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New Shape Function in the Free-Vibration Analysis of Antisymmetric Angleply Composite Laminates

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Abstract: The paper analyzes the problem of free vibrations in antisymmetric crossply laminates. A new shape function which is used in higher order shear deformation theories has been introduced. The comparative analysis was performed with the known shape functions. The procedure for obtaining dynamic equations of motion in the Matlab software package has been developed. For theoretical considerations, the module with the symbolic variable has been used. The paper shows which of the existing shape functions are applicable in the free vibration analysis of antisymmetric angleply laminates. The advantages and disadvantages of the newly developed shape function are clearly highlighted. Analytical procedures have been used to obtain the results of partial differential equations, based on Navier's solutions. Numerical integration procedures were used as an integral part of the developed Matlab codes for those shape functions where it was necessary. The results are presented in a table and figures. The procedure itself has been verified by comparison with the reference results from the literature.

Keywords: shape function, high order shear deformation theories, cross-ply composite laminates

1 Introductory considerations and theoretical assumptions

With the development of computer technologies and numerical methods for solving the problems of classical and complex analysis, preconditions have been created for the application of somewhat more complex theories than classical plate theory and first order shear deformation theories. The shortcomings of these theories, such as influence coefficients and the like, are eliminated by introducing shape functions.By introducing these theories, it is possible to reduce the degree of approximation of real problems. Similar to lower order theories, higher order shear deformation theories are also based on assumed fieldsof displacement:

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$$\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) - z\tfrac{\partial w}{\partial x}(x,y,t) + f(z)\,\theta_x,\\
v(x,y,z,t) &= v_0(x,y,t) - z\tfrac{\partial w}{\partial y}(x,y,t) + f(z)\,\theta_y,\\
w(x,y,z,t) &= w_0(x,y,t).
\end{aligned} \tag{1}$$

u_0, v_0, w_0 – displacements of the middle plane of the laminate,
$\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$ – angles of rotation of the normal in relation to the vertical axis due to bending,
θ_x, θ_y – displacement due to transverse shear,
$f(z)$ – shape functions.

Lo, Christensen, and Wu, [12], [13], combined the notations which had been used to express different higher-order shear deformation theories.They presented the development of HSDT theories according to the functional dependence of displacement on the degree of independently variable z, which in the deformation theory represents a coordinate axis perpendicular to the middle plane in the undeformed configuration of the laminate plate.

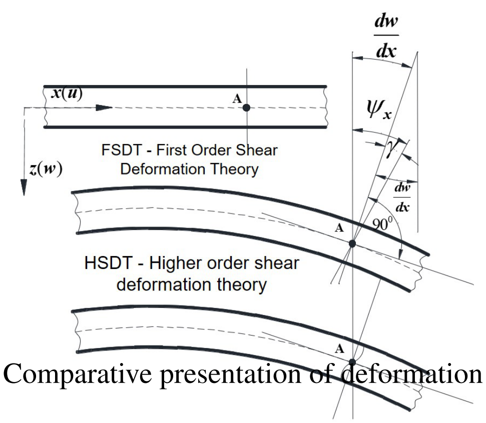


Fig. 1: Comparative presentation of deformation theories

Theories based on functions that are not polynomial can be exponential, trigonometric, hyperbolic, etc. The implementation of these theories has been the subject of analysis of many authors. Due to the large number of those theories there have been several review papers [26], [6], [1] which aim to help researchers in further development, as well as to point out advantages and disadvantages of the previously developed theories. In this paper, a new form function is proposed:

$$f(z) = z\left(\cosh\left(\frac{z}{h}\right) - 1.388\right). \tag{2}$$

Analytical methods have been used to implement the newly introduced shape function in the problem of free vibrations of angleply antisymmetric laminates. Analytical methods increase the accuracy of the results and provide easier access to the physics of the problem. For complex engineering structures, the use ofnumerical methods is necessary. Verification of results is performed by comparison with analytically obtained solutions or experimental results. The problems of this analysis have been addressed by the authors in [10], [24], [25], [8], [17], [20]. Matlab codes have been written where the procedure of combining symbolic and numerical forms of variables were implemented. Using the known relations between deformations and displacements, as well as between stresses and strains in the region of linear elasticity, load projections per unit width are defined, as well as the shapes of matrices that define the stacking of layers in the laminate. Load projections per unit width are:

