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Homoclinic Orbits in 3D Dissipative Systems

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The paper deals with a variational system corresponding to a three-dimensional dynamic system. The characteristic equation of the variational system depends on partial solutions. The matrix of the right-hand part of the variational system is a sum of two matrices. One matrix contains the spectrum of the linear system and the other one represents partial solutions. Two theorems, determining the sign of the sum of the characteristic exponents of a point on a homoclinic trajectory, are proved.

Key words: **dynamic system, dissipative system, variational systems, homoclinic orbit.**

Statement of the problem

THE problem of periodic orbit generations from homoclinic loops in three-dimensional systems was discussed in [1–12].Consider the system

$$dx/dt = F(x,\mu),\ \ x(t)\in R^n,\ \ \mu\in R^m, \tag{1}$$

where $n=3$, $F(x,\mu)$ is a smooth function and R^m is a parameter space. We introduce into consideration a small deviation in the neighborhood of partial solutions $\overline{x}_i (i=1,2,\dots,n)$, $\delta x_i = x_i(t)-\overline{x}_i(t)$ $(i=1,2,\dots,n)$ of equation (1). Consider δx_i to be new coordinates. The linear system corresponding to system (1) in the coordinates δx_i

$$d\delta x/dt = A(\overline{x})\delta x,\ \ \delta x\in R^n, \tag{2}$$

where $A(\overline{x}) = \partial F/\partial x|_{x=\overline{x}}$, is called a system of variational equations [13]. By means of the analysis of the roots of the characteristic equation of the matrix $A(\overline{x})$, one can study the mechanism of the formation of periodic and complex motions. We present the matrix $A(\overline{x})$ of system (2) in the form of a sum of two matrices

$$A(\overline{x}) = N + M(\overline{x}), \tag{3}$$

where the matrix N corresponds to the spectrum of linear system (2) which does not contain its partial solutions. The matrix $M(\overline{x})$ corresponds to a part of the spectrum of equations (2) which contains partial solutions $\overline{x}_1,\overline{x}_2,\overline{x}_3$.

We cite some definitions and a theorem from [2].

We designate, by $\gamma,\lambda_1,\lambda_2$, the characteristic exponents of the saddle equilibrium state of the point O at the origin of system (1) so that

$$\gamma > 0 > Re\lambda_{1,2}.$$

The unstable manifold W^u of the saddle O is one-dimensional and the stable manifold W^s is two-dimensional. The unstable manifold consists of three orbits: the saddle O itself, and two separatrices Γ_1 and Γ_2. We assume that the system has a separatrix loop, i.e. Γ_1 tends to O as $t\to\infty$.. The parameter μ controls the loop splitting. The parameter sign is opposite to the sign of the saddle value, which equals to

$$\sigma = \gamma + Re\lambda_1 + Re\lambda_2.$$

Theorem 13.6 (Shilnikov [2]). If the saddle value σ is negative, a single stable periodic orbit L is generated from the homoclinic loop for $\mu > 0$. The separatrix Γ_1 tends to L as $t\to+\infty$. For $\mu\le 0$ there are no periodic orbits in a small neighborhood U of the homoclinic loop. The trajectories tend either to L (or to the loop Γ for $\mu=0$) or to O, or leave U as $t\to+\infty$.

This paper also deals with a homoclinic loop. By means of one approach, the signs of the saddle values are determined and the attraction of all points on the loop is established.

On the existence of a homoclinic trajectory

We assume system (1) as follows.

Proposition 2.1. System (1) possesses three singular points. The singular point $O(0,0,0)$ (a saddle knot with a negative saddle value) has the characteristic exponents $\lambda_1,\lambda_2,\lambda_3$, where $\lambda_1>0$, $\lambda_2<0$, and $\lambda_3<0$. There exists a neighborhood of the point O that, according to the Grobman-Hartman theorem [1], is filled with saddle-knot points which go into the knot-focus ones so that $Re\lambda_1<0$, $Re\lambda_2<0$, $\lambda_3<0$.

