SCIENTIFIC PUBLICATIONS OF THE STATE UNIVERSITY OF NOVI PAZAR
SER. A: APPL. MATH. INFORM. AND MECH. vol. 13, 1 (2021), 43-60.

C-class and pair upper class functions and other kind of contractions in fixed point theory

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Abstract: In 2014 was introduced C-class and pair upper Class functions that cover more papers before and after that .base on them some other ideas like :1-1-upclass functions,multiplicative C-class functions,inverse-C-class functions,CF -simulation functions were planed.In this glance we look for some condition that can use them or can not.

1 Introduction and mathematical preliminaries

The contraction mapping principle,presenteded in Banach's Ph.D. dissertation and published in 1922 [4], is the source of metric fixed point theory. This basic principle was largely used in dealing with various theoretical and practical problems, arising in a number of branches of mathematics. This potentiality attracted many researchers and hence the literature is reach in fixed point results.

Definition 1.1. *Let (X,d) be a metric space. Then a map $T : X \to X$ is called a contraction mapping on X if there exists $k \in [0,1)$ such that*

$$d(T(x),T(y)) \leq kd(x,y) \tag{1.1}$$

for all x, y in X .

Theorem 1.1. *(Banach Fixed Point Theorem). Let (X,d) be a non-empty complete metric space with a contraction mapping $T : X \to X$. Then T admits a unique fixed-point x^* in X (i.e. $T(x^*) = x^*$). Furthermore, x^* can be found as follows: If we start with an arbitrary element x_0 in X and define a sequence $\{x_n\}$ by $x_n = T(x_{n-1})$, then $x_n \to x^*$.*

Manuscript received September 21, 2020; accepted January 12, 2021.

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Example 1.1. *Let* $X = \mathbb{R}, T : X \to X$,

$$T(x) = (x-2)(x-3)(x-4) - 3(x-1)(x-3) - 4(x-2)(x-4) + 5(x-2)(x-3) - 1$$

then

$$T(2) = 2 \quad \& \quad T(3) = 3$$

note that

$$36 = |T(2) - T(0)| > |2-0| \; = 2$$

Example 1.2. *Let* $X = \mathbb{R}, T : X \to X$,

$$T(x) = \frac{x+2}{2}$$

then

$$T(2) = 2$$

note that

$$|T(x) - T(y)| \le k|x-y| \; \text{, for all} \quad \frac{1}{2} < k < 1$$

The concept Geraghty contraction type maps was introduced by Geraghty [19] in 1973 for generalization of the Banach contraction principle by an contorol function.

Let $\mathfrak{S}$ denote the class of all real functions $\beta : [0, +\infty) \to [0, 1)$ satisfying the condition

$$\beta(t_n) \to 1 \text{ implies } t_n \to 0, \text{ as } n \to \infty.$$

In order to generalize the Banach contraction principle, Geraghty proved the following.

Theorem 1.2. [19] *Let* (X, d) *be a complete metric space, and let* $f : X \to X$ *be a self-map. Suppose that there exists* $\beta \in \mathfrak{S}$ *such that*

$$d(fx, fy) \le \beta(d(x,y))d(x,y)$$

holds for all $x, y \in X$. *Then f has a unique fixed point* $z \in X$ *and for each* $x \in X$ *the Picard sequence* $f^n x$ *converges to z.*

Definition 1.2. [2] *A map A will be called weakly contractive on a closed convex set* Ω *in the Banach space B if there exists a continuous and nondecreasing function defined on* R^+ *such that* ψ *is positive on* $R^+ \backslash \{0\}, \psi(0) = 0$, $lim_{t \to +\infty} \psi(t) = +\infty$ *and* $\forall x, y \in \Omega$,

$$\|A(x) - A(y)\| \le \|x-y\| - \psi(\|x-y\|) \tag{1.2}$$

for all x, y *in* X .

If $\psi(t) = (1-k)t$ where $0 < k < 1$, then (1.2) reduces to (1.1) .

Theorem 1.3. [2] *If A is a weakly contractive map on $\Omega \subset H$ then it has a unique fixed point $x^* \in \Omega$.*

Khan et al. [16] introduce a new control function (altering distance function) which are very useful in fixed point theory.

Definition 1.3. [16] *A function $\psi : [0,+\infty) \to [0,+\infty)$ is called a altering distance function if the following properties are satisfied:*

(i) $\psi(0) = 0$,

(ii) ψ *is continuous and monotonically non-decreasing.*

We denote by Ψ the set of all altering distance functions.

Theorem 1.4. [16] *Let (X,d) be a complete metric space, let ψ be an altering distance function,and let $f : X \to X$ be a self-mapping which satisfies the following inequality:*

$$\psi(d(T(x),T(y))) \leq c\psi(d(x,y)) \tag{1.3}$$

for all $x,y \in X$ and for some $0 < c < 1$. Then f has a unique fixed point.

Rhoades [22] considered this class of mappings in metric spaces .We can see the work of Rhoades in the following.

Definition 1.4. [22] *A mapping $T : X \to X$, where X, d is a metric space, is said to be weakly contractive if,*

$$d(T(x),T(y)) \leq d(x,y) - \varphi(d(x,y)) \tag{1.4}$$

for all x,y in X and $\varphi : [0,+\infty) \to [0,+\infty)$ is a continuous and nondecreasing function such that $\varphi(0) = 0$ if and only if $t = 0$.

Theorem 1.5. [22] *Let (X,d) be a complete metric space, T a weakly contractive map . then T has a unique fixed point x in X .*

Theorem 1.6. [8] *Let (X,d) be a complete metric space and let $T : X \to X$ be a self-mapping satisfying the inequality*

$$\psi(d(T(x),T(y))) \leq \psi(d(x,y)) - \varphi(d(x,y)) \tag{1.5}$$

where $\psi,\varphi : [0,+\infty) \to [0,+\infty)$ are both continuous and monotone nondecreasing functions with $\psi(0) = \varphi(0) = 0$, if and only if $t = 0$. Then T has a unique fixed point.