$$\begin{gathered}
N=\int_{h^-}^{h^+}\sigma dz=\sum_{l=1}^{n}\left(\int_{h_l^-}^{h_l^+}Q^{(l)}k_0dz+\int_{h_l^-}^{h_l^+}Q^{(l)}k_1zdz+\int_{h_l^-}^{h_l^+}Q^{(l)}k_2f(z)\,dz\right),\\
M=\int_{h^-}^{h^+}\sigma zdz=\sum_{l=1}^{n}\left(\int_{h_l^-}^{h_l^+}Q^{(l)}k_0zdz+\int_{h_l^-}^{h_l^+}Q^{(l)}k_1z^2dz+\int_{h_l^-}^{h_l^+}Q^{(l)}k_2zf(z)\,dz\right),\\
P=\int_{h^-}^{h^+}\sigma f(z)\,dz=\sum_{l=1}^{n}\left(\int_{h_l^-}^{h_l^+}Q^{(l)}k_0f(z)\,dz+\int_{h_l^-}^{h_l^+}Q^{(l)}k_1zf(z)\,dz+\int_{h_l^-}^{h_l^+}Q^{(l)}k_2\left(f(z)\right)^2dz\right),\\
R=\int_{h^-}^{h^+}Tf'(z)\,dz=\sum_{l=1}^{n}\int_{h_l^-}^{h_l^+}Q_S^{(l)}k_S\left(f'(z)\right)^2dz,
\end{gathered}\tag{3}$$

where

$$Q=\begin{bmatrix}\mathrm{Q}_{11}&\mathrm{Q}_{12}&\mathrm{Q}_{16}\\\mathrm{Q}_{12}&\mathrm{Q}_{22}&\mathrm{Q}_{26}\\\mathrm{Q}_{16}&\mathrm{Q}_{26}&\overline{\mathrm{C}}_{66}\end{bmatrix},Q_S=\begin{bmatrix}\mathrm{Q}_{44}&\mathrm{Q}_{45}\\\mathrm{Q}_{45}&\mathrm{Q}_{55}\end{bmatrix},$$

$$\sigma=\left\{\begin{matrix}\sigma_{xx}&\sigma_{yy}&\sigma_{xy}\end{matrix}\right\}^T,\ \tau=\left\{\begin{matrix}\tau_{xz}&\tau_{yz}\end{matrix}\right\}^T,$$

$$N=\left\{\begin{matrix}N_{xx}&N_{yy}&N_{xy}\end{matrix}\right\}^T,M=\left\{\begin{matrix}M_{xx}&M_{yy}&M_{xy}\end{matrix}\right\}^T,P=\left\{\begin{matrix}P_{xx}&P_{yy}&P_{xy}\end{matrix}\right\}^T,$$

$$R=\left\{\begin{matrix}R_y&R_x\end{matrix}\right\}^T,k_0=\left\{\begin{matrix}\frac{\partial u_0}{\partial x}&\frac{\partial v_0}{\partial y}&\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\end{matrix}\right\}^T,$$

$$k_1=\left\{\begin{matrix}-\frac{\partial^2w_0}{\partial x^2}&-\frac{\partial^2w_0}{\partial y^2}&-2\frac{\partial^2w_0}{\partial x\partial y}\end{matrix}\right\}^T,\ k_2=\left\{\begin{matrix}\frac{\partial\theta_x}{\partial x}&\frac{\partial\theta_y}{\partial y}&\frac{\partial\theta_x}{\partial y}+\frac{\partial\theta_y}{\partial x}\end{matrix}\right\}^T,$$

$$k_S=\left\{\begin{matrix}\theta_x&\theta_y\end{matrix}\right\}^T.$$

Layer stacking matrices are defined as:

$$(A_{ij},B_{ij},\,\mathrm{D}_{ij},\,\mathrm{E}_{ij},\,F_{ij},\,G_{ij})=\sum_{l=1}^{n}\int_{h_l^-}^{h_l^+}Q_{ij}^{(l)}\left(1,z,f(z),z^2,zf(z),(f(z))^2\right)dz,\,i,j=(1,2,6),$$

$$H_{ij}=\sum_{l=1}^{n}\int_{h^-}^{h^+}Q_{ij}^{(l)}\left(f'(z)\right)^2dz,\ (i,j)=(4,5).$$

