR	-8,36%	-1,56%	-1,49%	-0,47%
Parametric CVaR	-13,04%	-1,90%	-1,95%	-0,60%
Empirical CVaR	-14,03%	-2,32%	-2,30%	-0,68%

Izvor: Računica autora

prvom koraku izračunati su dnevni povraćaji za instrument koji se uključuje u proces optimizacije. U narednom koraku, koristeći Markowitz metodologiju portfolija, izračunat je efikasan portfolio, odnosno optimalan portfolio, koji nosi najmanji rizik izražen standardnom devijacijom. Ograničenja koja su postavljena prilikom izračunavanja optimalnog portfolija su sledeća:

- Udeo svakog instrumenta je određen između 0% do 100%. To znači da je prilikom simulacija udeo svakog instrumenta mogao da se kreće u ovom intervalu;
- Cilj optimizacije bio je dvostruk: da se iznađe portfolio sa minimalnim rizikom, minimalnom standardnom devijacijom i da se odredi agresivniji portfolio koji nosi veći stepen povraćaja;
- Prodaje na kratko nisu dozvoljene;
- Bezrizična kamatna stopa je određena na 0%.

Optimizacija portfolija je izračunata nad dnevnim podacima za sva tri posmatrana perioda. U prvom koraku dat je pregled

iznad 0%, dok Bitcoin nosi prosečan dnevni povraćaj od 0,72%. Međutim, i standardna devijacija, kao mera rizika, je najveća kod Bitcoin-a, što je potvrda da je ovaj instrument najrizičniji. Zlato i indeks S&P imaju skoro isti nivo rizika, ukoliko se posmatra standardna devijacija.

Ukoliko rizik značajno varira tokom vremena, standardna devijacija daje manje pouzdane procene rizika. Stoga je za procenu rizika korisno primeniti i metod VaR. Merom VaR se procenjuje koliko investitor može da očekuje da će izgubiti investirajući sredstva u neku aktivu, pod normalnim tržišnim uslovima, unutar određenog vremenskog okvira. Regulatori, poput centralnih banaka kao i drugih kreatora politika u finansijskom sektoru, koriste ovu meru kako bi ocenili koji deo aktive treba biti pokriven od potencijalnog gubitka za dati nivo rizika. U ovoj analizi izračunati su parametarski i empirijski VaR, kao i CvaR, uz interval poverenja od 95%. Rezultati parametarskog i empirijskog VaR-a za Bitcoin za sva tri posmatrana perioda pokazuju da

investments;

- Global Bond Index (global bonds maturity 1-10y) denominated in USD, which represents bond market investments;
- Bitcoin denominated in USD.

All instruments were observed on a daily basis from 30.7.2010 until 30.6.2019. In the first step, the daily returns for each instrument were calculated. In the next step, using Markowitz's portfolio methodology, the efficient portfolio was calculated, i.e. the optimal portfolio that carried the least amount of risk expressed through the standard deviation. The limitations

1. Descriptive statistics show the mean of returns for all instruments, standard deviation, frequency of negative returns and VaR.

Daily returns show that returns on traditional instruments are very modest, earning just above 0%, while Bitcoin carries an average daily return of 0.72%. However, standard deviation, as a measure of risk, is the highest with Bitcoin, which is to say that this instrument is the riskiest one. Gold and the S&P index have almost the same level of risk if the standard deviation is observed.

If the risk varies significantly over time,

Table 1. Descriptive statistics: daily data from 19.7.2010 to 30.6.2019

	Bitcoin	S&P Index	Gold	1-10 Year Global Government Index
Mean	0.72%	0.04%	0.01%	0.00%
StDev	6.67%	0.94%	0.95%	0.29%
Freq<0	43.03%	45.62%	48.42%	48.68%
Parametric VaR	-10.25%	-1.50%	-1.55%	-0.48%
Empirical VaR	-8.36%	-1.56%	-1.49%	-0.47%
Parametric CVaR	-13.04%	-1.90%	-1.95%	-0.60%
Empirical CVaR	-14.03%	-2.32%	-2.30%	-0.68%

Source: Authors' calculations

that were set when calculating the optimal portfolio were as follows:

- The share of each instrument is set between 0% and 100% That means that, during the simulations, the share of each instrument could change within this interval;
- The aim of the optimisation was twofold: to end up with a minimal risk portfolio, with a minimal standard deviation and to set a more aggressive portfolio with a higher yield;
- Short selling is not allowed;
- The risk-free rate is 0%.