We make the following assumption on the matrix $M(\overline{x})$

Proposition 2.2. The characteristic equation of the matrix $M(\overline{x})$ possesses the following eigenvalues: one zero and two imaginary ones. The sum of the eigenvalues equals to zero: $\overline{\lambda}_1(\overline{x})+\overline{\lambda}_2(\overline{x})+\overline{\lambda}_3(\overline{x}) = 0$.

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Theorem 2.3. For differential system (1) let the conditions of Proposition 2.1 and 2.2 be satisfied. Then, in the neighbourhood of the singular point $O(0,0,0)$ of system (1) there exists a field of saddle-knot points which go into the knot-focus ones. The saddle value $\sigma = \lambda_1 + \lambda_2 + \lambda_3$ obtained in correspondence with certain parameter values will be one and the same for all the points of three-dimensional space including all singular points.

Proof. Under the conditions of Proposition 2.2 the saddle value of the trajectory points is determined by the matrix $A(\bar{x})$ according to the equation

$$|A(\bar{x}) - \lambda E| = |N - \lambda E| = 0,$$

where E is an identity matrix. The bifurcation process in the field of three-dimensional system (1) occurs in accordance with the Grobman-Hartman theorem [1]. The neighborhood, filled with the saddle-knot points so that $\sigma = \lambda_1 + \lambda_2 + \lambda_3 < 0$, goes into the knot-focus continuum. For the knot-focus domain $Re\lambda_1 < 0,\ Re\lambda_2 < 0,\ \lambda_3 < 0$. The knot-focus loop has a value $\sigma = Re\lambda_1 + Re\lambda_2 + \lambda_3 < 0$ at all points of the trajectory. This value is specified by the roots of a linear system, then $\sigma = \sigma_O = \sigma_A = \sigma_B$, where σ_A and σ_B are the values of any points *A* and *B*, including the singular ones.

Corollary 2.4. For differential system (1) let the conditions of Proposition 2.1 and 2.2 be satisfied and in system (1) a loop be formed. If this loop embraces all singular points, then a limiting cycle is generated from the loop.

Proof. Theorem 2.3 shows that the loop has a negative value σ at all points. From the homoclinic loop with a negative value σ, such that $Re\lambda_1 < 0,\ Re\lambda_2 < 0, \lambda_3 < 0$, a stable periodic orbit is generated.

Corollary 2.5. For differential system (1) let the conditions of Proposition 2.1 and 2.2 be satisfied and in system (1) two loops be formed. Then limiting cycles are generated from the loops, provided the loop orbits do not intersect.

On system (1) we shall assume as follows.

Proposition 2.6. The singular point $O(0,0,0)$ of system (1) is a saddle-focus with a saddle value equal to zero $(\sigma_O = 2Re\lambda_{1,2} + \lambda_3 = 0)$.

Proposition 2.7. The saddle value of the points of system (1) specified by the characteristic equation $|M(\bar{x}) - \lambda E| = 0$, is negative for $\bar{x} \neq 0$ and all the points are of attractive character.

Proposition 2.8. For $\bar{x} \neq 0$ differential system (1) forms a loop.

Theorem 2.9. For differential system (1) let the conditions of Proposition 2.6, 2.7 and 2.8 be satisfied. Then, in the neighbourhood of the singular point $O(0,0,0)$ of system (1) a closed integral curve exists.

Proof. Under the conditions of Proposition 2.6 and 2.7 the matrix $A(\bar{x})$ has eigenvalues satisfying the equation $|A(\bar{x}) - \lambda E| = |M(\bar{x}) - \lambda E| = 0$. If the conditions of Proposition 2.8. are satisfied, a loop is formed with a negative value σ and the attraction at every point. Such a loop generates a stable periodic orbit.