In antisymmetric angleply composite laminates, the coupling matrices A, B, D, E, F,

G, H are defined as:

$$\mathbf{A}=\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix};\ \mathbf{B}=\begin{bmatrix} 0 & 0 & B_{16} \\ 0 & 0 & B_{26} \\ B_{16} & B_{26} & 0 \end{bmatrix};\ \mathbf{D}=\begin{bmatrix} 0 & 0 & D_{16} \\ 0 & 0 & D_{26} \\ D_{16} & D_{26} & 0 \end{bmatrix};$$
$$\mathbf{E}=\begin{bmatrix} E_{11} & E_{12} & 0 \\ E_{12} & E_{22} & 0 \\ 0 & 0 & E_{66} \end{bmatrix};\ \mathbf{F}=\begin{bmatrix} F_{11} & F_{12} & 0 \\ F_{12} & F_{22} & 0 \\ 0 & 0 & F_{66} \end{bmatrix}; \quad (4)$$
$$\mathbf{G}=\begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{12} & G_{22} & 0 \\ 0 & 0 & G_{66} \end{bmatrix};\ \mathbf{H}=\begin{bmatrix} H_{14} & 0 \\ 0 & H_{55} \end{bmatrix}.$$

Using Hamilton's principle for dynamic problems [19] we obtain equations in the form:

$$\begin{aligned}
&\delta u_0:\quad N_{xx,x}+N_{xy,y}=I_1\ddot{u}-I_2\ddot{w}_{,x}+I_4\ddot{\theta}_x,\\
&\delta v_0:\quad N_{yy,y}+N_{xy,x}=I_1\ddot{v}-I_2\ddot{w}_{,y}+I_4\ddot{\theta}_y,\\
&\delta w_0:\quad M_{xx,xx}+2M_{xy,xy}+M_{yy,yy}=I_1\ddot{w}+I_2\left(\ddot{u}_{,x}+\ddot{v}_{,y}\right)-I_3\left(\ddot{w}_{,xx}+\ddot{w}_{,yy}\right)+I_5\left(\ddot{\theta}_{x,x}+\ddot{\theta}_{y,y}\right),\\
&\delta\theta_x:\quad P_{xx,x}+P_{xy,y}-R_x=I_4\ddot{u}-I_5\ddot{w}_{,x}+I_6\ddot{\theta}_x,\\
&\delta\theta_y:\quad P_{xy,x}+P_{yy,y}-R_y=I_4\ddot{v}-I_5\ddot{w}_{,y}+I_6\ddot{\theta}_y.
\end{aligned} \quad (5)$$

where I_i ($i=1,2,3,4,5$ and 6) are inertiaelements, defined as:

$$\begin{aligned}
&I_1=\int_{-h/2}^{h/2}\rho(z)dz, && I_2=\int_{-h/2}^{h/2}\rho(z)zdz,\\
&I_3=\int_{-h/2}^{h/2}\rho(z)f(z)\,dz, && I_4=\int_{-h/2}^{h/2}\rho(z)z^2dz,\\
&I_5=\int_{-h/2}^{h/2}\rho(z)zf(z)\,dz, && I_6=\int_{-h/2}^{h/2}\rho(z)\left(f(z)\right)^2dz.
\end{aligned} \quad (6)$$

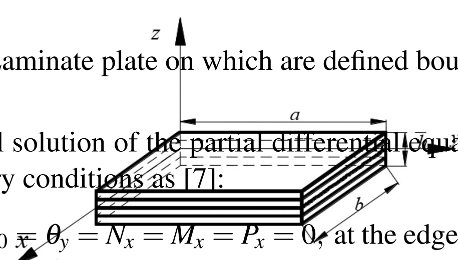


Fig. 2: Laminate plate on which are defined boundary conditions

For the analytical solution of the partial differential equation of motion, it is necessary to define the boundary conditions as [7]:

$$\begin{aligned}
&v_0=w_0=\theta_y=N_x=M_x=P_x=0,\ \text{at the edges } x=0,\ x=b,\\
&u_0=w_0=\theta_x=N_y=M_y=P_b=0,\ \text{at the edges } y=0,\ y=a.
\end{aligned}$$

