Fixed point theorem for (α, β) -admissible mappings in metric-like space with respect to simulation function

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Abstract: In this paper, we have introduced some new type simulation function of (α, β) -admissible mappings with respect to Z -contraction in the complete metric-like space and our results improved several recent results in the metric-like space. Also, we supported example for justification of theorem.

Keywords: Simulation function, Admissible mapping, Fixed point, Contraction mapping.

1 Introduction and Preliminaries

It is well known that the Banach contraction principle [4, 16] is a fundamental result in the fixed point theory. Which have been extended in many directions. Also there are several generalization of usual metric spaces such that as quasi metric space, partial metric space, metric-like space etc.

In 2012 Samet et al [3] introduced the concept of α -contraction and α -admissible and established various fixed point theorems for such class of mappings defined on complete metric spaces. There after the existence of fixed point of α -admissible contraction type mappings in complete metric space has been studied by several researchers (see [6, 21, 22, 23, 30]) and references cited there in.

Hitzler and Seda [24] are the first who considered the concept of metric-like (or dislocated metric) spaces. Later Amini-Harandi [1] established some fixed point results in the class of metric-like space. Very recently many fixed point results on metric-like spaces have been provided. We have used some helpful article related to this papers [8, 12, 13, 17 – 20, 25, 26, 27, 31].

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Definition 1.1. [1, 9, 20] A metric $\sigma : X \times X \rightarrow R^+$, where X is a nonempty set is said to be metric-like (or dislocated metric) on X , if for any $x, y, z \in X$ the following three conditions hold

- (i) $\sigma(x, y) = 0$ implies $x = y$.
- (ii) $\sigma(x, y) = \sigma(y, x)$
- (iii) $\sigma(x, z) \leq \sigma(x, y) + \sigma(y, z)$

The pair (X, σ) is called a metric-like space.

Then a metric-like on X satisfies all of the conditions of a metric except that $\sigma(x, x)$ may be a positive for $x \in X$. Each metric-like σ on X generates a topology τ_σ on X , whose base is the family of open σ -balls, then for all $x \in X$ and $\varepsilon > 0$

$$B_\sigma(X, \varepsilon) = \{y \in X : \sigma(x, y) - \sigma(x, x) < \varepsilon\}.$$

Then a sequence $\{x_n\}$ in the metric-like space (X, σ) converges to a point $x \in X$ if and only if

$$\lim_{n \rightarrow \infty} \sigma(x_n, x) = \sigma(x, x).$$

Let (X, σ) be metric-like space and let $f : X \rightarrow X$ be a continuous mapping. Then

$$\lim_{n \rightarrow \infty} x_n = x \text{ implies } \lim_{n \rightarrow \infty} f(x_n) = f(x).$$

A sequence $\{x_n\}_{n=0}^\infty$ of elements of X is called σ -cauchy if the $\lim_{n, m \rightarrow \infty} \sigma(x_n, x_m)$ exists and is finite. The metric-like space (X, σ) is called complete, if for each cauchy sequence $\{x_n\}_{n=0}^\infty$, there is some $x \in X$ such that

$$\lim_{n \rightarrow \infty} \sigma(x_n, x) = \sigma(x, x) = \lim_{m, n \rightarrow \infty} \sigma(x_m, x_n).$$

Every partial metric space and metric space is a metric-like space.

Example 1.1. Let $X = \{1, 2, 3\}$ and $\sigma(x, y) = \begin{cases} 3 : \text{if } x = y \\ 2 : \text{otherwise} \end{cases}$. Then (X, σ) is a metric-like space. It is neither a partial metric space ($\sigma(1, 1) = 3$) and ($\sigma(1, 2) = 2$) nor a metric space ($\sigma(1, 1) = 2 \neq 0$)

Remark 1.1. [15] A subset A of a metric-like space (X, σ) is bounded if there is a point $b \in X$ and a positive constant K such that $\sigma(a, b) \leq K$ for all $a \in A$

Remark 1.2. [1, 15], Let $X = \{0, 1\}$ such that $\sigma(x, y) = 1$ for each $x, y \in X$ and let $x_n = 1$ for each $n \in N$. Then it is easy to see that $x_n \rightarrow 0$ and $x_n \rightarrow 1$ and so in metric-like space, the limit of a convergence sequence is not necessarily unique.

