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Control Charts Based on Quantiles – New Approaches

Vesna Jevremović, Atif Avdović

Abstract: Contemporary development of Statistical quality control includes researches on different control charts, which could be easily implemented in production processes due to facilities offered by computers. New control charts give more information about production processes than the conventional ones, that's why there is a lot of investigation in this area of applied statistics. In this paper, we shall explain some new ideas concerning the construction of control charts based on quantiles, empirical distribution function and p-value.

Keywords: Quality control, control charts, quantiles, empirical distribution function, p-value

1 Introduction

The use of probability and statistics in quality control dates back to 1924 when W. Shewhart presented his control chart based on quantiles for normal distribution, more precisely on 6σ rule. From this time to nowadays control charts were used, improved, changed. Shewhart chart controlled the mean/target value of a product, assuming the normal distribution for the characteristic under control. Soon the R and S charts were added in quality control, followed by CUSUM and EWMA charts able to monitor small changes in the production process. We have listed some of the papers dealing with the key-words listed above: in [1] box plot and standard control charts were used, in [2] the idea of using quantiles for quality control for non-normal distribution is proved to be better than the transformation which led to a normal distribution, in [3] Q-Q plot was used for checking the distribution of the whole sample, in [4] there is a discussion about the need of (standard) quality control in radiology, in [5] the Analysis of Variance is used instead of control chart since ANOVA represents statistical test dealing with means of several subgroups of data, in [6] the empirical distribution function (EDF) is applied in quality control based on Kolmogorov's theorem. In our previous paper [7], we have introduced a procedure for quality control using EDF in a different way than the one given in [6]. And now we shall present some new control charts based on quantiles.

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2 Control Charts

2.1 Use of EDF for Each Sample

Let $x_1, x_2, \ldots, x_n$ be a random sample for a characteristic X, and let $F_n^*(x)$ be the corresponding empirical distribution function (EDF). Let also X have the normal $N(m, \sigma^2)$ distribution, with cumulative distribution function (CDF) $F(x)$. For the normal distributions $N(m-3\sigma, \sigma^2)$ and $N(m+3\sigma, \sigma^2)$ the CDFs are $F_{m-3\sigma}(x)$ and $F_{m+3\sigma}(x)$.

If the i-th element of the sample satisfies the condition:

$$F_{m+3\sigma}(x_i) \le F_n^*(x_i) \le F_{m-3\sigma}(x_i); i = 1, \ldots, n$$

we will take it as an element from the $N(m, \sigma^2)$ distribution. We can add the CDFs: $F_{m-\sigma(x)}, F_{m+\sigma}(x), F_{m-2\sigma}(x)$ and $F_{m+2\sigma}(x)$. First we plot the graphs of mentioned functions and then we plot the points $(x_i, F_n^*(x_i)); i = 1, \ldots, n$ and connect them. This way we can obtain a control chart, with 3 zones, given on the picture 1 where:

$$zone(x_i) = \begin{cases} 1; & F_{m+\sigma}(x_i) \le F_n^*(x_i) \le F_{m-\sigma}(x_i) \\ 2; & F_{m-2\sigma}(x_i) \le F_n^*(x_i) < F_{m-\sigma}(x_i) \vee F_{m+\sigma}(x_i) < F_n^*(x_i) \le F_{m+2\sigma}(x_i) \\ 3; & F_{m-3\sigma}(x_i) \le F_n^*(x_i) < F_{m-2\sigma}(x_i) \vee F_{m+2\sigma}(x_i) < F_n^*(x_i) \le F_{m+3\sigma}(x_i) \end{cases}$$

