INVESTMENT PROJECTS EVALUATION IN A FUZZY ENVIRONMENT USING THE SIMPLIFIED WISP METHOD

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Abstract

This paper examines the importance of investment activity for companies and the challenges they face when evaluating investment projects in a fuzzy environment, that is when decisions have to be made based on some predictions and uncertain or imprecise data. The study focuses on the usage of a new extension of the Simplified WISP (Weighted Sum Product) method, which allows the use of triangular fuzzy numbers, as a tool for evaluating investment projects and minimizing the risk associated with such decisions. Investment projects were evaluated based on the following criteria: Net Present Value, Internal Rate of Return, Profitability Index, Payback Period, and Risk of project failure. The proposed extension of the Simplified WISP method can be used to solve other complex decision problems associated with predictions and uncertainties. The paper highlights the benefits of using this MCDM technique in investment project evaluation and the potential to improve decision-making processes. The study also discusses the challenges associated with applying MCDM techniques in a fuzzy environment and proposes solutions to overcome them. It also provides valuable insights for academics, practitioners, and policymakers interested in investment evaluation and decision-making processes.

Keywords: MCDM, fuzzy, triangular fuzzy numbers, Simplified WISP, investment projects

1. INTRODUCTION

Multiple criteria decision-making (MCDM) enables the ranking of alternatives, or the selection of the most appropriate one, based on a set of often conflicting criteria. MCDM, i.e., MCDM methods, have so far been used for solving numerous different problems from various fields. As a result of intensive use for solving different problems, a number of MCDM methods such as: Technique for Order of Preference by
Similarity to Ideal Solution (TOPSIS) (Hwang & Yoon, 1981), Analytic Hierarchy Process (AHP) method (Saaty, 1978), and Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) (Brans, 1982) have been proposed.

In addition to the mentioned methods, it can also be observed tendency of developing and using newly proposed MCDM methods such as Weighted Aggregated Sum Product ASsessment (WASPAS) (Zavadskas et al., 2012), Evaluation Based on Distance from Average Solution (EDAS) (Keshavarz Ghorabaee et al., 2015), Additive Ratio Compromise Assessment (ARCAS) (Stanujkic et al., 2017), Combined Compromise Solution (CoCoSo) (Yazdani et al., 2018), Simple Weighted Sum Product (WISP) (Stanujkic et al., 2021), and so on. An essential characteristic of these methods is that their aggregation procedures combine the Weighted Sum (WS) and Weighted Product (WP) approaches, in different ways, to determine the overall utility of each alternative and propose the most acceptable one.

Many MCDM methods have been primarily intended for use with crisp numbers. However, many real-world decision problems involve the vagueness and inaccuracy of the data used to solve decision-making problems, and often predictions, which caused significant limitations for the use of ordinary MCDM methods. In order to enable solving decision-making problems related to inaccuracies, unreliability, and predictions many MCDM methods are extended to enable using fuzzy (Zadeh, 1965), intuitionistic fuzzy (Atanassov, 1986), interval-valued fuzzy (Turksen, 1986), single-valued neutrosophic (Smarandache, 1998; Smarandache, 1999) numbers, and so on. As important characteristic of these numbers can be mentioned in the possibility of making decisions based on imprecise data, as well as considering different scenarios such as pessimistic, realistic, and optimistic, i.e. scenarios that lie between strongly pessimistic to extremely optimistic.

MCDM methods have been used so far for the evaluation of investment projects, even though the number of published articles regarding the application of MCDM methods for evaluating investment projects is not so great. In addition, the following articles can be cited as some of the more significant research in this area: Dimova et al. (2006) applied the AHP method and fuzzy sets for the evaluation of investment projects, while Popovic et al. (2012) applied the COPRAS method and grey set theory in order to overcome the problems related to uncertainty and prediction during the evaluation of investment projects. Kilic and Kaya (2015) combined AHP and TOPSIS methods in a fuzzy environment, while Rudnik et al. (2021) combined AHP and WASPAS, also in a fuzzy environment, for evaluating investment projects.