2 *C*-class functions

In 2014 the observation $d(T(x),T(y)) \le cd(x,y) \le d(x,y)$, $d(fx,fy) \le \beta(d(x,y))d(x,y) \le d(x,y)$, $\psi(d(T(x),T(y))) \le \psi(d(x,y)) - \varphi(d(x,y)) \le \psi(d(x,y))$, guided to *C*-class function in [9] as following,

Definition 2.1. [9] *A mapping $f : [0,\infty)^2 \to \mathbb{R}$ is called C-class function if it is continuous and satisfies following axioms:*
(1) $f(s,t) \le s$;
(2) $f(s,t) = s$ implies that either $s = 0$ or $t = 0$ for all $s,t \in [0,\infty)$.

Note that for some f we have $f(0,0) = 0$.
We denote *C*-class functions as $\mathscr{C}$.

Example 2.1. [9] *The following functions $f : [0,\infty)^2 \to \mathbb{R}$ are elements of $\mathscr{C}$:*
(1) $f(s,t) = s-t$, $f(s,t) = s \Rightarrow t = 0$;
(2) $f(s,t) = ms$, $0<m<1$, $f(s,t) = s \Rightarrow s = 0$;
(3) $f(s,t) = \frac{s}{(1+t)^r}$; $r \in (0,\infty)$, $f(s,t) = s \Rightarrow s = 0$ or $t = 0$;
(4) $f(s,t) = \log(t+a^s)/(1+t)$, $a > 1$, $f(s,t) = s \Rightarrow s = 0$ or $t = 0$;
(5) $f(s,t) = \ln(1+a^s)/2$, $a > e$, $f(s,t) = s \Rightarrow s = 0$;
(6) $f(s,t) = (s+l)^{(1/(1+t)^r)} - l$, $l > 1, r \in (0,\infty)$, $f(s,t) = s \Rightarrow t = 0$;
(7) $f(s,t) = s\log_{t+a} a$, $a > 1$, $f(s,t) = s \Rightarrow s = 0$ or $t = 0$;
(8) $f(s,t) = s - (\frac{1+s}{2+s})(\frac{t}{1+t})$, $f(s,t) = s \Rightarrow t = 0$;
(9) $f(s,t) = s\beta(s)$, $\beta : [0,\infty) \to [0,1)$ and is continuous, $f(s,t) = s \Rightarrow s = 0$;
(10) $f(s,t) = s - \frac{t}{k+t}, f(s,t) = s \Rightarrow t = 0$;
(11) $f(s,t) = s - \varphi(s), f(s,t) = s \Rightarrow s = 0$, here $\varphi : [0,\infty) \to [0,\infty)$ is a continuous function such that $\varphi(t) = 0 \Leftrightarrow t = 0$;
(12) $f(s,t) = sh(s,t), f(s,t) = s \Rightarrow s = 0$,here $h : [0,\infty) \times [0,\infty) \to [0,\infty)$ is a continuous function such that $h(t,s) < 1$ for all $t,s > 0$;
(13) $f(s,t) = \sqrt[n]{\ln(1+s^n)}$, $f(s,t) = s \Rightarrow s = 0$.

Definition 2.2. [9] *A function $\varphi : [0,+\infty) \to [0,+\infty)$ is called an Ultra-altering distance function if φ is continuous, and $\varphi(0) \ge 0$, $\varphi(t) > 0$,$t > 0$.*

Definition 2.3. [9] *A mapping $h : [0,+\infty) \to [0,+\infty)$ is an A-class function if $h(t) \ge t, \forall t \ge 0$.*
We denote by $\mathscr{A}$ the set of all $\mathscr{A}$-class functions.

Example 2.2. *The following functions $h : [0,+\infty) \to [0,+\infty)$ are elements of $\mathscr{A}$:*

(1) $h(t) = a^t - 1$, $a > 1$, $t \in [0,+\infty)$;

(2) $h(t) = mt$, $m \ge 1$, $t \in [0,+\infty)$.

Definition 2.4. [9] *Let $T : X \to X$, then $F \subset X$ a subset of X invariant under T iff*

$$x \in F \Longrightarrow T(x) \in F$$

Theorem 2.1. [9] *Let T be a self-mapping defined on a complete metric space (X,d) satisfying the condition*

$$h(\psi(d(T(x),T(y))) \leq f(\psi(d(x,y)),\varphi(d(x,y))) \tag{2.1}$$

for $x,y \in F \subset X$, F subclosed of X and invariant under T , ψ and φ are the earlier described altering distance function(or an Ultra-altering distance function),f a function of C- class , h a function of A- class , Then T has a unique fixed point in F .

If let take $h(t) = t, f(s,t) = s - t, F = X$, then (2.1) reduces to (1.5) .

3 Some remarks for best case of contractions

Remark 3.1. *Let $h,g : [0,+\infty) \to [0,+\infty)$ with $t \leq g(t) \leq h(t)$ and if we have that*

$$h(\psi(d(T(x),T(y))) \leq f(\psi(d(x,y)),\varphi(d(x,y))) \tag{3.1}$$

and

$$g(\psi(d(T(x),T(y))) \leq f(\psi(d(x,y)),\varphi(d(x,y))) \tag{3.2}$$

so (3.2) is more general than (3.1) ,therefore ,

$$\psi(d(T(x),T(y)) \leq f(\psi(d(x,y)),\varphi(d(x,y))) \tag{3.3}$$

is the best case

Remark 3.2. *Let $h,g : [0,+\infty) \to [0,+\infty)$ with $g(t) \leq h(t)$ and if we have that*

$$\psi(d(T(x),T(y))) \leq g(\psi(d(x,y))) \tag{3.4}$$

and

$$\psi(d(T(x),T(y))) \leq h(\psi(d(x,y))) \tag{3.5}$$

so (3.5) is more general than (3.4) .Therefore if of the following

$$\psi(d(T(x),T(y))) \leq g(\psi(d(x,y)))$$

we obtain fixed point then the following contraction

$$\psi(d(T(x),T(y))) \leq f(g(\psi(d(x,y))),\varphi(d(x,y)))$$

where $f \in \mathscr{C}$, is not new . because

$$\psi(d(T(x),T(y))) \leq f(g(\psi(d(x,y))),\varphi(d(x,y))) \leq g(\psi(d(x,y))).$$

