Fixed point Theorems for Cyclic-Ćirić-Reich-Rus contraction mapping in quasi-partial b-metric spaces

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Abstract: In this paper, Ćirić-Reich-Rus cyclic contraction mapping is defined in the setting of quasi-partial b-metric spaces and fixed point results are proved. Some examples are given to validate our results.

Keywords: Quasi-Partial b-Metric space, Fixed point theorems, qp_b Ćirić-Reich-Rus Cyclic Contraction mapping.

1 Introduction

The French mathematician Maurice Frechet initiated the study of metric spaces [1] in 1905. In 1989, Bakhtin [2] introduced the concept of b-metric space and gave the contraction mapping which was the generalization of the Banach Contraction Principle. In 1993, Czerwik [3] extended this concept of b-metric spaces whereas Shukla introduced partial b metric [4] in 2014. The concept of Partial-metric spaces was introduced by Matthews [5] in 1994 as a generalization of standard metric spaces by replacing the condition $d(x, x) = 0$ with the condition $d(x, x) \leq d(x, y)$, for all x, y . As a further generalization for quasi-metric spaces and partial-metric spaces, Karapinar [6] introduced the notion of quasi-partial metric space and discussed the existence of fixed points of self-mappings T on quasi-partial metric spaces. Gupta and Gautam [7] further, generalized quasi-partial metric spaces to the class of quasi-partial b-metric spaces and proved some fixed point results ([8],[9]) in the setting of quasi-partial metric space.

In 1974, Ćirić [10] extend quasi-contraction mappings and stated some fixed point results in which it has shown that the condition of quasi-contractivity implies all conclusions

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of Banach's contraction principle:

Let (X, d) be a metric space. A mapping $T : X \rightarrow X$ is said to be a quasi-contraction mapping if there exists $\beta \in [0, 1)$ such that

$$d(Tx, Ty) \leq M(x, y)$$

for all $x, y \in X$, where

$$M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}.$$

Reich [11] generalized concepts of Kannan and Banach by introducing the following mapping:

A mapping $T : X \rightarrow X$ is said to be a Reich-contraction mapping if there are $\alpha, \beta, \gamma \in [0, 1)$ such that $\alpha + \beta + \gamma < 1$ and

$$d(Tx, Ty) \leq \alpha d(x, Tx) + \beta d(y, Ty) + \gamma d(x, y)$$

for all $x, y \in X$.

we also review the concept of cyclic mapping as follows:

Let A and B be non-empty subsets of a metric space (X, d) , $T : A \cup B \rightarrow A \cup B$ is called cyclic if $T(A) \subset B$ and $T(B) \subset A$.

In 2016, Fan [12] proved fixed point theorems for some special cyclic mappings satisfying Banach contraction condition in the setting of a quasi-partial b-metric space as follows: Let A and B be nonempty subsets of a quasi-partial b-metric space (X, qp_b) . A cyclic mapping $T : A \cup B \rightarrow A \cup B$ is said to be a qp_b -cyclic-Banach contraction mapping if there exists $\alpha \in [0, 1)$ such that if $s \geq 1$, $s\alpha < 1$ then

$$qp_b(Tx, Ty) \leq \alpha qp_b(x, y)$$

holds both for $x \in A, y \in B$ and $x \in B, y \in A$.

Fan[12] in the same paper proved Kannan contraction condition in the setting of a quasi-partial b-metric space which is stated as follows:

Let A and B be nonempty subsets of a quasi-partial b-metric space (X, qp_b) . A cyclic mapping $T : A \cup B \rightarrow A \cup B$ is said to be a qp_b -cyclic-Kannan contraction mapping if there exists $\beta \in [0, 1/2)$ such that if $s \geq 1$, $s\beta < 1/2$ then

$$qp_b(Tx, Ty) \leq \beta qp_b(x, Tx) + \beta qp_b(y, Ty)$$

holds both for $x \in A, y \in B$ and $x \in B, y \in A$.