We return to Theorem 13.6 [2]. In Theorem 13.6, a parameter μ is mentioned, governing the loop splitting. The theorem was discussed in [4]. In this paper, we consider a principle of determining the sign of the saddle value of a homoclinic loop and the parameter values. This is because of the attractive character of the trajectory points. The applications are presented.

Applications

Application of Theorem 2.3. Example 1. Consider the Lorentz system

$$\begin{cases} dx/dt = s(-x+y), \\ dy/dt = rx - y - xz, \\ dz/dt = -bz + xy. \end{cases} \tag{4}$$

where b, r and s are positive parameters $(r > 1)$. In system (4) we introduce the small deviations $\delta x,\ \delta y,$ and δz from the partial solutions $\bar{x}$, $\bar{y}$ and $\bar{z}$ and compile the variational equation

$$\begin{cases} \delta\dot{x} = -s\delta x + s\delta y, \\ \delta\dot{y} = (r-\bar{z})\delta x - \delta y - \bar{x}\delta z, \\ \delta\dot{z} = -b\delta z + \bar{y}\delta x + \bar{x}\delta y. \end{cases} \tag{5}$$

System (4) possesses the following singular points

$$O(0,0,0),$$
$$A(\sqrt{b(r-1)},\ \sqrt{b(r-1)},\ r-1),$$
$$B(-\sqrt{b(r-1)},\ -\sqrt{b(r-1)},\ r-1).$$

By the characteristic equation of system (5)

$$\begin{aligned} &\lambda^3 + \lambda^2(b+s+1) + \lambda(s(1-r+\bar{z}) + b(s+1) + \bar{x}^2) + \\ &+s(b(1-r+\bar{z}) + \bar{x}(\bar{x}+\bar{y})) = 0 \end{aligned} \tag{6}$$

The characteristic exponents of the points in the field of three-dimensional space of system (4) can be determined. At point O, equation (6) becomes

$$(\lambda+b)(\lambda^2 + \lambda(1+s) + s(1-r)) = 0.$$

From it, we find

$$\begin{aligned} &\lambda_{1,2} = -(s+1)/2 \pm \sqrt{((s+1)/2)^2 + s(r-1)}, \\ &\lambda_3 = -b(7) \end{aligned} \tag{7}$$

The singular point *O* is a saddle-knot with the saddle value $\sigma = -(s+1) - b$. We write the matrix equality in terms of variational equation (5). Correlation (3) is presented as

$$\begin{pmatrix} -s & s & 0 \\ r-\bar{z} & -1 & -\bar{x} \\ \bar{y} & \bar{x} & -b \end{pmatrix} = \begin{pmatrix} -s & s & 0 \\ r & -1 & 0 \\ 0 & 0 & -b \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ -\bar{z} & 0 & -\bar{x} \\ \bar{y} & \bar{x} & 0 \end{pmatrix}. \tag{8}$$

There is a matrix of the variational system (5) in the left-hand part of equality (8). The first matrix in the right-hand part of equality (8) represents a spectrum of the linear system corresponding to system (4). The second matrix $M(\bar{x})$ corresponds to the part of equations (5) which contains the partial solutions $\bar{x}, \bar{y}$ and $\bar{z}$ and has the roots of the characteristic equation

$$\bar{\lambda}_{1,2} = \pm i\sqrt{\bar{x}^2}, \quad \bar{\lambda}_3 = 0. \tag{9}$$

Note that the partial solutions $\bar{x}, \bar{y},$ and $\bar{z}$ of the system of equations (5) are unknown. In system (4), there are several attractors. In Fig.1, a phase portrait of the limiting cycle of system (4) is shown for the parameter values $(b,r,s)=(8/3;153;10)$. By means of equation (6) and the numerical solution of system (4), one can specify the domains of the points of the saddle-knot character in the zero neighborhood. In Fig.1, the thick lines mark the totalities of the trajectory points of the saddle-knot character, and the thin lines indicate the knot-focus points for which $Re\lambda_1<0$, $Re\lambda_2<0$, $\lambda_3<0$. Fig.2 shows two limiting cycles (the parameter values $(b,r,s)=(8/3;100;10)$. The spectrum of the characteristic Lyapunov exponents for the limiting cycle satisfies the inequality $\Lambda_1+\Lambda_2+\Lambda_3<0$.

