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FINITE-TIME STABILITY OF DISCRETE-TIME SYSTEMS WITH TIME-VARYING DELAY

Finite-time stability can be used in all applications where large values of the state are not acceptable. In this paper, finite-time stability problem for a class of linear time-varying delay systems is studied. Based on Lyapunov-like functions method and using an appropriate model transformation of the original sysem, the sufficient delay-dependent finite-time stability conditions are derived. The criteria are presented in the form of LMIs, which are dependent on the minimum and maximum delay bounds. The numerical examples are presented to illustrate the applicability of the developed results.

Keywords: finite-time stability; time-delay discrete systems; time-varying delay; LMIs; Lyapunov-like method.

Classical stability concepts (*e.g.*, Lyapunov stability, BIBO stability) deal with systems operating over an infinite interval of time. These concepts require that system variables be bounded, whereby the values of the bounds are not prescribed. Therefore, classical stability concepts are not enough for practical applications, whereas there are some cases where large values of the state are not acceptable. In this sense it appears reasonable to define as stable a system whose state, given some initial conditions, remains within prescribed bounds in the fixed-time interval. The term *practical stability* was introduced for systems operating over an infinite time interval with prescribed bounds in the text of LaSalle and Lefschetz [1], published in 1961. Somewhat earlier, papers [2,3] were published in the Russian literature, dealing with both prescribed bounds and finite-time intervals, under the title of *finite-time stability*. Therefore, the concepts practical stability and finite-time stability have in common the specification of bounds, but differ in the size of the time interval of interest.

Many valuable results have been obtained for this type of stability; see for instance [1-15] for continuous and [16-26] for discrete systems. More recently, the concept of finite-time stability has been revisited in the light of linear matrix inequality (LMI) theory, which has allowed finding less conservative conditions guaranteeing finite-time stability and stabilization [7,9,16,18,19,22-26].

In the existing literature, finite-time stability of system can be classified into following two categories: a) (ordinary) finite-time stability - FTS (given a bound on the initial condition, the system state does not exceed a certain threshold during a specified time interval) [8,9,12,16-19,21,25], and b) attractive finite-time stability - AFTS (the system state reaches the system equilibrium in a finite time) [10,13-15]. In our paper, we consider only FTS problems.

Furthermore, FTS in the presence of exogenous input leads to the concept of finite-time boundedness (FTB) [7-9,18,22-24] (a system is said to be FTB if, given a bound on the initial condition and characterization of the set of admissible inputs, the state variables remain below the prescribed limit for all inputs in the set).

Time delays often occur in many continuous industrial systems (chemical process, biological systems, population dynamics, neural networks, large-scale systems, etc.). It has been shown that the existence of delay is the source of instability and poor performance of control systems. To the best of the authors' knowledge, little work has been done for the finite-time stability and stabilization of continuous time-delay systems. Some early results on finite time stability of time-delay systems can be found in the literature [27-29]. The methods in these papers give

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conservative results because they are based on the majorization of the system response using certain inequalities. Recently, based on linear matrix inequality theory, some results have been obtained for FTS [27-31,33,34] and FTB [32,33,35-37] for particular classes of time-delay systems. The papers [31,35-37] consider the problem of FTS [31] and FTB [33-35] of the delayed neural networks. Finite-time boundedness of switched linear systems with time-varying delay and exogenous disturbances are studied [32,33]. Some papers [30,34] investigate the finite-time control problem for networked control systems with time-varying delay. A particular linear transformation is introduced [34] to convert the original time-delay system into a delay-free form. Attractive finite-time stability and stabilization of retarded-type nonlinear functional differential equations are developed [38].

Unlike continuous time-delay systems, much less attention is devoted to discrete systems with delays. This is mainly due to the fact that such systems can be transformed into augmented systems without delay. This augmentation of the system is, however, inappropriate for systems with unknown delays and for systems with time-varying delay, which are the subject of analysis in this work. According to the author's knowledge, there is no result available yet in the literature about finite-time stability and stabilization for a class of linear discrete systems with time-varying delay. The aim of our paper is the presentation of new sufficient conditions of the finite-time stability for this class of systems. In this sense, we started from the results [39] which deal with the asymptotic stability for discrete-time systems with time-varying delay. To solve the problem of FTS we used the Lyapunov-like method. The sufficient conditions are expressed in the form of LMIs which are dependent on the minimum and maximum delay bounds. Numerical examples are used to illustrate the applicability of the developed results. Two cases are considered: a) when the system is asymptotically stable and b) when the system is unstable. In both cases it was shown that the systems are finite-time stable.

