

APPLICATION OF TWO-BIT DITHERED DISCRETE FOURIER BASIS FUNCTIONS IN ELECTRICITY MEASUREMENT

PRIMENA DVOBITNIH DITEROVANIH FURIJEOVH BAZISNIH FUNKCIJA U MERENJU ELEKTRIČNE ENERGIJE

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ABSTRACT

The paper defines an algorithm for generating two-bit dithered discrete Fourier basis functions (DDFBF) used in a Stochastic Digital Discrete Fourier Transformation (SDDFT) processor. Based on the theoretical criterion of upper limit precision of norm and orthogonality, the orthonormality of DFT with 32 harmonics is explored by simulation and experimentally. The experiment was detailed and comprehensive, both for norm and for both types of orthogonality. It is performed in 236,8 million points in each of three variants of orthonormality. The matching of theoretical and experimental precision is very acceptable and it can be said with great confidence that the proposed algorithm for generating DDFBF is correct. The DDFBF approach plays a key role in electricity measurement, which is emphasized in the paper.

Keywords: Fourier transformation; Basis functions; Dither; Stochastic digital measurement method

REZIME

U radu je definisan i detaljno opisan algoritam generisanja digitalnih dvobitnih Furijeovih bazisnih funkcija (DDFBF) koje se koriste u dvobitnom stohastičkom digitalnom procesoru diskretne Furijeove transformacije (DFT) visokog reda. Na osnovu teorijskog kriterijuma granične preciznosti i norme i ortogonalnosti, simulacijom i eksperimentalno je potvrđena ortonormiranost DFT sa 32 harmonika. Simulacija je izvedena u Python-u, dok je eksperimentalna potvrda dobijena primenom dvostrukog trofaznog analizatora snage tipske oznake MM2 i četverostrukog trofaznog analizatora snage tipske oznake MM4, koji su vršili nadzor tokova električne snage i energije i detektovali neregistrovanu električnu energiju. Time je teorijski i eksperimentalno dokazana korektnost opisanog algoritma generisanja DDFBF. Eksperiment je bio veoma detaljan i sveobuhvatan, i za normiranost, i za oba tipa ortogonalnosti. Izveden je u po 236,8 miliona tačaka u svakoj od tri varijante ortonormiranosti. Slaganje teorijske i eksperimentalne preciznosti je vrlo prihvatljivo i sa velikom pouzdanošću se može reći da je predloženi algoritam generisanja DDFBF korektan. Njime generisane DDFBF se, bez ikakvih ograničenja, mogu koristiti u merenju električne snage i energije po najnaprednijem svetskom standardu – „IEEE Std. 1459 2000 – 2010“. Elektronskih brojila, koja mere električnu energiju po ovom standardu, na našem tržištu još uvek nema, ali su ona – „pred vratima“. Brojila koriste diskretnu Furijeovu transformaciju (DFT) visokog reda, koju mogu da izvrše u realnom vremenu, na bar tri fazna napona i na tri faze struje i struju u nuli, samo još uvek vrlo skupi i komercijalno teško dostupni mikroprocesori i mikrokontroleri. Dvobitni pristup tom problemu otvara nove i velike mogućnosti pojednostavljenja i pojeftinjenja njegovog rešenja.

Ključne reči: Furijeova transformacija; Bazisne funkcije; Diter; Stohastička digitalna merna metoda

INTRODUCTION

Orthogonal transformations have great importance in measurement and signal processing, and the oldest and most widely used is Fourier transformation. The fast Fourier transform (FFT) is especially important due to large reductions in the number of necessary multiplications (the most critical arithmetic operations) and the memory needed for their execution.

Advances in computer component technology, especially in terms of processor speed and memory capacity, are gradually coming to the fore and DFT as an alternative that has several advantages over FFT:

- Fourier transformation coefficients are calculated independently of each other, i.e. separately, and, if necessary, even only one arbitrarily chosen;
- The calculation can be performed simultaneously with the measurement - when the measurement sequence is completed, in that same instant the DFT is completed;
- Simple parallelization in DFT calculation is possible. FFT has none of the above properties.

Recent developments in the application of DFT in power and energy measurements in the electricity distribution (ED) network have led to the realization of a dedicated stochastic digital DFT (SDDFT) processor whose optimal version is a two-bit SDDFT processor (Author3 et. al. 2018).

The key operation, MAC (Multiply and Accumulate), is now performed on simple hardware - the multipliers consist of 4 two-input "and" circuits and 2 two-input "or" circuits, while the corresponding set of up/down counters execute accumulations. The simple structure of the hardware for performing a two-bit MAC operation, especially if applied in the FPGA implementation of the SDDFT processor, allows high parallelization of DFT calculations and thus achieving high processing speeds of the analyzed signal. On the other hand, two-bit stochastic additive A/D conversion (SAADC) allows significantly simpler parallel measurements (Author4 et. al. 2019).