Inspired and motivated by the work of Fan [12], Ćirić [10] and Reich [11], in this paper we have introduced new contraction conditions for Ćirić-Reich-Rus contraction ([13]-[21]) for cyclic mappings in the setting of a quasi-partial b-metric space and discuss the existence and uniqueness of fixed points for such mappings when the underlying space is complete. Throughout this paper N, R and R^+ denote the set of all positive integers, set of real numbers and the set of all non-negative real numbers respectively. Here, we will recall some definition and lemma as discussed by Gupta and Gautam[9] in their paper:

2 Preliminaries and basic properties

Definition 1. ([7]). A *Quasi-Partial b-metric* on a non-empty set X is a mapping $qp_b : X \times X \rightarrow R^+$ such that for some real numbers $s \geq 1$ and for all $x, y, z \in X$

(QP_{b_1}) $qp_b(x, x) = qp_b(x, y) = qp_b(y, y)$ implies $x = y$

(QP_{b_2}) $qp_b(x, x) \leq qp_b(x, y)$

(QP_{b_3}) $qp_b(x, x) \leq qp_b(y, x)$

(QP_{b_4}) $qp_b(x, y) + qp_b(z, z) \leq s[qp_b(x, y) + qp_b(y, z)]$

A *Quasi-Partial b-metric space* is a pair (X, qp_b) , where X is a non-empty space and qp_b is *Quasi-Partial b-metric* on X . The number s is called *coefficient* of (X, qp_b) .

The following lemmas are instrumental in our result.

Lemma 1. ([9]). Let (X, qp_b) be a *Quasi-Partial b-metric space*. Then the following holds

(i) If $qp_b(x, y) = 0$ then $x = y$.

(ii) If $x \neq y$ then $qp_b(x, y) > 0$ and $qp_b(y, x) > 0$.

Lemma 2. ([9]). let (X, qp_b) be a *Quasi-Partial b-metric space*. Then

(i) A sequence $\{x_n\} \subset X$ converges to $x \in X$ if and only if

$qp_b(x, x) = \lim_{n \rightarrow +\infty} qp_b(x_n, x) = \lim_{n \rightarrow +\infty} qp_b(x, x_n)$

(ii) A sequence $\{x_n\} \subset X$ is called a *cauchy sequence* if and only if

$\lim_{n, m \rightarrow +\infty} qp_b(x_n, x_m) = \lim_{m, n \rightarrow \infty} qp_b(x_m, x_n)$ exist and finite.

(iii) The *Quasi-Partial b-metric space* (X, qp_b) is said to be *complete* if every sequence $\{x_n\} \subset X$ converges with respect to τ_{qp_b} to a point $x \in X$ such that

$qp_b(x, x) = \lim_{n, m \rightarrow +\infty} qp_b(x_n, x_m) = \lim_{m, n \rightarrow +\infty} qp_b(x_m, x_n)$.

Lemma 3. ([12]). Let (X, qp_b) be a *quasi-partial b-metric space* and $\{x_n\}_{n=1}^\infty$ be a sequence in X . If $x_n \rightarrow x, x_n \rightarrow y$ and $qp_b(x, x) = qp_b(y, y) = 0$, then $x = y$.

3 Main Result

In this section, we extend the fixed point theorem for Ćirić-Reich-Rus contraction mappings in standard metric spaces to qp_b -cyclic-Ćirić-Reich-Rus contraction mappings in the setting of quasi-partial b-metric spaces.

Definition 2. . Let A and B be nonempty subsets of a quasi-partial b-metric space (X, qp_b) . A cyclic mapping $T : A \cup B \rightarrow A \cup B$ is said to be a qp_b -cyclic- Ćirić-Reich-Rus contraction mapping if there exists $\alpha \in [0, 1)$ and $\beta \in [0, 1/2)$ such that if $s \geq 1$, $s(\alpha + 2\beta) < 1$ then

$$qp_b(Tx, Ty) \leq \alpha qp_b(x, y) + \beta qp_b(x, Tx) + \beta qp_b(y, Ty), \quad (1)$$

holds both for $x \in A, y \in B$ and $x \in B, y \in A$.