PROBLEM FORMULATION AND SOME PRELIMINARIES

The following notations will be used throughout the paper. $\Re^n$ and Z^+ denote the n-dimensional Euclidean space and positive integers. Notation $P>0$ ($P\geq 0$) means that matrix P is real symmetric and positive definite (semi-definite). $\lambda_{\min}(P)\triangleq\min\lambda(P)$ and $\lambda_{\max}(P)\triangleq\max\lambda(P)$ denote minimum and maximum of eigenvalues of symmetric matrix P. For real symmetric matrices P and Q, the notation $P>Q$ ($P\geq Q$) means that matrix P-Q is positive definite (positive semi-definite). I is an identity matrix with an appropriate dimension. Superscript "T" represents the transpose. In symmetric block matrices or complex matrix expressions, we use an asterisk (*) to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

Consider a linear discrete systems with time-varying delay in the state:

$$x(k+1)=Ax(k)+A_d x(k-h(k)) \tag{1}$$

with an associated function of initial state:

$$x(\theta)=\psi(\theta),\ \theta\in\{-h_M,-h_M+1,\dots,0\} \tag{2}$$

which satisfies:

$$\begin{aligned}&\left[\psi(\theta+1)-\psi(\theta)\right]^T\left[\psi(\theta+1)-\psi(\theta)\right]<\mu,\\&\theta\in\{-h_M,-h_M+1,\dots,-1\}\end{aligned} \tag{3}$$

where $x(k)\in\Re^n$ is the state at instant k, matrices $A\in\Re^{n\times n}$ and $A_d\in\Re^{n\times n}$ are constant matrices. $h(k)$ is the positive integer representing the time-varying and satisfies the following:

$$h_m\leq h(k)\leq h_M \tag{4}$$

where h_m and h_M are constant positive integer representing the minimum and maximum delays, respectively.

The assumption on the time delay $h(k)$ in Inequality (4) characterizes the real situation in many practical applications. A typical example containing time delays that can be characterized by Inequality (4) can be found in networked control systems, where the delays induced by the network transmission (either from sensor to controller or from controller to actuator) are actually time-varying, and can be assumed to have minimum and maximum delay bounds without loss of generality.

This paper studies the finite-time stability of the class of time-varying delay systems (1). Our aim is to develop a sufficient condition such that the system (1) is finite-time stable for any $h(k)$ satisfying $h_m\leq h(k)\leq h_M$. Before moving on, the following definition of finite-time stability for the time-delay system (1) is introduced.

Definition 1. Discrete time-delay system (1) with an associated function of initial state (2) is said to be finite-time stability (FTS) with respect to (α,β,N), where $0\leq\alpha<\beta$ if:

$$\begin{aligned}&\sup_{k\in[-h_M,-h_M+1,\dots,0]}\psi^T(k)\psi(k)<\alpha\Rightarrow x^T(k)x(k)<\beta,\\&\forall k\in[1,2,\dots,N]\end{aligned} \tag{5}$$

Remark 1. Lyapunov asymptotic stability (LAS) and FTS are independent concepts: a system which is FTS may be not asymptotically stable; conversely an LAS system could be not FTS if, during the transients, its state exceeds the prescribed bounds.

MAIN RESULTS

In this section, we aim to establish a finite-time stability criterion for the system given by Eq. (1) using the Lyapunov method combined with the LMI technique. We start from the results presented in [39], which gives some asymptotic delay-dependent stability criteria, and derive a sufficient condition for FTS.

