Generalized 2–proximal C –contraction mappings in complete ordered 2–metric space and their best proximity points

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Abstract: In this paper, we extend the concept of best proximity point to 2–metric spaces and prove the existence of such points for contraction type non-self mappings in the setting of complete 2–metric spaces. Also, we presented an example to support our results.

Keywords: 2–proximal C –contraction mapping, best proximity point, 2-metric space.

1 Introduction

There are many kinds of fixed point theorems in the literature. Some of fixed point results for generalized contractions have been studied by L.N.Mishra et al ([4, 16, 17, 18, 21]). In 2011, Harjani et al [11] have presented some fixed point result for weakly C – contraction mappings in an ordered complete metric space.

Let $T : A \longrightarrow B$ be a non-self mapping where A, B are non-empty subsets of a metric space (X, d) .

Let $T : A \longrightarrow B$ be a non-self mapping where A, B are non-empty subsets of a metric space (X, d) . Then T may not have a fixed point. In this case, $d(x, Tx) > 0$ and it is important that we find an element $x \in A$ such that $d(x, Tx)$ is minimum in some sense. For example the best approximation problem and best proximity problem are investigated in this regard.

An element $x \in A$ is said to be the best proximity point of T if $d(x, Tx) = d(A, B)$ where

$$d(A, B) = \inf\{d(x, y) : x \in A, y \in B\}$$

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It is easy to check that if T is self-mapping then the best proximity problem reduces to the fixed point problem. There are several various of contractions that guarantee the existence of a best proximity point.

In 2013 C.Mongkolkeha, Y.J.Cho and P.Kumam [19] extended the notion of weakly C -contraction to the case of non-self mapping and established a best proximity theorem for this class.

In 1960's Gähler [9] have introduced the concept of 2-metric space and extend various notions in metric space to 2-metric space. A 2-metric on non-empty set X is a non-negative real function σ on $X \times X \times X$ satisfying the following conditions:

(2M₁) For two different elements a and b in X there is element c in X such that $\sigma(a, b, c) \neq 0$,

(2M₂) $\sigma(a, b, c) = 0$ when two of the three elements are equal,

(2M₃) $\sigma(a, b, c) = \sigma(a, c, b) = \sigma(b, c, a)$

(2M₄) $\sigma(a, b, c) \leq \sigma(a, b, d) + \sigma(a, d, c) + \sigma(d, b, c)$ for any d in X (rectangle inequality).

The pair (X, σ) is called 2-metric space. In the prototypical example, $\sigma(a, b, c)$ is the area of triangle spanned by a, b, c .

A sequence (x_n) in 2-metric space (X, σ) is called convergent to x if for any $a \in X$,

$$\lim_{n \rightarrow +\infty} \sigma(x_n, x, a) = 0.$$

(x_n) is called Cauchy if for any $a \in X$, $\lim_{n, m \rightarrow +\infty} \sigma(x_n, x_m, a) = 0$. We say that X is complete if every Cauchy sequence in X is a convergent sequence.

Several authors studied the question of fixed point theorems for mapping on 2-metric spaces (see [1], [2], [5], [6], [7], [10], [12], [20] and [26]). In 2013, Nguyen Van Dung and Vo Thi Le Hang [7] gave a sufficient condition for the existence of a fixed point of a weak C -contraction on a 2-metric space. Also, in 2014 Tran Van An, Nguyen Van Dung and Vo Thi Le Hang [2] stated a form of a fixed point theorem on metric and 2-metric space.

Let A and B are non-empty subset of 2-metric space (X, σ) and $x \in X$ then we define $\sigma(A, B, x)$ as follow

$$\sigma(A, B, x) = \inf\{\sigma(a, b, x) \mid a \in A, b \in B\}.$$

Let X be a non-empty set such that $(X, \preceq)$ is partially ordered set and let (X, σ) be 2-metric space. Let A and B be non-empty subsets of 2-metric space (X, σ) . Now, we define A_0 and B_0 as following;

$$A_0 = \{a \in A \mid \text{there exists } b \in B, \text{ such that for any } x \in X; \sigma(a, b, x) = \sigma(A, B, x)\}$$

$$B_0 = \{b \in B \mid \text{there exists } a \in A, \text{ such that for any } x \in X; \sigma(a, b, x) = \sigma(A, B, x)\}.$$

It is easy to check that if $A \cap B \neq \emptyset$ then A_0 and B_0 are non-empty. A mapping $T : A \rightarrow B$ is said to be increasing if

$$a_1 \preceq a_2 \text{ implies } T(a_1) \preceq T(a_2) \quad \text{for all } a_1, a_2 \in A.$$

T is said to be proximally order-preserving if it satisfies $a_1 \preceq a_2$, $\sigma(u, T(a_1), x) = \sigma(A, B, x)$ and $\sigma(v, T(a_2), x) = \sigma(A, B, x)$ then $u \preceq v$, for all $u, v, a_1, a_2 \in A$ and $x \in X$.