Portfolio optimization is calculated for daily data. Firstly, the descriptive statistics are computed for all analyzed instruments. Secondly, the correlations between the analyzed instruments are computed and after that, the portfolio optimization was performed.

3.1. Descriptive Statistics

Descriptive statistics of the sample data for daily time horizons are summarized in Table

the standard deviation gives less reliable risk estimates. Therefore, the VaR method is also useful for risk assessment. The VaR measures how much an investor can expect to lose by investing in an asset, under normal market conditions, within a specified time frame. Regulators, such as central banks and other policymakers in the financial sector, use this measure to gauge what portion of assets should be covered by a potential loss for a given level of risk. In this analysis, parametric and empirical VaR as well as CvaR were calculated, with a confidence interval of 95%. The results of parametric and empirical VaR for Bitcoin for all three observed periods show that this instrument carries a higher risk than other instruments. Data for VaR, parametric VaR, suggest that the investor could lose up to 10.2% of their investment in Bitcoin on a daily basis over the period under review. Bitcoin is known to be a less stable instrument and therefore parametric VaR may not show a true level of risk. In order to overcome these obstacles, the

ovaj instrument nosi veći rizik od ostalih instrumenata. Podaci za VaR, parametarski VaR, sugerišu da bi investitor mogao izgubiti do 10,2% ulaganja u Bitcoin na dnevnoj osnovi tokom posmatranog perioda. Poznato je da je Bitcoin manje stabilan instrument, te stoga parametarski VaR možda ne pokazuje istinitu sliku rizika. Kako bi se prevazišle ove prepreke, primenjuje se uslovni VaR, CvaR, koji obuhvata i modifikuje određene nedostatke parametarskog modela. VaR procenjuje najgori scenario, najveći stepen gubitka za datu verovatnoću i za dati vremenski okvir. Podatak za parametarski i empirijski VaR za dnevne povraćaje pokazuje da investitor može izgubiti do 14,03% investicije u Bitcoin u celom posmatranom vremenskom periodu investiranja.

3.2. Optimizacija portfolija

Optimizacija portfolija koju je razvio Markowitz sugeriše da dobra diversifikacija portfolija treba da uključi one instrumente koji nisu međusobno visoko korelisani. Polazeći od tog prvog uslova, izračunata je korelacija za analizirane instrumente, što pokazuje tabela broj 2.

Tabela 2. Matrica korelacije, dnevni povraćaji (19.7.2010. do 30.6.2019)

	Bitcoin	S&P Index	Zlato	1-10 g globalni indeks obveznica
Bitcoin	1			
S&P Index	0,003917	1		
Zlato	0,036688	0,049956	1	
1-10 g globalni indeks obveznica	-0,01537	0,023119	0,035422	1

Izvor: Računica autora

Rezultati pokazuju da je korelacija veoma niska između posmatranih instrumenata, skoro da i ne postoji, dok je blago negativna između Bitcoin-a i obveznica (-0,01537). Ovakav rezultat pokazuje da analizirani instrumenti mogu kreirati diversifikovan portfolio. Kako bi se ispitali uloga i doprinos Bitcoin-a u portfoliju analizirana su dva portfolija. Jedan koji ima za cilj minimalizovanje rizika investitora, odnosno kreiranje portfolija koji nosi prihvatljiv nivo rizika uz određeni nivo povraćaja i drugi, agresivniji portfolio, koji investitoru nosi veći stepen povraćaja na dnevnom nivou. Tabela broj 3 pokazuje diversifikovanu strukturu portfolija kojem je cilj minimalizovanje rizika

Tabela 3. Diversifikacija portfolija, portfolio sa minimalnim rizikom, dnevni podaci od 19.7.2010. do 30.6.2019.

