MATHEMATICAL MODELLING OF THE THIN-LAYER DRYING KINETICS OF SOME FRUITS

MATEMATIČKO MODELIRANJE KINETIKE SUŠENJA NEKIH VOĆA U TANKOM SLOJU

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ABSTRACT

The mathematical modelling of the food drying process is of significant importance to scientific and engineering calculations. Thin-layer drying models represent valuable tools for modelling the drying curve and estimating the drying time. These models have wide application due to the ease of use and requirement of less data compared to complex mathematical models. In this paper, the thin-layer drying kinetics of some fruits (namely, pear and quince) was studied. Using an experimental setup designed to simulate an industrial convective dryer, the experimental results were obtained at five drying air temperatures (30, 40, 50, 60 and 70 °C) and three drying air velocities (1, 1.5 and 2 ms⁻¹). For the approximation of the experimental data with regard to the moisture ratio, a new thin-layer model was developed. The performed statistical analysis shows that this model has the best performance features compared to other well-known thin-layer drying models found in the scientific literature.

Key words: mathematical model, drying kinetics, pear, quince.

REZIME

Modeliranje procesa sušenja prehrambenih materijala značajno je za naučne i inženjerke proračune. Tankoslojni modeli važan su alat za matematičko modeliranje krive sušenja i procenu vremena sušenja. Ovi modeli imaju široku primenu zbog jednostavnosti korišćenja, a zahtevaju i manji broj podataka za razliku od složenih matematičkih modela. U ovom radu proučavana je kinetika sušenja voćnih vrsta kruške i dunje. Prikazani su eksperimentalni podaci kinetike sušenja u tankom sloju za pet temperature vazduha sušenja 30, 40, 50, 60 i 70°C i tri brzine vazduha za sušenje 1, 1.5 i 2 ms⁻¹. Dobijeni su rezultati za obavljanje eksperimente, na eksperimentalnom laboratorijskom modelskom postrojenju za ispitivanje kinetike konvektivnog sušenja, koja je projektovana tako da simulira industrijsku sušaru. Za aproksimaciju eksperimentalnih rezultata kinetike sušenja korišćen je novorazvijeni matematički model. Podaci iz eksperimenta sušenja o odnosu trenutne i početne vlažnosti materijala aproksimovani su novim generisanim modelom sušenja tankog sloja, a relevantan uticaj određen je statističkim pokazateljom (ϕ) i testom χ -kvadrat vrednosti (χ^2). Rezultati sprovedene statističke analize pokazuju da sopstveni model ima bolje statističke pokazatelje nasuprot poznatije tenkoslojnih modela koji se danas sa uspehom koriste, a prikazani su u dostupnoj naučnoj literaturi. Dobijeni model može biti koristan pri projektovanju sušara za voće sa ciljem pouzdanog predviđanja toka i ukupnog vremenasušenja kriški kruške i dunje.

Ključne reči: matematički model, kinetika sušenja, kruška, dunja.

INTRODUCTION

Although several drying methods are commercially used to remove moisture from food products, convective hot air drying is the most widely used method. From the mathematical perspective, convective drying is a complex process of simultaneous heat and mass transfer within dried material and from its surface to the surroundings caused by a number of transport mechanisms. There are several different methods of describing the complex simultaneous heat and moisture transport processes within the drying material. However, there is no single theory for wet material drying prediction which encompasses all transfer mechanisms. In the approach initially proposed by Philip and De Vries (Philip and De Vries, 1957) and Luikov (Luikov, 1968) the moisture and temperature fields in the drying material are described by a system of two coupled partial differential equations. The system of equations incorporates coefficients that are functions of temperature and moisture contents, thus making it a non-linear system. Such a system has

been used for certain applications. However, for many practical calculations, the influence of temperature and moisture content on all transport coefficients has often been neglected and the resulting system of two linear partial differential equations has been used (Kanevce et al., 2007).

Nevertheless, thin-layer drying models are important tools for the mathematical modelling of drying curves. Owing to their simplicity, they are used to describe the drying kinetics of some products, mainly when the geometry of the material is unknown.

In the scientific literature, there are many researches on the experimental studies and mathematical modelling of the drying behaviour of various fruits such as apples (Cruz et al., 2012), bananas (Doymaz, 2010), cherries (Doymaz, 2011), grapes (Doymaz, 2002), kiwis (Doymaz, 2009), quinces (Babić et al., 2007; Tzempelikos et al., 2014), pears (Guiné et al., 2007; Doymaz, 2012) and plums (Doymaz, 2004).

The objectives of this study were as follows:

(a) experimental investigation of the drying kinetics of some fruits: pears and quinces under specific drying conditions (drying air temperatures of 30, 40, 50, 60 and 70°C and drying

air velocities of 1, 1.5 and 2 ms⁻¹; absolute air humidity of 0.0154 kg water kg⁻¹ dry air), and

(b) development of a new thin-layer drying model as a function of the drying conditions and comparison of their goodness of fit with the existing models found in the literature.