Theorem 1. . Let A and B be two nonempty closed subsets of a complete quasi-partial b -metric space (X, qp_b) and T be a cyclic mapping which is a qp_b -cyclic-Ćirić-Reich-Rus mapping. Then $A \cap B$ is nonempty and T has a unique fixed point in $A \cap B$.

Proof. . Let $x \in A$, considering condition (1), we have

$$\begin{aligned} qp_b(Tx, T^2x) &\leq \alpha qp_b(x, Tx) + \beta qp_b(x, Tx) + \beta qp_b(Tx, T^2x) \\ (1 - \beta)qp_b(Tx, T^2x) &\leq (\alpha + \beta)qp_b(x, Tx) \\ qp_b(Tx, T^2x) &\leq \frac{(\alpha + \beta)}{(1 - \beta)}qp_b(x, Tx) \end{aligned} \quad (2)$$

Using (2) we get

$$\begin{aligned} qp_b(T^2x, Tx) &\leq \alpha qp_b(Tx, x) + \beta qp_b(Tx, T^2x) + \beta qp_b(x, Tx) \\ qp_b(T^2x, Tx) &\leq \alpha qp_b(Tx, x) + \beta \left(\frac{\alpha + \beta}{1 - \beta} \right) qp_b(x, Tx) + \beta qp_b(x, Tx) \end{aligned}$$

Take, $\gamma = \text{Max}\{qp_b(x, Tx), qp_b(Tx, x)\}$, then we have

$$\begin{aligned} qp_b(Tx, T^2x) &\leq \frac{(\alpha + \beta)}{(1 - \beta)}\gamma, \text{ and} \\ qp_b(T^2x, Tx) &\leq \frac{(\alpha + \beta)}{(1 - \beta)}\gamma. \end{aligned}$$

Similarly,

$$\begin{aligned} qp_b(T^3x, T^2x) &\leq \left(\frac{\alpha + \beta}{1 - \beta} \right)^2 \gamma, \text{ and} \\ qp_b(T^2x, T^3x) &\leq \left(\frac{\alpha + \beta}{1 - \beta} \right)^2 \gamma. \end{aligned}$$

Hence,

$$qp_b(T^n x, T^{n+1} x) \leq \left(\frac{\alpha + \beta}{1 - \beta} \right)^n \gamma \quad \text{and} \quad qp_b(T^{n+1} x, T^n x) \leq \left(\frac{\alpha + \beta}{1 - \beta} \right)^n \gamma, \quad (3)$$

for every $n \in N$.

Let $m, n \in N$ and $m < n$, using (3) and QP_{b_4} we obtain

$$\begin{aligned} qp_b(T^m x, T^n x) &\leq s[qp_b(T^m x, T^{m+1} x) + qp_b(T^{m+1} x, T^n x)] - qp_b(T^{m+1} x, T^{m+1} x) \\ qp_b(T^m x, T^n x) &\leq s[qp_b(T^m x, T^{m+1} x) + qp_b(T^{m+1} x, T^n x)] \\ qp_b(T^m x, T^n x) &\leq sqp_b(T^m x, T^{m+1} x) + s^2 qp_b(T^{m+1} x, T^{m+2} x) + s^2 qp_b(T^{m+2} x, T^n x) \\ qp_b(T^m x, T^n x) &\leq sqp_b(T^m x, T^{m+1} x) + s^2 qp_b(T^{m+1} x, T^{m+2} x) + \dots + s^{n-m} qp_b(T^{n-1} x, T^n x). \end{aligned}$$

Let $(\frac{\alpha+\beta}{1-\beta}) = \delta$, then

$$\begin{aligned} qp_b(T^m x, T^n x) &\leq (s\delta^m + s^2\delta^{m+1} + \dots + s^{n-m}\delta^{n-1})\gamma \\ qp_b(T^m x, T^n x) &\leq s\delta^m(1 + s\delta + (s\delta)^2 + \dots + (s\delta)^{n-m-1})\gamma \\ qp_b(T^m x, T^n x) &\leq s\delta^m(\frac{1 - (s\delta)^{n-m}}{1 - s\delta})\gamma. \end{aligned}$$

