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MATHEMATICAL MODELLING OF FAR-INFRARED VACUUM DRYING PROCESSES

MATEMATIČKO MODELIRANJE PROCESA TERMORADIJACIONOG VAKUUM SUŠENJA

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ABSTRACT

In this paper, a mathematical model of far-infrared vacuum drying of shrinkage body is presented. The system of two coupled partial differential equations for heat and mass transfer with appropriate initial and boundary conditions are solved numerically with used of the finite difference method. On the basis of the numerical solutions a computer program for calculation of temperature profiles, transient moisture content, mid-plane temperature, and the volume averaged moisture content changes for given drying regime was developed. For verification of the mathematical model a series of numerical drying experiments of potato slices were realized.

Key words: *far-infrared vacuum drying, mathematical model, potato.*

REZIME

U radu je prikazan matematički model termoradijacionog vakuum sušenja tela promenlivih dimenzija. Nelinearni sistem povezanih parcijalnih diferencijalnih jednačina prenosa toplote i mase sa odgovarajućim početnim i graničnim uslovima rešen je numerički primenom metode konačnih razlika. Na osnovu numeričkog rešenja napisan je računarski program koj omogućava proračun profila temperature i vlažnosti u materijalu kao i proračun promene srednje temperature i srednje vlaznosti u materijalu za odbrani režima sušenja. Za verifikaciju modela realizovana je serija numeričkih proračuna sušenja listića kropmira.

Ključne reči: *termoradijaciono vakuum sušenje, matematički model, krompir.*

INTRODUCTION

The mathematical modelling of the drying process of food materials is important for scientific and engineering calculations. The existing mathematical models which are used to model the drying process can be classified into several groups:

- first group consists of models that are used for modeling of drying kinetic curves using the phenomenological models i.e. using thin-layer drying equation (*Mitrevski et al., 2013; Mitrevski et al., 2013a; Mitrevski et al., 2014, Lutovska et al., 2015*),
- in the second group are the so-called diffusion models that assume conductive heat transfer for energy and diffusive transport for moisture. In the case when there is available experimental data of drying kinetics, diffusion equations based on the second Fick's law is used for modeling of mass transfer. In this case, the diffusivity is the main mechanism for mass transport. Then, the equation of second Fick's law can be solved analytically (*Karathanos et al., 1990; da Silva et al., 2009*) or numerically (*Daud et al., 1997; Park et al., 2007*) depending on whether the diffusivity is taken as a constant or variable value,
- in the third group are categorized models which are based on the numerical solution of the equation of second Fick's law with constant or variable value of diffusivity, whereby in the calculation are taken outside the mass transport and shrinkage of materials (*Zogzas et al., 1996; Park et al., 2007*),
- the fourth group consists of models that are based on the simultaneous processes of heat and mass transfer. The modelling of drying process is defined by a system of two coupled partial differential equations of second order with appropriate initial and boundary conditions, and the model take into account or not the shrinkage of the dry material (*Kanevce et al., 2004; Kanevce et al., 2006; Kanevce et al., 2007; Mitrevski et al., 2009; Afolabi and Agarry, 2014*),
- in the fifth group are classified models that are based on the theory of porous media, and when it is applied equilibrium approach (*Yamsaengsung and Moreira, 2002; Dinčov et al., 2004*), and
- the sixth group consists of models based on the theory of non-equilibrium approach (*Ousegui et al., 2010; Halder et al., 2011*).

In this paper, a mathematical model that is used for modeling far-infrared vacuum drying of shrinkage body is presented. The mathematical model is based on a system of partial differential equations for heat and mass transfer with appropriate initial and boundary conditions, where such parameters include thermophysical properties of materials that are functions of temperature and moisture content.

