GRANIČNA NOSIVOST PRITISNUTE GREDE SA IMPERFEKCIJAMA

LIMIT LOAD CAPACITY OF COMPRESSED BEAM WITH IMPERFECTIONS

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ORIGINALNI NAUČNI RAD
ORIGINAL SCIENTIFIC PAPER
UDK: 624.072.2.046
doi: 10.5937/GRMK1802003M

1 UVOD

Konstrukcije napravljene spajanjem ravnih ploča na njihovim podužnim krajevima veoma su česte. Važan podskup tih konstrukcija koje su glavni predmet ovog rada u suštini jesu prizmatične forme, ali one mogu imati i poprečna ukrućenja koja se koriste u sandučastim nosačima, ukrućenim pločama i pločastim nosačima. Opterećenje je uglavnom takvo da su najveći naponi u pravcu dužine, na primer, aksijalno opterećenje ili uzdužno savijanje.

Analiza ponašanja pločastih konstrukcija odvijala se na nekoliko različitih načina. Jedan od načina bio je sprovođenje sveobuhvatnog istraživanja jednog tipa pločastih konstrukcija, kao što je sandučasti stub, tako da se testira cela familija modela u laboratoriji [1]. Drugi načini bili su numerički, primenom metoda konačnih elemenata (MKE) koji mogu uključiti komplikovane geometrije konstrukcija [2]. Cilj ovog rada jeste da istraži lom grede s početnim imperfekcijama, pojednostavljenim modelima koji se koriste u mehanici, radi poređenja dobijenih rezultata.

Iako je korištenje MKE trenutno dominantno u analizi pločastih konstrukcija, nije tako jednostavno postaviti problem. Za tačnost je poželjno da se koriste manji elementi u zonama gde se čelik izvija lokalno i postaje plastičan, ali nije uvek poznato unapred gde te zone treba de se pojave. Takođe, u analizi ponašanja pločastih konstrukcija može se primeniti i metod konačnih traka (MKT). MKT se zasniva na svojstvenim funkcijama koje su izvedene iz rešenja diferencijalne jednačine

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1 INTRODUCTION

The structures, which are made by joining flat plates at their longitudinal edges are very common. An important sub-set of these structures, and which are the main concern of this paper, are those essentially of prismatic form but which can have some transverse stiffening such as is used in box girders, stiffened plates and plate girders. The loading is generally such that the greatest stresses are in the longitudinal direction, e.g. axial loading or longitudinal bending.

Analysis of the behavior of plate structures has been approached in several different ways. One way of carrying out a comprehensive investigation of a single type of plated structures, such as a box-column, would be to test a whole family of models in the laboratory [1]. Other methods were numerically using finite element method (FEM), which may include a complicated geometry of the structures [2]. The aim of this paper is to investigate the fracture of beam with initial imperfections simplified models used in mechanics, in order to compare the results.

Although the use of FEM currently dominant in the analysis of plate structures, the problem set is not so simple. For accuracy it is desirable to use smaller elements in the regions where the steel buckles locally and becomes plastic but it is not always known beforehand where these plastic zones will occurs. Also, the analysis of the behavior of plate structures may be approached using the finite strip method (FSM). The FSM is based on eigen functions, which are derived from

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poprečnih vibracija grede, a pokazao se kao efikasan alat za analiziranje velikog broja konstrukcija kod kojih se i geometrija i svojstva materijala mogu smatrati konstantnim duž podužnog pravca [2]. Ipak, ako se geometrija nosača i opterećenje komplikuju - MKE ima prednost.

Ako se analizira jednostavna konstrukcija – kao što je prosta greda – pojednostavljeni modeli koji se pojavljuju u mehanici korisni su jer mogu dati rešenja kada je problem komplikovan. Takva rešenja mogu dati dovoljno informacija projektantima pri projektovanju konstrukcija. U ovom radu predlaže se jednostavna i vrlo efikasna analiza postavljenog problema granične nosivosti pri neelastičnom izvijanju. Polazi se od činjenice da su tankozidne konstrukcije veoma osetljive na početne imperfekcije, pa je njihovo postojanje polazna pretpostavka. Elastično rešenje problema dobro je poznato [1]. Međutim, elastično rešenje može se primeniti samo do linije granične nosivosti. Rešenje problema granične nosivosti u području neelastičnosti postignuto je primenom RDA. Primena RDA je jednostavna, jer ona transformiše komplikovan materijalno-nelinearan problem u jednostavan linearno-dinamički problem [3].

2 ODREĐIVANJE LINIJE GRANIČNE NOSIVOSTI PRIMENOM RDA

Faktori koji utiču na graničnu nosivost pri izvijanju mogu biti podeljeni u dve grupe. Prva grupa uključuje geometriju pritisnutog elementa, kao što su poprečni presek i dužina, uslovi oslanjanja, mehanička svojstva materijala, uključujući pritom i čvrstoću, kao i uslove okoline, trajanje opterećenja, i tako dalje. Druga grupa faktora koji utiču na graničnu nosivost uključuju geometrijske i materijalne imperfekcije i njihove varijacije. RDA je viskoelastoplastična teorija, nezavisna od teorije plastičnosti ili nelinearne mehanike loma koja je uspešno primenjena u istraživanju krivih izvijanja stubova [4]. U ovom radu RDA se koristi u istraživanju granične nosivosti tankozidne grede s početnim imperfekcijama.

2.1 RDA – kratak pregled

Mikropukotine i deformacije u plastičnom materijalu jesu posledica delovanja spoljašnjih sila na noseći element, što uzrokuje njegovo oštećenje ili lomljenje. Razmotrimo slučaj viskoelastoplastične (VEP) deformacije slobodno oslonjenog stuba predstavljenog na slici 1(a). U istraživanju materijala i napon $\sigma(t)$ i neelastična deformacija $\varepsilon^*(t)=\varepsilon_{ve}(t)+\varepsilon_{vp}(t)$ jesu funkcije vremena.

Ako je ukupna VEP deformacija $\varepsilon(t)=\varepsilon_{el}+\varepsilon^*(t)$ predstavljena kao zbir elastične (trenutne) ε_{el}, viskoelastične (VE) $\varepsilon_{ve}(t)$, i viskoplastične (VP) $\varepsilon_{vp}(t)$, komponente, svaki jednovremeni dijagram napon–deformacija prizmatičnog stuba (npr. s kvadratnim ili kružnim poprečnim presekom A_0) može se precizno aproksimirati reološkim modelom materijala H-K-(StV|N), sastavljenim od pet elemenata. Reološki model prikazan je na slici 1(b), koristeći sledeće simbole: N – za Newton-ov model, StV – za Saint-Venant-ov model, H – za Hooke-ov model, "|" – za paraleno spajanje modela i "—" za spajanje modela u nizu. Zbog toga što su Hooke-

the solution of the beam differential equation of transverse vibration, and proved to be an efficient tool for analyzing a great deal of structures for which both geometry and material properties can be considered as constant along a longitudinal direction [2]. However, if the geometry and loading are complicated FEM has the advantage.

If we analyze a simple structure, such as a simple beam, simplified models which are present in mechanics are useful because they can provide solutions when the problem is complicated. Such solutions can provide enough information to designers in the design of structures. In this paper we propose a simple and very efficient analysis of the above problem of the limit load capacity regarding inelastic buckling. The starting point is the fact that the thin-walled structures are very sensitive to initial imperfections, and is the premise of their existence. Elastic solution of the problem is well known [1]. However, the elastic solution can be applied only to the line of limit load capacity. Solving the problem of the limit load capacity in the inelasticity is achieved by using RDA. Application is simple because RDA transforms a complicated material non-linear problem to a simple linear dynamic problem [3].