Let A and B be two non-empty subsets of a 2-metric space (X, σ) and $T : A \rightarrow B$ be a non-self mapping. Clearly, if $A \cap T(A) = \emptyset$; the fixed point equation $Tx = x$ has no solution. In this case, for every $a \in A$ there exists $x \in X$ such that $\sigma(a, Ta, x) > 0$. The aim of best proximity theory is to find $a \in A$ such that $\sigma(a, Ta, x)$ is minimum for all $x \in X$ and so to guarantee the existence a best proximal- point of T , named $a \in X$ that is, $\sigma(A, B, x) = \sigma(a, Ta, x)$ for all $x \in X$. In view of the fact that $\sigma(a, T(a), x) \geq \sigma(A, B, x)$ for all $a \in A$, $x \in X$, it can be observed that the global minimum of the mapping $a \mapsto \sigma(a, Ta, x)$ is attained from a 2-best proximity point. It is easy to see that if underlying mapping T is a self-mapping then the best proximity point problem reduces to fixed point problem in 2-metric space. In this paper, we extend the notions of generalized proximal C -contraction non-self mapping and best proximity point in metric space to the case of 2-metric space and establish a best proximity theorem for this class. An example is given to validate the results.

2 The main results

In this section, we define the generalized 2-proximal C -contraction mapping for 2-metric space and prove a best proximity theorem for this class.

Throughout this paper, X is complete 2-metric space that is equipped with a partial order $\preceq$.

Definition 1. Let A and B be non-empty subsets of X . A point $a \in A$ is called a 2-proximity point of mapping $T : A \rightarrow B$ if

$$\sigma(a, T(a), x) = \sigma(A, B, x)$$

for all $x \in X$.

Definition 2. Let A and B be non-empty subsets of X and $T : A \longrightarrow B$ be a map such that $\sigma(u, T(a_1), x) = \sigma(A, B, x)$ and $\sigma(v, T(a_2), x) = \sigma(A, B, x)$ whenever $a_1, a_2, u, v \in A, a_1 \preceq a_2$ and $x \in X$. Then T is said to be generalized 2-proximal C -contraction if

$$\sigma(u, v, x) \leq \frac{1}{2}(\sigma(a_1, v, x) + \sigma(a_2, u, x)) - \eta(\sigma(a_1, v, x), \sigma(a_2, u, x)). \quad (1)$$

where $\eta : [0, +\infty) \times [0, +\infty) \longrightarrow [0, +\infty)$ is continuous and non-decreasing function such that $\eta(x, y) = 0$ if and only if $x = y = 0$

Lemma 1. Let A and B be non-empty closed subsets of X such that A_0 and B_0 are non-empty. Let $T : A \longrightarrow B$ satisfy the following conditions:

- (1) T is a continuous, proximally order-preserving and generalized 2-proximal C -contraction such that $T(A_0) \subset B_0$;
- (2) There exists element a_0 and a_1 in A such that $a_0 \preceq a_1$ and $\sigma(a_1, Ta_0, x) = \sigma(A, B, x)$, for every $x \in X$.

Then there exists a sequence $\{a_n\}$ in A such that $\sigma(a_{n-1}, a_n, a_{n+1}) = 0$.

Proof. By the hypothesis (2), there exist $a_0, a_1 \in A_0$ such that $a_0 \preceq a_1$ and $\sigma(a_1, Ta_0, x) = \sigma(A, B, x)$ For all $x \in X$. Since $T(A_0) \subset B_0$ there exists a point $a_2 \in A_0$ such that for any $x \in X$

$$\sigma(a_2, Ta_1, x) = \sigma(A, B, x). \quad (2)$$

By the proximally order-preserving property of T , we get $a_1 \preceq a_2$. continuing this process, we can find a sequence a_n in A_0 such that $a_{n-1} \preceq a_n$ and for every $x \in X$

$$\sigma(a_n, Ta_{n-1}, x) = \sigma(A, B, x). \quad (3)$$

Having found the point a_n , one can choose a point $a_{n+1} \in A_0$ such that $a_n \preceq a_{n+1}$ and for any $x \in X$