Nomenclature

a [-]	water activity
c [J/K/kg db]	specific heat (dry basis)
a_m [m^2/s]	moisture diffusivity
a_{m0} [m^2/s]	Arrhenius factor,
E_0 [J/kg]	activation energy
j_m [kg/m^2s]	mass flux
j_q [W/m^2]	heat flux
L [m]	flat plate thickness
m [kg]	mass
p [Pa]	pressure
R [J/K/mol]	absolute gas constant
r [J/kg]	latent heat of vaporization
t [^{o}C]	temperature
T_k [K]	temperature
v [m^3/kg]	specific volume
V [m^3]	volume
x [m]	distance from the mid-plane
u [kg/kg db]	moisture content (dry basis)

Greek symbols

β' [-]	shrinkage coefficient
δ [1/K]	thermo-gradient coefficient
σ [W/m^{2}K^4]	Stefan-Boltzman constant
ε [-]	phase conversion factor
ε_r [-]	emissivity of the material
λ [W/m/K]	thermal conductivity
ρ [kg/m]	density
τ [s]	time
φ [-]	relative humidity
ψ [-]	dimensionless coordinate

Subscripts

0	initial
b0	fully dried body
eq	equlibrium
h	heater
m	monolayer
w	water
s	dry solid

MATERIALS AND METHODS

Mathematical modeling of far-infrared vacuum drying processes

The physical problem involves a single slice of a potato of thickness 2L initially at uniform temperature and uniform moisture content (Fig. 1).

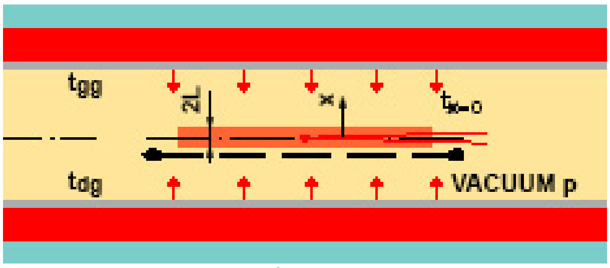

Fig. 1. Scheme of the drying experiment

In the case of an infinite flat plate when the influence of thermodiffusion is small, δ = 0, the the unsteady temperature, t(x, τ), and moisture content, u(x, τ), fields in the drying body are expressed by the following system of coupled nonlinear partial differential equations for energy and moisture transport (*Kanevce et al., 2006*)

$$c\rho_s \frac{\partial t}{\partial \tau} = \lambda \frac{\partial^2 t}{\partial x^2} + \varepsilon r \frac{\partial(\rho_s u)}{\partial \tau} \qquad (1)$$

$$\frac{\partial(\rho_s u)}{\partial \tau} = \frac{\partial}{\partial x}\left(a_m \rho_s \frac{\partial u}{\partial x}\right). \qquad (2)$$

The shrinkage effect was incorporated through the changes of the specific volume of the drying body. In this paper the linear relationship between the specific volume, v_s, and the moisture content, u, has been used (*Mitrevski, 2005*)

$$v_s = \frac{1}{\rho_s} = \frac{V}{m_s} = \frac{1+\beta' u}{\rho_{b0}} \qquad (3)$$

The problem of the moving boundaries due to the changes of the dimensions of the body during the drying was resolved by introducing the dimensionless coordinate

$$\psi = \frac{x}{L(t)}. \qquad (4)$$

The resulting system of equations for the temperature and moisture content prediction becomes (*Mitrevski, 2005*)

$$\frac{\partial t}{\partial \tau} = \frac{\lambda}{\rho_s c}\frac{1}{L^2}\frac{\partial^2 t}{\partial \psi^2} + \frac{\psi}{L}\frac{\partial L}{\partial \tau}\frac{\partial t}{\partial \psi} + \frac{\varepsilon r}{c}\frac{\rho_s}{\rho_{\beta 0}}\left(\frac{\partial u}{\partial \tau} - \frac{\psi}{L}\frac{\partial L}{\partial \tau}\frac{\partial u}{\partial \psi}\right) \qquad (5)$$

$$\frac{\partial u}{\partial \tau} = a_m \frac{\rho_{\beta 0}}{\rho_s}\frac{1}{L^2}\frac{\partial^2 u}{\partial \psi^2} + \left[\frac{\rho_{\beta 0}}{\rho_s^2}\frac{1}{L^2}\frac{\partial(a_m \rho_s)}{\partial \psi} + \frac{\psi}{L}\frac{\partial L}{\partial \tau}\right]\frac{\partial u}{\partial \psi} \qquad (6)$$