Rizik i povraćaj			Struktura portfolija			
Prosečan povraćaj	St. devijacija	Šarp racio	Bitcoin	S&P	Zlato	Državne obveznice
0,00%	0,29%	0,0117	0,00%	0,00%	0,00%	100,00%
0,01%	0,28%	0,0180	0,00%	2,82%	6,90%	90,28%
0,01%	0,27%	0,0296	0,17%	7,51%	6,98%	85,34%
0,05%	0,46%	0,1084	5,33%	20,47%	5,63%	68,57%
0,08%	0,69%	0,1154	9,02%	29,75%	4,67%	56,55%
0,10%	0,86%	0,1165	11,48%	35,94%	4,03%	48,55%
0,12%	1,03%	0,1168	13,94%	42,13%	3,39%	40,54%
0,15%	1,29%	0,1167	17,65%	51,45%	2,42%	28,48%
0,17%	1,46%	0,1165	20,11%	57,64%	1,78%	20,47%
0,20%	1,72%	0,1162	23,79%	66,89%	0,82%	8,50%
0,22%	1,90%	0,1161	26,26%	73,15%	0,31%	0,28%
0,25%	2,16%	0,1156	30,68%	69,32%	0,00%	0,00%
0,27%	2,35%	0,1151	33,64%	66,36%	0,00%	0,00%
0,30%	2,63%	0,1143	38,11%	61,89%	0,00%	0,00%
0,32%	2,81%	0,1137	41,06%	58,94%	0,00%	0,00%
0,35%	3,10%	0,1129	45,51%	54,49%	0,00%	0,00%

Izvor: Računica autora

Table 2. Correlation matrix, daily returns (19.7.2010 to 30.6.2019)

	Bitcoin	S&P Index	Gold	1-10 Year Global Government Index
Bitcoin	1			
S&P Index	0.003917	1		
Gold	0.036688	0.049956	1	
1-10 Year Global Government Index	-0.01537	0.023119	0.035422	1

Source: Authors' calculations

to include financial instruments that are not highly correlated with each other. Considering this, correlations analysis is carried out and the results are outlined below (See Table 2).

The results show that the correlation is very low between all instruments and a mildly negative correlation was noted between Bitcoin and bonds (-0.01537) can be observed. These results show that all analysed instruments are good for diversification. In order to explore

implementation refers to the conditioned VaR, CvaR, which captures and modifies certain disadvantages of the parametric model. VaR estimates the worst-case scenario, the highest degree of loss for a given probability and for a given time frame. Data for parametric and empirical VaR for daily returns shows that an investor can lose up to 14.03% of an investment in Bitcoin over the entire investment time period.

3.2. Portfolio Optimization

Markowitz's portfolio theory suggests that, for good portfolio diversification, it is necessary

the role and benefit of Bitcoin in investment portfolios we created two portfolios. One which is aimed at creating portfolios that carry an acceptable level of risk with a certain level of return and a second, more aggressive portfolio, which gives the investor a higher level of return on a daily basis. Table 3 shows the diversified portfolio structure aimed at minimizing the risk, expressed through standard deviation.

Table 3 shows the different portfolio structures for a given degree of risk and return. The results show that if the portfolio does not include Bitcoin in its structure and aims to minimize the risk, such portfolio carries

Table 3. Portfolio diversification, minimum variance portfolio, daily data from 19.7.2010 to 30.6.2019.

Risk and return			Portfolio structure			
Return	St. deviation	Sharp ratio	Bitcoin	S&P	Gold	1-10 Year Global Government Index
0.00%	0.29%	0.0117	0.00%	0.00%	0.00%	100.00%
0.01%	0.28%	0.0180	0.00%	2.82%	6.90%	90.28%
0.01%	0.27%	0.0296	0.17%	7.51%	6.98%	85.34%
0.05%	0.46%	0.1084	5.33%	20.47%	5.63%	68.57%
0.08%	0.69%	0.1154	9.02%	29.75%	4.67%	56.55%
0.10%	0.86%	0.1165	11.48%	35.94%	4.03%	48.55%
0.12%	1.03%	0.1168	13.94%	42.13%	3.39%	40.54%
0.15%	1.29%	0.1167	17.65%	51.45%	2.42%	28.48%
0.17%	1.46%	0.1165	20.11%	57.64%	1.78%	20.47%
0.20%	1.72%	0.1162	23.79%	66.89%	0.82%	8.50%
0.22%	1.90%	0.1161	26.26%	73.15%	0.31%	0.28%
0.25%	2.16%	0.1156	30.68%	69.32%	0.00%	0.00%
0.27%	2.35%	0.1151	33.64%	66.36%	0.00%	0.00%
0.30%	2.63%	0.1143	38.11%	61.89%	0.00%	0.00%
0.32%	2.81%	0.1137	41.06%	58.94%	0.00%	0.00%
0.35%	3.10%	0.1129	45.51%	54.49%	0.00%	0.00%

Source: Authors' calculations

izraženog kroz standardnu devijaciju.