4 1-1-upclass functions

Definition 4.1. [7] *The pair of functions (ψ,ϕ) is a pair of generalized altering distance where $\psi,\phi:[0,+\infty)\to[0,+\infty)$ if the following hypotheses hold:*

(a1) ψ *is continuous and non-decreasing;*

(a2) $\lim_{n\to\infty}\phi(t_n)=0\Rightarrow\lim_{n\to\infty}t_n=0$.

Definition 4.2. [7] *Let X be a set, and let $\mathscr{R}$ be a binary relation on X. A mapping $T:X\to X$ is an $\mathscr{R}$-preserving mapping if $x,y\in X: x\mathscr{R}y\Rightarrow Tx\mathscr{R}Ty$.*

In the sequel, let $\mathbb{N}$ denote the set of all non-negative integers, let $\mathbb{R}$ denote the set of all real numbers.

Definition 4.3. [7] *Let $N\in\mathbb{N}$. $\mathscr{R}$ is N-transitive on X if $x_0,x_1,...,x_{N+1}\in X: x_i\mathscr{R}x_{i+1}$ for all $i=\{0,1,...,N\}\Rightarrow x_0\mathscr{R}x_{N+1}$.*

The following remark is a consequence of the previous definition.

Definition 4.4. [7] *Let (X,d) be a metric space and $\mathscr{R}_1$, $\mathscr{R}_2$ two binary relations on X. A metric space (X,d) is $(\mathscr{R}_1,\mathscr{R}_2)$-regular if for every sequence $\{x_n\}$ in X such that $x_n\to x\in X$ as $n\to+\infty$, and $x_n\mathscr{R}_1x_{n+1},x_n\mathscr{R}_2x_{n+1}$ for all $n\in\mathbb{N}$, there exists a subsequence $\{x_{n(k)}\}$ such that $x_{n(k)}\mathscr{R}_1x,x_{n(k)}\mathscr{R}_2x$. for all $k\in\mathbb{N}$.*

Definition 4.5. [7] *A subset D of X is $(\mathscr{R}_1,\mathscr{R}_2)$-directed if for all $x,y\in D$, there exists $z\in X$ such that $(x\mathscr{R}_1z)\wedge(y\mathscr{R}_1z)$ and $(x\mathscr{R}_2z)\wedge(y\mathscr{R}_2z)$.*

Definition 4.6. [7] *Let X be a set and $\alpha,\beta:X\times X\to[0,+\infty)$ are two mappings. We define two binary relations $\mathscr{R}_1$ and $\mathscr{R}_2$ on X by*

$$x\mathscr{R}_1y\Longleftrightarrow\alpha(x,y)\le 1\quad\textit{and}\quad x\mathscr{R}_2y\Longleftrightarrow\beta(x,y)\ge 1, \tag{4.1}$$

for all $x,y\in X$.

Definition 4.7. [7] *Let (X,d) be a metric space. A mapping $T:X\to X$ is $(\alpha\psi,\beta\phi)$-contractive mappings if there exists a pair of generalized distance (ψ,ϕ) such that*

$$\psi(d(Tx,Ty))\le\alpha(x,y)\psi(d(x,y))-\beta(x,y)\phi(d(x,y))\ \textit{for all}\ x,y\in X, \tag{4.2}$$

where $\alpha,\beta:X\times X\to[0,+\infty)$.

Theorem 4.1. [7] *Let (X,d) be a complete metric space, $N\in\mathbb{N}\backslash\{0\}$, and $T:X\to X$ be an $(\alpha\psi,\beta\phi)$-contractive mapping satisfying the following conditions:*

(A1) $\mathscr{R}_i$ *is N-transitive for $i=1,2$;*

(A2) T *is* $\mathscr{R}_i$*-preserving for* $i = 1,2$*;*

(A3) *there exists* $x_0 \in X$ *such that* $x_0 \mathscr{R}_i T x_0$ *for* $i = 1,2$*;*

(A4) T *is continuous.*

Then, T *has a fixed point, that is, there exists* $x^* \in X$ *such that* $Tx^* = x^*$.

Definition 4.8. [12] *A mapping* $f : [0,\infty)^4 \to \mathbb{R}$ *is a 1-1-upclass function if the following conditions hold for all* $u,v,s,t \in [0,\infty)$

1. $f(1,1,s,t)$ *is continuous;*
2. $0 \le u \le 1, v \ge 1 \Rightarrow f(u,v,s,t) \le f(1,1,s,t) \le s$*;*
3. $f(1,1,s,t) = s \Rightarrow s = 0$ *or* $t = 0$.

We denote $\mathscr{C}_1$ *the set of all 1-1-upclass functions.*

Note that for some f we have $f(1,1,0,0) = 0$.