$$\sigma(a_{n+1}, Ta_n, x) = \sigma(A, B, x). \quad (4)$$

Now we prove that

$$\sigma(a_{n-1}, a_{n+1}, a_n) = 0.$$

By the condition $(2M_3)$ of definition of 2-metric space we have

$$\sigma(a_{n-1}, a_{n+1}, a_n) = \sigma(a_n, a_{n+1}, a_{n-1})$$

by (3), (4) and proximally order-preserving property of T , we have

$$\begin{aligned} \sigma(a_n, a_{n+1}, a_{n-1}) &\leq \frac{1}{2}(\sigma(a_{n-1}, a_{n+1}, a_{n-1}) + \sigma(a_n, a_n, a_{n-1})) \\ &\quad - \eta(\sigma(a_{n-1}, a_{n+1}, a_{n-1}), \sigma(a_n, a_n, a_{n-1})) \end{aligned}$$

and so

$$\sigma(a_n, a_{n+1}, a_{n-1}) \leq 0$$

therefore

$$\sigma(a_n, a_{n+1}, a_{n-1}) = 0. \tag{5}$$

□

Theorem 1. *Let A and B be non-empty closed subsets of X such that A_0 and B_0 are non-empty. Let $T : A \rightarrow B$ satisfy the conditions of Lemma 1. Then there exists a point $a \in A$ such that for every $x \in X$, $\sigma(a, Ta, x) = \sigma(A, B, x)$ (a is best proximity point for T).*

Proof. Let (a_n) be the sequence that we construct in proof of Lemma 1. Choose $x \in X$ arbitrary and fix it. We have $\sigma(a_{n+1}, Ta_n, x) = \sigma(A, B, x)$.

Since T is a generalized 2-proximal C -contraction, for each $n \in \mathbb{N}$ we have

$$\begin{aligned} \sigma(a_n, a_{n+1}, x) &\leq \frac{1}{2}(\sigma(a_{n-1}, a_{n+1}, x) + \sigma(a_n, a_n, x)) - \eta(\sigma(a_{n-1}, a_{n+1}, x), \sigma(a_n, a_n, x)) \\ &= \frac{1}{2}(\sigma(a_{n-1}, a_{n+1}, x) - \eta(\sigma(a_{n-1}, a_{n+1}, x), 0)) \leq \frac{1}{2}(\sigma(a_{n-1}, a_{n+1}, x) \leq \frac{1}{2}(\sigma(a_{n-1}, a_n, x) \\ &\quad + \sigma(a_n, a_{n+1}, x) + \sigma(a_{n-1}, a_{n+1}, a_n)) - \eta(\sigma(a_{n-1}, a_{n+1}, x), 0). \end{aligned}$$

Therefore

$$\begin{aligned} \sigma(a_n, a_{n+1}, x) &\leq \frac{1}{2}(\sigma(a_{n-1}, a_{n+1}, x) \leq \frac{1}{2}(\sigma(a_{n-1}, a_n, x) + \\ &\quad \sigma(a_n, a_{n+1}, x) + \sigma(a_{n-1}, a_{n+1}, a_n)) - \eta(\sigma(a_{n-1}, a_{n+1}, x), 0). \end{aligned} \tag{6}$$

By proof of Lemma 1 we have; $\sigma(a_{n-1}, a_{n+1}, a_n) = 0$. and so by (6) we have

$$\sigma(a_n, a_{n+1}, x) \leq \frac{1}{2}(\sigma(a_{n-1}, a_n, x) + \sigma(a_n, a_{n+1}, x)) - \eta(\sigma(a_{n-1}, a_n, x), 0)$$

so

$$\frac{1}{2}\sigma(a_n, a_{n+1}, x) \leq \frac{1}{2}\sigma(a_{n-1}, a_n, x) - \eta(\sigma(a_{n-1}, a_n, x), 0) \leq \frac{1}{2}\sigma(a_{n-1}, a_n, x),$$

consequently $\sigma(a_n, a_{n+1}, x) \leq \sigma(a_{n-1}, a_n, x)$. Therefore $\sigma_n = \sigma(a_n, a_{n+1}, x)$ is non-increasing sequence. Since σ_n is bounded below, there exists $r_x \geq 0$ such that

$$\lim_{n \rightarrow +\infty} \sigma(a_n, a_{n+1}, x) = r_x \quad (7)$$

Taking $n \rightarrow +\infty$ in (6), we have

$$r_x \leq \lim_{n \rightarrow +\infty} \sigma(a_n, a_{n+1}, x) \leq \frac{1}{2}(r_x + r_x) = r_x$$