For the defined boundary conditions, Navier's forms of assumed solutions are:

$$\begin{aligned}
u_0(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn} \sin\tfrac{m\pi x}{a} \cos\tfrac{n\pi y}{b} e^{i\omega t},\\
v_0(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn} \cos\tfrac{m\pi x}{a} \sin\tfrac{n\pi y}{b} e^{i\omega t},\\
w_0(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} \sin\tfrac{m\pi x}{a} \sin\tfrac{n\pi y}{b} e^{i\omega t},\\
\theta_x(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} T_{xmn} \cos\tfrac{m\pi x}{a} \sin\tfrac{n\pi y}{b} e^{i\omega t},\\
\theta_y(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} T_{ymn} \sin\tfrac{m\pi x}{a} \cos\tfrac{n\pi y}{b} e^{i\omega t}.
\end{aligned} \tag{7}$$

If the assumed forms of the solution are replaced in the dynamic equations of motion, we obtain:

$$\left\{ \underbrace{\begin{bmatrix} L_{11} & L_{11} & L_{13} & L_{14} & L_{15} \\ L_{12} & L_{22} & L_{23} & L_{24} & L_{25} \\ L_{13} & L_{23} & L_{33} & L_{34} & L_{35} \\ L_{14} & L_{24} & L_{34} & L_{44} & L_{45} \\ L_{15} & L_{25} & L_{35} & L_{45} & L_{55} \end{bmatrix}}_{\mathbf{L}} - \omega^2 \underbrace{\begin{bmatrix} I_1 & 0 & -\alpha I_2\Delta_2 & I_4\Delta_2 & 0 \\ 0 & I_4 & -\beta I_2\Delta_1 & 0 & I_4\Delta_1 \\ \alpha I_2\Delta & \beta I_2\Delta & -I_3(\alpha^2+\beta^2)+I_1 & -\alpha I_5 & -I_5\beta \\ I_4 & 0 & -\alpha I_5 & I_6 & 0 \\ 0 & I_4\Delta_2 & -\beta I_5 & 0 & I_6 \end{bmatrix}}_{\mathbf{I}} \right\} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ T_{xmn} \\ T_{ymn} \end{Bmatrix} = 0, \tag{8}$$

where $\Delta = \cot(\alpha x)\cot(\beta y)$, $\Delta_1 = \tan(\alpha x)/\tan(\beta y)$, $\Delta_2 = \tan(\beta x)/\tan(\alpha y)$, while the coefficients of the matrix L are:

$$\begin{gathered}
L_{11} = \alpha^2 A_{11} + \beta^2 A_{66}, L_{12} = \alpha\beta(A_{12}+A_{66}), L_{13} = -3B_{16}\alpha^2\beta - B_{26}\beta^3, L_{14} = 2D_{16}\alpha\beta,\\
L_{15} = \alpha^2 D_{16} + \beta^2 D_{26}, L_{22} = \alpha^2 A_{66} + \beta^2 A_{22}, L_{23} = -B_{16}\alpha^3 - 3B_{26}\alpha\beta^2,\\
L_{24} = \alpha^2 E_{16} + \beta^2 E_{26}, L_{25} = 2\alpha\beta E_{26}, L_{33} = \alpha^4 E_{11} + 2\alpha^2\beta^2 E_{12} + 4\alpha^2\beta^2 E_{66} + \beta^4 E_{22},\\
L_{34} = -\alpha^3 F_{11} - \alpha\beta^2 F_{12} - 2\alpha\beta^2 F_{66}, L_{35} = -\alpha^2\beta F_{12} - 2\alpha^2\beta F_{66} - \beta^3 F_{22},\\
L_{44} = H_{44} + \alpha^2 G_{11} + \beta^2 G_{66}, L_{45} = \alpha\beta(G_{12}+G_{66}), L_{55} = H_{55} + \alpha_2 G_{66} + \beta^2 G_{22}.
\end{gathered} \tag{9}$$

