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Analytical study of the motion of particles with variable electric charge

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Abstract: The paper is the short review of a problem of motion of bodies with variable electric charge in space plasma medium. The basic attention is given the results of the author on an analytical method of the solution of such problems. Also in the paper, the basic ways of the further researches are offered.

Keywords: variable electric charge, the canonical equations, the near Earth space, manmade microparticles

1 Introduction

Development of space researches has raised the question about the motion of bodies in a plasma environment. It is known that a body which moves in magnetosphere's plasma get electric charge which depends on density and temperature of surrounding plasma, a stream of sunlight and therefore on position and orientation of bodies during its moving. For the first time the problem of the motion of a body with variable electric charge has been stated in paper [10] in connection with consideration of the concept of new engine systems. Studying influence of the value of acceleration of Lorentz force on character of the motion of the charged space vehicle in superposition of central gravitational and dipole magnetic fields, Nesterenko has found possible kinds of trajectories and critical values of acceleration at which the kind of trajectories varies.

Last years there has been growing the new ecological application of this problem which is bound on studying of features of the motion of manmade microparticles (MP) in the near Earth space (NES) that are appeared during the work of solid-fuel rocket engines (the bibliography on this problem is possible to see in [6]). The similar problem arises in connection with studying of mechanisms of formation of dust rings in a vicinity of other heavenly bodies possessing strong magnetic fields (Jupiter, Saturn, etc.) [2, 4] (the bibliography is possible to see in [4]). At last, the problem of dynamics of microparticles with a variable

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charge in space plasma arises in connection with research of a problem of electrostatic levitation of dust particles near a surface of nonatmospheric heavenly bodies [8, 12, 14–16]. A more detailed list of papers on this problem it is possible to see, for example, in [12, 16].

All facts mentioned above has allowed us to confirm in [9] about appearance of new object of dynamic research - bodies of constant mass and variable electric charge.

For a great number of physical factors which generally can affect on dynamics of the charged bodies in the NES, the basic method of theoretical research of features of orbital motion of these bodies in a vicinity of the Earth is the method (it was considered, for example, in [6]) of numerical modelling based on the joint numerical solution of the equations of motion in the NES and the equation of the charging in plasm of circumterraneous space. However, physical assessments and results of numerical experiments show that for certain interesting cases the number of factors that influence on orbital dynamics decrease essentially and it simplifies statement of a problem of ballistics in the NES and does possible to use the conforming analytical methods for definition of features of orbital motion near the Earth.

Generally the description of evolution of such body is given by system of odd number of the differential equations of the first order: six equations of the motion and the equation of the charging. Qualitative analysis of similar system is very combined and consequently it pays to consider cases when the set of equations becomes simpler and known methods of the analysis of the motion of mechanical systems can be used.

As mentioned above in this paper, a charge of a body varies during its motion in the near Earth space, and character of this change depends on the area of the NES where the motion occur. For bodies with a low photoemission yield there are situations for that body's electric charge is locally equilibrium, i.e. the electric charge depends only on particle's position on a trajectory.

$$Q = Q(\mathbf{r}) \tag{1}$$

where $\mathbf{r}$ – radius-vector of current position of a body. In this case we had been offered following approaches in [9]:

1) the using of function of Hamilton and the application of methods of analysis of Hamilton systems;

2) the generalization of the theory of drift motion of the charged particles on a case of a variable charge.

2 The Hamilton function for bodies with variable electric charge

In [7] conditions of applicability of the canonical formulation of a problem of the motion of a body with locally equilibrium electric charge in the NES have been defined.

A general condition of correctness of canonical formulation of the problem of microparticle's motion in NES is the possibility to represent all forces acting upon MP in the form

$$\mathbf{F}^j = -\nabla V^j \quad or \quad F_i^j = -\frac{\partial U^j}{\partial x_i} + \frac{d}{dt}\frac{\partial U^j}{\partial \dot{x}_i} \quad (i = 1, 2, 3)$$

where $V^j = V^j(x_1, x_2, x_3, t)$ is the usual potential, and $U^j = U^j(x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3, t)$ is the generalized potential. When MP moves at sufficiently high altitudes at which one can neglect the drag force of the background gas of the upper atmosphere, the basic forces acting upon MP are represented by the gravitational force $\mathbf{F}^{gr}$, solar pressure force $\mathbf{F}^{sp}$, and the Lorentz force $\mathbf{F}^{L}$. The gravitational force and the force of solar pressure do not depend on velocity (for spherical MP) and are represented by usual potentials V^{gr} and V^{sp} . At the same time we showed in [7] that the Lorentz force acting upon variable electric charge of MP from the side of magnetic and electric fields of the near-Earth space admits to be represented with the help of generalized potential V^{sp} , if appropriate constraints are imposed on the character of variation of the charge in the process of its orbital motion and also on geometrical features of the magnetic and electric fields of the NES. The first of these constraints consists in validity of the condition of quasi-equilibrium (1) of electric charge which is determined by local values of parameters of the ambient plasma and depends only on current coordinates of body.