and so

$$\lim_{n \rightarrow +\infty} \sigma(a_n, a_{n+1}, x) = 2r_x. \quad (8)$$

Again, taking $n \rightarrow +\infty$ in (6) and using (7), (8) and the continuity of η , we get

$$r_x \leq \frac{1}{2}(2r_x) = r_x - \eta(2r_x, 0) \leq r_x \quad (9)$$

and hence $\eta(2r_x, 0) = 0$. therefore

$$\lim_{n \rightarrow +\infty} \sigma(a_{n+1}, a_n, x) = 0 \quad (10)$$

$$\lim_{n \rightarrow +\infty} \sigma(Ta_n, a_n, x) = 0. \quad (11)$$

Now we claim that (a_n) is Cauchy sequence in 2-metric space X . Suppose that (a_n) is not Cauchy sequence. Then there exists $\varepsilon > 0$ such that for every $k \in \mathbb{N}$, there exist n_k, m_k such that

$$n_k, m_k \geq k, \quad r_k := \sigma(a_{m_k}, a_{n_k}, x) \geq \varepsilon, \quad \sigma(a_{n_k-1}, a_{m_k}, x) < \varepsilon.$$

For each $n \in \mathbb{N}$, let $\alpha_n = \sigma(a_{n+1}, a_n, x)$ then

$$\varepsilon \leq r_k \leq \sigma(a_{n_k-1}, a_{m_k}, x) + \sigma(a_{n_k}, a_{n_k-1}, x) + \sigma(a_{n_k}, a_{m_k}, a_{n_k-1})$$

and so

$$\varepsilon \leq r_k < \varepsilon + \alpha_{n_k-1} + \sigma(a_{n_k}, a_{n_k-1}, a_{m_k}). \quad (12)$$

Taking $K \rightarrow +\infty$ in (12); we have

$$\lim_{k \rightarrow +\infty} r_k = \varepsilon. \quad (13)$$

On the other hand; we have

$$\begin{aligned}
 r_k &= \sigma(a_{n_k}, a_{m_k}, x) \leq \sigma(a_{m_k+1}, a_{m_k}, x) + \sigma(a_{n_k}, a_{m_k+1}, x) + \sigma(a_{n_k}, a_{m_k}, a_{m_k+1}) = \\
 &= n\alpha_{m_k} + \sigma(a_{n_k}, a_{m_k+1}, x) + \sigma(a_{n_k}, a_{m_k}, a_{m_k+1}) \leq \sigma(a_{m_k}, a_{m_k+1}, x) + \sigma(a_{n_k}, a_{m_k}, x) \\
 &+ \sigma(a_{n_k}, a_{m_k+1}, a_{m_k}) + \alpha_{m_k} + \sigma(a_{m_k}, a_{m_k+1}, a_{n_k}) = \sigma(a_{m_k}, a_{m_k+1}, x) + \\
 &\sigma(a_{n_k}, a_{m_k}, x) + 2\sigma(a_{m_k}, a_{m_k+1}, a_{n_k}) + \alpha_{m_k} = 2\alpha_{m_k} + r_k + 2\sigma(a_{m_k}, a_{m_k+1}, a_{n_k})
 \end{aligned} \tag{14}$$

Taking $k \rightarrow +\infty$ in (14), we get

$$\varepsilon \leq \sigma(a_{n_k}, a_{m_k+1}, x) \leq \varepsilon$$

and so

$$\lim_{k \rightarrow +\infty} \sigma(a_{n_k}, a_{m_k+1}, x) = \varepsilon. \tag{15}$$

In the same way, we can prove that

$$\lim_{k \rightarrow +\infty} \sigma(a_{m_k}, a_{n_k+1}, x) = \varepsilon. \tag{16}$$

On the other hand, by the construction of a_n , we may assume that $a_{m_k} \leq a_{n_k}$ such that

$$\sigma(a_{n_k+1}, a_{n_k}, x) = \sigma(A, B, x), \tag{17}$$

$$\sigma(a_{m_k+1}, a_{m_k}, x) = \sigma(A, B, x). \tag{18}$$

By rectangle inequality, (17), (18) and the generalized 2-proximal C -contraction of T , we have