Nomenclature:

A,B,C,D,E,F,G	- parameter
M (kg/kg)	- moisture content
MR	- moisture ratio
MRD	- mean relative deviation
R ²	- coefficient of determination
RMSE	- root mean squared error
t (°C)	- temperature of dry air
v (ms ⁻¹)	- velocity of dry air
Z ₁ , Z ₂	-statistic for testing the skewness and kurtosis of the residual population

Greek symbols

φ	- statistic performance index
χ ²	- statistic for testing the normality of the moisture residual
τ (min ⁻¹)	- drying time

Subscripts

0	- initial
eq	- equilibrium

MATERIAL AND METHOD

Fresh pears of the 'Williams' cultivar and fresh quinces of the 'Champion' cultivar were used as raw material in the experimental part of the research. Prior to processing, the fruits were stored in a cold chamber at a temperature of 4°C and a relative air humidity of 75%. The pears and quinces were washed, peeled and sliced manually in order to obtain uniform samples. The spherical samples with thickness of 4±10⁻¹, obtained from the central medulla region where the cell structure is more uniform, were used in the drying experiments. Several measurements were made using a calliper, and only the samples with a tolerance of ±5 % were used in the research.

The experimental data on the thin-layer drying kinetics of pear and quince slices were obtained using an experimental setup designed to simulate an industrial convective dryer (Pavkov, 2012). The dryer consists of three basic units: a fan providing the desired drying air velocity, an electrical heater for the control of the drying air temperature and a drying chamber. The dryer unit was started 1 h before each experiment in order to achieve the desired steady state conditions of the drying air flow.

The measurement of changes in the sample mass was conducted continually, without interruptions to the drying process, using tray carriers attached to sensors. A mass measuring sensor was connected to a measuring acquisition system, which recorded mass changes during the drying process. The material temperature was measured using a micro-thermocouple K-type inserted in the mid-plane of each of the three slices. The drying experiments were performed at drying air temperatures of 30, 40, 50, 60 and 70 °C, drying air velocities of 1, 1.5 and 2 ms⁻¹, whereas the absolute air humidity remained constant at 0.0154 kg water kg⁻¹ dry air. By measuring changes in the mass of the samples during convective drying, the basic data on drying kinetics were collected. The drying experiments were stopped when there was no change in the mass of drying pear or quince slices.

The initial moisture content M₀ was measured in each of the experiments. The initial moisture content of fresh pear and

quince slices and the final moisture content of dried samples were determined gravimetrically using the hot air oven method at 105°C and atmospheric pressure for a period of 24 h. The drying experiments involving pear slices were stopped when the moisture content of the samples decreased to 0.14 kg kg⁻¹ d.m., from an initial value of 4.99 kg water kg⁻¹ d.m. However, the drying experiments involving quince samples were stopped when the moisture content decreased to 0.22 kg kg⁻¹ d.m. from an initial value of 4.86 kg water kg⁻¹ d.m. The experiments were replicated three times at each drying air temperature and drying air velocity, and the average value of the moisture ratio was used for constructing the drying curves.

Mathematical modelling of the drying curves

The experimental moisture content data on pear and quince slices obtained at different drying air temperatures and different drying air velocities were converted to the moisture ratio (MR), and subsequently fitted to the thin-layer drying models given in Table 1 (M01 and M02).

Table 1. Thin-layer mathematical drying models

Model	Name of model	Equation	References
M01	Midilli	MR = Aexp(-k ₁ τ ^B) + Cτ	Midilli et al., 2002
M02	Hii	MR = Aexp(-k ₁ τ ^B) + Cexp(-Dτ ^B)	Hii et al., 2008

In these models, the moisture ratio (MR) is defined by the following equation:

$$MR = (M - M_{eq}) / (M_0 - M_{eq}) \quad (1)$$

The values of M_{eq} are relatively little compared to those of M or M₀, so the error involved in the simplification is negligible.

Therefore, the moisture ratio was calculated as follows:

$$MR = M / M_0 \quad (2)$$

According to previous statistical analyzes, the best model describing the thin-layer drying characteristics of pear or quince slices has to be selected on the basis of a higher value of the performance index φ and a lower χ² value (Mitrevski et al., 2013). According to the results obtained from previous statistical analyzes, it was concluded that the Midilli model can be used successfully for predicting the moisture of pear slices (Lutovska et al., 2016), whereas the Hii model can be used successfully for predicting the moisture of quince slices (Mitrevski et al., 2015), at any time of the drying process between drying air temperatures of 30 and 70 °C and drying air velocities of 1 to 2 ms⁻¹.

In this paper, a new generalized thin-layer model was developed as the effect of drying air temperature and drying air velocity on the empirical parameters was not included in the Midilli and Hii models.

$$MR = (A \cdot \text{EXP}(-B \cdot \tau^C \cdot v^D \cdot E \cdot t)) + (-F \cdot \text{EXP}(-G \cdot \tau^C \cdot v^D \cdot E \cdot t)) \quad (3)$$

where: A,B,C,D,E,F,G are the model constants, τ is the drying time, v is the velocity of drying air, and t is the temperature of drying air. The statistical performance features of this model were assessed on the basis of the calculated values of the performance index φ and the χ², chi-squared value. The value of the performance index φ was calculated on the basis of the calculated values of the coefficient of determination (R²), the

root mean squared error (RMSE) and the mean relative deviation (MRD) (Ruiz-Lopez and Lara, 2009):

$$\phi = \frac{R^2}{\text{RMSE} \cdot \text{MRD}} \quad (4)$$

In this study, the χ^2 chi-squared value was calculated on the basis of the D'Agostino-Pearson test of normality.