Tabela broj 3 prikazuje različite strukture portfolija za dati stepen rizika i povraćaja. Rezultati pokazuju da ukoliko portfolio ne uključuje u svojoj strukturi Bitcoin, a za cilj ima minimalizovanje rizika, takav portfolio nosi povraćaj od 0,01% i standardnu devijaciju od 0,28%. Ipak, ukoliko se određeni udeo Bitcoin-a uključi u strukturu portfolija, moguće je blago smanjiti ukupan rizik. Portfolio koji nosi minimalan rizik jeste onaj koji uključuje veoma mali udeo Bitcoin-a od 0,17%, S&P indeks sa udelom od 7,51%, zlato sa udelom od 6,98% i državne obveznice dospeća od 1 do 10 godina sa udelom od 85,34%. Takav portfolio investitoru nudi dnevni povraćaj od 0,01% uz rizik od 0,27%. Međutim, optimalan portfolio ne mora da bude, i najčešće nije, onaj koji nosi minimalan rizik, nego onaj koji nudi veći pokazatelj Šarp racija. Onaj portfolio koji nudi veći Šarp racio ujedno nudi i bolje performanse portfolija koji je prilagođen za rizik.

indeksa S&P, 3,39% zlata i 40,54% državnih obveznica. Ovakav portfolio investitoru nudi dnevni povraćaj od 0,12% i rizik od 1,03%. Na osnovu prikazane strukture portfolija zaključuje se da Bitcoin nudi mogućnost diversifikacije i da je poželjeno određeni udeo ovog instrumenta uključiti u portfolio, bez obzira na to da li investitor želi minimalizovati rizik ili želi da razvije portfolio koji maksimizuje Šarp racio. Uzimajući u obzir da je Bitcoin veoma volatilna i samim tim rizičan instrument, povraćaj ovog instrumenta je takođe visok. Takve karakteristike ovog instrumenta investitoru mogu da osiguraju mnogo veće nivoe povraćaja u odnosu na tradicionalne instrumente investiranja. Stoga je konstruisan portfolio koji je označen kao „agresivni portfolio“ a koji nudi investitoru visoki nivo dnevnog povraćaja, značajno veći u odnosu na povraćaje koje nosi portfolio kojem je u fokusu minimalizovanje rizika. Struktura takvog portfolija prikazana je tabelom broj 4.

Tabela 4. Diversifikacija portfolija, agresivni portfolio sa većim stepenom dnevnog povraćaja, dnevni podaci od 19.7.2010. do 30.6.2019.

Rizik i povraćaj			Struktura portfolija			
Prosečan povraćaj	St. devijacija	Šarp racio	Bitcoin	S&P	Zlato	Državne obveznice
0,60%	1,84%	0,3265	0,61%	46,94%	0,00%	52,45%
1,00%	2,64%	0,3795	2,54%	46,00%	0,00%	51,46%
2,00%	5,51%	0,3632	7,36%	43,65%	0,00%	48,99%
3,00%	8,62%	0,3479	12,17%	41,29%	0,00%	46,53%
4,00%	11,79%	0,3392	16,99%	38,94%	0,00%	44,07%
5,00%	14,32%	0,3491	20,12%	73,68%	6,21%	0,00%
6,00%	17,48%	0,3433	24,94%	71,32%	3,74%	0,00%
7,00%	20,64%	0,3392	29,72%	69,93%	0,00%	0,36%
8,00%	23,87%	0,3352	34,63%	65,37%	0,00%	0,00%
9,00%	27,11%	0,3320	39,57%	60,43%	0,00%	0,00%
10,00%	30,36%	0,3294	44,51%	55,49%	0,00%	0,00%
11,00%	33,61%	0,3272	49,44%	50,56%	0,00%	0,00%
12,00%	36,87%	0,3254	54,38%	45,62%	0,00%	0,00%
13,00%	40,14%	0,3239	59,31%	40,69%	0,00%	0,00%
14,00%	43,39%	0,3226	64,25%	35,75%	0,00%	0,00%
15,00%	46,66%	0,3214	69,19%	30,81%	0,00%	0,00%