On the basis of the Simple WISP method Stanujkic (2022) and Stanujkic et al. (2022) proposed a Simplified variant of the WISP method, Simplified WISP method. Stanujkic et al. (2023) also prove that the Simplified WISP method gives ranking orders similar to ranking orders obtained using prominent MCDM methods. Although Simple WISP and Simplified WISP are recently proposed, these methods are already used to solve some decision-making problems, such as: Selecting the optimal naval ship drainage system (Kirmizi et al., 2023), pallet truck selection (Ulutaş et al. 2022), conctractor selection (Zavadskas et al. 2022) electric vehicles evaluation (Ivanov and Stanujkic,
2022), selecting a tourist destination (Stanujkic at al. 2022). Until now, no appropriate extensions have been proposed for the Simplified WISP method that enables the application of the Simplified WISP method for solving complex decision-making problems, i.e. extensions that enable the use of fuzzy, intuitionistic, or single-valued neutrosophic numbers. Therefore, in this article, an extension of the Simplified WISP method, which enables the application of triangular fuzzy numbers, was proposed and considered, and its applicability is considered in the case of investment projects evaluation under conditions of uncertainty and predictions. By using the proposed extension, the Simplified WISP method can be used for solving complex decision-making problems, that is, decision-making problems related to predictions and the use of imprecise data.

Therefore, the article is organized as follows: In Section 2, some basic elements of the fuzzy set theory, as well as some topics relevant to the proposed approach, are discussed. In Section 3, a fuzzy extension of the Simple WISP method is proposed, while in section 4 criteria for investment projects evaluation are considered. The usability of the proposed approach is presented in Section 5 Finally, the conclusions are given at the end of the article.

2. PRELIMINARIES

Some significant elements of fuzzy set theory, essential for the development of the fuzzy extension of the Simplified WISP method, are presented in this section.

2.1. Basic concepts and definitions of the fuzzy sets

Definition 1. Let $X$ be a non-empty set. A fuzzy subset $\tilde{A}$ of $X$ is defined by its membership function $\mu_{\tilde{A}}(x)$ as follows:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\},$$

where $x \in X$ denotes the belonging of $x$ to the non-empty set $X$, and $\mu_{\tilde{A}}(x): X \rightarrow [0,1]$.

Definition 2. A fuzzy number $\tilde{A}$ is a triangular fuzzy number (TFN) if its membership function is as follows (Cheng, 2004):

$$\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x - l}{m - l} & l \leq x < m \\
1 & x = m \\
\frac{u - x}{u - m} & m < x \leq u \\
0 & \text{otherwise}
\end{cases}$$

where $l$, $m$, and $u$ denote the left endpoint, mode, and right endpoint, respectively. TFNs can also be expressed by their triplets $(l, m, u)$, as shown in Figure 1.

![Figure 1. Triangular fuzzy number with different spreads](image-url)
Definition 3. Let $\tilde{A} = (a_l, a_m, a_u)$ and $\tilde{B} = (b_l, b_m, b_u)$ be two positive TFNs. The arithmetic operations of these TFNs are as follows (Chen & Hwang, 1992):

\[
\tilde{A} \oplus \tilde{B} = (a_l + b_l, a_m + b_m, a_u + b_u),
\]

\[
\tilde{A} \ominus \tilde{B} = (a_l - b_u, a_m - b_m, a_u - b_l),
\]

\[
\tilde{A} \otimes \tilde{B} = (a_l \cdot b_l, a_m \cdot b_m, a_u \cdot b_u),
\]

\[
\tilde{A} \odot \tilde{B} = (a_l/b_u, a_m/b_m, a_u/b_l).
\]

Definition 4. Let $\tilde{A} = (a_l, a_m, a_u)$ be a positive TFN, and $k$ be a positive crisp number. The multiplication of fuzzy numbers and a positive crisp number is as follows (Chen & Hwang, 1992):

\[
\tilde{A} \cdot k = (a_l \cdot k, a_m \cdot k, a_u \cdot k).
\]

2.2. Defuzzification of triangular fuzzy numbers

Ranking fuzzy numbers are not simple, that is why fuzzy numbers are often converted into corresponding crisp numbers before ranking, and this procedure is called defuzzification. So far, a number of procedures have been proposed for the defuzzification of TFNs of which two procedures are shown below.

Opricovic and Tzeng (2003) proposed a very simple defuzzification procedure as follows:

\[
df(\tilde{A}) = \frac{1}{3}(l + m + u)
\]

In the above procedure, all three points that form a fuzzy number are equally important. The defuzzification procedure proposed by Liou and Wang (1992) provides more significant possibilities for analysis that could be realized by applying different values of the coefficient $\lambda$, and it can be expressed as follows:

\[
df(\tilde{A}) = \frac{1}{2}[(1 - \lambda)l + m + \lambda u]
\]

where $\lambda$ denotes the index of optimism, and $\lambda \in [0,1]$.

3. FUZZY EXTENSION OF THE SIMPLIFIED WISP METHOD

The procedure of the Simplified WISP method adapted for using triangular fuzzy numbers can be precisely explained as follows:

Step 1. Construct a fuzzy initial decision-making matrix and determine criteria weights.