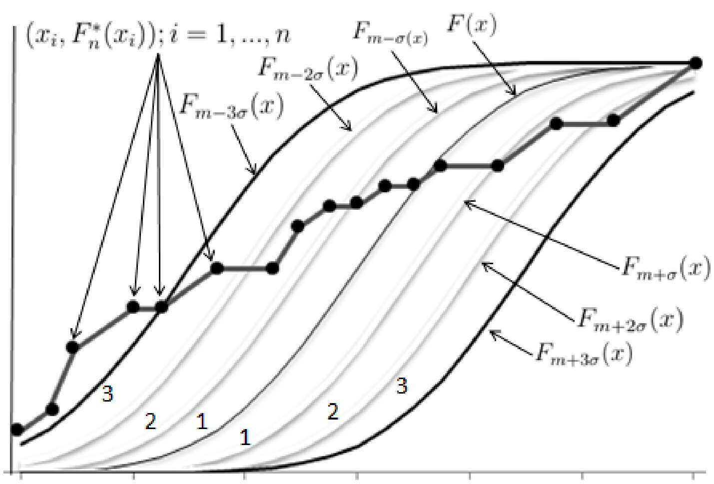

Fig. 1. CDF control chart for one sample

To have a criterion for acceptance of a sample we will take:

$$T = 1 - \frac{1}{n} \sum_{i=1}^{n} I(F_{m+3\sigma}(x_i) \le F_n^*(x_i) \le F_{m-3\sigma}(x_i))$$

and compare it with the threshold of significance α: if $T < \alpha$, we accept that the distribution of the sample to be normal, otherwise not.

The integer value $V = \left[\frac{1}{n}\sum_{i=1}^{n} zone(x_i)\right]$ could be taken as an estimator of goodness-of-fit of data with the normal distribution of interest: the lower V is, the better fit we have.

2.2 Use of EDF for More than One Sample

Let's have k samples of size n: $x_{i1}, x_{i2}, ..., x_{in}; i = 1, 2, ...k$, with means $\bar{x}_{in}; i = 1, 2, ..., k$, and let $\bar{F}_k^*(x)$ be EDF for a series of these means.

Let $\bar{F}(x)$ denotes the CDF for the normal $N(m, \frac{\sigma^2}{n})$ distribution, and $\bar{F}_{m-3\frac{\sigma}{\sqrt{n}}}(x)$, $\bar{F}_{m+3\frac{\sigma}{\sqrt{n}}}(x)$ be the CDFs for normal distributions $N(m - 3\frac{\sigma}{\sqrt{n}}, \frac{\sigma^2}{n})$ and $N(m + 3\frac{\sigma}{\sqrt{n}}, \frac{\sigma^2}{n})$. If the i-th mean of the series of means satisfies the condition:

$$\bar{F}_{m-3\frac{\sigma}{\sqrt{n}}}(\bar{x}_{in}) \leq \bar{F}_k^*(\bar{x}_{in}) \leq \bar{F}_{m-3\frac{\sigma}{\sqrt{n}}}(\bar{x}_{in}); i = 1, ..., k$$

then this mean belongs to the normal $N(m, \frac{\sigma^2}{n})$ distribution, and the corresponding sample to the $N(m, \sigma^2)$ distribution. This way we have the control chart with upper (UCL) and lower (LCL) control limits to be $\bar{F}_{m-3\sigma}(x)$ and $\bar{F}_{m+3\sigma}(x)$. We can also plot graphs of functions $\bar{F}_{m-2\frac{\sigma}{\sqrt{n}}}(x), \bar{F}_{m+2\frac{\sigma}{\sqrt{n}}}(x), \bar{F}_{m-\frac{\sigma}{\sqrt{n}}}(x)$ and $\bar{F}_{m+\frac{\sigma}{\sqrt{n}}}(x)$, which can be useful in making conclusions. Instead of EDF $\bar{F}_k^*(\bar{x}_{in})$ we can only plot the points $(\bar{x}_{in}, \bar{F}_k^*(\bar{x}_{in})); i = 1, ..., k$ and see which ones are between UCL and LCL, as it can be seen on figure 2.