Theorem 1. System (1) with time-varying delay is FTS with respect to (α,β,T), $\alpha<\beta$, if there exist positive definite symmetric matrices P, Q and Z, positive scalars θ_1, θ_2, θ_3 and θ_4 and scalar $\gamma \ge 1$, such that the following LMIs hold:

$$\Gamma = \begin{bmatrix} \Gamma_{11} & (A+A_d)^T PA_d & -A^T PA_d & h_M(A-I)^T Z \\ * & -Q & -A_d^T PA_d & h_M A_d^T Z \\ * & * & -Z & 0 \\ * & * & * & -Z \end{bmatrix} < 0$$

$$\Gamma_{11} = -\gamma\left(P+P^T\right)+(h_M-h_m+1)Q+A^TP(A+A_d)+ +(A+A_d)^T PA \tag{6}$$

$$\begin{aligned} &\theta_1 I < P < \theta_2 I \\ &0 < Q < \theta_3 I \\ &0 < Z < \theta_4 I \end{aligned} \tag{7}$$

$$\begin{bmatrix} -\gamma^{-N}\beta\theta_1 & \sqrt{\alpha}\theta_2 & \sqrt{\delta}\theta_3 & \sqrt{\varepsilon}\theta_4 \\ * & -\theta_2 & 0 & 0 \\ * & * & -\theta_3 & 0 \\ * & * & * & -\theta_4 \end{bmatrix} < 0$$

$$\begin{aligned} &\delta = \alpha\left[h_M+(h_M-1)(h_M-2)/2-(h_m-1)(h_m-2)/2\right] \\ &\varepsilon = \mu h_M(h_M-1)/2 \end{aligned} \tag{8}$$

Proof. Let:

$$\begin{aligned} \eta(m) &\triangleq x(m+1)-x(m) \\ &= Ax(m)+A_d x\left(m-h(m)\right)-x(m) \\ &= (A-I)x(m)+A_d x\left(m-h(m)\right) \end{aligned} \tag{9}$$

and:

$$\begin{aligned} x\left(k-h(k)\right) &= x(k)-\left(x(k)-x\left(k-h(k)\right)\right) \\ &= x(k)-\sum_{m=k-h(k)}^{k-1}\left(x(m+1)-x(m)\right) \\ &= x(k)-\sum_{m=k-h(k)}^{k-1}\eta(m) \end{aligned} \tag{10}$$

Then, the system given by Eq. (1) can be transformed into:

$$\begin{aligned} x(k+1) &= Ax(k)+A_d x(k-h(k)) \\ &= Ax(k)+A_d\left(x(k)-\sum_{m=k-h(k)}^{k-1}\eta(m)\right) \end{aligned}$$

i.e.,

$$x(k+1) = (A+A_d)x(k)-A_d\sum_{m=k-h(k)}^{k-1}\eta(m) \tag{11}$$

Choose a discrete-time counterpart Lyapunov-Krasovskii functional candidate as [39]:

$$\begin{aligned} V(k) &= V_1(k)+V_2(k)+V_3(k)+V_4(k) \\ V_1(k) &= x^T(k)Px(k) \\ V_2(k) &= \sum_{i=k-h(k)}^{k-1} x^T(i)Qx(i) \\ V_3(k) &= \sum_{j=-h_M+2}^{-h_m+1}\sum_{i=k+j-1}^{k-1} x^T(i)Qx(i) \\ V_4(k) &= \sum_{i=-h_M}^{-1}\sum_{j=k+i}^{k-1}\eta^T(j)Z\eta(j) \end{aligned} \tag{12}$$

where P, Q and Z are positive definite matrices to be determined.

Taking the forward difference $\Delta V = V(k+1) - V(k)$ along the solutions of system given by Eq. (1) and transformed system given by Eq. (11), we can obtain [39]:

$$\Delta V(k) \le \xi^T(k)\Omega\xi(k) \tag{13}$$

where:

$$\xi(k) = \left[x^T(k)x^T\left(k-h(k)\right)\sum_{m=k-h(k)}^{k-1}\eta^T(m)\right]^T$$

$$\Omega = \begin{bmatrix} \Omega_{11} & \frac{1}{2}(A+A_d)^T PA_d + h_M(A-I)^T ZA_d & -\frac{1}{2}A^T PA_d \\ * & -Q+h_M A_d^T ZA_d & -\frac{1}{2}A_d^T PA_d \\ * & * & -\frac{1}{h_M}Z \end{bmatrix} \tag{14}$$