It has been proven in (Author3 et. al. 2018) that for the application of two-bit SAADC, it is optimal to apply two-bit dithered Fourier basis functions. The problem of generating two-bit dithered Fourier basis functions (DDFBF) is not elaborated in detail in previous works. With high order SDDFT (over 1000) this

problem becomes critical and requires special attention. The aim of this paper is to solve the problem of generating these functions and to prove their correctness and usability.

MATERIAL AND METHOD

Algorithm for generating two-bit dithered discrete Fourier basis functions (AG2BF)

The algorithm for generating two-bit dithered discrete Fourier basis functions (AG2BF) has the following steps:

1. Samples of Fourier basis functions (FBF) in maximal precision floating-point format (FP) are in memory. They need to be converted to an integer format (IF). We choose the number of bits of an integer format - say 32 bits.
2. Since one bit is the sign bit, we multiply the selected FBF sample in FP format by 231, apply the ROUND function and place the sample in LongInt (LI) format. Thus, we prepared a sample of FBF for dithering.
3. The corresponding uniform noise (UN) sample then has 31 bits, and we get it from the Random function in FP format in the range (0, 1),
4. From a sample of UN, we subtract 0.5 in FP format, multiply by 230 and apply the ROUND function.
5. We add these two integer samples. The result has 31 bits + the sign bit.
6. We detect the sign bit in the sum and apply the ABS function to the result.
7. If the most significant bit (of 31st order) of the absolute value of the sum is 1, the corresponding absolute value of DDFBF is 1, and if it is not - then the absolute value of 2BDDFBF is equal to zero (0).
8. The sign bit then must be returned to the absolute value and we have a complete two-bit sample of the DDFBF.
9. The complete set of DDFBF samples in measurement time interval can be used in Discrete Fourier Transformation.

Experimental Confirmation of DDFBF Orthonormality

In (Author5 et al., 2020), it is proved that:

$$\sigma_{\bar{e}}^2(2) = \frac{1}{N} \cdot \frac{(2g)^2}{T} \int_0^T |f_1(t) \cdot f_2(t)| dt - \frac{1}{N} \cdot \frac{1}{T} \int_0^T f_1^2(t) \cdot f_2^2(t) dt \tag{1}$$

where $\sigma_{\bar{e}}$ represents the precision of measuring the mean value of the product of two functions, $y_1 = f_1(t)$ and $y_2 = f_2(t)$, by two-bit stochastic digital measurement method (SDMM) over the time

interval $[0, T]$. In an ED network, T is the period of the mains voltage.

The normality of the DDFBF is experimentally confirmed, as shown in Table 1 and Figure 1. Theoretically, σ in that figure is, in fact, $\sigma = \sigma_{\bar{e}}(2)$ for $g = 0.5$, $f_1(t) = f_2(t) = \sin(n\omega t)$ or $\cos(n\omega t)$, $\{n = 1, 2, \dots, 32\}$, at 50 Hz with 2048 samples per period, i.e. $N = 2048$. Here, $\sigma_{\bar{e}}$ is the averaged σ on a complete set of 64 norms in 236,8 million points on the time axis. Also, \bar{e} is the mean value of the error of each product measured at 236,8 million points. This is the case A.

The orthogonality of the DDFBF function is experimentally confirmed which is shown in Table 2 and Figure 2. Theoretically, σ in this figure is, in fact, $\sigma = \sigma_{\bar{e}}(2)$ for $g = 0.5$, $f_1(t) = \sin(n\omega t)$, $f_2(t) = \sin(m\omega t)$ or $f_1(t) = \cos(n\omega t)$, $f_2(t) = \cos(m\omega t)$, $n, m \in \{1, 2, \dots, 32\}$ and $n \neq m$ at 50 Hz, with 2048 samples per period, i.e. $N = 2048$. In this case, $\sigma_{\bar{e}}$ is the averaged σ on the complete set of $2 \cdot \left[\binom{32}{2} - 32 \right] = 1742$ orthogonal combinations in 236,8 million points on the time axis. Also, \bar{e} is the mean value of the error of each product measures in 236,8 million points. This is the case B.

The orthogonality of the DDFBF was also confirmed experimentally, as shown in Table 3 and Figure 3. Here, theoretically, σ is $\sigma = \sigma_{\bar{e}}(2)$ for $g = 0.5$, $f_1(t) = \sin(n\omega t)$, $f_2(t) = \cos(m\omega t)$, $n, m \in \{1, 2, \dots, 32\}$ and $n \neq m$, at 50 Hz, with 2048 samples per periods, i.e. $N = 2048$. Also, $\sigma_{\bar{e}}$ is the averaged σ on a complete set of $64^2 - 2 \cdot \left[\binom{32}{2} - 32 \right] - 64 = 2290$ orthogonal combinations at 236,8 million points on the time axis, while \bar{e} is the mean value of error of each product of the measurements at 236,8 million points. This is the case C.