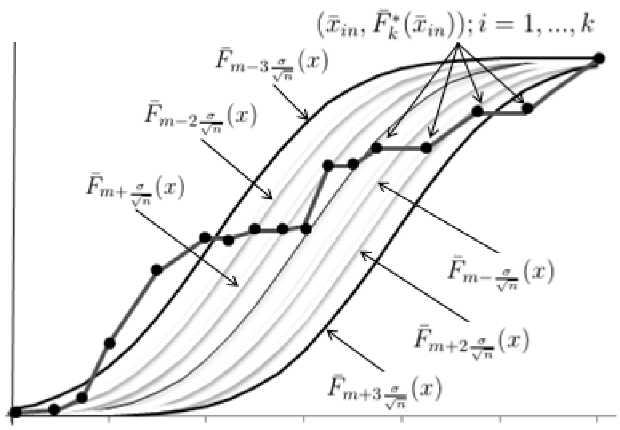

Fig. 2. CDF control chart for series of means

Analysis and interpretation of results are again similar to the procedure (see in [8]) for standard X-bar charts but using EDF we obtain more information about the production process than with X-bar charts.

2.3 Use of Q-Q Plot in Quality Control

Q-Q plot is used in statistics to visually represent the compatibility of given data and some distribution of interest, or the compatibility of two distributions/samples. Using small modifications, we can construct a control chart based on the Q-Q plot.

Let's have k samples of size n: $x_{i1},...,x_{in}; i = 1,2,...k$, with means $\bar{x}_{in}; i = 1,2,...,k$. We order these values in non-decreasing series: $\bar{x}_{(1)n} \leq \bar{x}_{(2)n} \leq ... \leq \bar{x}_{(k)n}$ and we calculate $\bar{F}^{-1}(\frac{j}{k}), j = 1,2,...,k-1$ and $\bar{F}^{-1}(\frac{k-0.5}{k})$. $\bar{F}(x)$ is the CDF for normal $N(m, \frac{\sigma^2}{n})$ distribution, and $\bar{F}^{-1}$ its inverse. Since $\bar{F}^{-1}(1) = \infty$, the correction factor for the last value is needed. Our control chart consists of the central line $y = \frac{\sigma}{\sqrt{n}}x + m$, control lines $y = \frac{\sigma}{\sqrt{n}}x + m + \frac{\sigma}{\sqrt{n}}, y = \frac{\sigma}{\sqrt{n}}x + m + 2\frac{\sigma}{\sqrt{n}}, y = \frac{\sigma}{\sqrt{n}}x + m - \frac{\sigma}{\sqrt{n}}, y = \frac{\sigma}{\sqrt{n}}x + m - 2\frac{\sigma}{\sqrt{n}}$, and control limits $y = \frac{\sigma}{\sqrt{n}}x + m + 3\frac{\sigma}{\sqrt{n}}$ and $y = \frac{\sigma}{\sqrt{n}}x + m - 3\frac{\sigma}{\sqrt{n}}$. On this graph, see figure 3, we plot the points: $(\bar{x}_{(j)n}, \bar{F}^{-1}(\frac{j}{k})); j = 1,2,....,k-1$ and $(\bar{x}_{(k)n}, \bar{F}^{-1}(\frac{k-0.5}{k}))$.

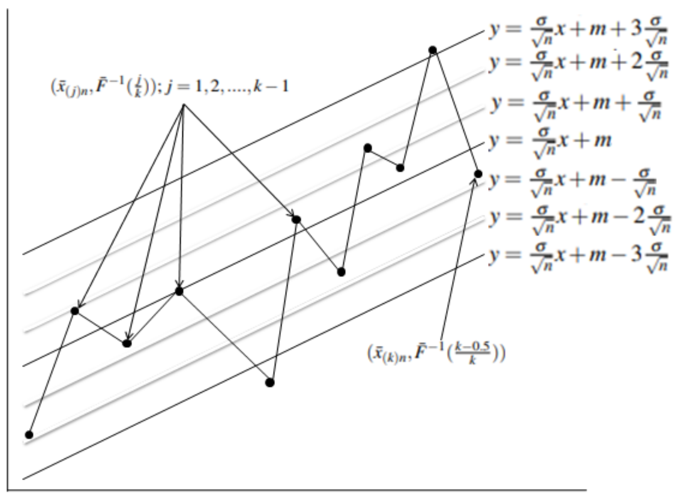

Fig. 3. Q-Q plot in Quality control.