$$\Omega_{11} = A^TP(A+A_d)-P+(h_M-h_m+1)Q+h_M(A-I)^T Z(A-I)$$

Let:

$$\hat{\Gamma} \triangleq \Omega - \begin{bmatrix} (\gamma-1)P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \hat{\Gamma}_{11} & \frac{1}{2}(A+A_d)^T PA_d + h_M (A-I)^T ZA_d & -\frac{1}{2}A^T PA_d \\ * & -Q + h_M A_d{}^T ZA_d & -\frac{1}{2}A_d{}^T PA_d \\ * & * & -\frac{1}{h_M}Z \end{bmatrix} \tag{15}$$

where:

$$\begin{aligned} \hat{\Gamma}_{11} &= A^T P(A+A_d) - P + (h_M - h_m + 1)Q + \\ &+ h_M (A-I)^T Z(A-I) - (\gamma-1)P = \\ &= A^T P(A+A_d) - \gamma P + (h_M - h_m + 1)Q + \\ &+ h_M (A-I)^T Z(A-I) \\ &\gamma \geq 1 \end{aligned}$$

If:

$$\hat{\Gamma} < 0 \tag{16}$$

then:

$$\begin{aligned} \Delta V(x(k)) &= \xi(k)^T \Omega \xi(k) = \\ &= \xi(k)^T \left\{ \hat{\Gamma} + \begin{bmatrix} (\gamma-1)P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \xi(k) = \\ &= \xi(k)^T \hat{\Gamma} \xi(k) + \xi(k)^T \begin{bmatrix} (\gamma-1)P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xi(k) < \\ &< \xi(k)^T \begin{bmatrix} (\gamma-1)P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xi(k) = (\gamma-1)x(k)^T Px(k) < \\ &< (\gamma-1)V(x(k)) \end{aligned} \tag{17}$$

i.e.,

$$V(x(k)) < \gamma V(x(k-1)) \tag{18}$$

Applying iterative procedure on Relation (18), we can have:

$$\begin{aligned} V(x(k)) &< \gamma V(x(k-1)) = \\ &= \gamma^2 V(x(k-2)) = \cdots = \gamma^k V(x(0)) \end{aligned} \tag{19}$$

Further:

$$\begin{aligned} V(x(0)) &= x^T(0)Px(0) + \sum_{i=-h(k)}^{-1} x^T(i)Qx(i) + \\ &+ \sum_{j=-h_M+2}^{-h_m+1} \sum_{i=j-1}^{-1} x^T(i)Qx(i) + \sum_{i=-h_M}^{-1} \sum_{j=i}^{-1} \eta^T(j)Z\eta(j) < \\ &< \lambda_{\max}(P)x^T(0)x(0) + \lambda_{\max}(Q) \sum_{i=-h_M}^{-1} x^T(i)x(i) + \\ &+ \lambda_{\max}(Q) \sum_{j=-h_M+2}^{-h_m+1} \sum_{i=j-1}^{-1} x^T(i)x(i) + \\ &+ \lambda_{\max}(Z) \sum_{i=-h_M}^{-1} \sum_{j=i}^{-1} \eta^T(j)\eta(j) \end{aligned}$$

$$\begin{aligned} V(x(0)) &< \lambda_{\max}(P)\alpha + \lambda_{\max}(Q) \sum_{i=-h_M}^{-1} \alpha + \\ &+ \lambda_{\max}(Q) \sum_{j=-h_M+2}^{-h_m+1} \sum_{i=j-1}^{-1} \alpha + \lambda_{\max}(Z) \sum_{i=-h_M}^{-1} \sum_{j=i}^{-1} \mu = \\ &= \lambda_{\max}(P)\alpha + \lambda_{\max}(Q)h_M\alpha + \lambda_{\max}(Q)\alpha \sum_{j=h_m}^{h_M-1} j + \\ &+ \lambda_{\max}(Z)\mu \sum_{j=1}^{h_M} j = \lambda_{\max}(P)\alpha + \lambda_{\max}(Q)h_M\alpha + \\ &+ \lambda_{\max}(Q)\alpha \left(\sum_{j=1}^{h_M-1} j - \sum_{j=1}^{h_m-1} j \right) + \lambda_{\max}(Z)\mu \sum_{j=1}^{h_M} j \end{aligned}$$