Example 4.1. [12] *The following functions* $f : [0,\infty)^4 \to \mathbb{R}$ *are elements of* $\mathscr{C}_1$ *for all* $u,v,s,t \in [0,\infty)$*:*

1. $f(u,v,s,t) = us - vt$, $f(1,1,s,t) = s \Rightarrow t = 0$*;*
2. $f(u,v,s,t) = \dfrac{us - vt}{1+vt}$, $f(1,1,s,t) = s \Rightarrow t = 0$*;*
3. $f(u,v,s,t) = \dfrac{us}{1+vt}$, $f(1,1,s,t) = s \Rightarrow s = 0$ *or* $t = 0$*;*
4. $f_a(u,v,s,t) = \log_a \dfrac{ut + a^{us}}{1+vt}$, $a > 1$, $f_a(1,1,s,t) = s \Rightarrow s = 0$ *or* $t = 0$*;*
5. $f(u,v,s,t) = \ln \dfrac{u + e^{us}}{1+v}$, $f(1,1,s,1) = s \Rightarrow s = 0$*;*
6. $f_a(u,v,s,t) = (us + a)^{\frac{1}{1+vt}} - a$, $a > 1$, $f_a(1,1,s,t) = s \Rightarrow t = 0$*;*
7. $f_a(u,v,s,t) = us \log_{a+vt} a$, $a > 1$, $f_a(1,1,s,t) = s \Rightarrow s = 0$ *or* $t = 0$

Definition 4.9. [12] *Let* (X,d) *be a metric space. A mapping* $T : X \to X$ *is* (CAB)*-contractive mapping if there exists a pair of generalized altering function* (ψ,ϕ)*,* $h \in \mathscr{A}$ *and* $f \in \mathscr{C}_1$ *such that*

$$h(\psi(d(Tx,Ty))) \le f(\alpha(x,y), \beta(x,y), \psi(d(x,y)), \phi(d(x,y))) \text{ for all } x,y \in X, \tag{4.3}$$

where $\alpha, \beta : X \times X \to [0,+\infty)$.

Theorem 4.2. [12] *Let (X,d) be a complete metric space, $N \in \mathbb{N}\backslash\{0\}$, and $T : X \to X$ be an (CAB)-contractive mapping satisfying the following conditions:*

(A1) $\mathscr{R}_i$ is N-transitive for $i = 1,2$;

(A2) T is $\mathscr{R}_i$-preserving for $i = 1,2$;

(A3) there exists $x_0 \in X$ such that $x_0 \mathscr{R}_i T x_0$ for $i = 1,2$;

(A4) T is continuous.

Then, T has a fixed point, that is, there exists $x^* \in X$ such that $Tx^* = x^*$.

5 Cone C-class functions

Let E be a real Banach space with the zero vector θ and P a nonempty subset of E. P is called a cone if and only if:
(i) P is closed, non-empty and $P \neq \{\theta\}$,
(ii) $ax + by \in P$ for all $x, y \in P$ and non-negative real numbers a, b,
(iii) $P \cap (-P) = \{\theta\}$.

Given a cone $P \subset E$, we define a partial ordering $\preceq$ with respect to P by $x \preceq y$ if and only if $y - x \in P$. We shall write $x \prec y$ if $x \preceq y$ and $x \neq y$; we shall write $x \ll y$ if $y - x \in intP$, where $intP$ denotes the interior of P.

Definition 5.1. [13] *Let $\psi, \phi : IntP \cup \{\theta\} \to IntP \cup \{\theta\}$ be two continuous and monotone increasing functions satisfying*
(a) $\psi(t) = \phi(t) = \theta$ if and only if $t = \theta$,
(b) $t - \psi(t) \in P \cup \{\theta\}, \phi(t) \ll t$, for $t \in intP$.

Definition 5.2. [13] *A mapping $F : P^2 \to P$ is called cone C-class function if it is continuous and satisfies following axioms:*
(1) $F(s,t) \preceq s$;
(2) $F(s,t) = s$ implies that either $s = \theta$ or $t = \theta$; for all $s, t \in P$.

We denote C-class functions as $\mathscr{C}_{co}$.

Example 5.1. [13] *The following functions $F : P^2 \to P$ are elements of $\mathscr{C}_{co}$, for all $s, t \in P$:*
(1) $F(s,t) = s - t$, $F(s,t) = s \Rightarrow t = \theta$;
(2) $F(s,t) = ks$, $0<k<1$, $F(s,t) = s \Rightarrow s = \theta$;
(3) $F(s,t) = s\beta(s)$, $\beta : P \to [0,1)$, $F(s,t) = s \Rightarrow s = \theta$;
(4) $F(s,t) = s - \varphi(s), F(s,t) = s \Rightarrow s = \theta$, here $\varphi : P \to P$ is a continuous function such that $\varphi(t) = \theta \Leftrightarrow t = \theta$;
(5) $F(s,t) = s - h(s,t)$, $F(s,t) = s \Rightarrow t = \theta$, here $h : P \times P \to P$ is a continuous function such that $h(s,t) = \theta \Leftrightarrow t = \theta$ for all $t, s \succ \theta$.

Theorem 5.1. [13] *Let (X,d) be a complete cone metric space with regular and solid cone P such that $d(x,y) \in intP$, for $x,y \in X$ with $x \neq y$. Let $T : X \to X$ be a mapping satisfying the inequality*

$$\psi(d(Tx,Ty)) \preceq F(\psi(d(x,y)), \phi(d(x,y))) \text{ for all } x,y \in X \tag{5.1}$$

where F is element of $\mathscr{C}_{co}$,ψ, ϕ are as in Definition 5.1 and they satisfy
(i) ψ is a continuous and strongly monotone increasing ($\psi(x) \preceq \psi(y) \Leftrightarrow x \preceq y$)
(ii) either $\phi(t) \preceq d(x,y)$ or $d(x,y) \ll \phi(t)$, for $t \in intP \cup \{\theta\}$ and $x,y \in X$. Then T has a unique fixed point in X.

Remark 5.1. *because operator exp,rational etc in cone do not mean, we can not freely use C-class functions in cone.*

6 Multiplicative C-class functions

Definition 6.1. [3] *A mapping $F : [1,\infty)^2 \to \mathbb{R}$ is called multiplicative C-class function if it is continuous and satisfies following axioms:*
(a) $F(x,y) \leq x$;
(b) $F(x,y) = x$ implies that either $x = 1$ or $y = 1$; for all $x,y \in [1,\infty)$.