Table 1: Dimensionless frequency values $\overline{\omega}$ for antisymmetric angleply laminate $[\theta/-\theta/\theta/-\theta/\theta/-\theta]$, $\theta = 30°$ at a variable ratio a/h, fixed ratio $E_1/E_2 = 40$ and adopted engineering constants defined by material 1

	a/h, $m=1$, $n=1$				
Author	2	4	10	50	100
Present study	5.122	9.984	17.931	23.054	23.292
Ambartsumain[2]	5.133	9.988	17.931	23.053	23.291
Kaczkowski, Pancand Reissner[21]	/	/	/	/	/
Levy, Stein, Touratier [9]	5.213	10.02	17.934	23.053	23.291
Mantari[17]	/	/	/	/	/
Viola [23]	5.989	11.748	19.424	23.168	23.321
Mantari[16]	5.131 /	9.987 /	17.931 /	23.053 /	23.291 /
Karama, Aydogdu [11], [4]	5.309	10.078	17.949	23.054	23.291
Mantari[14]	5.321	10.084	17.951	23.054	23.291
Meiche[18]	5.321	10.084	17.951	23.054	23.292
Soldatos [22]	5.126	9.985	17.931	23.054	23.292
Mantari[15]	/	/	/	/	/
Akavci and Tanrikulu [3]	4.999	9.96	17.968	23.058	23.293
Akavci and Tanrikulu [3]	8.732	13.233	17.740	23.180	23.324
Grover [7]	/	/	/	/	/
Mechab[5]	5.139	9.990	17.931	23.053	23.292

From Tables 1–6, it is clear that not all of the presented shape functions are applicable in the analysis of free vibrations of antisymmetric angleply composite laminates. It can also be noticed that in the case of thick and moderately thick plates, among the shape functions which can be used to obtain a solution, there is a large number of those which would not provide satisfactory solutions.Therefore, it is not difficult to conclude that not all shape functions are applicable to all types of macromechanical analysis of laminate plates.The difference between the obtained results is especially outstanding in the ratio $E_1/E_2 \geq 40$. The proposed shape function gives satisfactory results in all types of analysis, which is clearly seen from the tables shown.

Table 2: Dimensionless frequency values $\overline{\omega}$ for antisymmetric angleply laminate $[\theta/-\theta/\theta/-\theta/\theta/-\theta]$, $\theta = 45°$ at a variable ratio a/h, fixed ratio $E_1/E_2 = 40$ and adopted engineering constants defined by material 1

	$a/h, m = 1, n = 1$				
Author	2	4	10	50	100
Present study	5.265	10.397	18.926	24.480	24.739
Ambartsumain [2]	5.277	10.401	18.925	24.480	24.739
Kaczkowski, Pancand Reissner [21]	/	/	/	/	/
Levy, Stein, Touratier [9]	5.365	10.441	18.929	24.479	24.739
Mantari [17]	/	/	/	/	/
Viola [23]	6.169	12.288	20.545	24.605	24.771
Mantari [16]	5.275 /	10.400 /	18.926 /	24.408 /	24.793 /
Karama, Aydogdu [11], [4]	5.275	10.500	18.945	24.480	24.739
Mantari [14]	5.483	10.507	18.947	24.480	24.739
Meiche[18]	5.483	10.507	18.947	24.480	24.739
Soldatos [22]	5.270	10.399	18.926	24.480	24.739
Mantari [15]	/	/	/	/	/
Akavci and Tanrikulu [3]	5.128	10.373	18.967	24.484	24.740
Akavci and Tanrikulu [3]	6.883	13.765	20.884	24.617	24.774
Grover [7]	/	/	/	/	/
Mechab [5]	5.284	10.404	18.925	24.480	24.739

In the comparative analysis in Table 3, it can be clearly seen that if we compare the values of the dimensionless frequency obtained by using two arbitrary functions (for example, the function defined by Karama and others, and the function defined by Soldatos) $E_1/E_2 = 3$ the difference will amount to 0.001 or less, while in the ratio $E_1/E_2 = 50$ this difference has value 0.03.