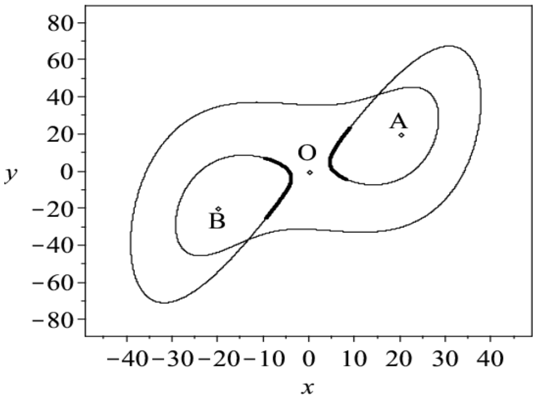

Figure 1. Limiting cycle of the Lorentz system

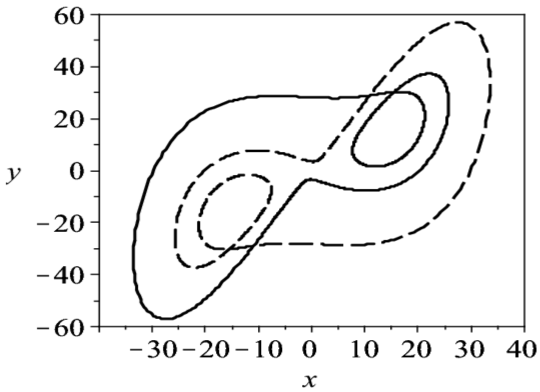

Figure 2. Two limiting cycles

The attractor alternative is a strange attractor emerging under the orbital loss of motion stability by the image point with respect to one of the singular points at transition (jump) to the motion with respect to the other singular point (points A and B).

Application of Theorem 2.9. Example 1. Consider a system of three differential nonlinear equations (a generator with quadratic nonlinearity [4])

$$\begin{cases} \dfrac{dx}{dt}=mx-xz+y, \\ \dfrac{dy}{dt}=-x, \\ \dfrac{dz}{dt}=-b(z-x^2), \end{cases} \tag{10}$$

where m and b are positive parameters. The system has one singular point $O(0,0,0)$. Introduce small deviations $\delta x,\delta y,\delta z$ from the partial solutions $\bar{x},\bar{y}$ and $\bar{z}$ of system (10) and compile the variational equation

$$\begin{cases} \dfrac{d\delta x}{dt}=(m-\bar{z})\delta x+\delta y-\bar{x}\delta z, \\ \dfrac{d\delta y}{dt}=-\delta x, \\ \dfrac{d\delta z}{dt}=-b(\delta z-2\bar{x}\delta x). \end{cases}$$

We write the characteristic equation of the system

$$\lambda^3+\lambda^2(b-m+\bar{z})+\lambda(b(-m+\bar{z}+2\bar{x}^2)+1)+b=0.$$

At point $O(0,0,0)$ the characteristic equation becomes

$$(\lambda+b)(\lambda^2-\lambda m+1)=0.$$

The characteristic exponents of point $O(0,0,0)$ are

$$\lambda_{1,2}=m/2\pm\sqrt{(m/2)^2-1},\ \lambda_3=-b.$$

We prescribe the following values of the parameters

$$(m,b)=(1,1). \tag{11}$$

The point O is of the saddle-focus type with the saddle value $\sigma=0$. The matrix

$$M(\bar{x},\bar{z})=\begin{pmatrix} \bar{z} & 0 & \bar{x} \\ 0 & 0 & 0 \\ -2b\bar{x} & 0 & 0 \end{pmatrix}$$