Step 2. Construct a normalized fuzzy decision-making matrix as follows:

\[
\tilde{r}_{ij} = \tilde{x}_{ij} \frac{1}{\max_i \tilde{x}_{ij}}
\]

where $\tilde{x}_{ij}$ denote fuzzy rating $\tilde{r}_{ij}$ and denotes a normalized fuzzy rating of alternative $i$ in regards to criterion $j$, respectively.

Step 3. Calculate the values of two fuzzy utility measures $\tilde{u}_i^{sd}$ and $\tilde{u}_i^{pr}$, as follows:

\[
\tilde{u}_i^{sd} = \Sigma_{j \in \Omega_{max}} \tilde{r}_{ij} w_j - \Sigma_{j \in \Omega_{min}} \tilde{r}_{ij} w_j,
\]

\[
\tilde{u}_i^{pr} = \frac{\Pi_{j \in \Omega_{max}} \tilde{r}_{ij} w_j}{\Pi_{j \in \Omega_{min}} \tilde{r}_{ij} w_j}.
\]
where $\Omega_{\text{max}}$ and $\Omega_{\text{min}}$ denote a set of beneficial and a set of non-beneficial criteria, respectively.

Step 4. Recalculate values of two utility measures, as follows:

$$
\tilde{\nu}_i^{sd} = \frac{1 + u_i}{1 + \max_i u_i^{sd}},
$$

$$
\tilde{\nu}_i^{pr} = \frac{1 + u_i}{1 + \max_i u_i^{pr}},
$$

where $\tilde{\nu}_i^{sd}$ and $\tilde{\nu}_i^{pr}$ denote recalculated values of $u_i$, and $u_i$, respectively, and $\max_i u_i^{sd}$ and $\max_i u_i^{pr}$ denote the maximum values of the right endpoints of two fuzzy utility measures, respectively.

Step 5. Determine the overall fuzzy utility $\tilde{\nu}_i$ of each alternative as follows:

$$
\tilde{\nu}_i = \frac{1}{4} (\tilde{\nu}_i^{sd} + \tilde{\nu}_i^{pr}).
$$

Step 6. Determine the crisp overall utility $\nu_i$ of each alternative. Compared to the ordinary Simple WISP method, the fuzzy extension of this method has one step more, in which fuzzy numbers are transformed into crisp numbers, which can be done by applying Eq. (9) or Eq. (10)

Step 7. Rank the alternatives and select the most suitable one. The alternatives are ranked in descending order, and the alternative with the highest value of $\nu_i$ is the most preferred one.

4. CRITERIA FOR THE EVALUATION OF INVESTMENTS PROJECTS

As some of the commonly used criteria for evaluating investment projects, the following can be mentioned: Net Present Value ($NPV$), Internal Rate of Return ($IRR$), Profitability index ($PI$), Payback Period ($PBP$), and Risk of project failure ($R$). These criteria have already been used for the evaluation of investment projects (Dai et al. 2022, Kose et al. 2014, Hublin et al. 2014, Popovic et al. 2012, Bhandari, 1989), mostly with crisp numbers. In this case, the specified criteria are expressed using fuzzy numbers due to the uncertainty regarding the duration of the projects.

The $NPV$ represents the difference between the initial cash investment and the present value of the future net cash flows. In cases where the annual net cash flow is uniform, $NPV$ can be calculated as follows:

$$
NPV = \sum_{t=1}^{T} \frac{CF_t}{(1 + r)^t} - CF_0
$$

where $CF_0$ denotes the cost of investment, $CF_t$ denotes the cash flow from the investment in period $t$, $r$ denotes the required rate of return and $T$ denotes the duration of the project.

The $IRR$ is a discount rate that makes the present value of the future net cash flows equal to the initial cash investment, and it can be calculated as follows:

$$
\sum_{t=1}^{T} \frac{CF_t}{(1 + IRR)^t} - CF_0
$$

The $PI$ is the ratio between the discounted value of future net cash flows and initial cash investment, and it can be calculated as follows:

$$
PI = \sum_{t=1}^{T} \frac{CF_t}{(1 + r)^t}.
$$
The PBP represents the number of years required for an investment project to pay itself off, from the annual revenues that it generates. When the projected annual net cash flow is uniform, PBP can be calculated as follows:

\[ PBP = \frac{CF_0}{F} \]  

(19)

where \( F \) denotes the projected average annual cash flow from the investment.