$$\begin{aligned}
 \varepsilon &\leq r_k = \sigma(a_{m_k}, a_{n_k}, x) \leq \sigma(a_{m_k+1}, a_{n_k}, x) + \sigma(a_{m_k}, a_{m_k+1}, x) + \sigma(a_{m_k}, a_{n_k}, a_{m_k+1}) \\
 &= \alpha_{m_k} + \sigma(a_{m_k+1}, a_{n_k}, x) + \sigma(a_{m_k}, a_{n_k}, a_{m_k+1}) \leq \sigma(a_{n_k+1}, a_{n_k}, x) + \\
 &\sigma(a_{m_k+1}, a_{n_k+1}, x) + \sigma(a_{m_k+1}, a_{n_k}, a_{m_k+1}) + \alpha_{m_k} + \sigma(a_{m_k}, a_{n_k}, a_{m_k+1}) \leq \\
 &\alpha_{n_k} + \alpha_{m_k} + \sigma(a_{m_k}, a_{n_k+1}, a_{n_k+1}) + \sigma(a_{m_k}, a_{n_k+1}, a_{n_k+1}) + \sigma(a_{m_k}, a_{n_k}, a_{m_k+1}) + \\
 &\frac{1}{2}[\sigma(a_{n_k}, a_{m_k+1}, x) + \sigma(a_{m_k}, a_{n_k+1}, x)] - \eta(\sigma(a_{n_k}, a_{m_k+1}, x), \sigma(a_{m_k}, a_{n_k+1}, x)).
 \end{aligned}$$

Taking $k \rightarrow +\infty$ in above inequality and by (10), (11), (13), (14) and the continuity of η , we have

$$\varepsilon \leq \frac{1}{2}(\varepsilon + \varepsilon) - \eta(\varepsilon, \varepsilon) \leq \varepsilon$$

so

$$\eta(\varepsilon, \varepsilon) = 0$$

and hence

$$\varepsilon = 0$$

which is a contradiction with $\varepsilon > 0$.

Since A is a closed subset of the complete 2-metric space X , there exists $a \in A$ such that $a_n \rightarrow a$. Letting $n \rightarrow +\infty$ in (4), by continuity of T ; it follow that

$$\sigma(a_n, Ta_n, x) = \sigma(A, B, x).$$

□

Corollary 1. *Let A and B be non-empty closed subsets of X such that A_0 and B_0 are non-empty. Let $T : A \rightarrow B$ satisfies the following conditions:*

(1) *T is a continuous, increasing such that $T(A_0) \subset B_0$.*

(2) *If for all $a_1, a_2, u, v \in A$, and $x \in X$, it satisfies $a_1 \preceq a_2$, $\sigma(u, T(a_1), x) = \sigma(A, B, x)$ and $\sigma(u, T(a_1), x) = \sigma(A, B, x)$*

$$\sigma(u, v, x) \leq \alpha(\sigma(a_1, v, x) + \sigma(a_2, u, x)) \quad (19)$$

where $\alpha \in (0, \frac{1}{2})$.

(3) *For every $x \in X$ there exist elements a_0 and a_1 in A_0 such that $a_0 \preceq a_1$ and*

$$\sigma(a_1, Ta_0, x) = \sigma(A, B, x).$$

Then there exists a point $a \in A$ such that

$$\sigma(a, Ta, x) = \sigma(A, B, x).$$

Moreover, for any fixed point $a_0 \in A_0$, the sequence a_n defined by

$$\sigma(a_{n+1}, Ta_n, x) = \sigma(A, B, x)$$

convergence to point a .

Proof. $\alpha \in (0, \frac{1}{2})$ define η as follow

$$\eta(a, b) = (\frac{1}{2} - \alpha)(a + b)$$

it follows

$$\eta(a, b) = 0 \quad \text{if and only if} \quad a = b = 0$$

and we have

$$\frac{1}{2}(a + b) - \eta(a, b) = \frac{1}{2}(a + b) - (\frac{1}{2} - \alpha)(a + b) = \alpha(a + b).$$

So equation (1) becomes equation (19). So T satisfy the conditions of Theorem 1 and hence we obtain the Corollary 1. $\square$

The following example support the main result of our paper or the choice is of authors.

Example 1. Let $X = \{0, 2, 4\}$, $A = \{0, 2\}$, $B = \{0, 4\}$ and σ be a 2-metric on X defined by

$$\sigma(x, y, z) = \min\{|x - y|, |x - z|, |y - z|\}.$$

Suppose the mapping $T : A \longrightarrow B$ be defined as follow

$$T(0) = 0, \quad T(2) = 4.$$

It is easy to check that $A_0 = \{0\}$, $B_0 = \{0\}$. T satisfy in conditions of corollary 1 and we have

$$\sigma(0, T(0), x) = \sigma(A, B, x) = 0$$

for every $x \in X$. So 0 is best proximity point for T .

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