$$\begin{aligned} V(x(0)) &< \lambda_{\max}(P)\alpha + \lambda_{\max}(Q)h_M\alpha + \\ &+ \lambda_{\max}(Q)\alpha((h_M-1)(h_M-2) - \\ &-(h_m-1)(h_m-2))/2 + \\ &+ \lambda_{\max}(Z)\mu h_M(h_M-1)/2 \end{aligned} \tag{20}$$

$$V(x(k)) > x^T(k)Px(k) \geq \lambda_{\min}(P)x^T(k)Px(k) \tag{21}$$

Based on relations (19)-(21), if the following condition is satisfied:

$$\begin{aligned} &\lambda_{\max}(P)\alpha + \lambda_{\max}(Q)\alpha(h_M + (h_M-1)(h_M-2)/2 - \\ &-(h_m-1)(h_m-2)/2) + \lambda_{\max}(Z)\mu h_M(h_M-1)/2 < \\ &< \gamma^{-N}\lambda_{\min}(P)\beta \end{aligned} \tag{22}$$

we have:

$$x^T(k)x(k) < \beta, \forall k \in [1,2,\ldots,N] \tag{23}$$

i.e., system (1) is finite-time stable.

Let:

$$0<\theta_1<\lambda_{\min}(P),\theta_2>\lambda_{\max}(P),\theta_3>\lambda_{\max}(Q),$$
$$\theta_4>\lambda_{\max}(Z),\delta=\alpha[h_M+(h_M-1)(h_M-2)/2-$$
$$-(h_m-1)(h_m-2)/2],\varepsilon=\mu h_M(h_M-1)/2$$

Then from Relation (22) follows:

$$\begin{aligned}&\theta_1 I<P<\theta_2 I,0<Q<\theta_3 I,0<Z<\theta_4 I\\&-\gamma^{-N}\beta\theta_1+\alpha\theta_2+\delta\theta_3+\varepsilon\theta_4<0\end{aligned} \quad (24)$$

It is easy to check that conditions (24) are guaranteed by imposing the conditions (7)-(8).

From (16), introducing a substitution $P\leftrightarrow 2P$, we have:

$$\bar{\Gamma}=\begin{bmatrix}\bar{\Gamma}_{11} & (A+A_d)^T PA_d+h_M(A-I)^T ZA_d & -A^T PA_d\\ * & -Q+h_M A_d^T ZA_d & -A_d^T PA_d\\ * & * & -\dfrac{1}{h_M}Z\end{bmatrix}<0 \quad (25)$$

$$\bar{\Gamma}_{11}=-\gamma\left(P+P^T\right)+A^T P(A+A_d)+(A+A_d)^T PA+(h_M-h_m+1)Q+h_M(A-I)^T Z(A-I)$$

Further:

$$\begin{aligned}&\begin{bmatrix}\bar{\Gamma}_{11} & (A+A_d)^T PA_d+h_M(A-I)^T ZA_d & -A^T PA_d\\ * & -Q+h_M A_d^T ZA_d & -A_d^T PA_d\\ * & * & -\dfrac{1}{h_M}Z\end{bmatrix}=\\
&=\begin{bmatrix}\Gamma_{11} & (A+A_d)^T PA_d & -A^T PA_d\\ * & -Q & -A_d^T PA_d\\ * & * & -\dfrac{1}{h_M}Z\end{bmatrix}+\begin{bmatrix}h_M(A-I)^T Z(A-I) & h_M(A-I)^T ZA_d & 0\\ * & h_M A_d^T ZA_d & 0\\ * & * & 0\end{bmatrix}=\\
&=\begin{bmatrix}\Gamma_{11} & (A+A_d)^T PA_d & -A^T PA_d\\ * & -Q & -A_d^T PA_d\\ * & * & -\dfrac{1}{h_M}Z\end{bmatrix}-\begin{bmatrix}(A-I)^T Z\\ A_d^T Z\\ 0\end{bmatrix}\left(-\frac{1}{h_M}Z\right)^{-1}\begin{bmatrix}Z(A-I) & ZA_d & 0\end{bmatrix}<0\end{aligned} \quad (26)$$

where:

$$\Gamma_{11}=\bar{\Gamma}_{11}-h_M(A-I)^T Z(A-I)=-\gamma\left(P+P^T\right)+A^T P(A+A_d)+(A+A_d)^T PA+(h_M-h_m+1)Q \quad (27)$$