As initial conditions, uniform temperature and moisture content profiles are assumed

$$\tau = 0 \quad t(\psi, 0) = t_0 \quad u(\psi, 0) = u_0 \qquad (7)$$

The temperature and the moisture content boundary conditions on the surfaces of the drying slice are

$$-\lambda \frac{1}{L}\left(\frac{\partial t}{\partial \psi}\right)_{\psi=1} + j_q - r(1-\varepsilon) j_m = 0 \qquad (8)$$

$$u_{\psi=1} = u_{eq}(t_{\psi=1}, p).$$

The heat flux, $j_q(t)$, and mass flux, $j_m(t)$, on the surfaces of drying slice are

$$j_q = \varepsilon_r \sigma \left(T_h^4 - T_{\psi=1}^4\right) \qquad (9)$$

$$j_m = -\rho_s L \frac{du_{sr}}{d\tau}$$

The boundary conditions on the mid-plane of the drying slice are

$$\left(\frac{\partial t}{\partial \psi}\right)_{\psi=0} = 0, \quad \left(\frac{\partial t}{\partial \psi}\right)_{\psi=0} = 0 \cdot \qquad (10)$$

The system of nonlinear partial differential eqns (5)-(6) with initial eq (7) and boundary conditions (8) and (10), and eq (9) is the mathematical model of simultaneous heat and moisture transport within the material and from its surface to the surroundings in case of far-infrared vacuum drying of shrinkage body.

NUMERICAL SOLUTIONS

A complex dependence on thermophysical properties of materials from temperature and moisture content makes the resulting system of partial differential equations nonlinear. The same cannot be solved without linearization which leads to unpermitted simplification of the problem. For this reason, the numerical solutions were applied. In order to approximate the solution of eqs. (5) and (6), an explicit numerical procedure has been used. The derivatives with respect to time was represented using forward differencing at the grid point (i, j). All first and second order space derivatives were approximated at time level (j) using central differencing. The values of temperature and moisture content in the first term was assigned its value at the grid point (i, j). Central differencing was also applied to the boundary conditions space derivatives. The number of the space grid points was 101 in all the drying processes calculation scheme. It is defined a network of grid points with grid spacings

$$\Delta\psi = \frac{1}{M-1} \qquad \Delta\tau = R_{KR}(\Delta\psi)^2 \qquad (11)$$

If you insert the replaces

$$R_{KR} = \frac{RK}{a_m} \quad R_{KT} = A_T R_{KR}$$

$$DR_{KT} = 0.5 B_T \Delta\psi R_{KR} \tag{12}$$

$$R_{KU} = A_u R_{KR} \quad DR_{KU} = 0.5 B_U \Delta\psi R_{KR} \tag{13}$$

and if in equations (5) and (6) the partial derivatives are replacement with finite differences, will receive the differentiated equations for calculating the value of temperature and moisture content at different points in space networks in the next time level j + 1 according to the values of the previous.

$$t_{i,j+1} = (R_{KT} - D_{RKT})t_{i-1,j} + (1-2R_{KT})t_{i,j} + (R_{KT} + D_{RKT})t_{i+1,j} \tag{14}$$

$$u_{i,j+1} = (R_{KU} - D_{RKU})u_{i-1,j} + (1-2R_{KU})u_{i,j} + (R_{KU} + D_{RKU})u_{i+1,j} \tag{15}$$

The stability of explicit scheme is providing to fulfill the conditions (*Mitrevski, 2005*)

$$R_{KT} \le 0.5 \quad R_{KU} \le 0.5 \, . \tag{16}$$

For the determination the values of temperature and moisture content on the boundary surface of the dried materials the boundary conditions (17) and (18) transformed in the form of finite differences are used.