To znači da portfolio koji nosi veći Šarp racio nudi i veću korisnost investitoru posmatrano sa aspekta povraćaja i rizika. U ovom slučaju portfolio koji nosi najveći Šarp racio jeste onaj u koji se uključuje 13,94% Bitcoin-a, 42,13%

Ukoliko investitor u strukturu portfolija uključuje veći udeo Bitcoin-a, čime bi kreirao agresivan portfolio, postoji mogućnost da osigura veći nivo dnevnog povraćaja, kako je i prikazano tabelom broj 4. Međutim, ukoliko

a return of 0.01% and a standard deviation of 0.28%. However, if a certain percentage of Bitcoin is included in the portfolio structure, it is possible to slightly reduce the overall risk. The minimum risk portfolio is one that includes a very low Bitcoin share of 0.17%, S&P index with a share of 7.51%, gold with a share of 6.98% and government bonds with a maturity of 1-10 years with a share of 85.34%. Such a portfolio offers the investor a daily return of 0.01% with a risk of 0.27%. However, the optimal portfolio does not have to be, and mostly is not, the one that carries minimal risk, but one that offers a higher Sharp ratio. The portfolio that offers a higher Sharp ratio also offers better risk-adjusted portfolio performance.

This means that a portfolio that carries a larger Sharp ratio offers better portfolio performance, considering risk and returns. In this case, the portfolio that carries the largest Sharp ratio is the one that includes 13.94% of Bitcoin, 42.13% of S&P Index, 3.39% of

and it is desirable to include a certain share of this instrument in the portfolio, regardless of whether the investor wants to minimize the risk or wants to develop a portfolio that maximizes Sharp ratio. Considering that Bitcoin is very volatile and therefore a risky instrument, the return on this instrument is also high. Such features of this instrument can provide investors much higher levels of return than traditional instruments. Therefore, a portfolio was created that was marked as an "aggressive portfolio" and which offers the investor a high level of daily return, significantly higher than the returns brought by a portfolio which focuses on minimizing risk. The structure of such a portfolio is shown in Table 4.

If an investor incorporates a higher portion of Bitcoin into the portfolio, thereby creating an aggressive portfolio (higher risk), there is a possibility to provide a higher level of daily return, as shown in Table 4. However, if the degree of diversification utility was considered,

Table 4. Portfolio diversification, aggressive portfolio with higher daily returns, daily data from 19.7.2010 until 30.6.2019

Risk and return			Portfolio structure			
Return	St. deviation	Return	St. deviation	Return	St. deviation	Return
0.60%	1.84%	0.3265	0.61%	46.94%	0.00%	52.45%
1.00%	2.64%	0.3795	2.54%	46.00%	0.00%	51.46%
2.00%	5.51%	0.3632	7.36%	43.65%	0.00%	48.99%
3.00%	8.62%	0.3479	12.17%	41.29%	0.00%	46.53%
4.00%	11.79%	0.3392	16.99%	38.94%	0.00%	44.07%
5.00%	14.32%	0.3491	20.12%	73.68%	6.21%	0.00%
6.00%	17.48%	0.3433	24.94%	71.32%	3.74%	0.00%
7.00%	20.64%	0.3392	29.72%	69.93%	0.00%	0.36%
8.00%	23.87%	0.3352	34.63%	65.37%	0.00%	0.00%
9.00%	27.11%	0.3320	39.57%	60.43%	0.00%	0.00%
10.00%	30.36%	0.3294	44.51%	55.49%	0.00%	0.00%
11.00%	33.61%	0.3272	49.44%	50.56%	0.00%	0.00%
12.00%	36.87%	0.3254	54.38%	45.62%	0.00%	0.00%
13.00%	40.14%	0.3239	59.31%	40.69%	0.00%	0.00%
14.00%	43.39%	0.3226	64.25%	35.75%	0.00%	0.00%
15.00%	46.66%	0.3214	69.19%	30.81%	0.00%	0.00%

gold and 40.54% of government bonds. The portfolio that includes this share of instruments can offer a daily return of 0.12% and a risk of 1.03%. Considering this, it is concluded that Bitcoin offers the possibility of diversification