Using Schur complements, it is easy to see that the condition given by Inequality (26) is equivalent to:

$$\begin{bmatrix}\Gamma_{11} & (A+A_d)^T PA_d & -A^T PA_d & (A-I)^T Z\\ * & -Q & -A_d^T PA_d & A_d^T Z\\ * & * & -\dfrac{1}{h_M}Z & 0\\ * & * & * & -\dfrac{1}{h_M}Z\end{bmatrix}<0 \quad (28)$$

Introducing a substitution $h_M Z\leftrightarrow Z$, we obtain the LMI given by Inequality (6).

Remark 2. The conditions given by Inequalities (6)-(8) are LMI conditions, therefore can be easily checked by using standard numerical software. These conditions depend on both the maximum and minimum delay bounds.

Remark 3. From Theorem 1, for the constant delay case, $h_M=h_m=h$, we have the next result.

Corollary 1. System given by Eq. (1) with $h(k)=h$ is FTS with respect to (α,β,T), $\alpha<\beta$, if there exist positive define symmetric matrices **P**, **Q** and **Z**, positive scalars θ_1, θ_2, θ_3 and θ_4 and scalar $\gamma\geq 1$, such that the following LMIs hold:

$$\Gamma = \begin{bmatrix} \Gamma_{11} & (A+A_d)^T PA_d & -A^T PA_d & h(A-I)^T Z \\ * & -Q & -A_d{}^T PA_d & hA_d{}^T Z \\ * & * & -Z & 0 \\ * & * & * & -Z \end{bmatrix} < 0$$

$$\Gamma_{11} = -\gamma\left(P+P^T\right) + Q + A^T P(A+A_d) + \tag{29}$$
$$+(A+A_d)^T PA$$

$$\begin{aligned} &\theta_1 I < P < \theta_2 I \\ &0 < Q < \theta_3 I \\ &0 < Z < \theta_4 I \end{aligned} \tag{30}$$

$$\begin{bmatrix} -\gamma^{-N}\beta\theta_1 & \sqrt{\alpha}\theta_2 & \sqrt{\delta}\theta_3 & \sqrt{\varepsilon}\theta_4 \\ * & -\theta_2 & 0 & 0 \\ * & * & -\theta_3 & 0 \\ * & * & * & -\theta_4 \end{bmatrix} < 0 \tag{31}$$
$$\delta = \alpha h, \varepsilon = \mu h(h-1)/2$$

Proof. For $h_M = h_m = h$, the conditions (29)-(31) follow directly from the conditions (6)-(8).

RESULTS AND DISCUSSION

Example 1. Consider the following asymptotic stable discrete-time systems with time-varying delay in the state [39]:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.60 & 0.00 \\ 0.35 & 0.70 \end{bmatrix} x(k) + \\ &+ \begin{bmatrix} 0.10 & 0.00 \\ 0.20 & 0.10 \end{bmatrix} x(k-h(k)) \end{aligned} \tag{32}$$

i) Let us compute the upper limit of time-varying delay h_M using Theorem 1, so the system (32) is yet finite-time stable with respect $(\alpha,\beta,N) = (2.1,50,50)$ and with particular choice of $h_m = 2$ and:

$$\psi_{\theta\in\{-6,-5,\ldots,0\}} = \left[\psi(-6) \ \cdots \ \psi(0)\right] =$$
$$= \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

It is obvious:

$$\psi^T(\theta)\psi(\theta) \le 2 < \alpha = 2.1, \theta \in \{-6,-5,\ldots,0\}$$

$$\left[\psi(\theta+1)-\psi(\theta)\right]^T \left[\psi(\theta+1)-\psi(\theta)\right] \le 1 < \mu = 1.1,$$
$$\theta \in \{-6,-5,\ldots,-1\}$$