Table 3: Dimensionless frequency values $\overline{\omega}$ for antisymmetric angleply laminate $[\theta/-\theta/\theta/-\theta/\theta/-\theta]$, $\theta = 30^\circ$ at a variable ratio E_1/E_2, fixed ratio $a/h = 10$ and adopted engineering constants defined by material 1

	$E_1/E_2 \quad m=1, n=1$				
Author	3	5	10	20	50
Present study	7.595	8.916	11.331	14.429	19.064
Ambartsumain [2]	7.595	8.916	11.331	14.428	19.064
Kaczkowski, Pancand Reissner [21]	/	/	/	/	/
Levy, Stein, Touratier [9]	7.595	8.916	11.331	14.429	19.069
Mantari [17]	/	/	/	/	/
Viola [23]	7.687	9.071	11.669	15.166	20.887
Mantari [16]	7.595 /	8.916 /	11.331 /	14.428 /	19.064 /
Karama, Aydogdu [11], [4]	7.596	8.918	11.335	14.435	19.089
Mantari [14]	7.596	8.918	11.335	14.436	19.092
Meiche[18]	7.596	8.918	11.335	14.436	19.092
Soldatos [22]	7.595	8.916	11.331	14.428	19.064
Mantari [15]	/	/	/	/	/
Akavci and Tanrikulu [3]	7.597	8.920	11.341	14.449	19.106
Akavci and Tanrikulu [3]	7.705	9.100	11.725	15.292	21.318
Grover [7]	/	/	/	/	/
Mechab [5]	7.595	8.916	11.331	14.428	19.064

Table 4: Dimensionless frequency values $\overline{\omega}$ for antisymmetric angleply laminate $[\theta/-\theta/\theta/-\theta/\theta/-\theta]$, $\theta = 45°$ at a variable ratio E_1/E_2, fixed ratio $a/h = 10$ and adopted engineering constants defined by material 1

	E_1/E_2 $m = 1, n = 1$				
Author	3	5	10	20	50
Present study	7.718	9.190	11.838	15.188	20.124
Ambartsumain [2]	7.718	9.190	11.838	15.188	20.124
Kaczkowski, Pancand Reissner [21]	/	/	/	/	/
Levy, Stein, Touratier [9]	7.718	9.190	11.838	15.188	20.129
Mantari [17]	/	/	/	/	/
Viola [23]	7.812	9.353	12.201	15.987	22.100
Mantari [16]	7.718 /	9.190 /	11.838 /	15.188 /	20.124 /
Karama, Aydogdu [11], [4]	7.719	9.191	11.842	15.195	20.150
Mantari [14]	7.720	9.191	11.842	15.196	20.154
Meiche[18]	7.720	9.192	11.842	15.196	20.154
Soldatos [22]	7.718	9.190	11.838	15.188	20.124
Mantari [15]	/	/	/	/	/
Akavci and Tanrikulu [3]	7.721	9.194	11.849	15.211	20.170
Akavci and Tanrikulu [3]	7.831	9.381	12.258	16.118	22.565
Grover [7]	/	/	/	/	/
Mechab [5]	7.718	9.190	11.838	15.187	20.124

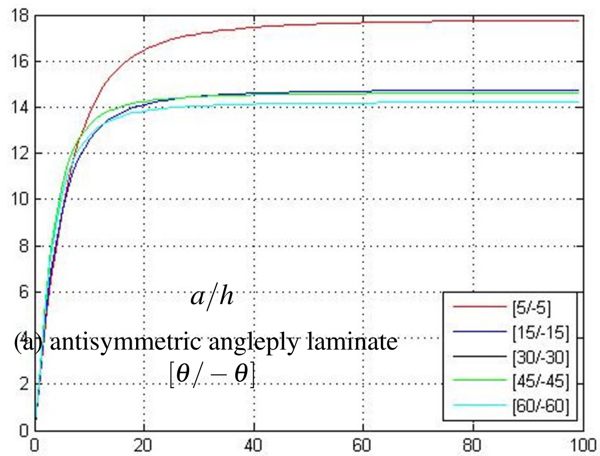
Table 5: Dimensionless frequency values $\overline{\omega}$ for antisymmetric angleply laminate $[\theta/-\theta]$, $\theta = 30^\circ$ at a variable ratio a/h, ficed ratio $E_1/E_2 = 40$ and adopted engineering constants defined by material 1