The second constraint (on geometrical characteristics of the fields) includes the validity of the conditions

$$\mathbf{B} \perp \nabla Q, \quad \mathbf{E} \| \nabla Q. \tag{2}$$

We showed in [7] that, if conditions (1)–(2) are met, the Lorentz force can be represented by the generalized potential

$$U^L = Y_1 - \frac{\mathbf{v} \cdot \mathbf{Y_2}}{c}, \tag{3}$$

where Y_1 and $\mathbf{Y_2}$ are the scalar and vector functions of coordinates that satisfy the equations

$$Y_1 = Q \cdot \nabla \varphi \qquad rot\mathbf{Y_2} = Q \cdot rot\mathbf{A} \tag{4}$$

In (4) φ and $\mathbf{A}$ are the scalar and vector potentials of electric and magnetic fields acting upon MP in the NES, respectively. Thus (2) follows directly from the introduction of the generalized charge-field potentials Y_1 and $\mathbf{Y_2}$.

As mentioned above in this paper, an important examples of bodies with variable electric charge are manmade microparticles. The further consideration in this paper we will make for such objects.

As we showed in [7] the Hamilton function describing the dynamics of a micro-particle with variable charge in the NES is determined by the formula

$$H = \frac{1}{2m} \sum_{i=1}^{3} \left(P_i - \frac{Y_{2i}}{c} \right)^2 + V^{gr} + V^{pr} + Y_1, \tag{5}$$

where P_i are components of the generalized momentum vector.

In [7] on the basis of numerical simulation's results we showed that for MP's motion in the Earth's plasmasphere the condition (1) is true for micro-particles with radii more than 0.01 microns if their orbits are situated on altitudes more than one and a half of Earth's radius from a surface of the Earth. The geometrical conditions (2) are satisfied due to specific features of plasma spatial distribution in the Earth's plasmasphere. This distribution is known to be described by the model of two-component plasma [3].

The equilibrium charge of a body, being a function of local density and temperature, does not depend on the longitudinal coordinate along the magnetic field line, which corresponds to the case of satisfied conditions (2). In this case $Q = Q(L)$, where L is the parameter of the local magnetic shell. The main contribution to the geo-electric field at altitudes of the plasmasphere is made, as is known, by the electric field of co-rotation $E_{cor} = \frac{1}{c}(\mathbf{r} \times \omega) \times \mathbf{B}$ where ω is the angular velocity vector of the Earth's rotation. It may be verified without difficulty that if we use as a model of the geomagnetic field the model of dipole magnetic field with the magnetic moment anti-parallel to the Earth's rotation axis then the electric field of co-rotation satisfy conditions (2).

In [7] we made a number of extra simplifying assumptions about the MP's sizes (its radii of the order of some hundredths of micrometer), their stuff and MP's orbits in plasmasphere. The efficiency of sunlight pressure on MP is small for particles with sizes mentioned above [5], and we can neglect the influence of solar pressure on micro-particles' motion in the NES. We consider MP from a material with a low photoemission yield. For such particles in plasmasphere the micro-particles' potentials (and accordingly their charges) varies insignificantly during transit through the Earth's shade. Thus, in this case we can also neglect the dynamic effect of a shadowing of a part of micro-particle orbit. At last we limited our study to the case of micro-particles' motion on weakly oblong orbits (with eccentricities $<$ 0.3 - 0.4) with perigee's altitudes of the order of the Earth's radius. We observe that on orbits mentioned above it is possible to neglect disturbance's influence of the electric field of convection on micro-particles' motion.

For these assumptions the considered problem becomes one of motion of MP with variable electric charge in superposition of the central field of gravitation and Lorentz force acting on micro-particle's electric charge from a dipole magnetic field of the Earth and the electric field of co-rotation.