5. NUMERICAL ILLUSTRATION

In this section, an example of the investment projects evaluation under uncertainty and prediction is presented. Due to simpler calculations, and a simpler presentation of the use of the proposed extension, it was assumed that the cash flow is constant during the duration of the projects. However, on the other hand, due to the impossibility of accurately predicting the average annual profit, the average annual profit is presented using triangular fuzzy numbers, where \( m \) represents the expected average annual profit and \( l \) and \( u \) represent pessimistic and optimistic expectations. In the considered example, pessimistic expectations are expressed as -7% and optimistic as +5% of the expected average annual profit.

The basic characteristics of investment projects, that is cost of investment \((CF_o)\), average annual profit \((CF_i)\), Project duration \((T)\), and Risk of project failure \((R)\) are shown in Table 1.

The investment projects were evacuated based on the criteria discussed in section four, that is:

- \( C_1 \), Net Present Value,
- \( C_2 \), Internal Rate of Return,
- \( C_3 \), Profitability index,
- \( C_4 \), Payback Period, and
- \( C_5 \), Risk of project failure.

The values of the evaluation criteria, determined based on the data from Table 1, are shown in Table 2. The same table also shows the weights of the criteria, as well as the optimization directions.

Table 1. The basic characteristics of investment projects (Authors’ calculations)

<table>
<thead>
<tr>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CF_0 )</td>
<td>300</td>
<td>350</td>
<td>400</td>
<td>450</td>
</tr>
<tr>
<td>( CF_i )</td>
<td>(697.50, 750, 787.50)</td>
<td>(701.22, 754, 791.70)</td>
<td>(925.35, 995, 1044.75)</td>
<td>(637.05, 685, 719.25)</td>
</tr>
<tr>
<td>( T )</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>( R )</td>
<td>4</td>
<td>3.7</td>
<td>3.9</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Table 2. The initial decision matrix for investment projects evaluation (Authors’ calculations)

<table>
<thead>
<tr>
<th>( w_j ) optimization</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
<th>( C_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>0.26</td>
<td>0.15</td>
<td>0.15</td>
<td>0.26</td>
<td>0.18</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(19.81, 247.11, 409.46)</td>
<td>(0.05, 0.08, 0.10)</td>
<td>(1.01, 1.08, 1.14)</td>
<td>4.00</td>
<td>4.0</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(59.18, 327.07, 518.43)</td>
<td>(0.06, 0.08, 0.09)</td>
<td>(1.02, 1.09, 1.15)</td>
<td>4.64</td>
<td>3.7</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>(6.28, 307.83, 523.22)</td>
<td>(0.05, 0.08, 0.10)</td>
<td>(1.00, 1.08, 1.13)</td>
<td>4.02</td>
<td>3.9</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>(28.04, 368.86, 612.30)</td>
<td>(0.05, 0.07, 0.08)</td>
<td>(1.01, 1.08, 1.14)</td>
<td>6.57</td>
<td>4.4</td>
</tr>
<tr>
<td>( A_6 )</td>
<td>(19.01, 396.78, 666.62)</td>
<td>(0.05, 0.07, 0.08)</td>
<td>(1.00, 1.08, 1.13)</td>
<td>5.99</td>
<td>3.8</td>
</tr>
</tbody>
</table>

1The values of \( CF_o \) and \( CF_i \) are given in thousands of euros
The normalized fuzzy decision matrix, constructed by applying Eq. (10), is shown in Table 3.

The values of the two utility measures, $\tilde{u}_i^{sd}$ and $\tilde{u}_i^{pr}$, calculated using Eqs. (11) to (12), and the recalculated values of these utility measures, $\tilde{v}_i^{sd}$ and $\tilde{v}_i^{pr}$, calculated using Eqs. (13) to (14), are shown in Table 4. The overall fuzzy utility of alternatives, calculated using Eq. (15), is also shown in Table 4.

The crisp overall utility of alternatives, obtained using Eq. (8), and the ranking order of the alternatives based on them are shown in Table 5.

As can be concluded from Table 5, the most acceptable alternative, or investment project, is the alternative denoted as $A_3$, where the second-placed alternative $A_2$ has only slightly lower overall utility compared to the first-placed alternative.

Compared to Eq. (8), Eq. (9) provides significantly greater possibilities for considering different scenarios. By varying the coefficient $\lambda$, in the interval $[0, 1]$, different scenarios can be examined, from strongly pessimistic to seriously optimistic. The results of one such analysis are summarized in Table 6 and in Figure 2.

From Table 6, that is, from Figure 2, it can be noticed that variations of the value of the $\lambda$ coefficient affected the ranking order of alternatives, that is, the selection of the most acceptable alternative. In the case of a pessimistic evaluation, alternative $A_2$ is the most acceptable, while in the case of a realistic and optimistic evaluation, alternative $A_3$ is identified as the most acceptable.