We denote multiplicative C-class functions as $\mathscr{C}_m$. Several examples of $\mathscr{C}_m$ functions can be find in [3].

Base on recent work [1] we state the following proposition.

Proposition 6.1. *There is a bijective mapping between $\mathscr{C}_m$ and $\mathscr{C}$*

Proof. for each $f \in \mathscr{C}$ consider $F(x,y) = e^{f(\ln x, \ln y)}$,where $x,y \geq 1$
for all $F \in \mathscr{C}_m$ consider $f(s,t) = \ln[F(e^s,e^t)]$,where $s,t \geq 0$
these show a bijective map between C-class function and multiplicative C-class function. □

Now in the following see some relations,

Example 6.1. *Following examples show related class $\mathscr{C}$ and $\mathscr{C}_m$:*

1. $f(s,t) = s - t$. $\Longleftrightarrow$ $F(x,y) = \frac{x}{y}$
2. $f(s,t) = ms$,for some $m \in (0,1)$. $\Longleftrightarrow$ $F(x,y) = x^m$; $m \in (0,1)$,
3. $f(s,t) = \frac{s}{(1+t)^r}$ for some $r \in (0,\infty)$. $\Longleftrightarrow$ $F(x,y) = x^{e^{\frac{1}{(1+\ln y)^r}}}$,for some $r \in (0,\infty)$
4. $f(s,t) = \ln(\frac{1+a^s}{2})$, for $e > a > 1$. $\Longleftrightarrow$ $F(x,y) = \frac{1+a^{\ln x}}{2}$, for $e > a > 1$
5. $f(s,t) = s - \frac{t}{k+t}$. $\Longleftrightarrow$ $F(x,y) = \frac{x}{y^{\frac{1}{1+\ln y}}}$
6. $f(s,t) = \frac{s}{(1+s)^r}$; $r \in (0,\infty)$. $\Longleftrightarrow$ $F(x,y) = x^{e^{\frac{1}{(1+\ln x)^r}}}$
7. $f(s,t) = \ln(1+s)$. $\Longleftrightarrow$ $F(x,y) = 1 + \ln x$

7 Inverse-C-class functions

Definition 7.1. [24] *A mapping $F : [0,\infty)^2 \to \mathbb{R}$ is called inverse-C-class function if it is continuous and satisfies following axioms:*

(1) $F(s,t) \geq s$;

(2) $F(s,t) = s$ *implies that either* $s = 0$ *or* $t = 0$; *for all* $s,t \in [0,\infty)$.

Note that for some F we have that $F(0,0) = 0$.
We denote collection of all inverse $C-$class functions as $\mathscr{C}_{inv}$.

Example 7.1. [24] *The following functions $F : [0,\infty)^2 \to \mathbb{R}$ are elements of $\mathscr{C}_{inv}$, for all $s,t \in [0,\infty)$:*

1. $F(s,t) = s+t$, $F(s,t) = s \Rightarrow t = 0$;
2. $F(s,t) = ms$, $1<m<\infty$, $F(s,t) = s \Rightarrow s = 0$;
3. $F(s,t) = s(1+t)^r$; $r \in (0,\infty)$, $F(s,t) = s \Rightarrow s = 0$ *or* $t = 0$;
4. $F(s,t) = \log_a(t+a^s)(1+t)$, $a > 1$, $F(s,t) = s \Rightarrow t = 0$;
5. $F(s,t) = \phi(s), F(s,t) = s \Rightarrow s = 0$,*here* $\phi : [0,\infty) \to [0,\infty)$ *is a upper semicontinuous function such that* $\phi(0) = 0$, *and* $\phi(t) > t$ *for* $t > 0$,
6. $f(s,t) = \vartheta(s)$; $\vartheta : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ *is a generalized Mizoguchi-Takahashi type function,* $f(s,t) = s \Rightarrow \quad s = 0$;

We will use the following control functions, defined as:
Let Φ denote the set of all functions $\varphi : [0,+\infty) \to [0,+\infty)$ that satisfy the following conditions:

1. φ is lower semi-continuous on $[0,+\infty)$,
2. $\varphi(0) = 0$,
3. $\varphi(s) > 0$ for each $s > 0$.

Let Φ_1 denote the set of all functions $\varphi : [0,+\infty) \to [0,+\infty)$ that satisfy the following conditions:

1. φ is lower semi-continuous on $[0,+\infty)$,
2. $\varphi(0) \geq 0$,
3. $\varphi(s) > 0$ for each $s > 0$

Let Ψ denote all the functions $\psi : [0,\infty) \to [0,\infty)$ which satisfy:

1. $\psi(t) = 0$ if and only if $t = 0$,
2. ψ is continuous and increasing.

Theorem 7.1. [24] *Let X be a set with a symmetric d. Suppose that f and g are owc self maps of X satisfying:*

$$\psi(d(fx, fy)) \geq F(\psi(m(x,y)), \varphi(m(x,y))), \tag{7.1}$$

where $m(x,y) = \min\{d(gx,gy), d(fx,gx), d(fy,gy)\}$. Then f and g have common fixed point in X.

8 C_F-simulation functions

In this section, we generalized the simulation function introduced by Khojasteh et al. [17] using the function of C-class as follows:

Definition 8.1. [20] *A mapping $F : [0,\infty)^2 \to \mathbb{R}$ has property C_F, if there exists an $C_F \geq 0$ such that*
(1) $F(s,t) > C_F \implies s > t$;
(2) $F(t,t) \leq C_F$, for all $t \in [0,\infty)$.