Analysis and interpretation of results are again similar to the procedure (see in [8]) for standard X-bar charts, but using Q-Q plot, we also obtain more accurate information about the production process distribution than with X-bar charts.

2.4 Use of p-value in Quality Control

In hypothesis testing, the use of p-value is very spread. Let's now see how p-value could be used in Quality control.

Let's have k samples of size n: $x_{i1}, x_{i2},...,x_{in}; i = 1,2,...k$, and let $\bar{F}_k^*(x)$ be EDF for a series of these means.

Since the sample sizes for Quality control are moderate or even small, p-values must be related to some powerful test. We will illustrate the procedure using Kolmogorov's test. This test uses the statistic:

$$D_{nj} = \sup_{x \in \mathbb{R}} |\bar{F}_{nj}^*(x) - F\bar{F}(x)|$$

where $F(x)$ is CDF prescribed by production standard. For each sample the realized value

d_{nj} of D_{nj} is calculated and then the p-values:

$$p_j = P(D_{nj} \geq d_{nj}); j = 1, ..., k.$$

A control chart has the lower limit equal $y = \alpha$; the upper limit is obviously 1, that's why it is not needed. We plot the points $(j, p_j); j = 1, ..., k$. The process is "in control" state if all the points are above the lower limit (see Figure 4.) Assumptions are the same as for 2.3.

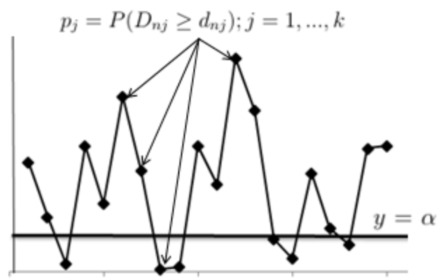

Fig. 4. p-value in Quality control.

3 Conclusion

We have presented some new approaches to control charts constructing. The procedures are easy to implement in production processes, giving more information about it than the widely used charts, X-bar, R chart etc.

Our future investigation will focus on simulation studies to obtain the rate of the false alarm, the comparison of efficiency with standard X-bar charts and the results of proposed control charts for non-normal distribution of characteristics of interest in the production processes.

References

[1] V PRAVEEN, T DELHI NARENDRAN, R PAVITHRAN, CHANDRASEGAR THIRUMALAI, *Data analysis using Box plot and Control Chart for Air Quality*, 2017, DOI: 10.1109/ICOEI.2017.8300877

[2] https://www.stat.fi/isi99/proceedings/arkisto/varasto/kanj0948.pdf

[3] HUI LIU, WEIJING WANG, CAIXIA ZHANG, CHUNSHENG XU, HAIPING DUAN, XIAOCAO TIAN, DONGFENG ZHANG, *Heritability and Genome-Wide Association Study of Plasma Cholesterol in Chinese Adult Twins*, 2018, DOI: 10.3389/fendo.2018.00677

[4] M. LEÃO, L. P. V. RIBEIRO, A. F. ABRANTES, J. P. PINHEIRO, R. P. P. ALMEIDA, K. B. AZEVEDO, S. RODRIGUES, *Implementation of a quality control program in a radiology department*, 2013, DOI: 10.1594/ecr2013/B-0602

[5] CORDUTA DURA, CLAUDIA ISAC, *The ANOVA method in quality audit*, Annals of the University of Petroşani, Mechanical Engineering, 8 (2006), 23-30

[6] SAAD BAKIR, *A non-parametric Shewhart-type quality control chart for monitoring broad changes in a process distribution*, Int. journal of quality statistics and reliability, 2012, DOI: 10.1155/2012/147520

[7] VESNA JEVREMOVIC, ATIF AVDOVIC, *Empirical Distribution Function as a Tool in Quality Control*, Scientific Publications of the State University of Novi Pazar, Series A, Applied Mathematics, Informatics and Mechanics, Volume 12, 1 (2020), 37-46

[8] MARIJA MINIC, *Quality control - control charts and their properties*, in Selected chapters of mathematical statistics, 2018, DUNP, red. V. Jevremovic, J. Malisic (in Serbian)