Example 1.2. [29] It is known that partial metric space is also a metric-like space. So, a trivial example of a metric-like space is the pair of (R_0^+, σ) , where $\sigma : R_0^+ \times R_0^+ \rightarrow R_0^+$ is defined as $\sigma(x, y) = \max\{x, y\}$.

In the following example, we give a metric-like, which is neither a metric nor a partial metric [12, 29].

Take $X = \{1, 2, 3\}$ and consider the metric-like $\sigma : X^2 \rightarrow R_0^+$ given by

$$\begin{aligned}\sigma(1, 1) &= 0, \sigma(2, 2) = 1, \sigma(3, 3) = \frac{2}{3}, \\ \sigma(1, 2) &= \sigma(2, 1) = \frac{9}{10}, \sigma(2, 3) = \sigma(3, 2) = \frac{4}{5}, \\ \sigma(1, 3) &= \sigma(3, 1) = \frac{7}{10}.\end{aligned}$$

Since $\sigma(2, 2) = 1 \neq 0$, so σ is not a metric space and $\sigma(2, 2) > \sigma(1, 2)$, so σ is not a partial metric.

Lemma 1.1. [1, 2, 9] Let (X, σ) be a metric-like space. Let $\{x_n\}$ be a sequence in X such that $x_n \rightarrow x$, where $x \in X$ and $\sigma(x, y) = 0$. Then for all $y \in X$ we have $\lim_{n \rightarrow \infty} \sigma(x_n, y) = \sigma(x, y)$.

Definition 1.2. [3, 14] Consider two mappings $f : X \rightarrow X$ and $\alpha : X \times X \rightarrow [0, \infty)$. Then f is called α -admissible mapping if for all $x, y \in X$ with $\alpha(x, y) \geq 1$ implies $\alpha(fx, fy) \geq 1$.

Definition 1.3. [5, 14] Let $T : X \rightarrow X$ and $\alpha : X \times X \rightarrow [0, \infty)$. Then T is called a triangular α -admissible mapping if

- (1) T is α -admissible
- (ii) $\alpha(x, z) \geq 1$ and $\alpha(z, y) \geq 1$ imply $\alpha(x, y) \geq 1$.

Chandok [28] who introduced the concept of (α, β) -admissible Geraghty type contractive mapping, which sufficient condition for the existence of a fixed point for such class of generalized non-linear contractive mapping in metric space proved some fixed point results. Defined as below

Definition 1.4. [28] Let X be a non empty set, $T : X \rightarrow X$ and $\alpha, \beta : X \times X \rightarrow R^+$, we say that T is an (α, β) -admissible mapping if $\alpha(x, y) \geq 1$ and $\beta(x, y) \geq 1$ implies $\alpha(Tx, Ty) \geq 1$ and $\beta(Tx, Ty) \geq 1$ for all $x, y \in X$.

Khojasteh, Shukla and Radenovic [7] presented the notion of Z-contraction with respect to ζ , that is a non-linear contraction involving a new class of mapping namely simulation

function. The Z-contraction generalized the Banach contraction and unify several known type of contractions involving the combination of $d(Tx, Ty)$ and $d(x, y)$ and satisfies some particular conditions in complete metric space. They studied the existence and uniqueness of fixed point for Z-contraction type operators.

Definition 1.5. [7] Let $\zeta : [0, \infty) \times [0, \infty) \rightarrow R$ be a mapping, then ζ is called a simulation function if it satisfies the following conditions.

$$(\zeta_1) \zeta(0, 0) = 0$$

$$(\zeta_2) \zeta(t, s) < s - t \text{ for all } s, t > 0$$

$$(\zeta_3) \text{ If } \{t_n\} \text{ and } \{s_n\} \text{ are sequences in } (0, \infty) \text{ such that } \lim_{n \rightarrow \infty} t_n = \lim_{n \rightarrow \infty} s_n > 0 \text{ then}$$

$$\limsup_{n \rightarrow \infty} \zeta(t_n, s_n) < 0.$$

We denote the set of all simulation function by Z .

Example 1.3. [7] Let $\zeta_i : [0, \infty) \times [0, \infty) \rightarrow R$, $i = 1, 2, 3$ be defined by

(i) $\zeta_1(t, s) = \psi(s) - \phi(t)$ for all $t, s \in [0, \infty)$, where $\phi, \psi : [0, \infty) \times [0, \infty) \rightarrow [0, \infty)$ are two continuous functions, such that $\psi(t) = \phi(t) = 0$ if and only if $t = 0$ and $\psi(t) < t \leq \phi(t)$ for all $t > 0$.