Example 8.1. [20] *The following functions $F : [0,\infty)^2 \to \mathbb{R}$ are elements of $\mathcal{C}$ that have property C_F, for all $s,t \in [0,\infty)$:*

$$\begin{aligned}
\textit{(1) } F(s,t) &= s-t, C_F = r,\ r \in [0,\infty)\\
\textit{(2) } F(s,t) &= \frac{s}{(1+t)^r}, r \in (0,\infty); C_F = 1\\
\textit{(3) } F(s,t) &= \frac{s}{1+kt}; k \geq 1, C_F = \frac{r}{1+k},\ r \in [2,\infty)\\
\textit{(4) } F(s,t) &= (s+l)^{\frac{1}{1+T}} - l, l > 1, C_F = 0,1\\
\textit{(5) } F(s,t) &= s - (\frac{2+t}{1+t})t; C_F = 0,\\
\textit{(6) } F(s,t) &= \frac{ks}{1+t}; 0 < k < 1, C_F = k,1\\
\textit{(7) } F(s,t) &= \frac{ks}{1+kt}; 0 < k, C_F = \frac{k+1}{k},1\\
\textit{(8) } F(s,t) &= \frac{s}{1+t}; 0 < k, C_F = 1,2
\end{aligned}$$

Definition 8.2. *A simulation function is a mapping $\zeta : [0,\infty) \times\ [0,\infty) \to \mathbb{R}$ satisfying the following axioms:*

(ζ_1) $\zeta(0,0) = 0$;

(ζ_2) $\zeta(t,s) < F(s,t)$ for all $t,s > 0$, here function $F : [0,\infty)^2 \to \mathbb{R}$ is element of $\mathscr{C}$;

(ζ_3) if $\{t_n\}$, $\{s_n\}$ are sequences in $(0,\infty)$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n > 0$, then $\limsup\limits_{n\to\infty} \zeta(t_n, s_n) < 0$.

The third condition is symmetric in both arguments of ζ but, in proofs, this property is not necessary. In fact, in practice, the arguments of ζ have different meanings and they play different roles. Then, we slightly modify the previous definition in order to highlight this difference and to enlarge the family of all simulation functions.

Definition 8.3. [20] *A C_F-simulation function is a mapping $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$ satisfying the following conditions:*

(ζ_a) $\zeta(0,0) = 0$;

(ζ_b) $\zeta(t,s) < F(s,t)$ for all $t,s > 0$; here function $F : [0,\infty)^2 \to \mathbb{R}$ is element of $\mathscr{C}$ which has property C_F

(ζ_c) if $\{t_n\},\{s_n\}$ are sequences in $(0,\infty)$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n > 0$, and $t_n < s_n$, then $\limsup\limits_{n\to\infty} \zeta(t_n, s_n) < C_F$.

Let Z_F be the family of all C_F-simulation functions $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$. Every simulation function as in Definition 8.2 is also a C_F-simulation function as in Definition 8.3, but the converse is not true, for this see Example 3.3 in [18] using C-class function $F(s,t) = s - t$.

Example 8.2. [18] *Let $k \in R$ be such that $k < 1$ and let $\zeta : [0,\infty) \times [0,\infty) \to R$ be the function defined by*

$$\zeta(t,s) \Longrightarrow \begin{cases} 5(s-t) & \text{if } s < t \\ ks - t & \text{otherwise} \end{cases}$$

Clearly, ζ verifies (ζ1), and (ζ2) follows from

$$t,s > 0 \ , \ \begin{cases} 0 < s < t \Longrightarrow & \zeta(t,s) = 5(s-t) < s-t \\ 0 < t < s \Longrightarrow & \zeta(t,s) = ks - t < s - t \end{cases}$$

If $\{t_n\},\{s_n\}$ are sequences in $(0,\infty)$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n = \delta > 0$, and $t_n < s_n$,then

$$\limsup_{n\to\infty} \zeta(t_n, s_n) = \limsup_{n\to\infty} (ks_n - t_n) = (k-1)\delta < 0,$$

Therefore, ζ is a simulation function in the sense of Definition 8.3. However, if we take $t_n = 5$ and $s_n = 5 - \frac{1}{n}$, for all $n \geq 1$,then we have that

$$\limsup_{n\to\infty} \zeta(t_n, s_n) = \limsup_{n\to\infty} 5[(5 - \frac{1}{n}) - 5] = \limsup_{n\to\infty} \frac{-5}{n} = 0,$$

that is, ζ does not verify axiom (ζ3) in Definition 8.2.

Definition 8.4. *Let (X,d) be a metric space and $T,g : X \to X$ be self-mappings. A mapping T is called a (Z_F,g)-contraction if there exists $\zeta \in Z_F$ such that*

$$\zeta(d(Tx,Ty),d(gx,gy)) \geq C_F \tag{8.1}$$

for all $x,y \in X$ such that $gx \neq gy$.

Now if let $F(s,t) = s-t$, we have the following Definition of [18]

Definition 8.5. *Let (X,d) be a metric space and $T,g : X \to X$ be self-mappings. A mapping T is called a (Z,g)-contraction if there exists $\zeta \in Z$ such that*

$$\zeta(d(Tx,Ty),d(gx,gy)) \geq 0$$

for all $x,y \in X$ such that $gx \neq gy$.

Theorem 8.1. *Let T be a $(Z_{F,d},g)$-contraction in a metric space (X,d) and suppose that there exists a Picard sequence $\{x_n\}_{n\geq 0}$ of (T,g). Also assume that, at least, one of the following conditions hold.*

(a) $(g(X),d)$ (or $(T(X),d)$) is complete.

(b) (X,d) is complete and T and g are continuous and compatible.

(c) (X,d) is complete and T and g are continuous and commuting.

Then T and g have, at least, a coincidence point. Furthermore, either the sequence $\{gx_n\}$ contains a coincidence point of T and g or, at least, one of the following properties holds.

In case (a), the sequence $\{gx_n\}$ converges to $u \in g(X)$ and any point $v \in X$such that $gv = u$ is a coincidence point of T and g.

In cases (b) and (c), the sequence $\{gx_n\}$ converges to a coincidence point of T and g.

In addition to this, if $x,y \in X$ are coincidence points of T and g, then $Tx = gx = gy = Ty$. And if g (or T) is injective on the set of all coincidence points of T and g (or, simply, it is injective), then T and g have a unique coincidence point.