In spherical coordinate system (r, ϑ, φ) with the origin in the Earth center and the polar axis which is passing through a southern magnetic pole, the Hamilton function (5) can be written as follows

$$H = \frac{1}{2m}\left[P_r^2 + P_\vartheta^2 + \frac{1}{r^2 \cdot sin^2\vartheta} \cdot \left(P_\varphi + \frac{Y_{2\varphi} \cdot r \cdot sin\vartheta}{c}\right)^2\right] - \frac{m\mu_E}{r} + Y_1 \tag{6}$$

where μ_E is gravitational parameter of the Earth.

Value of an electric charge $Q(L)$ is determined as a result of the numerical solution of the balance equation of charging currents

$$J_{tot}(\Phi_{eq}) \equiv J_p^e + J_p^i + J_{ph} + J_{f-em} = 0, \tag{7}$$

here $\Phi_{eq} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ is the equilibrium potential, J_p^e is the current of plasma electrons, J_p^i is the total current of plasma ions, J_{ph} is the current of photoelectrons and J_{f-em} is the field emission current, ε_0 is the electric constant, and R is particle radius. In the plasmasphere the equilibrium charge is negative therefore the charging currents in (7) are determined by the expressions of [7].

We approximate value of a charge by a following polynomial

$$Q(L) = Q_0 \left[1 + \sum_{k=1}^{n} \xi_k \left(\frac{L - L_0}{L} \right)^k \right] \tag{8}$$

here Q_0 is the charge for $L = L_0$. We receive expansion coefficients ξ_k by substitution of expression (8) in the equation (7). It has been shown in [17] the increase of polynomial degree essentially improves accuracy of approximation even for a greater interval of values L .

To receive expressions for $Y_{2\varphi}$ and Y_1 we use dipolar coordinates L, Φ, M and the formula (8). Then

$$Y_{2\varphi} h_\Phi = -Q_0 B_E R_E^2 \int \left[1 + \sum_{k=1}^{n} \xi_k \left(\frac{L - L_0}{L} \right)^k \right] \frac{1}{L^2} dL, \tag{9}$$

$$Y_1 = Q_0 h_L \int \left[1 + \sum_{k=1}^{n} \xi_k \left(\frac{L - L_0}{L} \right)^k \right] E_{cor} dL, \tag{10}$$

here h_Φ is Lame coefficient, B_E is a magnetic field induction in the plane of magnetic equator ($\vartheta = 0$) on the Earth surface, R_E is the Earth radius, $E_{cor} = -\frac{CR_E}{r^2}$ is the intensity of the co-rotation field, C =92 kV.

Using this way when the co-rotation field is not taken into account we showed that as well as for a case of a constant charge, the eternal affinity of such osculating elements as semimajor axis, eccentricity and an angle of inclination of an orbit to a plane of magnetic equator to initial values should take place (this fact for a case of a constant charge is proved in [13]).These conclusions are confirmed by results of the numerical simulation, some of them are given in [17].

In [18] the analysis of influence of a field co-rotation on the motion of MP in the Earth plasmasphereis carried out. In [19] we obtained the analytical expression for the increment of the semimajor axis over the orbital period for the motion in the plane of geomagnetic equator. Also in [19] we proved analytically that if initial angle of orbit inclination does not equal zero, there is the frequency of the oscillation of the semimajor axis, an eccentricity and the angle of orbit inclination. This frequency is significant less than the frequency of the motion round the Earth. This fact is bound on existence of shift of a perigee point in an orbit plane. Results of [18] confirm these conclusions by data of numerical calculations.

3 Generalized theory of drift motion

In a strong magnetic field the basic force acting on a body is Lorentz's force which leads a particle to rotate round a magnetic field line. Thus nonmagnetic forces can be considered as the perturbation which leads to drift motion. [9] is the first paper to generalize the theory of drift motion of particles with constant electric charge [11] on a case of a variable charge.

If we present an current radius-vector of a particle **r** in the form of the sum of a radius-vector of the leading center **R** and a vector connecting the leading center and a particle ρ and write electric charge which does not change along a magnetic field line in a form

$$Q = Q_0 (1 + \rho \cdot \nabla_\perp Q)$$

and also average obtained equation on Larmor period and use expression for acceleration of the leading center that was obtained in [11], we can obtain expression for speed of drift across a magnetic field [9]. New addend in this expression is named "charging drift" by us in [9]. It is possible to consider influence of change of a charge on the motion of a body along a magnetic field by similar way. The numerical calculations for a case of a constant uniform magnetic field have shown validity of expression for charging drift. Application of the generalized drift theory to supersmall a component of space debris allows to create programs of numerical simulation of the motion of supersmall particles in magnetic fields of the Earth and major planets which give the chance to carry out calculations much faster.

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