Solving LMIs (6)-(8) for fixed $\gamma = 1.0011$, we obtain the following feasible solution:

$$\sup(h_M) = 6, \ \mathbf{P} = \begin{bmatrix} 678.86 & 4.49 \\ 4.49 & 75.84 \end{bmatrix},$$

$$\mathbf{Q} = \begin{bmatrix} 36.83 & 4.91 \\ 4.91 & 2.94 \end{bmatrix}, \ \mathbf{Z} = \begin{bmatrix} 40.82 & 8.71 \\ 8.71 & 3.48 \end{bmatrix}$$

$$\theta_1 = 74.91, \theta_2 = 693.63, \theta_3 = 38.75, \theta_4 = 45.30$$

It is obvious that the following inequalities are valid:

$$\begin{aligned} &0 < \lambda_i(P) \in \{75.81, 678.90\} \\ &0 < \lambda_i(Q) \in \{2.24, 37.53\} \\ &0 < \lambda_i(Z) \in \{1.55, 42.75\} \end{aligned}$$

$$\begin{aligned} &\theta_1 = 74.91 < 75.81 = \lambda_{\min}(P) \\ &\theta_2 = 693.63 > 678.90 = \lambda_{\max}(P) \\ &\theta_3 = 38.75 > 37.53 = \lambda_{\max}(Q) \\ &\theta_4 = 45.30 > 42.75 = \lambda_{\max}(Z) \end{aligned}$$

Therefore, the system (32) with time varying delay $2 \le h(k) \le 6$ is finite-time stable with respect to (2.1,50,50).

ii) Let us repeat the finite-time stability test using now Corollary 1 and compute the upper limit of constant delay h so the system (32) is yet finite-time stable with respect (2.1,50,50) and:

$$\psi_{\theta\in\{-11,-10,\ldots,0\}} = \left[\psi(-11) \ \cdots \ \psi(0)\right] =$$
$$= \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

Solving LMIs (29)-(31) for fixed $\gamma = 1.0003$ and μ = 1.1, we obtain the following feasible solution:

$$\sup(h) = 11, \ \mathbf{P} = \begin{bmatrix} 8169.80 & -70.62 \\ -70.62 & 842.62 \end{bmatrix},$$

$$\mathbf{Q} = \begin{bmatrix} 559.15 & 67.41 \\ 67.41 & 53.70 \end{bmatrix}, \ \mathbf{Z} = \begin{bmatrix} 169.93 & 34.75 \\ 34.75 & 16.46 \end{bmatrix}$$

$$\theta_1 = 840.05, \theta_2 = 8211.54, \theta_3 = 571.96, \theta_4 = 178.98$$

Therefore, the system (32) with time constant delay $h \le 11$ is finite-time stable with respect to (2.1,50,50).

Comparing these two results we can make the following conclusions:

a) Theorem 1 takes into account the variability of time-delay, while Corollary 1 does not. Thus, when we examine the finite-time stability of the systems with time-varying delay, we must use Theorem 1.

b) Due to the constancy of time-delay, applying Corollary 1 we get a higher value for the upper limit of the time-delay:

$$\sup(h) = 11 > 6 = \sup(h_M)$$

Thus, when we check the finite-time stability of the systems with constant delay, it is better to apply Corollary 1.

Example 2. Consider the following discrete time-delay systems:

$$x(k+1)=\begin{bmatrix}0.60 & 0.00\\ 0.35 & 0.70\end{bmatrix}x(k)+ +\begin{bmatrix}0.20 & 0.25\\ 0.25 & 0.15\end{bmatrix}x(k-h(k)) \tag{33}$$

a) Let $h(k)$ = 2. Obviously, the system (33) is not asymptotic stable. Let us compute the upper limit of β so the system (33) is yet finite-time stable with respect to $(\alpha,\beta,N)=(2.1,\beta,10)$ and:

$$\psi_{\theta\in\{-2,-1,0\}}=\left[\psi(-2)\ \psi(-1)\ \psi(0)\right]=\begin{bmatrix}-1 & -1 & -1\\ 0 & 1 & 1\end{bmatrix}$$