Now if let $F(s,t) = s-t$, we have the following result of [18]

Corollary 8.1. *Let T be a (Z_{d},g)-contraction in a metric space (X,d) and suppose that there exists a Picard sequence $\{x_n\}_{n\geq 0}$ of (T,g). Also assume that, at least, one of the following conditions holds.*

(a) $(g(X),d)$ (or $(T(X),d)$) is complete.

(b) (X,d) is complete and T and g are continuous and compatible.

(c) (X,d) is complete and T and g are continuous and commuting.

Then T and g have, at least, a coincidence point. Furthermore, either the sequence $\{gx_n\}$ contains a coincidence point of T and g or, at least, one of the following properties holds.

In case (a), the sequence $\{gx_n\}$ converges to $u \in g(X)$ and any point $v \in X$ such that $gv = u$ is a coincidence point of T and g.

In cases (b) and (c), the sequence $\{gx_n\}$ converges to a coincidence point of T and g.

In addition to this, if $x, y \in X$ are coincidence points of T and g, then $Tx = gx = gy = Ty$. And if g (or T) is injective on the set of all coincidence points of T and g (or, simply, it is injective), then T and g have a unique coincidence point.

9 Pair upper Class functions

Definition 9.1. [26] *Let $T : X \to X$ and $\alpha : X \times X \to \mathbb{R}^+$. We say that T is an α-admissible mapping if $\alpha(x,\ y) \geq 1$ implies $\alpha(Tx,\ Ty) \geq 1,\ x,\ y\ \in\ X$.*

Theorem 9.1. [15] *Let (X,d) be a complete metric space and $T : X \to X$ be an α-admissible mapping. Assume that there exists a function $\beta : [0,\infty) \to [0,1)$ such that, for any bounded sequence $\{t_n\}$ of positive real , $\beta(t_n) \to 1$ implies $t_n \to 0$, such that*

$$(d(Tx,Ty)+l)^{\alpha(x,Tx)\alpha(y,Ty)} = \beta\left(d\left(x,y\right)\right)d\left(x,y\right)+l \tag{9.1}$$

for all $x,\ y\ \in X$ .Suppose that either
(a) T is continuous, or
(b) if $\{x_n\}$ is a sequence in F such that $x_n \to x$, $\alpha(x_n,x_{n+1}) \geq 1$, for all n, then $\alpha(x,Tx) \geq 1$.

Then T has a fixed point.

Theorem 9.2. [15] *Let (X,d) be a complete metric space and $T : X \to X$ be an α-admissible mapping. Assume that there exists a function $\beta : [0,\infty) \to [0,1)$ such that, for any bounded sequence $\{t_n\}$ of positive real , $\beta(t_n) \to 1$ implies $t_n \to 0$, such that*

$$(\alpha\left(x,Tx\right)\alpha\left(y,Ty\right)+1)^{d(Tx,Ty)} = 2^{\beta(d(x,y))d(x,y)} \tag{9.2}$$

for all $x,\ y\ \in X$ .Suppose that either
(a) T is continuous, or
(b) if $\{x_n\}$ is a sequence in X such that $x_n \to x$, $\alpha(x_n,x_{n+1}) \geq 1$, for all n, then $\alpha(x,Tx) \geq 1$.
If there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0) \geq 1$, then T has a fixed point.

Theorem 9.3. [15] *Let (X,d) be a complete metric space and $T : X \to X$ be an α-admissible mapping. Assume that there exists a function $\beta : [0,\infty) \to [0,1)$ such that, for any bounded sequence $\{t_n\}$ of positive real , $\beta(t_n) \to 1$ implies $t_n \to 0$, such that*

$$\alpha\left(x,Tx\right)\alpha\left(y,Ty\right)d(Tx,Ty) = \beta(d(x,y))d(x,y) \tag{9.3}$$

for all $x,\ y\ \in X$ .Suppose that either
(a) T is continuous, or
(b) if $\{x_n\}$ is a sequence in X such that $x_n \to x$, $\alpha(x_n,x_{n+1}) \geq 1$, for all n, then $\alpha(x,Tx) \geq 1$.
If there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0) \geq 1$, then T has a fixed point.

In 2014 the observation

$$(d(Tx,Ty)+l)^{\alpha(x,Tx)\alpha(y,Ty)} = \beta\left(d\left(x,y\right)\right)d\left(x,y\right)+l\,,$$

$$\left(\alpha\left(x,Tx\right)\alpha\left(y,Ty\right)+1\right)^{d(Tx,Ty)} = 2^{\beta(d(x,y))d(x,y)}\,,$$

$$\alpha\left(x,Tx\right)\alpha\left(y,Ty\right)d(Tx,Ty) = \beta(d(x,y))d(x,y)\,,$$

guided to upper class function in [10] that then reform definition(part words) in [11],as following.

Definition 9.2. [10], [11] *We say that the function $h\colon \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ is a function of subclass of type I, if $x \geq 1 \Longrightarrow h(1,y) \leq h(x,y)$ for all $y \in \mathbb{R}^+$.*

Example 9.1. [10], [11] *Define $h\colon \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ by:*

(a) $h(x,y) = (y+l)^x, l > 1$;

(b) $h(x,y) = (x+l)^y, l > 1$;

(c) $h(x,y) = x^n y$, $n \in \mathbb{N}$;

(d) $h(x,y) = y$;

(e) $h(x,y) = \frac{1}{n+1}\left(\sum_{i=0}^{n} x^i\right)y$, $n \in \mathbb{N}$;

(f) $h(x,y) = \left[\frac{1}{n+1}\left(\sum_{i=0}^{n} x^i\right)+l\right]^y, l > 1, n \in \mathbb{N}$

for all $x,y \in \mathbb{R}^+$. Then h is a function of subclass of type I.