<table>
<thead>
<tr>
<th>Table 3. The normalized decision matrix (Authors' calculations)</th>
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<tbody>
<tr>
<td>$\tilde{u}_i^{sd}$</td>
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<tr>
<td>---------------------</td>
</tr>
<tr>
<td>$A_1$</td>
</tr>
<tr>
<td>$A_2$</td>
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<tr>
<td>$A_3$</td>
</tr>
<tr>
<td>$A_4$</td>
</tr>
<tr>
<td>$A_5$</td>
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</table>

<table>
<thead>
<tr>
<th>Table 4. The values of utility measures (Authors' calculations)</th>
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<tr>
<td>$u_i^{sd}$</td>
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<tr>
<td>---------------------</td>
</tr>
<tr>
<td>$A_1$</td>
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<tr>
<td>$A_2$</td>
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<tr>
<td>$A_3$</td>
</tr>
<tr>
<td>$A_4$</td>
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<tr>
<td>$A_5$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5. The ranking order of alternatives (Authors' calculations)</th>
</tr>
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<tbody>
<tr>
<td>$v_i$</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>$A_1$</td>
</tr>
<tr>
<td>$A_2$</td>
</tr>
<tr>
<td>$A_3$</td>
</tr>
<tr>
<td>$A_4$</td>
</tr>
<tr>
<td>$A_5$</td>
</tr>
</tbody>
</table>
6. CONCLUSION

As investment projects often involve multiple criteria and various sources of uncertainties, so usage of MCDM method is crucial for ensuring accurate and reliable decision-making. However, it is important to note that different investment projects may require different MCDM methods depending on the characteristics of the project and the decision-making environment. Therefore, selecting an appropriate MCDM method that suits the specific needs of each investment project is crucial for achieving optimal outcomes.

The proposed extension of the Simplified WISP method that allows the use of triangular fuzzy numbers is a valuable addition to the existing MCDM techniques. This extension offers a practical solution to handle complex decision-making problems associated with uncertainties and predictions that are often encountered in investment projects. Moreover, the results obtained from the application of the proposed extension demonstrate the effectiveness of the Simplified WISP method in solving complex investment project evaluation problems.

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Table 6. Analysis of different scenarios (Authors' calculations)

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1)</td>
<td>0.850</td>
<td>0.870</td>
<td>0.889</td>
<td>0.909</td>
<td>0.929</td>
</tr>
<tr>
<td>(A_2)</td>
<td>0.860</td>
<td>0.881</td>
<td>0.903</td>
<td>0.924</td>
<td>0.946</td>
</tr>
<tr>
<td>(A_3)</td>
<td>0.856</td>
<td>0.881</td>
<td>0.905</td>
<td>0.930</td>
<td>0.954</td>
</tr>
<tr>
<td>(A_4)</td>
<td>0.802</td>
<td>0.822</td>
<td>0.842</td>
<td>0.862</td>
<td>0.882</td>
</tr>
<tr>
<td>(A_5)</td>
<td>0.828</td>
<td>0.851</td>
<td>0.875</td>
<td>0.899</td>
<td>0.922</td>
</tr>
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</table>

Figure 2. Analysis of different scenarios
**References**


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**ЕВАЛУАЦИЈА ИНВЕСТИЦИЈСКИХ ПРОЈЕКАТА У ФАЗИ ОКРУЖЕЊУ ПОМОЋУ ПОЈЕДНОСТАВЉЕНЕ WISP МЕТОДЕ**

Dragisa Stanujkic, Aleksandra Fedajev, Marcos Santos

**ИЗВОД**

Овај рад истражује важност инвестиционих активности за компаније и изазове са којима се сукобају приликом евалуације инвестиционих пројеката у нејасном окружењу. Рад се такође фокусира на употребу новог проширења поједностављене WISP методе, које омогућава употребу троугластих фази бројева, и њену примену као алата за процену успешности инвестиционих пројеката и минимизирања ризика повезаних са таквим одлукама. У раду се такође указује на предности коришћења ове MCDM технике у евалуацији инвестиционих пројеката и њеног потенцијала за побољшање процеса доношења одлука. Рад такође разматра изазове повезане с применом MCDM у фази окружењу и предлазе решења за њихово превазилажење, а такође пружа значајне уvide за академику, практичаре и креаторе политика заинтересованих за евалуацију инвестиција и процесе доношења одлука.

**Кључне речи: MCDM, фази, троугласти фази бројеви, поједностављени WISP, инвестициони пројекти**


Economics and Management, 18 (4), 599-618.