It should be noted that $+g$ is the positive threshold, $-g$ is the negative threshold in two-bit SAADC. The idea of SAADC is used for obtaining a two-bit sample from a high resolution (multi-bit) FBF sample (Author3 et al. 2018).

Figures 1, 2 and 3 strongly confirm even visually theoretical formulae for standard deviation in cases A, B and C. These figures are obtained using extensive corresponding simulations over 236,8 million points. This conclusion has great significance since "There is nothing more practical than a good theory" (Curt Levine). Having in mind above mentioned formulae one can predict the precision of a particular two-bit SDDFT processor in the design stage.

Table 1. Numerical confirmation of DDFBF normality - the case A

Number of samples	Case	Theoretical σ formula	Theoretical value of σ	Obtained by experiment σ	$\frac{\sigma}{\sigma_{\bar{e}}}$	\bar{e}	$\frac{\bar{e}}{\sigma_{\bar{e}}}$	Number of points
2048	A	$\sigma_{\bar{e}} = \frac{1}{4} \sqrt{\frac{2}{N}} = \sqrt{\frac{1}{8N}}$	0.0078 125	0.0078 14339	1.00023 5375	-5.37027 E-07	-6.87394 E-05	236800000

Table 2. Numerical confirmation of DDFBF orthogonality - the case B

Number of samples	Case	Theoretical σ formula	Theoretical value of σ	Obtained by experiment σ	$\frac{\sigma}{\sigma_{\bar{e}}}$	\bar{e}	$\frac{\bar{e}}{\sigma_{\bar{e}}}$	Number of points
2048	B	$\sigma_{\bar{e}} = \sqrt{\frac{8}{\pi} - 1} \cdot \sqrt{\frac{1}{8N}}$	0.00971543 1	0.00971 3238	0.99 9774 324	-6.28933 E-07	-6.4759 E-5	2368000 00

Table 3. Numerical confirmation of DDFBF orthogonality - the case C

Number of samples	Case	Theoretical σ formula	Theoretical value of σ	Obtained by experiment σ	$\frac{\sigma}{\sigma_{\bar{e}}}$	\bar{e}	$\frac{\bar{e}}{\sigma_{\bar{e}}}$	Number of points
2048	C	$\sigma_{\bar{e}} \leq \sqrt{2} \sqrt{\frac{8}{\pi} - 1} \cdot \sqrt{\frac{1}{8N}}$	0.01373969 4	0.00870872 9	0.63383 7169	1.22603 E-04	8.92326 E-03	1984000 00