considering the level of Sharp ratio, the investor who prefers a higher degree of risk should choose a portfolio structure that includes 20.12% of Bitcoin, 73.68% of S&P and 6.21% of gold, excluding government bonds. Such a

se posmatra stepen korisnosti diversifikacije, uzimajući u obzir visinu Šarp racija, investitor koji preferira veći stepen rizika trebalo bi da izabere strukturu portfolija koja uključuje 20,12% Bitcoin-a, 73,68% indeksa S&P te 6,21% zlata, bez uključivanja državnih obveznica. Takav portfolio bi investitoru nosio dnevni povraćaj od čak 5%, ali uz rizik od 14,32%. Diversifikacija portfolija koja uključuje veći udeo Bitcoin-a nosi mnogo veće dnevne povraćaje, međutim, upitno je koliko je prihvatljivo uključivanje visokog rizika u portfolio. To, naravno, zavisi od stepena rizika koji je investitor spreman da prihvati.

4. Testiranje diversifikacije

Nakon što je pokazana uloga Bitcoin-a u procesu diversifikacije portfolija, ispitan je značaj uključivanja ovog instrumenta u investicioni portfolio. Za testiranje Bitcoin-a u strukturi portfolija korišten je Wald test kojim je ispitan značaj parametra regresije. Testira se da li je parametar regresije, u ovom slučaju Bitcoin, značajan u modelu. Parametar je značajan ukoliko je različit od nule. Wald testom se ispituje

Tabela 5. Wald test, dnevni podaci

Wald test Hypothesis: $\alpha=0$; $\beta=1$	F-statistics	Verovatnoća	Hipoteza
Minimalizovanje rizika	0,00	0,00	Odbacuje se
Max Šap uz min. rizika	0,00	0,00	Odbacuje se
Agresivan portfolio	0,00	0,00	Odbacuje se

Izvor: Računica autora

osnovna hipoteza da je analizirani parametar ravan nuli. U prvom koraku izračunata je regresija, prema predloženoj metodologiji, a nakon toga je Wald testom ispitan značaj ovog parametra. Rezultati su prikazani tabelom broj 5 za posmatrane tri strukture portfolija:

- Portfolio koji nosi minimalan rizik izražen kroz standardnu devijaciju. Struktura portfolija: Bitcoin 0,17%, S&P 7,51%, zlato 6,98% i državne obveznice 85,34%;
- Portfolio koji nudi maksimalan Šarp racio sa ciljem nižeg rizika. Struktura portfolija: Bitcoin 13,94%, S&P 42,13%, zlato 3,39% i državne obveznice 40,54%;

- Agresivni portfolio koji nudi veći stepen dnevnog povraćaja. Struktura portfolija: Bitcoin 20,12% Bitcoin-a, S&P 73,68% i zlato 6,21%.

Rezultati Wald testa pokazuju da je tokom celog perioda posmatranja, od 19.7.2010. do 30.6.2019, Bitcoin nudio dobru osnovu za diversifikaciju i predstavljao je značajnu varijablu u strukturi portfolija. Bitcoin je kao instrument u portfoliju značajan kako u slučaju portfolija koji nosi minimalan rizik, tako i u slučaju portfolija koji maksimizuje Šarp racio, i u slučaju agresivnog portfolija.

5. Zaključak

U poslednjih nekoliko godina, Bitcoin je postao veoma atraktivan instrument kojem se pridaje sve veća pažnja. Investitori imaju različite stavove oko Bitcoin-a, u prvom redu u delu njegovog definisanja, ali i stava prema uključivanju ovog instrumenta u investicioni portfolio, s obzirom na to da se radi o špekulativnoj i visoko volatilnoj aktivni. U ovom istraživanju analizirana je uloga Bitcoin-a u procesu diversifikacije portfolija na primeru portfolija koji uključuje tradicionalnu aktivu poput obveznica, akcija i zlata. Optimizacija portfolija je određena primenom Markowitz teorije portfolija. Rezultati pokazuju da Bitcoin može biti dobar izvor diversifikacije portfolija, ukoliko se posmatra portfolio koji uključuje tradicionalne finansijske instrumente, posmatrano prema podacima na dnevnom nivou. Rezultati pokazuju i navode na zaključak da je povoljno uključiti u strukturu portfolija mali udeo Bitcoin-a. Takođe, ukoliko investitor ima veći apetit za rizik, može kreirati agresivniji portfolio u kojem je veće učešće Bitcoin-a i koji mu, zasigurno, može doneti veći dnevni povraćaj ali i značajno veći nivo rizika.