2 DETERMINATION OF LINE OF LIMIT LOAD CAPACITY USING THE RDA

The factors influencing the limit load capacity may divide into two groups. The first group involves the nominal geometry of the compresses member such as cross-section and length, the support conditions, the material properties including the strength, the surrounding climate, the load duration, etc. The second group of factors which influence to the limit load capacity of column involves geometric and material imperfections and their variations. RDA is inelastic theory independent of the theory of plasticity, or non-linear fracture mechanics, which was successfully applied in the study of buckling curves of columns [4]. In this paper, RDA is used in research of the limit load capacity of thin-walled beam with initial imperfections.

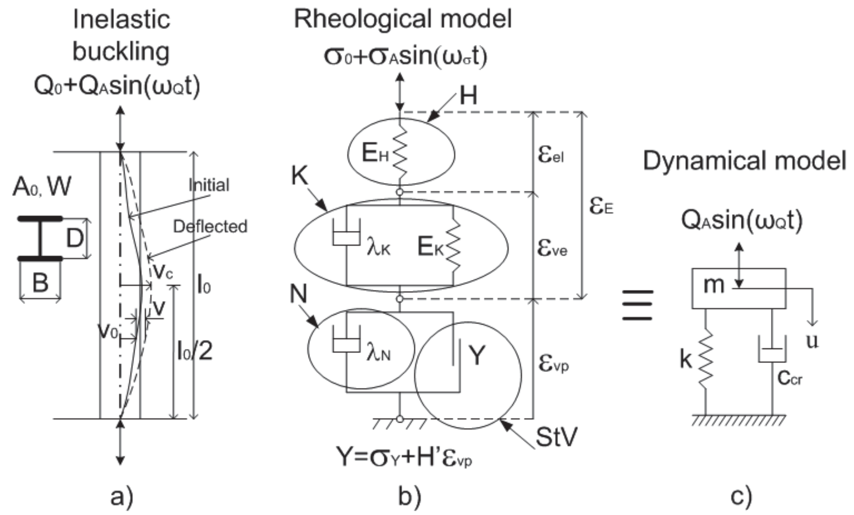
2.1 RDA – a short overview

Micro cracks and deformations in the plastic material are consequence of action of external forces to the carrying member, which leads to its damage or breakage. Consider the case of viscoelastoplastic (VEP) strain of a simple pin-ended column presented in Fig. 1(a). In research of the material, both a stress $\sigma(t)$ and inelastic strain $\varepsilon^*(t) = \varepsilon_{ve}(t)+\varepsilon_{vp}(t)$ are functions of time.

If the total VEP strain $\varepsilon(t) = \varepsilon_{el}+\varepsilon^*(t)$ is presented as a sum of elastic (instantaneous) ε_{el}, viscoelastic (VE) $\varepsilon_{ve}(t)$, and viscoplastic (VP) $\varepsilon_{vp}(t)$, component, each isochronous stress-strain diagram of a prismatic column (e.g., with a square or circular cross section A_0) can accurately be approximated by the rheological model of material H-K-(StV|N), consisting of five elements. The rheological model is shown in Fig. 1(b) using the following symbols: N for the Newton's model, StV for Saint-Venant's model, H for Hooke's model, "|" for a parallel connection of models and "—" for connection of models in a series. Since a Hooke's model, Kelvin's ov model, Kelvin-ov model (K=H|N) i VP model (StV|N) spojeni u nizu, napon $\sigma(t)$ u sva tri modela jeste jednak.

Diferencijalnu jednačinu reološkog modela na slici 1(b) već je izveo prvi autor [4]

model (K=H|N) and VP model (StV|N) are connected in a series, the stress $\sigma(t)$ in all models is equal. Differential equation of rheological model, Fig. 1(b) has already been derived in [4] by the first author



Slika. 1. Neelastično izvijanje slobodno oslonjenog stuba: (a) stub s početnim imperfekcijama; (b) reološki model materijala; (c) dinamički model stuba

Fig. 1. Inelastic buckling of simple pin-ended column: (a) column with initial imperfections; (b) rheological model of material; (c) dynamic model of column

$$\ddot{\varepsilon}(t)+\dot{\varepsilon}(t)\left(\frac{E_K}{\lambda_K}+\frac{H'}{\lambda_N}\right)+\varepsilon(t)\frac{E_K H'}{\lambda_K\lambda_N}=\frac{\ddot{\sigma}(t)}{E_H}+\dot{\sigma}(t)\left(\frac{E_K}{\lambda_K E_H}+\frac{H'}{\lambda_N E_H}+\frac{1}{\lambda_K}+\frac{1}{\lambda_N}\right)+ +\sigma(t)\left(\frac{E_K}{\lambda_K\lambda_N}+\frac{H'}{\lambda_K\lambda_N}+\frac{E_K H'}{\lambda_K\lambda_N E_H}\right)-\sigma_Y\frac{E_K}{\lambda_K\lambda_N} \tag{1}$$

gde je E_H Young-ov modul, a σ_Y napon tečenja. Kriterijum tečenja jeste

where E_H is the Young modulus and σ_Y is the uniaxial yield stress. The yield condition is

$$\sigma-\left(\sigma_Y+H'\varepsilon_{vp}\right)\geq 0\,. \tag{2}$$

Četiri svojstva materijala u fiksnom vremenskom intervalu jesu: koeficijent VE viskoznosti λ_K, koeficijent VP viskoznosti λ_N, modul viskoelastičnosti E_K i modul viskoplastičnosti H'. Međutim, ove konstante ne mogu se lako odrediti iz fizičkih eksperimenata, posebno Trouton-ovi koeficijenti viskoznosti λ_K i λ_N. Odgovarajuća homogena diferencijalna jednačina glasi

The four material properties in fixed step of time are: the coefficient of VE viscosity λ_K, coefficient of VP viscosity λ_N, VE modulus E_K and VP modulus H'. However, these constants cannot easily be determined in physical experiments, especially Trouton's coefficient of viscosity λ_K and λ_N. Corresponding homogeneous differential equation is as follows

$$\ddot{\varepsilon}(t)\lambda_K\lambda_N+\dot{\varepsilon}(t)\left(E_K\lambda_N+H'\lambda_K\right)+\varepsilon(t)E_K H'=0\,. \tag{3}$$

S druge strane, mehanički poremećaj (dilatacija) propagira kroz elastičnu sredinu konačnom brzinom $v_0=(E_H/\rho)^{1/2}$, gde je ρ gustina materijala. Vibracija proizvoljne tačke M zaostaje u fazi za izvorom talasa. Ako sa l_0 označimo rastojanje između krajeva stuba, vremenska razlika iznosi $t-t_0=T^D=l_0/v_0$. T^D tako predstavlja vreme kašnjenja za koje talas brzine v_0 prelazi rastojanje l_0. Kružna frekvencija dinamičkog modela na sl. 1(c) jeste

On the other hand, a mechanical disturbance (strain) propagates in an elastic medium at the finite velocity $v_0=(E_H/\rho)^{1/2}$, where ρ is the density. The vibration at an arbitrary point M lags in phase behind that at the source of the wave. If l_0 is the distance between two ends of the column, the time difference is $T^D=t-t_0=l_0/v_0$. So T^D represents a delay time for which a wave at the velocity v_0 takes to propagate the distance l_0. The natural angular frequency of dynamical model shown in Fig. 1(c) is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{E_H A_0}{l_0}\frac{1}{\rho A_0 l_0}} = \frac{v_0}{l_0} = \frac{1}{T^D} \tag{4}$$

gde je m masa stuba, a k njegova aksijalna krutost.