Since, $s(\alpha + 2\beta) < 1, s \geq 1$ then, $1 - s\beta \leq 1 - \beta$ and $s(\alpha + \beta) < 1 - s\beta$, therefore, $s(\frac{\alpha+\beta}{1-\beta}) < 1$, i.e. $s\delta < 1$. Furthermore,

$$qp_b(T^m x, T^n x) \leq (\frac{s\delta^m}{1 - s\delta})\gamma.$$

Taking limit as $m, n \rightarrow +\infty$ in the above inequality, we have

$$\lim_{n, m \rightarrow +\infty} qp_b(T^m x, T^n x) \leq 0.$$

Thus,

$$\lim_{n, m \rightarrow +\infty} qp_b(T^m x, T^n x) = 0. \quad (4)$$

Also,

$$\begin{aligned} qp_b(T^n x, T^m x) &\leq s[qp_b(T^n x, T^{m+1} x) + qp_b(T^{m+1} x, T^m x)] - qp_b(T^{m+1} x, T^{m+1} x) \\ qp_b(T^n x, T^m x) &\leq s[qp_b(T^n x, T^{m+1} x) + qp_b(T^{m+1} x, T^m x)] \\ qp_b(T^n x, T^m x) &\leq s^2 qp_b(T^n x, T^{m+2} x) + s^2 qp_b(T^{m+2} x, T^{m+1} x) + s qp_b(T^{m+1} x, T^m x) \\ qp_b(T^n x, T^m x) &\leq s^{n-m} qp_b(T^n x, T^{n-1} x) + s^{n-m-1} qp_b(T^{n-1} x, T^{n-2} x) + \dots + s qp_b(T^{m+1} x, T^m x) \\ qp_b(T^n x, T^m x) &\leq (s^{n-m}\delta^{n-1} + s^{n-m-1}\delta^{n-2} + \dots + s\delta^m)\gamma \\ qp_b(T^n x, T^m x) &\leq s\delta^m((s\delta)^{n-m-1} + (s\delta)^{n-m-2} + \dots + s\delta)\gamma \\ qp_b(T^n x, T^m x) &\leq s\delta^m(\frac{1 - (s\delta)^{n-m}}{1 - s\delta})\gamma. \end{aligned}$$

Taking limit as $m, n \rightarrow +\infty$ in the above inequality, we have

$$\lim_{n, m \rightarrow +\infty} qp_b(T^n x, T^m x) \leq 0.$$

Thus,

$$\lim_{n, m \rightarrow +\infty} qp_b(T^n x, T^m x) = 0. \quad (5)$$

Using (4) and (5) indicate that sequence $\{T^n x\}_{n=0}^{+\infty}$ is a Cauchy sequence.

Since (X, qp_b) is complete, therefore $\{T^n x\}_{n=0}^{+\infty}$ converges to some $u \in X$, that is,

$$\begin{aligned} qp_b(u, u) &= \lim_{n \rightarrow +\infty} qp_b(T^n x, u) = \lim_{n \rightarrow +\infty} qp_b(u, T^n x) = \lim_{n, m \rightarrow +\infty} qp_b(T^n x, T^m x) = \\ &= \lim_{n, m \rightarrow +\infty} qp_b(T^m x, T^n x) = 0. \end{aligned} \quad (6)$$

Observe that $\{T^{2n}x\}_{n=0}^{+\infty}$ is a sequence in A and $\{T^{2n-1}x\}_{n=1}^{+\infty}$ is a sequence in B in a way that both sequences converge to u . Note also that A and B are closed, we have $u \in A \cap B$

$$qp_b(T^n x, Tu) \leq \alpha qp_b(T^{n-1}x, u) + \beta qp_b(T^{n-1}x, T^n x) + \beta qp_b(u, Tu).$$