(ii) $\zeta_2(t, s) = s - \frac{f(t, s)}{g(t, s)}t$ for all $t, s \in [0, \infty)$, where $f, g : [0, \infty) \rightarrow [0, \infty)$ are two continuous functions with respect to each variable such that $f(t, s) > g(t, s)$ for all $t, s > 0$.

(iii) $\zeta_3(t, s) = s - \psi(s) - t$ for all $t, s \in [0, \infty)$, where $\psi : [0, \infty) \rightarrow [0, \infty)$ is a continuous function such that $\psi(t) = 0$ if and only if $t = 0$.

Then ζ_i for $i = 1, 2, 3$ are simulation function.

Definition 1.6. [7] Let (X, d) be a metric space, $T : X \rightarrow X$ a mapping and $\zeta \in Z$. Then T is called a Z-contraction with respect to ζ if the following condition is satisfied

$$\zeta(d(Tx, Ty), d(x, y)) \geq 0 \text{ for all } x, y \in X$$

A simple example of Z contraction is the Banach contraction, which can be obtained by taking $\lambda \in [0, 1)$ and $\zeta(t, s) = \lambda s - t$ for all $t, s \in [0, \infty)$ in above definition.

Remark 1.3. [7] It is clear from the definition of simulation function that $\zeta(t, s) < 0$ for all $t \geq s > 0$. Therefore, if T is a Z-contraction with respect to $\zeta \in Z$ then, for all distinct $x, y \in X$ such that

$$d(Tx, Ty) < d(x, y).$$

This shows that every Z-contraction mapping is contraction, therefore it is continuous.

Lemma 1.2. [2, 10] *Let (X, σ) be a metric-like space. Let $\{x_n\}$ be a sequence in X such that $x_n \rightarrow x$ where $x \in X$ and $\sigma(x, y) = 0$. Then for all $y \in X$, we have*

$$\lim_{n \rightarrow \infty} \sigma(x_n, y) = \sigma(x, y)$$

Lemma 1.3. [7] *Let (X, d) be a metric space and $f : X \rightarrow X$ be a Z -contraction with respect to a simulation function ζ , that is*

$$\zeta(d(fx, fy), d(x, y)) \geq 0 \quad \text{for all } x, y \in X.$$

Then T has a unique fixed point.

In this work, we introduce a new type of simulation function in the notion of (α, β) -admissible mapping Z -contraction with respect ζ and proved some fixed point results. Also, we supported example for the justification of the results.

2 Main results

Theorem 2.1. *Let (X, σ) be a complete metric-like space and a continuous mapping $f : X \rightarrow X$ be a (α, β) -admissible with respect to ζ simulation function satisfying as*

$$\zeta(\alpha(fx, fy)\beta(fx, fy)\sigma(fx, fy), \sigma(x, y)) \geq 0 \quad (2.1)$$

and there exist $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1, \beta(x_0, fx_0) \geq 1$. Then f has a unique fixed point $u \in X$.

Proof: Let $\{x_n\}$ be a sequence in X such that $x_{n+1} = fx_n$ for all $n = 0, 1, 2, \dots$. If $x_n = x_{n+1}$ then $fx_n = x_{n+1} = x_n$ i.e. x_n is a fixed point of f . So proof is trivial. Now, we consider $x_n \neq x_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$.

Since $\alpha(x_0, fx_0) \geq 1 \Rightarrow \alpha(x_0, x_1) \geq 1$ and f is an (α, β) -admissible, so

$$\alpha(fx_0, fx_1) \geq 1 \text{ implies } \alpha(x_1, x_2) \geq 1.$$

Continuing, we have for all $n \geq 0$

$$\alpha(x_n, x_{n+1}) \geq 1 \quad (2.2)$$

Similarly for all $n \geq 0$, we obtain

$$\beta(x_n, x_{n+1}) \geq 1. \quad (2.3)$$

From (2.1), we have

$$0 \leq \zeta(\alpha(fx_{n-1}, fx_n)\beta(fx_{n-1}, fx_n)\sigma(fx_{n-1}, fx_n), \sigma(x_{n-1}, x_n))$$

$$\begin{aligned}
&= \zeta(\alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}), \sigma(x_{n-1}, x_n)) \\
&< \sigma(x_{n-1}, x_n) - \alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}) \quad (\text{by } \zeta_2)
\end{aligned}$$

$$\alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}) < \sigma(x_{n-1}, x_n). \quad (2.4)$$

We know

$$\sigma(x_n, x_{n+1}) \leq \alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}) \quad (2.5)$$

since $\alpha(x_n, x_{n+1}) \geq 1$ and $\beta(x_n, x_{n+1}) \geq 1$.