Imajući u vidu jednačinu (3), izraz sličan (4) može se formulisati postavljanjem reološkog modela stuba u stanje kritičnog viskoznog prigušenja ($c=c_{cr}$)

where m is the mass of the column and k its axial stiffness.

Heaving in mind Eq. (3), we can formulate expression similar to Eq. (4), turning the rheological model into the state of critical viscous damping ($c = c_{cr}$)

$$\omega = \sqrt{\frac{E_K H'}{\lambda_K \lambda_N}} = \sqrt{\frac{1}{T_K T^*}} = \frac{1}{T^D} \tag{5}$$

gde su:

where:

$$E_K/\lambda_K = H'/\lambda_N\ ,\lambda_K = E_K T_K\ ,\lambda_N = H'T^*, T_K = T^* = T^D\ .$$

Ako zamenimo $\lambda_K \cdot \lambda_N$ sa $m \cdot \gamma$, $E_K \cdot \lambda_N + H' \cdot \lambda_K$ sa $c_{cr} \cdot \gamma$ i $E_K \cdot H'$ sa $k \cdot \gamma$, jednačina (3) postaje

Replacing $\lambda_K \cdot \lambda_N$ by $m \cdot \gamma$, $E_K \cdot \lambda_N + H' \cdot \lambda_K$ by $c_{cr} \cdot \gamma$ and $E_K \cdot H'$ by $k \cdot \gamma$, Eq. (3) becomes.

$$\ddot{\varepsilon}(t)m + \dot{\varepsilon}(t)c_{cr} + \varepsilon(t)k = 0 \tag{6}$$

gde su

where:

$$m = \frac{\lambda_K \lambda_N}{\gamma} = k \cdot T^{D2}, \quad c_{cr} = \frac{E_K \lambda_N + H' \lambda_K}{\gamma} = 2 \cdot k \cdot T^D, \quad k = \frac{E_K H'}{\gamma}\ . \tag{7}$$

γ je zapreminska težina materijala.

Prema navedenom, propagacija talasa kroz elastičnu sredinu predstavlja fizičku osnovu za postavljanje analogije između dva različita fizička fenomena – reološkog i dinamičkog, nazvana RDA. Na osnovu RDA, vrlo komplikovan materijalno nelinearan problem u području VEP deformacija može se rešiti kao jednostavan linearno-dinamički problem. RDA je izvedena da reši dinamičke probleme [4], ali može se koristiti i u analizi kvazistatičkih problema, imajući u vidu odgovarajuće granične vrednosti izvedenih analitičkih izraza. Na primer, svaki kvazistatički dijagram napon–deformacija može se dobiti korišćenjem RDA modul funkcije [3], uključujući čvrstoću na pritisak, što je ključni parametar za analizu energije loma.

γ is the specific gravity.

Accordingly, the elastic wave propagation constitutes the physical basis for setting up an analogy between two different physical phenomena, rheological and dynamical, called RDA. Based on RDA, a very complicated non-linear problem in the range of VEP strains can be solved as a simple linear dynamic problem. RDA is derived to solve the dynamic problems [4], but can be used also in the analysis of the quasi static problems taking into account the corresponding limit values of derived analytical expressions. For example, each quasi static stress-strain diagram can be obtained by using the RDA modulus function [3], including the compression strength, which is a key parameter for the analysis of fracture energy.

2.2 Strukturalno-materijalna konstanta

Korišćenje tangentnog modula umesto Young-ovog modula (na osnovu Engesser-ove pretpostavke $\sigma_{En} = \pi^2 E_T / \lambda^2$) pokazalo se realnim u slučaju neelastičnog izvijanja, a to su potvrdila i eksperimentalna istraživanja. U radu [4] prvi autor pokazao je da je RDA modul jednak tangentnom modulu ($E_R(t,t_0) = E_T$) u fiksnom vremenskom intervalu (t,t_0). Zbog ovoga se u razmatranje postavljenog problema uvodi RDA modul funkcija E_R. Ona je već korištena za formulaciju kvazistatičkog napon–deformacija dijagrama standardnog betonskog cilindra [3], kako sledi

2.2 Structural-material constant

The utilisation of the tangent modulus instead of Young modulus (according to the Engesser assumption $\sigma_{En} = \pi^2 E_T / \lambda^2$) proved to be realistic in the case of inelastic buckling, based on the and experimental researches. In [4], the first author has shown that RDA modulus is equal to the tangent modulus ($E_R(t,t_0) = E_T$) within a fixed time interval (t,t_0). Because of this, in consideration of the above problem is introduced RDA modulus function E_R. It has already been used for the formulation of the quasi-static stress-strain diagram of the standard concrete cylinder [3], as

$$\varepsilon = \frac{\sigma_{cr}}{E_R(0)} = \frac{\sigma_{cr}}{E(0)}(1+\varphi_{cr}) = \frac{\sigma_{cr}}{E(0)}(1+\sigma_{cr} K_E)\ . \tag{8}$$

gde je $E(0)$ modul elastičnosti materijala u inicijalnom, neoštećenom stanju, a K_E je strukturalno-materijalna konstanta. Na osnovu (8), sledi kvadratna jednačina

where $E(0)$ is the initial elastic modulus in undamaged state, and K_E is the structural-material constant. According to Eq. (8) the quadratic equation follows

$$\sigma_{cr}^2 K_E + \sigma_{cr} - E(0)\varepsilon = 0 . \tag{9}$$

Koren jednačine (9), koristeći početne uslove $\varepsilon(0)=0$ i $\sigma_{cr}(0)=0$, kritična je vrednost napona za odabranu deformaciju ε krive linije napon–deformacija. Tako je

The root of Eq. (9), using the initial conditions $\varepsilon(0)=0$ and $\sigma_{cr}(0)=0$, is the critical value of stress for the selected deformation ε of stress-strain curve. Thus

$$\sigma_{cr} = \frac{1}{2K_E}\left(\sqrt{1+4K_E E(0)\varepsilon} - 1\right). \tag{10}$$

Na granici elastičnosti nagib je jednak Young-ovom modulu E_H (poznata vrednost). Zbog toga, jeste $E_R(0) = E_H$, tako da sledi

At the limit of elasticity the slope is equal to the Young modulus E_H (a known value). Because of that $E_R(0) = E_H$, so we get

$$E(0) = E_H\left(1+\varphi^*\right). \tag{11}$$

gdje je φ^* strukturalno-materijalni koeficijent tečenja na granici elastičnosti. U radu [4] prvi autor na granici elastičnosti odredio je presek Euler-ove i RDA krive izvijanja, iz koga sledi vitkost na granici elastičnosti

where φ^* is the structural-material creep coefficient at the limit of elasticity. In [4], the first author has defined the intersection of the Euler and RDA buckling curves at the limit of elasticity, from which follows the slenderness at the limit of elasticity

$$\lambda_E = \pi^2 \frac{i^3}{I}\frac{1}{\gamma\varphi^*} . \tag{12}$$

gde je $i=\sqrt{I/A_0}$ minimalni radijus inercije. Nadalje, Euler-ov kritični napon slobodno oslonjenog stuba $\sigma_E = \pi^2 E_H / \lambda_E^2$ koristi se za izračunavanje strukturalno-materijalne konstante na granici elastičnosti, kako je pokazano u [3]

where $i=\sqrt{I/A_0}$ is the minimum radius of gyration. Further, Euler's critical stress of a simple pin-ended column $\sigma_E = \pi^2 E_H / \lambda_E^2$ is used to calculate the structural-material constant at the limit of elasticity, as explained in [3]

$$K_E = \frac{\varphi^*}{\sigma_E} . \tag{13}$$

Iza granice elastičnosti, koristi se zakon linearne promene kritičnog napona u odnosu na kritični koeficijent tečenja (zakon toka), kako je definisao Milašinović [3]