Pre nego što donesu odluku o investiranju u Bitcoin, investitori treba da procene svoj apetit za rizik i željeni povraćaj, te da saglasno tome ocene i udeo Bitcoin-a koji žele da uključe u portfolio. Struktura portfolija prema Markowitz teoriji, koja preferira oprezan stav prema riziku, pokazuje da je optimalno u portfolio koji uključuje tradicionalne instrumente uključiti do 14% Bitcoin-a. Takav udeo ovog instrumenta

portfolio could give a daily return of as much as 5%, but the risk could be 14.32%. Portfolio diversification involving higher shares of Bitcoin definitely carries much higher daily returns. However, it is questionable how acceptable it is to include high risk in the portfolio. That, of course, depends on the degree of risk the investor is willing to accept.

4. Diversification Test

In the previous section, it was shown that Bitcoin should be included in a portfolio that is aimed towards minimizing the risk and in an aggressive portfolio. In order to have a full analysis, it is necessary to test the portfolio diversification – diversification that includes Bitcoin. In order to test the importance of Bitcoin in portfolio diversification, we used a Wald test. Through the Wald test we examined the significance of the regression parameter. The regression parameter is significant if it is different from zero. The basic hypothesis in the Wald test is that the analyzed parameter is zero. In the first step, we ran the regression analysis and after that, we tested the significance of Bitcoin in the regression model. The results are shown in Table 5 for the three portfolio structures that were observed:

- A portfolio that carries minimal risk expressed through standard deviation. Portfolio Structure: Bitcoin 0.17%, S&P 7.51%, gold 6.98% and government bonds 85.34%;
- The portfolio that maximizes the Sharp ratio and minimizes the risk. Portfolio Structure: Bitcoin 13.94%, S&P 42.13%, gold 3.39% and government bonds 40.54%;
- An aggressive portfolio that carries a higher daily return. Portfolio Structure: Bitcoin 20.12% Bitcoin, S&P 73.68% and gold 6.21%.

The results show that during the whole period of observation, from 19.7.2010 until 30.6.2019, Bitcoin was a good source of diversification and it was a significant variable in the regression model, so we conclude that is useful to include Bitcoin in a model, in a portfolio. Bitcoin is significant in a both case: in

a portfolio that carries minimal risk and in the portfolio that maximizes Sharpe ratio as well as in the case of an aggressive portfolio.

5. Conclusion

Over the last few years Bitcoin has become a very attractive instrument that is receiving increasing attention. Investors have different opinions about Bitcoin and it appears to be a speculative asset with high volatility and lack of scalability. This paper explores the role of Bitcoin in portfolio diversification among traditional classes of assets such as bonds, stocks, and gold, using the portfolio optimization theory of Markowitz. The results show that Bitcoin can be a good source of diversification daily for a portfolio that consists of traditional financial instruments. However, it is advisable to include a small portion of Bitcoin in the portfolio because of its excessive volatility and lack of scalability. The results show that Bitcoin can be a good source of portfolio diversification if one looks at a portfolio that includes traditional financial instruments, as viewed on

Table 5. Wald test, Daily data

Wald test Hypothesis: $\alpha=0; \beta=1$	F-statistics	Probability	Hypothesis
Minimizing risk	0.00	0.00	Rejected
Max sharp with min. risk	0.00	0.00	Rejected
Aggressive portfolio	0.00	0.00	Rejected

Source: Authors' calculations

a daily basis. The results also indicate that it is advantageous to include a small share of Bitcoin in the portfolio structure. Also, if an investor has a higher appetite for risk, they can create a more aggressive portfolio with a higher Bitcoin share and which can certainly bring them a higher daily return but also a significantly higher level of risk. If an investor has a higher appetite for risk, they can create a more aggressive portfolio with higher Bitcoin share and which, of course, can bring them a higher daily return but also a significantly higher level of risk.