From (2.4) and (2.5) for all $n \geq 0$, we have

$$\sigma(x_n, x_{n+1}) \leq \alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}) < \sigma(x_{n-1}, x_n). \quad (2.6)$$

$$\sigma(x_n, x_{n+1}) < \sigma(x_{n-1}, x_n). \quad (2.7)$$

The sequence $\{\sigma(x_n, x_{n+1})\}$ is non increasing i.e. decreasing. So, there exist $r \geq 0$ such that

$$\lim_{n \rightarrow \infty} \sigma(x_{n-1}, x_n) = r.$$

We prove that

$$\lim_{n \rightarrow \infty} \sigma(x_{n-1}, x_n) = 0. \quad (2.8)$$

Now, we assume on the contrary such that $r > 0$. By (2.6), we have

$$\lim_{n \rightarrow \infty} \{\alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1})\} = r.$$

Since $r > 0$ and letting $s_n = \alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1})$ and $t_n = \sigma(x_n, x_{n+1})$ such that $\lim_{n \rightarrow \infty} s_n = \lim_{n \rightarrow \infty} t_n = r$.

then by (ζ_3)

$$\limsup_{n \rightarrow \infty} \zeta(s_n, t_n) < 0$$

Since $\zeta(s_n, t_n) \geq 0$, so

$$0 \leq \limsup_{n \rightarrow \infty} \zeta(s_n, t_n) < 0.$$

Which is contradiction. So our assumption is false.

Hence $r = 0$.

Again we show that $\{x_n\}$ is a cauchy sequence in (X, σ) i.e.

$$\lim_{n, m \rightarrow \infty} \sigma(x_n, x_m) = 0. \quad (2.9)$$

Suppose on the contrary i.e. $\{x_n\}$ is not a cauchy sequence. Then there exist $\varepsilon > 0$ for which we can assume subsequences $x_{n(k)}$ and $x_{m(k)}$ of x_n with $n(k) > m(k) > k$ such that for every k

$$\sigma(x_{n(k)}, x_{m(k)}) \geq \varepsilon. \quad (2.10)$$

and $n(k)$ is the smallest number such that (10) holds.

From (2.10), we get

$$\sigma(x_{n(k)-1}, x_{m(k)}) < \varepsilon. \quad (2.11)$$

Then by triangular inequality and (2.9), we have

$$\begin{aligned} \varepsilon &\leq \sigma(x_{n(k)}, x_{m(k)}) \leq \sigma(x_{n(k)}, x_{n(k)-1}) + \sigma(x_{n(k)-1}, x_{m(k)}) \\ &< \sigma(x_{n(k)}, x_{n(k)-1}) + \varepsilon. \end{aligned}$$

Taking $n \rightarrow \infty$ in above equation and applying (2.8), we get

$$\begin{aligned} \varepsilon &\leq \lim_{n \rightarrow \infty} \sigma(x_{n(k)}, x_{m(k)}) < \varepsilon \\ \lim_{n \rightarrow \infty} \sigma(x_{n(k)}, x_{m(k)}) &= \varepsilon. \end{aligned} \quad (2.12)$$

From the triangular inequality, we have

$$\sigma(x_{n(k)+1}, x_{m(k)}) \leq \sigma(x_{n(k)+1}, x_{n(k)}) + \sigma(x_{n(k)}, x_{m(k)})$$

taking limit $n \rightarrow \infty$ and using (2.8), (2.10) and (2.12) we have

$$\lim_{n \rightarrow \infty} \sigma(x_{n(k)+1}, x_{m(k)}) = \varepsilon. \quad (2.13)$$

Similarly, it is easy to show that

$$\lim_{n \rightarrow \infty} \sigma(x_{n(k)+1}, x_{m(k)+1}) = \varepsilon. \quad (2.14)$$