corresponds to the characteristic equation

$$\lambda(\lambda^2+\lambda\bar{z}+2b\bar{x}^2)=0,$$

whose roots are

$$\lambda_{1,2}=-\bar{z}/2\pm\sqrt{(\bar{z}/2)^2-2b\bar{x}^2},\ \lambda_3=0.$$

The saddle value is $\sigma=-\bar{z}$. Let us show that the matrix $M(\bar{x},\bar{z})$ corresponds to the dissipative oscillator. Consider a linear system

$$\begin{cases} \dfrac{dX}{dt}=-\bar{z}X-\bar{x}Z, \\ \dfrac{dZ}{dt}=-2b\bar{x}X, \end{cases}$$

which is identical to the dissipative oscillator

$$\frac{d^2Z}{dt^2}+\frac{dZ}{dt}+2b\bar{x}^2Z=0.$$

Then the saddle value $\sigma=-\bar{z}$ has a negative sign. According to Theorem 2.9, for the parameters chosen (11), there exists a periodic orbit in system (10). Figurse 3 and 4 present a closed curve of system (10) in the projection on the coordinate planes.

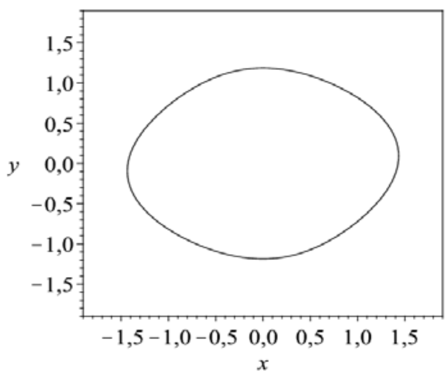

Figure 3. Projection of the limiting cycle of system (10) on the plane xy

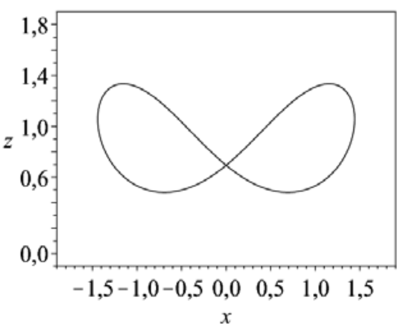

Figure 4. Projection of the limiting cycle of system (10) on the plane xz

Example 3. Consider a system of three nonlinear differential Chua equations

$$\begin{cases} dx/dt = \alpha(ax - bx^3 + y), \\ dy/dt = x - y + z, \\ dz/dt = -\beta y, \end{cases} \quad (12)$$

where a, b, α and β are positive parameters. Though the Chua chain dynamics is widely described in literature [2, 14, 15] we are intending to apply Theorem 2.9 to establish the existence of a homoclinic trajectory. System (12) possesses three equilibrium states: singular point $O(0,0,0)$ and singular points $A(x_A = \sqrt{a/b},\ y_A = 0,\ z_A = -\sqrt{a/b})$, and $B(x_B = -\sqrt{a/b},\ y_B = 0,\ z_B = \sqrt{a/b})$. We introduce small deviations $\delta x, \delta y$ and δz from the partial solutions $\bar{x}, \bar{y}$ and $\bar{z}$ of system (12) and compile the variational equations

$$\begin{cases} d\delta x/dt = \alpha(a\delta x - 3b\bar{x}^2\delta x + \delta y), \\ d\delta y/dt = \delta x - \delta y + \delta z, \\ d\delta z/dt = -\beta\delta y. \end{cases}$$

The characteristic equation corresponding to the variational system reads

$$\lambda^3 + \lambda^2(1 + \alpha(-a + 3b\bar{x}^2)) + \lambda(\beta - \alpha(1 + a - 3b\bar{x}^2)) + \\ +\alpha\beta(-a + 3b\bar{x}^2) = 0. \quad (13)$$