$$t_{M+1,j+1} = t_{M-i,j+1} + \frac{2\Delta\psi L}{\lambda}[\varepsilon_r \sigma (T_h^4 - T_{M,j+1}^4) + r(1-\varepsilon)\rho_s L \frac{\bar{u}_j - \bar{u}_{j-1}}{\Delta\tau}] \tag{17}$$

$$u_{M+1,j+1} = u_{eq}(t_{M+1,j+1}, \varphi_{M+1,j+1}) \tag{18}$$

RESULTS AND DISCUSSION

On the basis of the numerical solutions a computer program **TVDShr** for calculation of temperature profiles and transient moisture content, mid-plane temperature, and the volume averaged moisture content changes for given drying regime was developed. For the calculation of thermophysical properties the appropriate models with parameters for potato were taken. Moisture diffusivity of foods is often considered as an Arrhenius-type temperature function

$$a_m(T_k) = a_{m0} \exp(-\frac{E_0}{RT_k}) \tag{19}$$

In this paper, the values of the Arrhenius factor, a_{m0} = $8.63 \cdot 10^{-5}$ m^2/s and activation energy E_0 = 30.60 kJ/mol/K proposed in the (*Zogzas et al., 1996*) for potato were used. The heat capacity potato as equal to the sum of the heat capacity of solid matter and water absorbed by that solid

$$c = c_s + c_w u \tag{20}$$

Although the heat capacity of solid matter, c_s, and water, c_w, are functions of the temperature, constant values have been most widely used. The following values, proposed in reference (*Niesteruk, 1996*) for potatoes, were used: c_s = 1381 J/kg/K, and c_w = 4187 J/kg/K. For practical calculations the system of the two simultaneous partial differential equations could be used by treating the thermal conductivity as constant. A mean value from the results obtained in (*Donsi, 1996*) for the potato λ = 0.40 W/m/K was utilized in this paper. The influence of the phase conversion factor ($0 \le \varepsilon \le 1$) on the transient moisture content and temperature profiles is very small. A mean value, ε = 0.5 was used in the paper. In numerical calculations the values of density of fully dried body ρ_{b0} = 755 kg/m^3 and the shrinkage coefficient β' = 0.57 were used (*Mitrevski et al., 2005*). For calculation of equilibrium moisture content the GAB isotherm equation was used

$$u_{eq} = \frac{u_m CKa_w}{(1-Ka_w)(1-Ka_w + CKa_w)} \tag{20}$$

were the monolayer moisture, u_m, and the adsorption constants C and K are related as Arrhenius type equations

$$u_m = u_x \exp(\frac{\Delta H_x}{RT_k}) \quad C = C_0 \exp(\frac{\Delta H_c}{RT_k}) \quad K = K_0 \exp(\frac{\Delta H_k}{RT_k}) \tag{21}$$

GAB model parameters, C_0, ΔH_C, K_0, ΔH_K, X_{m0}, and ΔH_K can be estimated by different regression procedures from experimental isotherm data. The Gane experimental results for potatoes (*Rahman, 2009*) were used in this paper ($C_0 = 6.609 \cdot 10^{-1}$; ΔHc = 528.4 kJ/kg; K_0 = 0.606 ; $X_{m0} = 2.489 \cdot 10^{-2}$ kg/kg, ΔHk = 53.33 kJ/kg; ΔHx = 123.6 kJ/kg). For verification of a mathematical model a number of numerical calculations have been carried out with different experimental conditions. The temperature of heaters were varied between 120-200°C, while the value of pressure in vacuum chamber were varied between 20-80 kPa.