Beyond the limit of elasticity the law of linear changes of the critical stress level in relation to the critical creep coefficient (flow law) is used, as defined in Milašinović [3]

$$\sigma_{cr} = \frac{1}{K_E}\varphi_{cr} . \tag{14}$$

Na osnovu svega, funkcija RDA modula glasi

Accordingly, the RDA modulus function is as follows

$$E_R = \frac{1}{\dfrac{1}{E_H} + \dfrac{\varphi^*}{E_H} + \dfrac{1}{H'}} = \frac{E_H}{1+\varphi_{cr}} = \frac{E_H}{1+\sigma_{cr}K_E} . \tag{15}$$

gde je kritični koeficijent tečenja

where the critical creep coefficient is

$$\varphi_{cr} = \varphi^* + \frac{E_H}{H'} = \sigma_{cr}K_E . \tag{16}$$

i koji uključuje neelastični deo koeficijenta tečenja $\varphi_{ine} = E_H / H'$.

and which includes the inelastic part of creep coefficient $\varphi_{ine} = E_H / H'$.

2.3 Kritična varijabla oštećenja

Budući da rast mikropukotina smanjuje krutost materijala, oštećeno stanje materijala opisano je varijacijom modula elastičnosti E_H, Lemaitre [5]. Shodno tome, varijabla oštećenja D uvedena je na osnovu hipoteze o ekvivalenciji deformacije između oštećenog i neoštećenog materijala, kako sledi

2.3 Critical damage variable

Since that growth of micro cracks reduces the stiffness of the material, the damaged state of material is described by the variation of Young modulus E_H, Lemaitre [5]. Hence, the damage variable D is introduced based to the hypotheses of strain equivalence between the damaged and undamaged material, as follows

$$E(D) = (1 - D) E_H . \tag{17}$$

U radu [3] prvi autor ovog rada uveo je pretpostavku da je $E(D)$ jednak RDA modulu, na osnovu čega je definisana varijabla oštećenja D. Ova pretpostavka koristi se i u ovom radu. Kako je tačka C_1 na liniji granične nosivosti, koja je ujedno i linija kritičnih napona u neelastičnoj oblasti, varijabla oštećenja koja sledi jeste kritična

In [3], the first author of this paper has introduced the assumption that the $E(D)$ is equal to the RDA modulus on the basis of which the damage variable D is defined. This assumption is also used in this paper. As the point C_1 is on the line of limit load capacity, which is the line of critical stresses in the inelastic range of strain, damage variable below is the critical

$$(1 - D_{C1}) E_H = E_R = E_H \frac{1 + \varphi_{C1} + \delta^2}{(1 + \varphi_{C1})^2 + \delta^2} \Rightarrow D_{C1} = \frac{(1 + \varphi_{C1}) \varphi_{C1}}{(1 + \varphi_{C1})^2 + \delta^2} . \tag{18}$$

gde je φ_{C1} koeficijent tečenja u tački C_1 na osnovu zakona toka (14)

where φ_{C1} is the creep coefficient at the point C_1, according to the flow law (14)

$$\varphi_{C1} = \sigma_{C1} K_E . \tag{19}$$

$\delta = \omega_\sigma / \omega = \omega_\sigma T^D$ jeste relativna frekvencija u kojoj je ω_σ kružna frekvencija pobude. U slučaju kvazistatičkog opterećenja sledi $\delta \rightarrow 0$, pa je kritična varijabla oštećenja

$\delta = \omega_\sigma / \omega = \omega_\sigma T^D$ is the relative frequency where ω_σ is the angular frequency of excitation. In the case of quasi-static loading follows $\delta \rightarrow 0$, and critical damage variable is

$$D_{C1} = \frac{\varphi_{C1}}{1 + \varphi_{C1}} . \tag{20}$$

Na ovaj način, oštećenje na liniji granične nosivosti u slučaju kvazistatičkog opterećenja opisano je skalarnom veličinom D_{C1} koja uzima vrednosti između 0 i 1.

In this way, the damage at the line of limit load capacity is described by a scalar value D_{C1}, which takes a value between 0 and 1.

2.4 Linija granične nosivosti dobijena primenom RDA

Na graničnu nosivost značajno utiču geometrijske i materijalne imperfekcije greda, kao i njihove varijacije. Zbog toga je ovaj problem veoma komplikovan. U ovom radu razmatraju se čistogeometrijske imperfekcije. To podrazumeva da samo referentna geometrija zavisi od imperfekcija, a da naponsko stanje ne zavisi.

Prema RDA, neelastičan ugib grede u slučaju kvazistatičkog opterećenja jeste [3]

2.4 Line of limit load capacity obtained using the RDA

Geometric and material imperfections, as well as their variations can significantly affect to the limit load capacity of beams. This is why this problem is very complicated. Purely initial geometric imperfections are considered here. It implies that only the reference geometry is influenced by imperfection, not the stress state.

According to RDA, inelastic deflection o the beam in the case of quasi-static load is [3]

$$v_{c1} = a_1 \left(1 + \varphi^*\right) . \tag{21}$$

gde a_1 je početna imperfekcija u sredini dužine grede.

Opterećenje u tački C_1 odgovara graničnoj nosivosti za datu početnu imperfekciju a_1. U tankozidnim konstrukcijama, granična nosivost skoro je jednaka naponu tečenja materijala σ_Y. Stoga, opravdano je koristiti napon tečenja kao kriterijum loma. Zbog toga, za gredu izloženu aksijalnoj sili Q_{C1} i odgovarajućem momentu savijanja $Q_{C1} \cdot v_{c1}$ dobijamo

where a_1 is the initial imperfection at mid-length.

The load at point C_1 corresponds to the limit load capacity for the given initial imperfection a_1. In thin-walled structures, the limit stress is almost equal to the yield stress of the material σ_Y. Hence, it is justified to use of the yield stress as the criterion of failure. Therefore, for the beam subjected to an axial force Q_{C1} and an appropriate bending moment, $Q_{C1} \cdot v_{c1}$ we obtain as follows

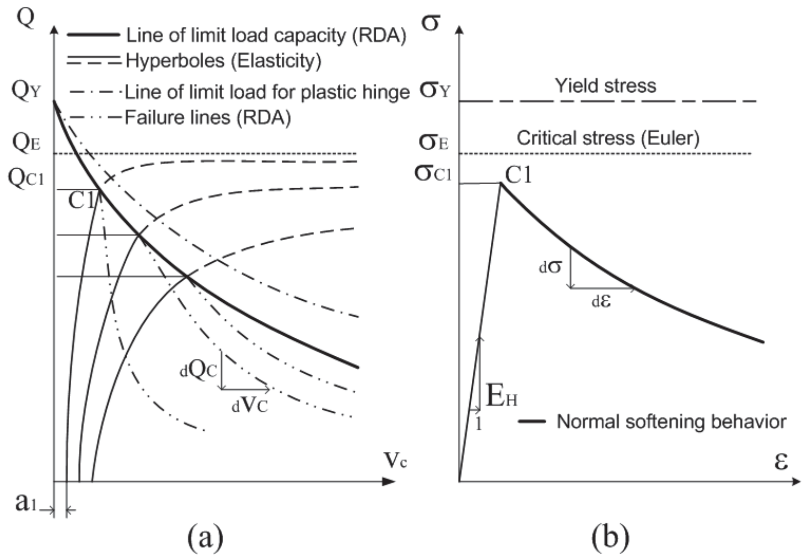
$$\sigma_{C1} = \frac{Q_{C1}}{A_0} + \frac{Q_{C1} \cdot v_{c1}}{W} = \sigma_Y \Rightarrow Q_{C1} = \frac{\sigma_Y A_0 W}{W + A_0 a_1 \left(1 + \varphi^*\right)} . \quad (22)$$

gde je *W* elastični otporni moment.