Taking limit as $n \rightarrow +\infty$ in the above inequality and using (6), we have

$$\begin{aligned} qp_b(T^n x, Tu) &\leq \beta qp_b(u, Tu) \\ \beta qp_b(u, Tu) &\leq \beta [sqp_b(u, T^n x) + sqp_b(T^n x, Tu) - qp_b(T^n x, T^n x)] \\ \beta qp_b(u, Tu) &\leq \beta [sqp_b(u, T^n x) + sqp_b(T^n x, Tu)]. \end{aligned} \quad (7)$$

Taking limit as $n \rightarrow +\infty$ in the above inequality, we have

$$\beta qp_b(u, Tu) \leq \beta sqp_b(T^n x, Tu). \quad (8)$$

Thus, using (7) and (8) we have

$$qp_b(T^n x, Tu) \leq \beta qp_b(u, Tu) \leq \beta sqp_b(T^n x, Tu).$$

Since, $s\beta < 1$

$$\lim_{n \rightarrow +\infty} qp_b(T^n x, Tu) = qp_b(u, Tu) = 0. \quad (9)$$

Similarly, it can be derived

$$\lim_{n \rightarrow +\infty} qp_b(Tu, T^n x) = qp_b(Tu, u) = 0. \quad (10)$$

In addition, by the contractive condition of theorem and in combination with (6) and (9), we get

$$qp_b(Tu, Tu) \leq \alpha qp_b(u, u) + 2\beta qp_b(u, Tu),$$

so,

$$qp_b(Tu, Tu) = 0. \quad (11)$$

Equations (9), (10) and (11) show that the sequence $\{T^n x\}_{n=0}^{+\infty}$ to Tu . Applying Lemma 3, we get

$$Tu = u.$$

For uniqueness, suppose there exist another fixed point in $A \cup B$ say w i.e. $Tw = w$, by contractive condition,

$$\begin{aligned} qp_b(u, w) &= qp_b(Tu, Tw) \leq \alpha qp_b(u, w) + \beta qp_b(u, Tu) + \beta qp_b(w, Tw) \\ qp_b(u, w) &\leq \alpha qp_b(u, w) + \beta qp_b(u, u) + \beta qp_b(w, w). \end{aligned} \quad (12)$$

Consider,

$$\begin{aligned}
 qp_b(u, u) &= qp_b(Tu, Tu) \\
 &\leq \alpha qp_b(u, u) + \beta qp_b(u, Tu) + \beta qp_b(u, Tu) \\
 &= (\alpha + 2\beta) qp_b(u, u) \\
 (1 - \alpha - 2\beta) qp_b(u, u) &\leq 0.
 \end{aligned}$$

Since

$$\alpha + 2\beta < 1, \quad qp_b(u, u) = 0. \quad (13)$$

In a similar manner,

$$qp_b(w, w) = 0. \quad (14)$$

Combining (12), (13) and (14) we get

$$qp_b(u, w) \leq \alpha qp_b(u, w)$$

Since, $\alpha \leq 1$ and $(1 - \alpha) qp_b(u, w) \leq 0$, therefore, $qp_b(u, w) = 0$. So we have $qp_b(u, u) = qp_b(w, w) = qp_b(u, w) = 0$, that $u = w$. □

Example 1. Let $X = [0, 1]$, $A = [0, \frac{1}{2}]$ and $B = [0, \frac{1}{3}]$, $T : A \cup B \rightarrow A \cup B$ as $Tx = \frac{x}{2}$. Define the quasi partial b-metric as

$$qp_b(x, y) = |x - y| + |x|$$

for all $x, y \in X$. Here (X, qp_b) is a quasi-partial b-metric space with $s = 1$. We will verify that the mapping T is qp_b -cyclic-Ćirić-Reich-Rus contraction mapping.

$T(A) = [0, \frac{1}{4}] \subset B$ and $T(B) = [0, \frac{1}{6}] \subset A$. Hence the mapping is cyclic on X .