The bifurcation process in the system is associated with the variation of the coordinate x, since the characteristic equation depends only on the partial solution $\bar{x}$. The characteristic exponents of the point O are determined in terms of the equation $\lambda^3 + \lambda^2(1 - \alpha a) + \lambda(\beta - \alpha(1 + a)) - \alpha\beta a = 0$. We prescribe the parameter values

$$(a, \alpha) = (1/6; 6); \quad b = a; \beta \in (7, ..., 10,1). \quad (14)$$

The requirement to parameters (14) is that the saddle value of point $O(0,0,0)$ equals to zero. In order that the initial perturbations generate a closed curve with respect to the point O in system (12), the initial conditions should be chosen in accordance with the following estimates

$$|x(0)| > |x_A|, \quad |y(0)| \geq 0, \quad |z(0)| > |z_A|. \quad (15)$$

Such a choice excludes the effect of the singular points A and B, which, similarly to the point O, form some motion of system (12). The choice of the initial conditions can be refined numerically (in the framework of inequalities (15)). The characteristic equation $\lambda^2(\lambda + 3\alpha b\bar{x}^2) = 0$ of the matrix $M(\bar{x})$ possesses the eigenvalues $\lambda_1 = \lambda_2 = 0,\ \lambda_3 = -3\alpha b\bar{x}^2$. Zero roots are simple. Under the conditions of parameters (14) and the choice of the initial conditions (15), there exists an attractive periodic orbit in system (12) (according to Theorem 2.9).

Note that the saddle value $\sigma = 2Re\lambda_{1,2} + \lambda_3$, calculated in terms of equation (13) depends only on the coordinate x. In Fig.5, the dependence of $\sigma(x)$ is shown. At point O, the saddle value equals to zero. Fig.6 presents a trajectory closed with respect to the point O under the initial perturbations $x(0) = -1,7$; $y(0) = 0$; $z(0) = -1,7$.

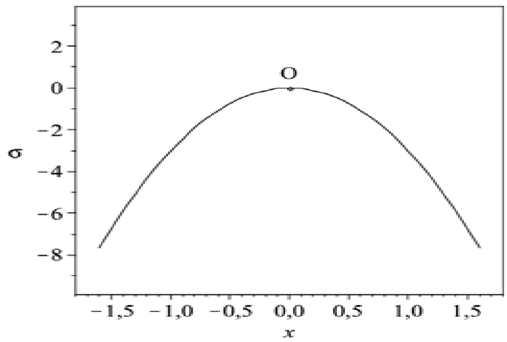

Figure 5. The graf $\sigma(x)$ of the saddle value of the Chua system

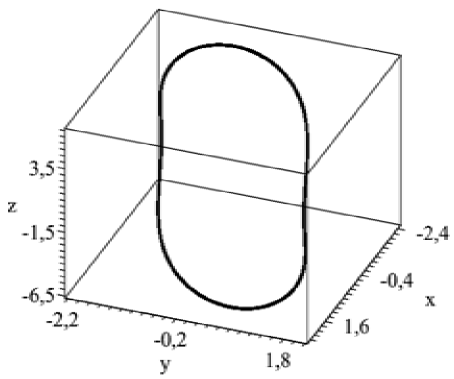

Figure 6. Limiting cycle of the Chua system

Consider the behavior of the solutions in the Chua system, which are due to the singular points A and B. The points A and B can form closed curves excluding the point O. The points A and B are skew-symmetric. Therefore, consider motion with respect to one point only. The parameter β can influence the location of the trajectories relatively to the points A and B. The main requirement is that the trajectories do not intersect. We relate with the point A a frame of reference $Avyw$ and compile the motion equations in new coordinates

$$\begin{cases} dv/dt = \alpha(-2av - bv^2(3\sqrt{a/b} + v) + y), \\ dy/dt = v - y + w, \\ dw/dt = -\beta y, \end{cases}$$

where $v = x - \sqrt{a/b},\ w = z + \sqrt{a/b}$. In the frame of reference $Avyw$, the singular point O has the coordinates: $v_0 = -\sqrt{a/b}, \quad y_0 = 0, \quad w_0 = \sqrt{a/b}$. Under the initial perturbations

$$|v(0)| < \sqrt{a/b}, \quad |y(0)| \geq 0, \quad |w_O(0)| < \sqrt{a/b} \quad (16)$$

the motion under the effect of the singular point A is dominating in the system. The estimates of the initial conditions can also be refined numerically (in the framework of inequalities (16)). At the motion of the image point in the neighborhood of the point A (or B), all points of the trajectory are attractive and with a negative saddle value.