The D'Agostino-Pearson test of normality is the most effective procedure for assessing a goodness of fit for a normal distribution (Sheskin, 2011). This test is based on the individual statistics for testing the population of skewness z_1 and kurtosis z_2 , respectively.

The test statistic for the D'Agostino-Pearson test of normality was computed using the following equation (Ruiz-Lopez and Lara, 2009):

$$\chi^2 = z_1^2 + z_2^2 \quad (5)$$

RESULTS AND DISCUSSIONS

The experimental moisture content data obtained at different drying air temperatures and different drying air velocities were converted to the moisture ratio (MR) and subsequently fitted with the newly developed thin-layer model.

Considering that the regression method, the estimation method, the initial step size, the start parameter values, the convergence criterion and the form of the function exert a significant influence on the accuracy of the parameters estimated, a large number of numerical experiments were performed (Mitreviski et al., 2015). The method of multiple indirect non-linear regression analysis was adopted, including all the experimental data from the drying kinetics of pear or quince slices. The estimation methods of Quasi-Newton, Simplex, Simplex and quasi-Newton, Hooke-Jeeves pattern moves, Hooke-Jeeves pattern moves and quasi-Newton, Rosenbrock pattern search, Rosenbrock pattern search and quasi-Newton, Gauss-Newton and Levenberg-Marquardt from computer program StatSoft Statistica (Statsoft Inc., Tulsa, OK, <http://www.statsoft.com>), were used in the numerical experiments.

Using the newly developed thin-layer drying model and the thin-layer pear and quince drying data converted to the moisture ratio, the values of the coefficient of determination (R^2), the root mean squared error (RMSE), the mean relative deviation (MRD), the performance index ϕ and χ^2 were calculated (Table 2).

Table 2. Statistical summary of the regression parameters

Model / Fruit	R^2	RMSE	MRD	ϕ	χ^2
Midilli / Pear	0.928	0.099	0.837	11.12	31.11
Midilli / Quince	0.942	0.088	0.439	24.38	76.30
Hii / Pear	0.932	0.095	0.640	15.33	54.42
Hii / Quince	0.937	0.091	0.542	19.00	41.78
Mitreviski / Pear	0.997	0.019	0.203	258.5	8.858
Mitreviski / Quince	0.994	0.027	0.221	166.6	14.72

Table 2 clearly indicates that the newly developed model of Mitreviski et al. has the highest value of average performance index ($\phi = 258.4$ for pear and $\phi = 166.6$ for quince) and the lowest average chi-squared value ($\chi^2 = 8.858$ for pear and $\chi^2 = 14.72$ quince) in comparison with the model of Midilli and Hii model.

In accordance with the statistical criteria, this model correlated with the experimental values of the drying kinetics of pear slices with a RMSE of 1.9 %, whereas a RMSE of 2.7% was obtained relative to the experimental values of the drying kinetics of quince slices. The parameter values estimated using the newly generated model of Mitreviski et al. are presented in Table 3.

Table 3. Non-linear regression parameters

Fruit	A	B	C	D	E	F	G
Pear	-1.863	0.022	0.654	0.231	0.123	-2.852	0.014
Quince	-5.594	0.083	0.645	0.188	0.031	-6.592	0.007

In Fig. 1, the experimental and calculated values of the moisture ratio and drying time at drying air temperatures of 30, 40, 50, 60 and 70°C for pear slices at drying air velocities of 2 ms⁻¹ are shown. It is evident from Fig. 1 that there is a good match between the experimental and calculated values of the moisture ratio of pear slices.

Fig.1. Experimental and predicted moisture ratio at different drying air temperatures and drying air velocities $v = 2 \text{ ms}^{-1}$ approximated with Mitreviski et al. model-pear

The experimental and calculated values of the moisture ratio and drying time of quince slices at drying air temperatures of 30, 40, 50, 60 and 70°C, and drying air velocities of 1 and 1.5 ms⁻¹ are shown in Fig. 2. It is evident from Fig. 2 that there is a good match between the experimental and calculated values of the quince slice moisture ratio.

Fig. 2. Experimental and predicted moisture ratio for different drying air temperatures and drying air velocities approximated with Mitreviski et al. model-quince

CONCLUSIONS

In the present study, the convective hot-air drying kinetics of pear and quince slices was investigated. The experimental drying data relative to the moisture ratio were approximated with a newly generated thin-layer drying model, and the goodness of fit was determined using the performance index (ϕ) and chi-squared value (χ^2). In accordance with the statistical criteria, the newly developed model of Mitrevski et al. correlated with the experimental values of the drying kinetics of pear slices with a RMSE of 1.9%, whereas a RMSE of 2.7% was obtained relative to the experimental values of the drying kinetics of quince slices.

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