	$a/h, m=1, n=1$				
Author	2	4	10	50	100
Present study	5.163	9.002	12.789	14.171	14.222
Ambartsumain [2]	/	/	/	/	/
Kaczkowski, Pancand Reissner [21]	/	/	/	/	/
Levy, Stein, Touratier [9]	5.287	9.093	12.821	14.172	14.222
Mantari [17]	5.816	9.492	12.960	14.179	14.224
Viola [23]	5.280	9.324	12.965	14.181	14.224
Mantari [16]	5.175 5.003	9.010 8.887	12.792 12.750	14.171 14.169	14.222 14.221
Karama, Aydogdu [11], [4]	/	/	/	/	/
Mantari [14]	/	/	/	/	/
Meiche[18]	/	/	/	/	/
Soldatos [22]	5.169	9.006	12.791	14.171	14.222
Mantari [15]	4.905	8.936	12.802	14.172	14.222
Akavci and Tanrikulu [3]	/	/	/	/	/
Akavci and Tanrikulu [3]	5.401	9.183	12.853	14.174	14.222
Grover [7]	/	/	/	/	/
Mechab [5]	5.187	9.019	12.795	14.171	14.222

Table 6: Dimensionless frequency values $\overline{\omega}$ for antisymmetric angleply laminate $[\theta/-\theta]$, $\theta = 45^{\circ}$ at a variable ratio a/h, fixed ratio $E_1/E_2 = 40$ and adopted engineering constants defined by material 1

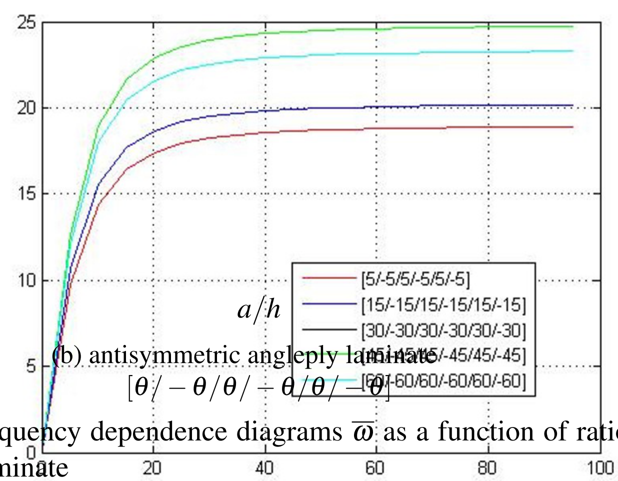
	$a/h, m=1, n=1$				
Author	2	4	10	50	100
Present study	5.346	9.371	13.224	14.572	14.621
Ambartsumain [2]	/	/	/	/	/
Kaczkowski, Pancand Reissner [21]	/	/	/	/	/
Levy, Stein, Touratier [9]	5.481	9.471	13.259	14.574	14.621
Mantari [17]	6.057	9.907	13.407	14.581	14.623
Viola [23]	5.426	9.651	13.373	14.580	14.623
Mantari [16]	5.358 5.169	9.380 9.243	13.228 13.181	14.572 14.570	14.620 14.621
Karama, Aydogdu [11], [4]	/	/	/	/	/
Mantari [14]	/	/	/	/	/
Meiche[18]	/	/	/	/	/
Soldatos [22]	5.351	9.375	13.226	14.572	14.621
Mantari [15]	5.043	9.266	13.217	14.575	14.621
Akavci and Tanrikulu [3]	/	/	/	/	/
Akavci and Tanrikulu [3]	5.605	9.571	13.294	14.572	14.622
Grover [7]	/	/	/	/	/
Mechab [5]	5.372	9.390	13.231	14.572	14.621

Diagrampresentations of the change in the value of the dimensionless frequency as a function of the change in the ratio a/h are given in the Figure 3. It should be emphasized that the theory of Mechab and others was chosen for the diagram because all the theories that are applicable to this type of problem give quite similar results, so the curves would overlap in the diagram. Another reason for choosing this function to present the results lies in the fact that this theory provides solutions without the use of numerical integration procedures. Therefore, the degree of approximation is lower and the accuracy of the results is higher.