Fig.7 presents two limiting cycles in the frame of the reference $Oxyz$ with the initial conditions $x(0)=0,2;\ y(0)=0;\ z(0)=-0,2$ (a cycle with respect to the point A); $x(0)=-0,2;\ y(0)=0;\ z(0)=0,2$ (a cycle with respect to the point B), the parameter $\beta=9$.

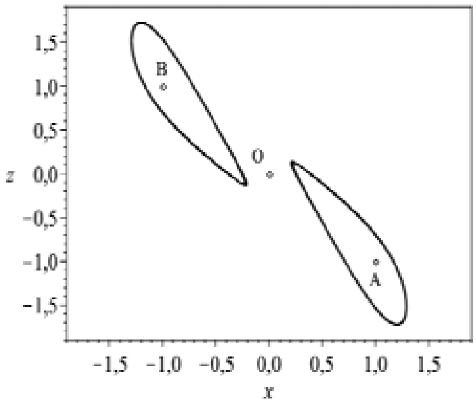

Figure 7. Two limiting cycles

Concluding remarks

The paper deals with the bifurcation processes in three-dimensional systems. Two theorems presented are associated with the problem of limiting cycle generations from homoclinic loops in three-dimensional systems. The first theorem establishes the existence of homoclinic trajectories in systems having in common a certain property of the topology of a three-dimensional system space: the saddle values are the same for all the points. The second theorem is associated also with variational equations. The two theorems are of sufficient character. A mathematical Chua model is considered as an example.

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Homokliničke orbite u 3D disipativnim sisteminma

U radu se proučavaju varijacioni sistemi koji odgovaraju trodimenzionalnim dinamičkim sistemima. Karakteristična jednačina varijacionog sistema zavisi of parcilanih rešenja. Matrica desne strane varijacionog sistema je suma dve matrice. Jedna matrica sadrži spektar linearnog sistema, dok druga reprezentuje parcijalno rešenje. Dokazane su dve teoreme, kojima se odredjuje znak sume karakterisičnih eksponenata tačaka homokloničke trajektorije.

Ključne reči: **dinamički sistem, disipativni system, varijacioni sistemi, homoklinička orbita.**

Гомоклинические орбиты в 3D диссипативных системах

В этой статье мы изучаем системы уравнений в вариациях, которые соответствуют трёхмерным динамическим системам. Характеристическое уравнение системы в вариациях зависит от частных решений. Матрица правой стороны системы в вариациях является суммой двух матриц. Одна матрица соответствует линейной системе, в то время как вторая матрица представляет частное решение. Доказанны две теоремы, которые определяют знак суммы характерных показателей точек гомоклинической траектории.

Ключевые слова: **динамическая система, диссипативная система, система в вариациях, гомоклиническая орбита.**

Orbites homo cliniques dans les systèmes 3D dissipatifs

Dans ce travail on étudie les systèmes de variations qui correspondent aux systèmes dynamiques à trois dimensions. L'équation caractéristique du système de variation dépend des solutions partielles. La matrice de la partie gauche du système de variation est la somme de deux matrices. Une matrice comporte le spectre du système linéaire tandis que l'autre représente la solution partielle. On a prouvé deux théorèmes par lesquelles se détermine le signe de la somme des exposants caractéristiques pour les points de la trajectoire homo clinique.

Mots clés: **système dynamique, système dissipatif, système de variations, orbite homo clinique.**