Before deciding whether to invest in Bitcoin, investors should evaluate their risk appetite, desirable returns, and evaluate the proportion

može da ponudi dobar i prihvatljiv balans između rizika i povraćaja, posmatrano na dnevnom nivou. Agresivniji portfolio sugerise uključivanje ovog instrumenta do 20% u ukupnoj strukturi portfolija. Test diversifikacije je pokazao da i veoma mali udeo Bitcoin-a u portfoliju nudi dobru osnovu za diversifikaciju.

Uprkos ovim rezultatima, važno je naglasiti

da se za Bitcoin vezuje nestabilnost i visoki stepen volatilnosti, pri čemu ovaj instrument nudi slabe osnove očuvanja vrednosti i pre služi kao sredstvo razmene za novac nego kao stvarni medijum razmene. U takvim okolnostima, uključivanje većeg udela Bitcoin-a u strukturu portfolija značajno pojačava rizik portfolija, što investitori uvek moraju da imaju u vidu.

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of Bitcoins they wish to include in the portfolio accordingly. The portfolio structure according to Markowitz theory, which prefers a cautious attitude towards risk, shows that it is optimal to include up to 14% of Bitcoin in a portfolio that includes traditional instruments. Such a share of this instrument can offer a good and acceptable risk-return ratio, observed on a daily basis. A more aggressive portfolio suggests including this instrument up to 20% in the overall portfolio structure. The diversification test showed that a very small share of Bitcoin

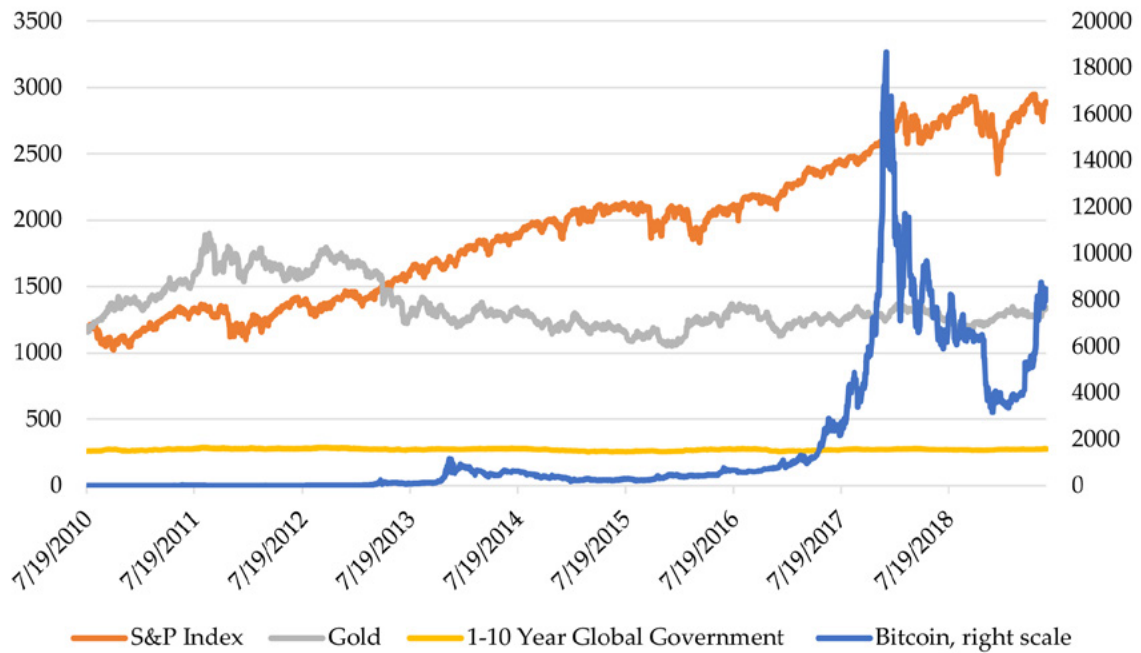
in the portfolio also offers a good basis for diversification.

Despite these results, it is important to emphasize that Bitcoin is bound by volatility and high volatility, with this instrument offering weak fundamentals of preserving value and serving more as a medium of exchange for money than an actual medium of exchange. In such circumstances, incorporating a larger share of Bitcoin into the portfolio structure significantly increases the risk of the portfolio, which investors always have to keep in mind.

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Prilog

Slika 1. Dnevni povraćaj za analizirane instrumente od 19.7.2010. do 30.6.2019.



Appendix

Figure 1. Daily values for analysed instruments from 19.7.2010 until 30.6.2019