a/h

(a) antisymmetric angleply laminate $[\theta/-\theta]$

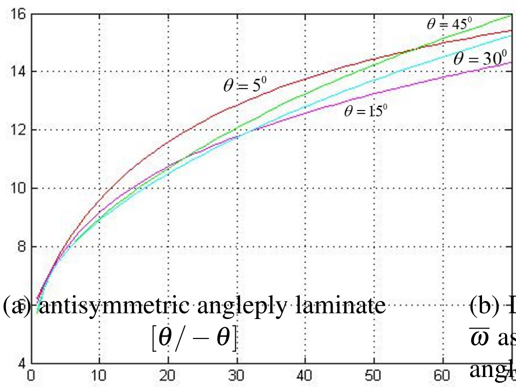


a/h

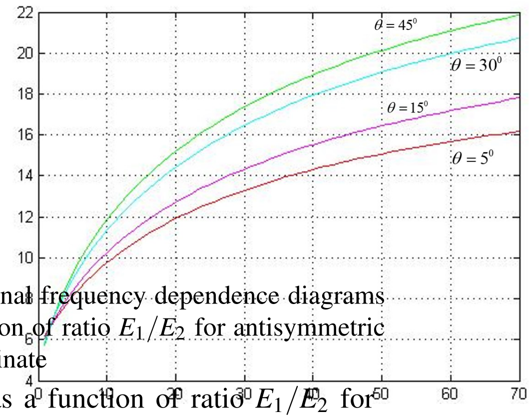
(b) antisymmetric angleply laminate $[\theta/-\theta/\theta/-\theta/\theta/-\theta]$

Fig. 3: Dimensional frequency dependence diagrams $\overline{\omega}$ as a function of ratio a/h for antisymmetric angleply laminate

In Figure 3 it can be seen that with the change of the ratio a/h there is an asymptotic approach to the maximum value of the dimensionless frequency. From the tables, and in the figure itself, it is clear that with large values of the ratio $a/h > 20$, the influence of the shape function loses its significance, so satisfactory results can be obtained with theories in which the mathematical procedure is much simpler.The simplicity of the mathematical procedure affects the time required for numerical calculations. The proposed shape function



(a) antisymmetric angleply laminate $[\theta/-\theta]$



(b) Dimensional frequency dependence diagrams $\overline{\omega}$ as a function of ratio E_1/E_2 for antisymmetric angleply laminate

Fig. 4: Dimensional frequency dependence diagrams $\overline{\omega}$ as a function of ratio E_1/E_2 for antisymmetric angleply laminate

gave good results in this consideration as well.

In the Figure 3a it can be noticed that the values of the dimensionless frequency at small values of the angle of orientation of the fibers, for example 5° for a laminate plate composed of two layers, deviate significantly from the values at orientation angles in the range of $15° - 60°$. For the mentioned range of orientation angles, it is clear that the differences in values $\overline{\omega}$ are minimal, which causes a large convergence of the curves, almost to the point of overlapping.

Figure 3b shows that the largest value $\overline{\omega}$ for a laminate plate composed of six layers is reached at the angles of orientation of the layers $\pm 45°$, while the lowest value is at the orientation angles $\pm 5°$. It is also clear that, unlike a plate composed of two layers, there is no great convergence, i.e. overlapping of curves for different angles of orientation of the layers.

Figure 4 shows the dependence of the dimensionless frequency on the change in the ratio E_1/E_2 for the adopted same shape function as in the previously mentioned analysis. From the diagram it is noticeable that with the increase of the ratio E_1/E_2 there is also the increase in the difference between the maximum values of the dimensionless frequency for different, fixed angles of layer orientation. Unlike changing ratio a/h here there is no asymptotic approximation of a value, but a constant increase in value.

2 Conclusion

In the analysis of the free vibrations of crosply and angleply laminates, it is possible to perform an analytical procedure to obtain the value of dimensionless frequency. The procedure is similar to the procedures used for bending and buckling problems. It has been observed that HSDT theories based on shape functions give good results at small ratios E_1/E_2, while with the increase in the value of this ratio, the differences increase.It has also been observed that some functions have larger deviations in the values of the dimensionless frequency $\overline{\omega}$, therefore, their use is limited to thin plates, similar to lower-order theories. When considering the problem of the free vibration of a simple supported angleply laminate plate, significant limitations of the use of some shape functions have been observed. As many as five of the proposed fifteen shape functions cannot be used because they cannot give results using analytical methods.The other negative side is that the use of the remaining functions does not allow obtaining acceptable results for $\overline{\omega}$.

The proposed shape function met all the necessary criteria and its application was verified by comparative analysis with existing shape functions. The advantages of this shape function are:

• analytical integrability;
• great accuracy of results;
• short calculation time;
• applicability on thick and moderately thick plates.

Having in mind everything aforesaid, it is not difficult to conclude that the use of the proposed shape function is fully justified and as such can be applied in numerical calculations of complex structures.

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