Since f is an (α, β) -admissible Z-contraction with respect to ζ and using (ζ_3)

$$0 \leq \limsup_{n \rightarrow \infty} \zeta(\alpha(fx_{n(k)}, fx_{m(k)})\beta(fx_{n(k)}, fx_{m(k)})\sigma(fx_{n(k)}, fx_{m(k)}), \sigma(x_{n(k)}, x_{m(k)}))$$

$$0 \leq \limsup_{n \rightarrow \infty} \zeta(\alpha(x_{n(k)+1}, x_{m(k)+1})\beta(x_{n(k)+1}, x_{m(k)+1})\sigma(x_{n(k)+1}, x_{m(k)+1}), \sigma(x_{n(k)}, x_{m(k)})) < 0.$$

Which is contradict due to our assumption. So $\{x_n\}$ is a cauchy sequence.

Since (X, σ) be a complete metric-like space, then there exist $x \in X$ and using (2.9) such that

$$\lim_{n \rightarrow \infty} \sigma(x_n, x) = \sigma(x, x) = \lim_{n, m \rightarrow \infty} \sigma(x_n, x_m) = 0. \quad (2.15)$$

We show that x is fixed point of f . Since f is continuous and $x_n \rightarrow x$ as $n \rightarrow \infty$. So, from (2.15)

$$\lim_{n \rightarrow \infty} \sigma(x_{n+1}, fx) = \lim_{n \rightarrow \infty} \sigma(fx_n, fx) = \sigma(fx, fx) = 0. \quad (2.16)$$

Using lemma (1.1) d (2.16) , we have

$$\lim_{n \rightarrow \infty} \sigma(x_{n+1}, fx) = \sigma(x, fx). \quad (2.17)$$

From (2.16) and (2.17)

$$\sigma(x, fx) = \sigma(fx, fx) = 0. \quad (2.18)$$

Hence $fx = x$, that is x is a fixed point of f .

Now, we shall show that x is unique . We argue by contrary. Assume that there exist $y \in X$ such that $fy = y$ and $x \neq y$.

Now,

$$\begin{aligned} 0 &\leq \zeta(\alpha(fx, fy)\beta(fx, fy)\sigma(fx, fy), \sigma(x, y)) \\ &\leq \sigma(x, y) - \alpha(fx, fy)\beta(fx, fy)\sigma(fx, fy) \\ &= \sigma(x, y) - \alpha(x, y)\beta(x, y)\sigma(x, y) \\ &= \sigma(x, y)[1 - \alpha(x, y)\beta(x, y)] < 0 \end{aligned}$$

since $\alpha(x, y) \geq 1, \beta(x, y) \geq 1$.

Which is contradiction. So $x = y$. Hence f has a unique fixed point

Example 2.1. Consider $X = \{a, b, c\}$ for all $a, b, c \in R^+$ and metric space $\sigma : X \times X \rightarrow R^+$ defined as following

$$\begin{aligned} \sigma(a, c) &= \frac{1}{2} = \sigma(c, a), \sigma(a, a) = 0 \\ \sigma(a, b) &= \frac{3}{2} \sigma(b, a), \sigma(b, b) = 1 \\ \sigma(b, c) &= \frac{2}{3} = \sigma(c, b), \sigma(c, c) = \frac{2}{3}. \end{aligned}$$

So (X, σ) be a metric-like space. Consider a mapping $f : X \rightarrow X$ such that $f(a) = a, f(b) = c, f(c) = b$ and $\alpha, \beta : X \times X \rightarrow R^+$, such that $\alpha(x, y) \geq 1$ and $\beta(x, y) \geq 1$ imply $\alpha(fx, fy) \geq 1$ & $\beta(fx, fy) \geq 1$ i.e. f is an (α, β) -admissible mapping defined as

$$\alpha(x, y) = \begin{cases} \frac{5}{3} : if x, y \in X \\ 0 : otherwise \end{cases} \quad \beta(x, y) = \begin{cases} \frac{7}{2} : if x, y \in X \\ 0 : otherwise. \end{cases}$$

Consider $\zeta(t, s) = \lambda s - t$, where $\lambda \in R^+$, taking $\lambda = 35$.