Kriva linija Q_C-v_c dobija se pretpostavljanjem niza početnih imperfekcija a_1. Ovo je linija granične nosivosti dobijena primenom RDA. Ova linija pokazuje normalno omekšavajuće ponašanje grede, slika 2(a).

where *W* is the elastic section modulus.

The curve line Q_C-v_c is obtained by assuming a series of the initial imperfections a_1. This is the line of limit load capacity obtained using the RDA. This line shows normal softening behavior of beam, Fig. 2 (a).



Slika 2. (a) Linija granične nosivosti i linije loma, dobijene primenom RDA; (b) dijagram napon-deformacija normalnog omekšavajućeg ponašanja materijala

Fig. 2. (a) Line of limit load capacity and failure lines obtained using the RDA; (b) stress-strain diagram of normal softening behavior of material

3 ODREĐIVANJE LINIJA LOMA

Na početku poglavlja 2 opisani su faktori koji utiču na graničnu nosivost konstrukcija. Međutim, kombinacija ovih faktora može dovesti do veoma različitih oblika lomova grede, koji se kreću od krtog do izrazito duktilnog. Oblik loma usko je povezan s linijama loma koje se pojavljuju u postkritičnom stanju, nakon što je granična nosivost dostignuta. U ovom poglavlju, daju se rešenja linija loma prema teoriji elastičnosti i teoriji plastičnosti, s ciljem pojašnjenja linija granične nosivosti, dobijenih primenom RDA.

3.1 Elastične linije loma

Ako analiziramo gredu poprečnog preseka A_0 i s početnom imperfekcijom a_1, elastična linija loma (hiperbola) dobro je poznato rešenje problema granične nosivosti – slika 2(a). Asimptota rešenja jeste Euler-ovo kritično opterećenje Q_E, [1]

3 DETERMINATION OF FAILURE LINES

At the beginning of Section 2 are described factors which affect the limit load capacity. However a combination of these factors can lead to a substantially different shape of fracture of the beam, which range from brittle to extremely ductile. The shape of the fracture is closely associated with failure lines that appear in post-critical state, after the limit load capacity is reached. In this section, solutions are given for the failure lines according to the theory of elasticity and plasticity theory in order to compare with solutions obtained by the RDA.

3.1 Elastic failure lines

If we analyze the beam with a constant cross-section A_0 and initial imperfection a_1, elastic failure line (hyperbola) is well known solution for elastic problem of the limit load capacity, Fig. 2(a). The asymptote of solution is Euler's critical load Q_E, [1]

$$v_{c1} = \frac{a_1}{1 - \frac{Q_{C1}}{Q_E}} . \quad (23)$$

Napon u tački C_1 jeste suma od aksijalnog napona i napona savijanja. Mi izjednačavamo ovaj napon s

The stress at point C_1 is the summation of the axial stress and bending stress. We equates this stress with naponom tečenja σ_Y, pod pretpostavkom plastičnog loma grede

yield stress σ_Y, assuming plastic failure of the beam

$$\sigma_{C1} = \frac{Q_{C1}}{A_0} + \frac{Q_{C1} v_{c1}}{W} = \frac{Q_{C1}}{A_0} + \frac{Q_{C1}}{W} \frac{a_1}{1 - \dfrac{Q_{C1}}{Q_E}} = \sigma_Y \, . \tag{24}$$

Tako, elastična granična nosivost glasi

Thus, the elastic carrying capacity is as follows

$$Q_{C1} = \frac{\left(Q_E W + a_1 Q_E A_0 + \sigma_Y A_0 W\right) - \sqrt{\left(Q_E W + a_1 Q_E A_0 + \sigma_Y A_0 W\right)^2 - 4W^2 \sigma_Y Q_E A_0}}{2W} \, . \tag{25}$$

Hiperbola Q-v_c može biti konstruisana izborom niza sila Q. Međutim, treba imati u vidu i to da se kada je reč o realnom materijalu hiperbola može primeniti samo do linije granične nosivosti, dobijene primenom RDA u poglavlju 2.4, odnosno do granične nosivosti Q_{C1}.

Ako je opterećenje veće od opterećenja Q_{C1}, onda je moguće da se pojave linije loma na osnovu novih ravnotežnih stanja. Kao što je pomenuto u [1], u tankozidnim konstrukcijama postoje dva važna razloga za formiranje linija loma, slika 2(a). Prvi razlog odnosi se na nagib linije granične nosivosti, a drugi razlog jeste to što se u tankozidnim konstrukcijama ne razvijaju jednostavni plastični zglobovi. Jednostavan plastični zglob javlja se samo pod pravim uglom u odnosu na neutralnu osu grede izložene čistom savijanju.

Hyperbola Q-v_c can be constructed by selecting a series of force Q. However, it should be noted that in a real material a hyperbola can be applied only to the line of limit load capacity, which is obtained by the application of RDA in Section 2.4, that is, up to the critical load Q_{C1}.

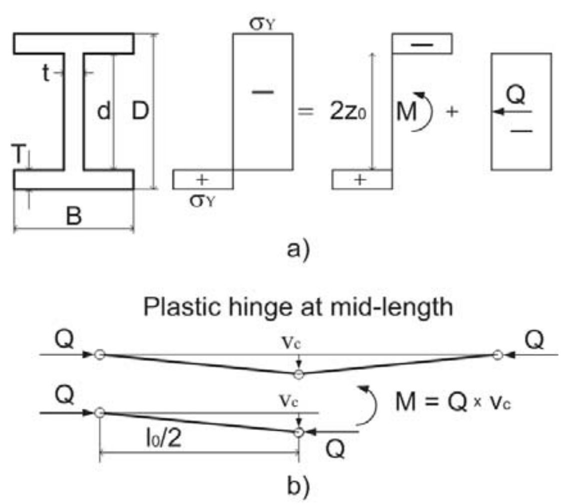
If load is greater than load Q_{C1} then it is possible to appear failure lines based on the new equilibrium states. As mentioned in [1], in the thin-walled structures there are two important reasons for the formation of the failure lines, Fig. 2(a). The first reason is related to the slope of the line of limit load capacity, while the second reason is that in thin-walled structures can not be developed a simple plastic hinges. Simple plastic hinge occurs only at a right angle to the neutral axis of a beam subjected to pure bending.

3.2 Plastična linija loma za jednostavan plastični zglob

Linija loma za jednostavan plastični zglob može se aproksimirati metodom objašnjenom u [6], slika 3(a).

3.2 Plastic failure line for simple plastic hinge

Failure line for simple plastic hinge can be approximated by the method explained in [6], Fig. 3(a).