Definition 9.3. [10], [11] *Let $h,\mathscr{F} : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$, then we say that the pair $(\mathscr{F},h)$ is an upper class of type I, if h is a function of subclass of type I and: (i) $0 \leq s \leq 1 \Longrightarrow \mathscr{F}(s,t) \leq \mathscr{F}(1,t)$, (ii) $h(1,y) \leq \mathscr{F}(1,t) \Longrightarrow y \leq t$,for all $t,y \in \mathbb{R}^+$.*

Example 9.2. [10], [11] *Define $h,\mathscr{F} : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ by:*

(a) $h(x,y) = (y+l)^x, l > 1$ *and* $\mathscr{F}(s,t) = st+l$;

(b) $h(x,y) = (x+l)^y, l > 1$ *and* $\mathscr{F}(s,t) = (1+l)^{st}$;

(c) $h(x,y) = x^m y$, $m \in \mathbb{N}$ *and* $\mathscr{F}(s,t) = st$;

(d) $h(x,y) = y$ *and* $\mathscr{F}(s,t) = t$;

(d) $h(x,y) = \frac{1}{n+1}\left(\sum_{i=0}^{n} x^i\right)y$, $n \in \mathbb{N}$ *and* $\mathscr{F}(s,t) = st$;

(e) $h(x,y) = \left[\frac{1}{n+1}\left(\sum_{i=0}^{n} x^i\right)+l\right]^y, l > 1, n \in \mathbb{N}$ *and* $\mathscr{F}(s,t) = (1+l)^{st}$

for all $x,y,s,t \in \mathbb{R}^+$. Then the pair $(\mathscr{F},h)$ is an upper class of type I.

Definition 9.4. [10], [11] *We say that the function $h : \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ is a function of subclass of type II, if $x, y \geq 1 \Longrightarrow h(1,1,z) \leq h(x,y,z)$ for all $z \in \mathbb{R}^+$.*

Example 9.3. [10], [11] *Define $h\colon \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ by:*

(a) $h(x,y,z) = (z+l)^{xy}, l > 1$;

(b) $h(x,y,z) = (xy+l)^z, l > 1$;

(c) $h(x,y,z) = z$;

(d) $h(x,y,z) = x^m y^n z^p, m,n,p \in \mathbb{N}$;

(e) $h(x,y,z) = \frac{x^m + x^n y^p + y^q}{3} z^k, m,n,p,q,k \in \mathbb{N}$

for all $x,y,z \in \mathbb{R}^+$. Then h is a function of subclass of type II.

Definition 9.5. [10], [11] *Let $h\colon \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ and $\mathscr{F} : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$, then we say that the pair $(\mathscr{F}, h)$ is an upper class of type II, if h is a subclass of type II and: (i) $0 \leq s \leq 1 \Longrightarrow \mathscr{F}(s,t) \leq \mathscr{F}(1,t)$, (ii) $h(1,1,z) \leq \mathscr{F}(s,t) \Longrightarrow z \leq st$ for all $s,t,z \in \mathbb{R}^+$.*

Example 9.4. [10], [11] *Define $h\colon \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ and $\mathscr{F}\colon \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ by:*

(a) $h(x,y,z) = (z+l)^{xy}, l > 1, \mathscr{F}(s,t) = st + l$;

(b) $h(x,y,z) = (xy+l)^z, l > 1, \mathscr{F}(s,t) = (1+l)^{st}$;

(c) $h(x,y,z) = z, F(s,t) = st$;

(d) $h(x,y,z) = x^m y^n z^p, m,n,p \in \mathbb{N}, \mathscr{F}(s,t) = s^p t^p$

(e) $h(x,y,z) = \frac{x^m + x^n y^p + y^q}{3} z^k, m,n,p,q,k \in \mathbb{N}, \mathscr{F}(s,t) = s^k t^k$

for all $x,y,z,s,t \in \mathbb{R}^+$. Then the pair $(\mathscr{F}, h)$ is an upper class of type II.

Definition 9.6. [10] *Let (X,d) be a metric space and $T : X \to X$, a nonempty subset F of X is valled invariant under the T if $Tx \in F$ for every $x \in F$.*

Definition 9.7. [10] *Let $T : X \to X$ and $\alpha : F \times F \to \mathbb{R}^+, (F \subset X)$. We say that T is an α_F-admissible mapping if $\alpha(x,\ y) \geq 1$ implies $\alpha(Tx,\ Ty) \geq 1,\ x,\ y\ \in\ F$.*

Note: A mapping T is called an α-admissible mapping (see [26]) if we take $F = X$ in Definition 9.7.

Definition 9.8. [10] *Let (X,d) be a metric space, F a nonempty subset of X, $T : X \to X$ and $\alpha : F \times F \to \mathbb{R}^+$. A mapping T is said to be α_β-contractive mapping if there exists a $\beta : [0,\infty) \to [0,\ 1]$ such that for any bounded sequence $\{t_n\}$ of positive reals, $\beta(t_n) \to 1$ implies $t_n \to 0$, such that for all $x,y \in F$, following condition holds:*

$$h(\alpha(x,Tx), \alpha(y,Ty), \psi d(Tx,Ty)) \leq \mathscr{F}(\beta(d(x,y)), \psi d(x,y)), \tag{9.4}$$

where pair $(\mathscr{F}, h)$ is a upclass of type II and $\psi \in \Psi$.

Theorem 9.4. [10] *Let (X,d) be a complete metric space, F a nonempty closed subset of X, $T:X\to X$ is an α_F-admissible mapping and F is invariant under T. Further assume that T is an α_β-contractive mapping. Suppose that there exists $x_0\in F$ such that $\alpha(x_0,Tx_0)\geq 1$ and either of the following conditions hold:*
(a) T is continuous, or
(b) if $\{x_n\}$ is a sequence in F such that $x_n\to x$, $\alpha(x_n,x_{n+1})\geq 1$, for all n, then $\alpha(x,Tx)\geq 1$.
Then T has a fixed point.

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