Now, we consider the following cases,

Case-I At (a, a)

$$\begin{aligned} & \zeta(\alpha(fa, fa)\beta(fa, fa)\sigma(fa, fa), \sigma(a, a)) \\ &= \zeta(\alpha(a, a)\beta(a, a)\sigma(a, a), \sigma(a, a)) \\ &= \zeta(0, \frac{5}{2} \cdot \frac{7}{2}, 0) = \zeta(0, 0) = 0 \geq 0. \end{aligned}$$

Case-II At (b, b)

$$\begin{aligned} & \zeta(\alpha(fb, fb)\beta(fb, fb)\sigma(fb, fb), \sigma(b, b)) \\ &= \zeta(\alpha(c, c)\beta(c, c)\sigma(c, c), \sigma(b, b)) \\ &= \zeta(\frac{2}{3} \cdot \frac{5}{2} \cdot \frac{7}{2}, 1) = \zeta(\frac{35}{6}, 1) = 35 \cdot 1 - \frac{35}{6} \geq 0. \end{aligned}$$

Case-III At (c, c)

$$\begin{aligned} & \zeta(\alpha(fc, fc)\beta(fc, fc)\sigma(fc, fc), \sigma(c, c)) \\ &= \zeta(\alpha(b, b)\beta(b, b)\sigma(b, b), \sigma(c, c)) \\ &= \zeta(1, \frac{5}{2} \cdot \frac{7}{2}, \frac{2}{3}) = \zeta(\frac{35}{4}, \frac{2}{3}) = 35 \cdot \frac{2}{3} - \frac{35}{4} \geq 0. \end{aligned}$$

Case-IV At (a, b)

$$\begin{aligned} & \zeta(\alpha(fa, fb)\beta(fa, fb)\sigma(fa, fb), \sigma(a, b)) \\ &= \zeta(\alpha(a, c)\beta(a, c)\sigma(a, c), \sigma(a, b)) \\ &= \zeta(\frac{5}{2} \cdot \frac{7}{2} \cdot \frac{1}{2}, \frac{3}{2}) = \zeta(\frac{35}{8}, \frac{3}{2}) = 35 \cdot \frac{3}{2} - \frac{35}{8} \geq 0. \end{aligned}$$

Case-V At (a, c)

$$\begin{aligned} & \zeta(\alpha(fa, fc)\beta(fa, fc)\sigma(fa, fc), \sigma(a, c)) \\ &= \zeta(\alpha(a, b)\beta(a, b)\sigma(a, b), \sigma(a, c)) \\ &= \zeta(\frac{5}{2} \cdot \frac{7}{2} \cdot \frac{3}{2}, \frac{1}{2}) = \zeta(\frac{105}{8}, \frac{1}{2}) = 35 \cdot \frac{3}{2} - \frac{105}{8} \geq 0. \end{aligned}$$

Case-VI At (b, c)

$$\zeta(\alpha(fb, fc)\beta(fb, fc)\sigma(fb, fc), \sigma(b, c))$$

$$\begin{aligned}
&= \zeta(\alpha(c, b)\beta(c, b)\sigma(c, b), \sigma(b, c)) \\
&= \zeta(\frac{5}{2} \cdot \frac{7}{2} \cdot \frac{2}{3}, \frac{2}{3}) = \zeta(\frac{35}{6}, \frac{2}{3}) = 35 \cdot \frac{2}{3} - \frac{35}{6} \geq 0.
\end{aligned}$$

Thus f is a an (α, β) -admissible mapping Z -contraction with respect to ζ . Hence all conditions of the main result are satisfied and hence f has a unique fixed point, which is a .

Corollary 2.1. *In the main result, if we have choose any one of the ζ simulation given below, we have the same result and proof are similar to these corollary.*

$$\zeta(\alpha(x, fx)\beta(y, fy)\sigma(fx, fy), \sigma(x, y)) \geq 0 \quad (2.19)$$

$$\zeta(\alpha(x, y)\beta(fx, fy)\sigma(fx, fy), \sigma(x, y)) \geq 0 \quad (2.20)$$

$$\zeta(\alpha(x, y)\beta(x, y)\sigma(fx, fy), \sigma(x, y)) \geq 0 \quad (2.21)$$

$$\zeta(\alpha(fx, fy)\beta(x, y)\sigma(fx, fy), \sigma(x, y)) \geq 0 \quad (2.22)$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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