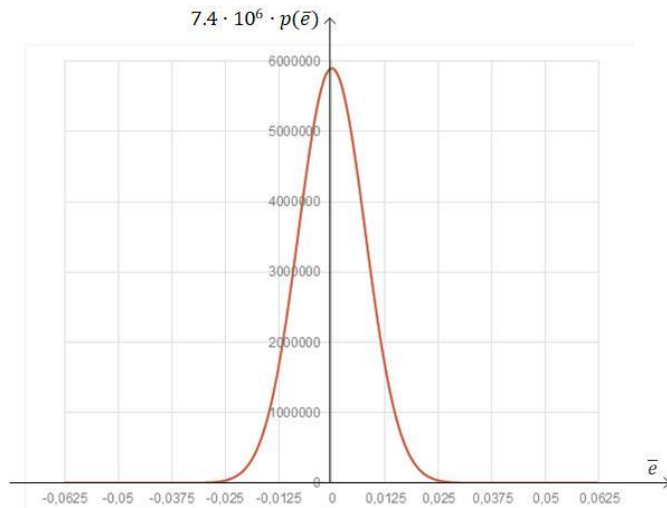


Fig.1. Graphical confirmation of DDFBF normality - the case A

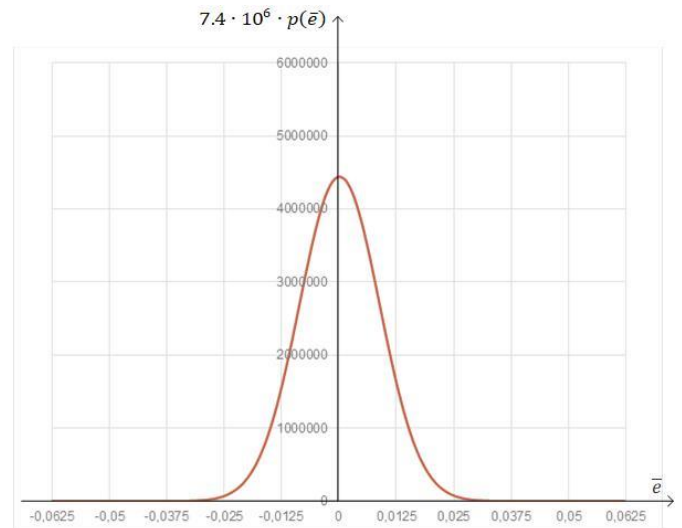


Fig.3. Graphical confirmation of DDFBF orthogonality - the case C

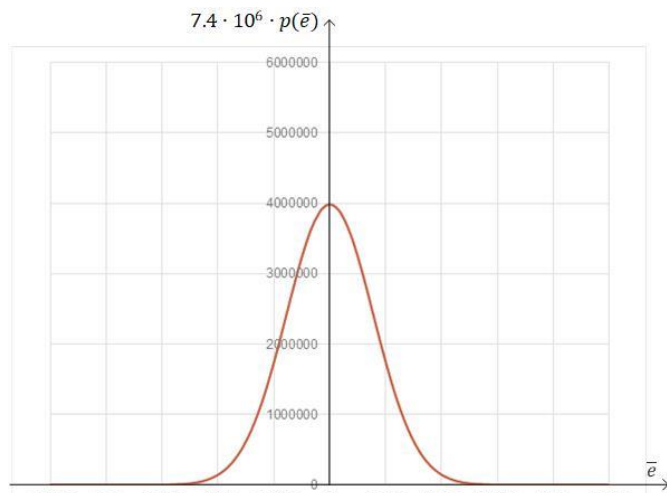


Fig.2. Graphical confirmation of DDFBF orthogonality - the case B

RESULTS AND DISCUSSION

Principles of application of two-bit dithered discrete Fourier basis functions in electric power engineering

In (Author3 et. al. 2018), the SDDFT processor is proposed and its application in reactive power measurements in an electricity distribution (ED) network has been presented. Reactive energy is an integral in the time of the reactive power.

In (Author5 et. al., 2019), it is shown how the set of SDDFT processors enables on-line detection of the most significant higher harmonic in the network in transient situations. This is important for on-line measurement of current electrical power in such sub-sampling mode (Ghanavati, A. et. al. 2018).

In the modern ED network, consumers are often also producers of electricity, so the number of transient events has increased significantly, especially at lower voltage levels. Measuring voltage and current phasors is practically impossible without the use of FBF. Chosen set of DDFBF, if the sampling frequency is high enough, in addition to high accuracy, also enables very precise measurements of all electrical power parameters. In (Author4 et. al. 2019), it is shown how simple the hardware is for measuring and processing in this case. Simple hardware for measurement and processing allows, on the one hand, simple and cheap parallelization, while, on the other hand, due to simplicity, it has a small number of sources of systematic error. Then the sources are identified easily, and a systematic error is corrected. Therefore, chosen set DDFBF is the basis of very accurate measurements in the ED network, especially of all electrical power, i.e. energy, components.

The main purpose of this paper is to present the algorithm for generating DDFBF and to determine the theoretical dependence of precision of orthonormality on the size of the computational sample, practically - on the length of the measurement time interval. The fundamental role here was played by relation (1), strictly derived in (Author5 et. al., 2020). Extensive simulation performed in Python has well confirmed the precision and of both the normality and orthogonality of DDFBF, obtained by the proposed algorithm.

Experimental verification of correctness is confirmed by devices MM2 and MM4, shown in (Author3 et. al. 2018; Author4 et. al. 2019). They are a dual two-bit three-phase power analyzer (MM2) and a quadruple two-bit three-phase power analyzer (MM4). These devices effectively monitored power and energy flows and detected and measured unregistered electricity consumption. The application of DDFBF was, therefore, performed before the theoretical and simulation analysis of the

precision of their orthonormality. Calibration of MM2 and MM4 practically confirmed their correctness.

This paper, therefore, filled this gap and provided a more complete insight into the properties of DDFBF. This is important because of the assessment of their usability in other areas of science and technology.

CONCLUSION

The paper defines an algorithm for generating digital two-bit dithered discrete Fourier basis functions (DDFBF). Based on the theoretical criteria for the precision limit of norm and orthogonality, the orthonormality of DFT with 32 harmonics is experimentally confirmed. Thus, the correctness of the described algorithm development is theoretically and experimentally proven. The experiment was very detailed and comprehensive for both, normality and orthogonality (both types of orthogonality). It was carried out in 236,8 million points in each of the three variants of orthonormality. The differences between corresponding theoretical and experimental precisions are negligible and the claim that the proposed algorithm for generating DDFBF is correct.

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