Slika 3. (a) metod objašnjen u [6]; (b) jednostavan plastični zglob
Fig. 3. (a) method explained in [6]; (b) A simple plastic hinge

Prema radu [6], pretpostavlja se da rebro prihvata aksijalnu silu Q, dok preostali deo poprečnog preseka prihvata moment savijanja M. Kada se neutralna osa nalazi u rebru ($z_0 \le d/2$), polovina dubine plastične zone i moment savijanja računaju se prema izrazima

According to [6] it is assumed that the web accepts axial force Q, while the remainder part of cross-section accepts bending moment M. When the neutral axis is in the web ($z_0 \le d/2$), half the depth of plastic zone of and bending moment are calculated according to equations

$$z_0 = \frac{Q}{2\sigma_Y t}, \qquad M = \left\{ BT(D-T) + \left[\left(\frac{d}{2}\right)^2 + z_0^2 \right] t \right\} \sigma_Y . \tag{26}$$

Kada je neutralna osa u flanšama ($d/2 \le z_0 \le d/2+T$), dubina plastične zone i moment savijanja računaju se prema izrazima

When the neutral axis is in the flanges ($d/2 \le z_0 \le d/2+T$), the depth of plastic zone and bending moment are calculated according to equations

$$z_0 = \frac{Q - \sigma_Y td}{2B\sigma_Y} + \frac{d}{2}, \qquad M = \left[\left(\frac{D}{2}\right)^2 - z_0^2 \right] B\sigma_Y .. \tag{27}$$

Tako, imajući u vidu vrednosti *Q* i *M*, ugib u sredini dužine glasi

Thus, taking into account the values of Q and M, the deflection at mid-length is as follows

$$v_c = \frac{M}{Q} . \tag{28}$$

Linija loma Q-v_c za jednostavan plastični zglob može biti konstruisana izborom niza sila *Q*.

Failure line Q-v_c for simple plastic hinge may be constructed by selecting a series force *Q*.

3.3 Linije loma – dobijene primenom RDA

3.3 Failure lines obtained using the RDA

Ako je opterećenje veće od opterećenja Q_{C1} greda je u novom, izvijenom ravnotežnom stanju. U slučaju normalnog omekšavanja, samo granična nosivost Q_{C1} mora opadati, dok se ugib grede povećava. Primena RDA polazi od granice elastičnosti. Zbog toga, za odabranu početnu imperfekciju a_1 konstruiše se hiperbola, te se sračunava elastična granična nosivost Q_{C1}. Za odgovarajući napon $\sigma_{C1} = Q_{C1}/A_0$ vitkost grede jeste

If the load is greater than load Q_{C1}, a beam is in the new buckling equilibrium. In the case of normal softening the ultimate carrying capacity Q_{C1} must decreases only, while the deflection increases. Application of RDA starts from the limit of elasticity. Therefore, for the selected initial imperfection a_1 a hyperbole may be constructed and the limit load capacity Q_{C1} calculated. For the corresponding stress $\sigma_{C1} = Q_{C1}/A_0$, the slenderness of a beam is

$$\lambda_{C1} = \pi \sqrt{\frac{E_H}{\sigma_{C1}}} . \tag{29}$$

Prema radu [4], koeficijent tečenja φ_{C1} jeste

According to [4], the creep coefficient φ_{C1} is

$$\varphi_{C1} = \pi^2 \frac{i^3}{I} \frac{1}{\gamma \lambda_{C1}} . \tag{30}$$

φ_{C1} jeste ključni parametar za testiranje omekšavajućeg ponašanja, slika 2(b). Na osnovu zakona toka (14), strukturalno-materijalna konstanta jeste

φ_{C1} is a key parameter for the testing of the softening behavior of the material, Fig. 2(b). Based on the flow law (14) the structural-material constant is as follows

$$K_{C1} = \frac{\varphi_{C1}}{\sigma_{C1}} . \tag{31}$$

RDA modul u prvoj iteraciji (1) jeste

RDA modulus in the first iteration (1) is

$$E_{RC1}^{(1)} = \frac{E_H}{1+\varphi_{C1}} . \tag{32}$$

Zatim, napon u prvoj iteraciji jeste

Then the stress in the first iteration (1) is

$$\sigma_{C1}^{(1)} = \frac{\pi^2 E_{RC1}^{(1)}}{\lambda_{C1}^2} . \tag{33}$$

RDA modul u drugoj iteraciji (2) jeste

RDA modulus in the second iteration (2) is

$$E_{RC1}^{(2)} = \frac{E_H}{1+\varphi_{C1}^{(1)}}. \quad (34)$$

gde je

where

$$\varphi_{C1}^{(1)} = \sigma_{C1}^{(1)} K_E. \quad (35)$$

Iterativni postupak objašnjen u [4] nastavlja se sve do konvergencije u rešavanju problema, to jest kada novi RDA modul više ne menja napon. Ugibi u sredini grede moraju se računati putem iteracija – kako sledi

Iterative method, which is explained in [4] continues until convergence in solving the problem, i.e. when the new RDA modulus does not change the stress. Deflection in the mid-length of beam must be calculated through iterations as follows

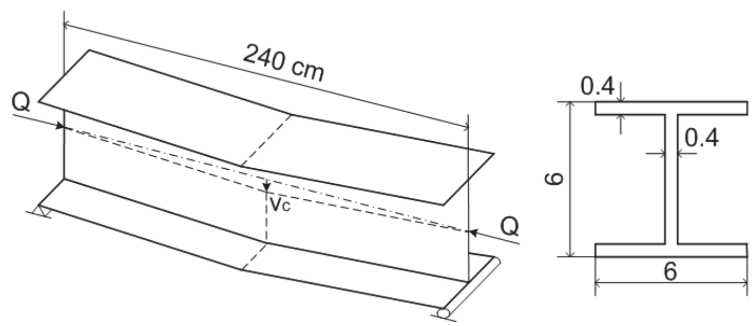
$$v_{c1}^{(i)} = v_{c1}\left(1+\varphi^{(i-1)}\right). \quad (36)$$

4 NUMERIČKA ANALIZA

Numerička analiza sprovodi se na prostoj gredi s tankozidnim poprečnim presekom. Početne imperfekcije a_1 = 0.1, 1.5 i 5 mm izmerene su na sredini visine grede [1]; greda se savija oko jače ose. Detalji poprečnog preseka uzeti su iz [1] i prikazani su na slici 4. Greda je napravljena od čelika sledećih mehaničkih karakteristika E_H = 206 GPa, μ = 0.3 i σ_Y = 250 MPa. Pretpostavljeno je da savijena greda formira jednostavan plastični zglob u sredini dužine.

4 NUMERICAL ANALYSIS

Numerical analysis is carried out in a simple beam with the thin-walled cross-section. The initial imperfections of a_1 = 0.1, 1.5 and 5 mm are measured at mid-height of beam [1] and beam bending takes place about the stronger axis. Details the cross-section was taken from [1] and is shown in Fig. 4. The beam is made of steel of the following mechanical characteristics of E_H = 206 GPa, μ = 0.3 i σ_Y = 250 MPa. It is assumed that bending beam forms a simple plastic hinge in the mid-length.



Slika 4. Greda s jednostavnim plastičnim zglobom u sredini dužine
Fig. 4. Beam with a simple plastic hinge in the mid-length

Slika 5 prikazuje Q-v_c linije granične nosivosti za tri izmerene imperfekcije prema [1]. Linija granične nosivosti, dobijena primenom RDA, nalazi se u elasto-plastičnoj oblasti, ispod plastične linije loma za jednostavan plastični zglob.

Tečenje počinje kada napon u spoljnim vlaknima poprečnog preseka grede dostigne napon tečenja σ_Y. Linija početka tečenja (slika 6) izračunava se prema Bernoulli–Euler-ovoj teoriji savijanja.