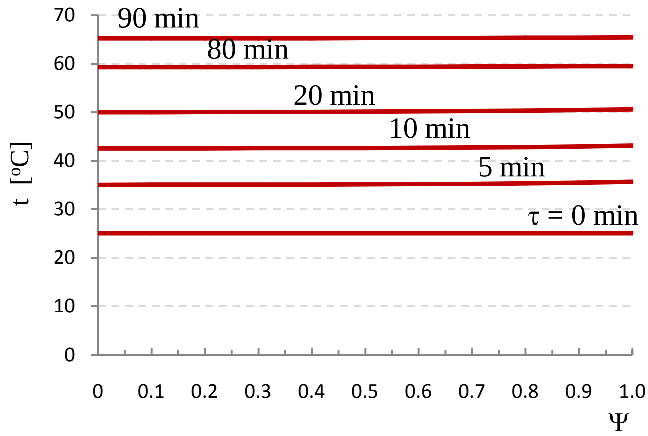

Fig. 2 Temperature profiles

In Fig.2 and Fig.3 the temperature profiles and transient moisture content for experimental conditions similar to those in the experiment E1.10 of far-infrared vacuum dried potatoes: t_0 = 25.07 °C, u_0 = 5.20 kg/kg, $2L_0$ = 3 mm, t_h = 160 °C and p = 40 kPa (*Bundalevski, 2015*) are shown.

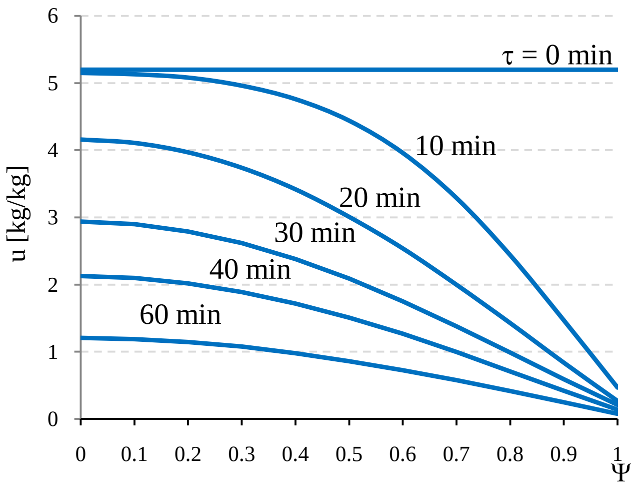

Fig. 3 Transient moisture content

In Fig.4 the mid-plane temperature, $t_x = 0$, and the volume-averaged moisture content, u, are shown.

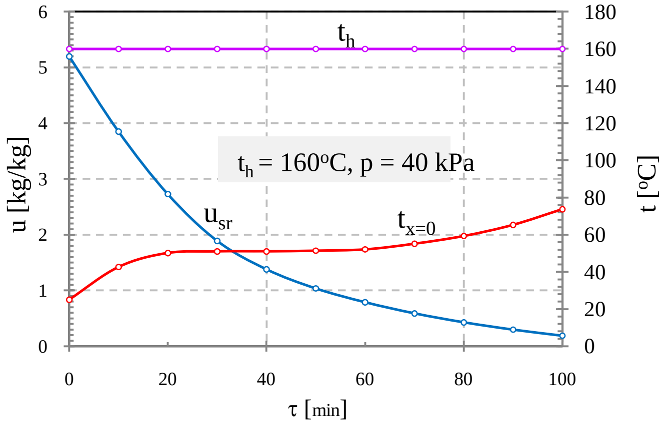

Fig. 4 The mid-plane temperature, $t_{x=0}$, the temperature of the heaters, t_h, and the volume averaged moisture content, u, changes during the drying of a potato slice

CONCLUSION

A mathematical model of far-infrared vacuum drying of shrinking bodies is presented. On the basis of the numerical solutions a computer program **TVDShr** for calculation of temperature profiles and transient moisture content, mid-plane temperature, and the volume averaged moisture content changes for given drying regime was developed. For verification of a mathematical model a number of numerical calculations have been carried out with different experimental conditions. As a representative drying vegetable product, the potato has been chosen.

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