Taking $\alpha = \frac{1}{2}, \beta \leq \frac{1}{2}, s(\alpha + 2\beta) \leq 1$

$$\begin{aligned}
 \frac{1}{2} qp_b(x, y) + \beta \{qp_b(x, Tx) + qp_b(y, Ty)\} &= \frac{1}{2} \{|x - y| + |x|\} + \beta \left| \frac{x}{2} \right| + |x| + \left| \frac{y}{2} \right| + |y| \geq \\
 &\geq \frac{1}{2} \{|x - y| + |x|\} \geq \left\{ \left| \frac{x}{2} - \frac{y}{2} \right| + \left| \frac{x}{2} \right| \right\} = \\
 &= qp_b(Tx, Ty)
 \end{aligned}$$

Therefore, all conditions of Theorem 1 are satisfied and so T has a fixed point (which is $0 \in A \cap B$).

Example 2 Let $X = [-\frac{\pi}{2}, \frac{\pi}{2}]$, $A = [-\frac{\pi}{2}, 0]$ and $B = [0, \frac{\pi}{2}]$, $T : A \cup B \rightarrow A \cup B$ as $Tx = -\frac{\sin x}{2}$. Define the quasi partial b- metric as

$$qp_b(x, y) = |x - y| + |x|$$

for all $x, y \in X$. Here (X, qp_b) is a quasi-partial b-metric space with $s = 1$. We will verify that the mapping T is qp_b -cyclic-Ćirić-Reich-Rus contraction mapping. $T(A) = [0, 1] \subset B$ and $T(B) = [-1, 0] \subset A$. Hence the mapping is cyclic on X .

Taking $\alpha = \frac{1}{2}, \beta \leq \frac{1}{2}, s(\alpha + 2\beta) \leq 1$

$$\begin{aligned} \frac{1}{2}qp_b(x, y) + \beta\{qp_b(x, Tx) + qp_b(y, Ty)\} &= \frac{1}{2}\{|x - y| + |x|\} + \beta|x + \frac{\sin x}{2}| + |x| + |y + \frac{\sin y}{2}| + |y| \geq \\ &\geq \frac{1}{2}\{|x - y| + |x|\} \geq \frac{1}{2}\{|\sin x - \sin y| + |\sin x|\} \geq \\ &\geq \{| - \frac{\sin x}{2} + \frac{\sin y}{2}| + | - \frac{\sin x}{2}|\} = qp_b(Tx, Ty) \end{aligned}$$

Therefore, all conditions of Theorem 1 are satisfied and so T has a fixed point (which is $0 \in A \cap B$).

Corollary 1. *Let A and B be two nonempty closed subsets of a complete quasi-partial b-metric space (X, qp_b) and T be a cyclic mapping which is a qp_b -cyclic-Kannan mapping. Then $A \cap B$ is nonempty and T has a unique fixed point in $A \cap B$.*

Proof. In theorem 1, Taking $\alpha = 0$. □

Corollary 2. *Let A and B be two nonempty closed subsets of a complete quasi-partial b-metric space (X, qp_b) and T be a cyclic mapping which is a qp_b -cyclic-Banach mapping. Then $A \cap B$ is nonempty and T has a unique fixed point in $A \cap B$.*

Proof. In theorem 1, Taking $\beta = 0$. □

Corollary 3. *Let (X, d) is a complete metric space and $T : X \rightarrow X$ be a self-map satisfying,*

$$d(Tx, Ty) \leq \alpha d(x, y) + \beta d(x, Tx) + \beta d(y, Ty)$$

for all $x, y \in X, (\alpha + 2\beta) \in [0, 1)$. Then T has a unique fixed point in X .

Proof. Since every metric space is also a quasi-partial b-metric space; with $s = 1$, so as a particular case of theorem 1, if the underlying space is taken to be a metric space; sets $A = B = X$; then the corresponding corollary 3 follows. It is worth observing that the

condition of sets A and B being closed is dropped in the above corollary by not assuming the space to be closed. This is justified as in the proof of the theorem, the closeness of sets A and B guaranteed that the fixed point thus obtained belongs to both A and B . Since $A = B = X$, so the fixed point obtained naturally belongs to X . $\square$

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