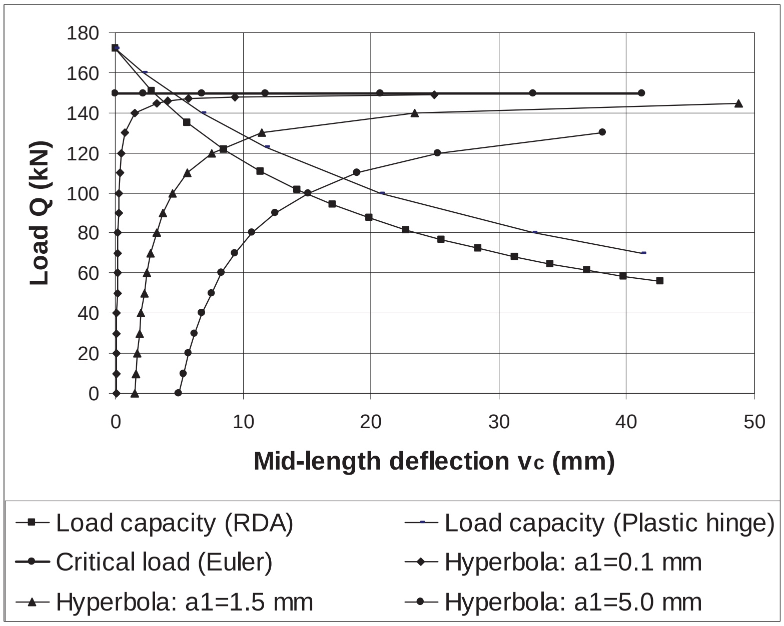
Slika 6 prikazuje linije loma dobijene primenom RDA, koje se pojavljuju u postkritičnom stanju grede, nakon što je granična nosivost dostignuta. U slučaju početne imperfekcije a_1 = 0.1 mm, lom se dešava u elastičnoj oblasti, jer linija loma leži ispod linije početka tečenja. Zbog toga, greda se ponaša krto iako je čelik duktilan materijal. Ovakvo krto ponašanje grede nije poželjno. U druga dva slučaja (a_1 = 1.5 i 5 mm), lom grede je

Fig. 5 presents Q-v_c lines of limit load capacity for the three measured imperfections [1]. Line of load capacity that obtained by RDA is located in the elastic-plastic failure zone under the plastic failure line for simple plastic hinge.

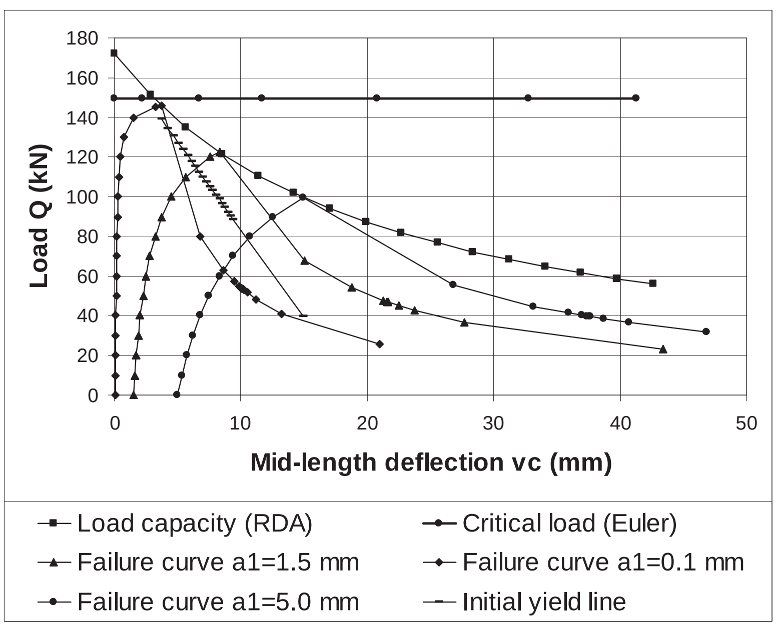
Yield of cross-section starts when stress in the outermost fiber reaches the yield stress σ_Y. Initial yield line, Fig. 6 is calculated according to the Bernoulli-Euler's bending theory.

Fig. 6 shows the failure lines obtained using the RDA, such as occur in a post-critical state of beam, after the limit load capacity is reached. In the case of initial imperfection of a_1 = 0.1 mm failure occurs in the elastic zone, because the failure line lies below the initial yield line. Because of this beam behaves brittle although the steel is ductile material. This brittle behavior of the beam is not desirable. In other two cases (a_1 = 1.5 and 5 mm) duktilan u elasto-plastičnoj oblasti. Duktilnost je veća kod većih početnih imperfekcija. Linije loma opisuju normalno omekšavajuće ponašanje. To je slučaj negativnih ugiba ($dQ_C/dv_c<0$), slika 2(a). To znači da se granična nosivost smanjuje, a raste ugib grede. Isti omekšavajući efekat već se dobio putem napon–deformacija relacije u radu [7].

the failure of the beam is ductile in the elastic-plastic zone. Ductility is greater at higher initial imperfections. Failure lines describe the normal softening behavior. This is the case of negative slope ($dQ_C/dv_c<0$), Fig. 2(a). This means that the limit load capacity decreases, while the deflection increases. The same softening effect has already been obtained through the stress-strain relation in the paper [7].



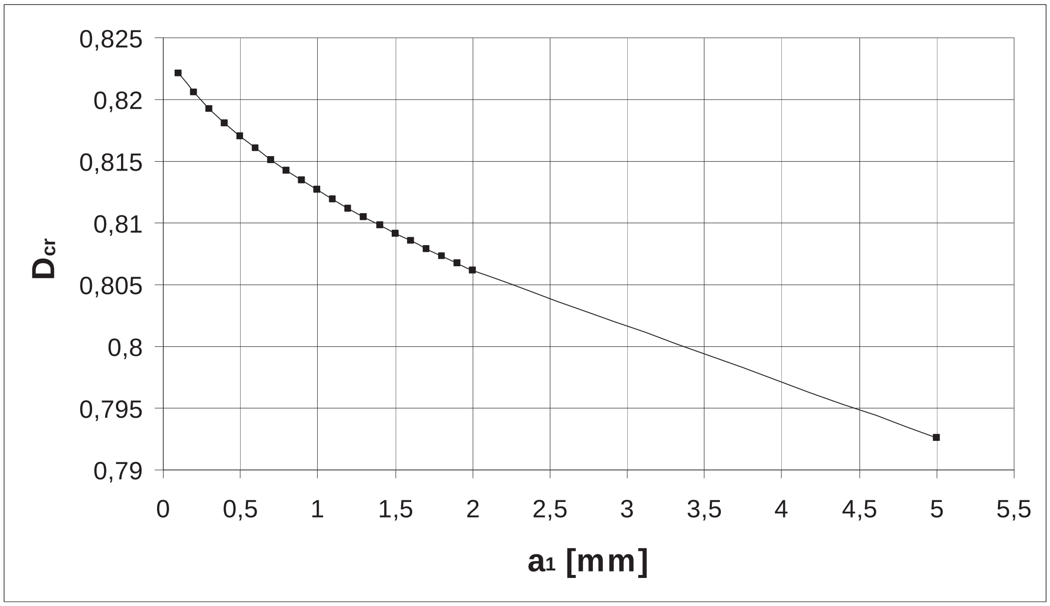
Slika 5. Uticaj početnih imperfekcija na linije granične nosivosti
Fig. 5. Influence of initial imperfections on the lines of limit load capacity



Slika 6. Uticaj početnih imperfekcija na linije početka tečenja i linije loma
Fig. 6. Influence of initial imperfections on the initial yield line and failure lines

Slika 7 prikazuje kritične varijable oštećenja, sračunate na liniji granične nosivosti za skup pretpostavljenih početnih imperfekcija. Velike vrednosti kritičnih varijabli oštećenja pokazuju da su tankozidne konstrukcije veoma osetljive na početne imperfekcije, što je dobro poznata činjenica potvrđena eksperimentalno.