It is obvious:

$$\psi^T(\theta)\psi(\theta)\le 2<\alpha=2.1,\theta\in\{-2,-1,0\}$$

$$\left[\psi(\theta+1)-\psi(\theta)\right]^T\left[\psi(\theta+1)-\psi(\theta)\right]\le 1<\mu=1.1,$$
$$\theta\in\{-2,-1,0\}$$

Using Corollary 1 for fixed $\gamma=1.7258$, we obtain the following feasible solution:

β = 2425.04

$$\mathrm{P}=\begin{bmatrix}80539.40 & -8705.52\\ -8705.52 & 44064.95\end{bmatrix},$$
$$\mathrm{Q}=\begin{bmatrix}27308.33 & 24437.48\\ 24437.48 & 23245.63\end{bmatrix},$$
$$\mathrm{Z}=\begin{bmatrix}26533.28 & 23870.36\\ 23870.36 & 22094.39\end{bmatrix}$$

$$\theta_1=42093.69,\theta_2=82510.68,\theta_3=\ 49798.76,$$
$$\theta_4=48287.20$$

Hence, the system (33) is FTS stable with respect to $(\alpha,\beta,N)=(2.1,2425.04,10)$, but not asymptotically stable (see Remark 1).

b) Let $h_m=1\le h(k)\le 2=h_M$ and $h(0)$ = 1. Solving LMIs (6)-(8) for fixed $\gamma=2.227$, N = 7 and:

$$\psi_{\theta\in\{-2,\,-1,\,0\}}=\left[\psi(-2)\ \psi(-1)\ \psi(0)\right]=\begin{bmatrix}-1 & -1 & -1\\ 0 & 1 & 1\end{bmatrix},$$

we obtain the following feasible solution:

β = 2082.9,

$$\mathrm{P}=\begin{bmatrix}1307.72 & -98.19\\ -98.19 & 902.12\end{bmatrix},\ \mathrm{Q}=\begin{bmatrix}434.04 & 377.52\\ 377.52 & 361.30\end{bmatrix},$$

$$\mathrm{Z}=\begin{bmatrix}347.69 & 308.53\\ 308.53 & 280.75\end{bmatrix},$$

$$\theta_1=879.59,\theta_2=1330.27,\theta_3=\ 776.94,\theta_4=624.61$$

Therefore, the system (33) with time varying delay $1\le h(k)\le 2$ is finite-time stable with respect to $(\alpha,\beta,N)=(2.1,2082.9,7)$.

CONCLUSION

In this paper, finite-time stability problem has been investigated for a class of linear discrete time-varying delay systems. The sufficient conditions, which can guarantee finite-time stability of these class systems, are proposed. The criteria are presented in the form of LMIs, which are dependent on the minimum and maximum delay bounds.

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NAUČNI RAD

STABILNOST NA KONAČNOM VREMENSKOM INTERVALU DISKRETNIH SISTEMA SA PROMENLJIVIM VREMENSKIM KAŠNJENJEM

U slučajevima kada velike vrednosti stanja sistema nisu prihvatljive, može se analizirati stabilnost sistema na konačnom vremenskom intervalu. U radu je razmatran problem stabilnosti na konačnom vremenskom intervalu za klasu diskretnih sistema sa promenljivim vremenskim kašnjenjem. Na osnovu metode koja je slična Ljapunovoj metodi stabilnosti izvedeni su dovoljni uslovi stabilnosti na konačnom vremenskom intervalu koristeći odgovarajuće transformacije modela originalnog sistema. Kriterijumi stabilnosti, koji zavise od donje i gornje granice vremenskog kašnjenja, iskazani su u obliku linearnih matričnih nejednakosti. Dati su numerički primeri kojima se ilustruje primenljivost izvedenih rezultata.

Ključne reči: stabilnost na konačnom vremenskom intervalu, diskretni sistemi sa vremenskim kašnjenjem, vremenski promenljivo kašnjenje, linearne matrične nejednakosti, metoda slična Ljapunovoj metodi stabilnosti.