Fig. 7 shows the critical damage variables calculated at the line of limit load capacity for a set of assumed initial imperfections. Large values of critical damage variables show that the thin-walled structures are very sensitive to the initial imperfections, which is a well known fact proved experimentally.



Slika 7. Uticaj početnih imperfekcija na kritičnu varijablu oštećenja
Fig. 7. Influence of initial imperfections on the critical damage variable

Varijabla oštećenja je najveća za imperfekciju a_1 = 0.1 mm, dokazujući tako da su tankozidne konstrukcije osetljivije za manje početne imperfekcije. Ovaj zaključak u potpunosti je u skladu s ranijim zaključkom da se za ovu veličinu imperfekcije greda krto lomi, iako je napravljena od duktilnog materijala. Zanimljivo je napomenuti i to da u radu [5], Lemaitre tvrdi da varijabla oštećenja pri lomu elemenata u slučaju metala jeste u granicama $0.2 \le D_{cr} \le 0.8$.

Identifikacija parametara za slučaj merene imperfekcije a_1 = 1.5 mm daje

• Euler-ova kritična sila

Damage variable is the greatest for the imperfection a_1 = 0.1 mm, thus proving that the thin-walled constructions more sensitive for the less initial imperfections. This conclusion is in full compliance with the earlier conclusion that for this size of imperfection beam brittle failures, although made of ductile material. It is interesting to note that in the paper [5], Lemaitre claimed that damage variable for breaking in the case of metal elements is within the limits $0.2 \le D_{cr} \le 0.8$.

Parameters identification for the case of the measured imperfection a_1 = 1.5 mm gives

• Euler's critical force

$$Q_E = \frac{\pi^2 E_H I}{l_0^2} = \frac{\pi^2 \cdot 20600 \cdot 42.38}{240^2} = 149.60 kN$$

• Granična nosivost u tački C_1, jednačina (25) i odgovarajući napon

• Ultimate carrying capacity at point C_1, Eq. (25) and corresponding stress

$$Q_{C1} = 122.53 kN\ , \qquad \sigma_{C1} = \frac{Q_{C1}}{A_0} = \frac{122.53}{6.88} = 17.81 kN/cm^2$$

• Ugib u sredini dužine grede

• Deflection in the mid-length of beam

$$v_{c1} = \frac{a_1}{1 - \frac{Q}{Q_E}} = \frac{1.5}{1 - \frac{122.53}{149.6}} = 8.29 mm$$

- Vitkost

- Slenderness

$$\lambda_{C1}=\pi\sqrt{\frac{E_H}{\sigma_{C1}}}=\pi\sqrt{\frac{206000}{178.10}}=106.84$$

- Koeficijent tečenja

- Creep coefficient

$$\varphi_{C1}=\pi^2\frac{i^3}{I}\frac{1}{\gamma\lambda_{C1}}=\pi^2\cdot 0.36076\cdot\frac{1000}{7.86\cdot 106.84}=4.24$$

- Ugib u sredini dužine grede u prvoj iteraciji

- Deflection in the mid-length of beam in the first iteration

$$v_{c1}^{(1)}=v_{c1}\left(1+\varphi_{C1}\right)=8.29\cdot\left(1+4.24\right)=43.44mm$$

- RDA modul u prvoj iteraciji

- RDA modulus in the first iteration

$$E_{RC1}^{(1)}=\frac{E_H}{1+\varphi_{C1}}=\frac{206000}{1+4,24}=39316.2MPa$$

- Napon i nosivost u prvoj iteraciji nakon što je granična nosivost dostignuta

- Stress and load capacity in the first iteration, after the limit load capacity is reached

$$\sigma_{C1}^{(1)}=\frac{\pi^2 E_{RC1}^{(1)}}{\lambda_{C1}^2}=\frac{\pi^2\cdot 39316.2}{106.84^2}=33.99MPa\,,\qquad Q_{C1}^{(1)}=\sigma_{C1}^{(1)}A_0=3.399\cdot 6.88=23.39kN\,.$$

- Kritična varijabla oštećenja u prvoj iteraciji

- Critical damage variable in the first iteration

$$D_{C1}^{(1)}=\frac{\varphi_{C1}^{(1)}}{1+\varphi_{C1}^{(1)}}=\frac{4.24}{1+4.24}=0.80916$$

- Efektivna nosivost prema mehanici oštećenja

- Effective load capacity according to the damage mechanics

$$\tilde{Q}_{C1}^{(1)}=\frac{Q_{C1}^{(1)}}{1-D_{C1}^{(1)}}=\frac{23.39}{1-0.80916}=122.5kN\,.$$

Zbog toga što je efektivna nosivost (nosivost neoštećene grede) $\tilde{Q}_{C1}^{(1)}$ jednaka graničnoj nosivosti Q_{C1}, proračuni obavljeni primenom RDA pokazuju saglasnost s hipotezama mehanike oštećenja.

Because that effective load capacity (capacity of undamaged beam) $\tilde{Q}_{C1}^{(1)}$ is equal to the limit load capacity Q_{C1}, calculations which are made by using the RDA are consistent with the hypothesis of damage mechanics.

5 ZAKLJUČCI

U radu je teorijski istraživan komplikovan problem granične nosivosti tankozidne grede s početnim geometrijskim imperfekcijama, pod pretpostavkama elastičnosti, plastičnosti i primenom RDA. RDA je neelastična (viskoelastoplastična) teorija i obuhvata obe prethodno pomenute. Radi prezentovanja primenjivosti RDA, urađeno je poređenje s numeričkim i eksperimentalnim rezultatima jedne čelične grede iz literature [1] od Murray-a. Zavisno od polaznih pretpostavki o materijalu, pojam granične nosivosti varira i zbog toga nije moguće dati jedinstvenu formulaciju za graničnu nosivost.

Pojam granične nosivosti, pod pretpostavkama idealno elastičnog materijala na analiziranom primeru

5 CONCLUSINS

The paper was theoretically investigated the problem involved ultimate bearing capacity of thin-walled beam with initial geometrical imperfections under the conditions of elasticity, plasticity and using RDA. RDA is inelastic (viscoelastoplastic) theory and includes both previously mentioned. In order to present applications of RDA the comparison is done with the numerical and experimental results of a steel beam from the literature of Murray [1]. Depending on the assumptions about the material the concept of limit load capacity varies and is therefore not possible to give a unique formulation for the ultimate bearing capacity.

The term limit load capacity under the conditions of ideal elastic material in the case of analyzed beam with grede sa zadanim početnim imperfekcijama, pokazuje da granična nosivost konvergira ka kritičnoj Euler-ovoj sili. Konvergencija je sporija što je veća početna imperfekcija. Prema teoriji elastičnosti, nije moguće dobiti omekšavajuće efekte pri opterećenjima bliskim kritičnom, koji se uočavaju eksperimentalno. Međutim, ključna mana ovog modela jeste to što u punom smislu reči elastičan materijal ne postoji, odnosno on je samo hipotetički zamišljen.

Pojam granične nosivosti pod pretpostavkom idealne plastičnosti daje u analiziranom primeru gornju graničnu nosivost. Ova teorija opisuje omekšavajuće efekte, jer daje manju graničnu nosivost pri većim početnim imperfekcijama. Ključna mana ovog modela jeste to što ne objašnjava kritične napone pri izvijanju.

Granična nosivost, dobijena primenom RDA, uvek se nalazi između goreopisanih teorija. RDA teorija opisuje kritične napone pri neelastičnom izvijanju. Razlog za to jeste to što RDA uključuje neelastična svojstva materijala pri analizi izvijanja. U ovom radu, analizirano je samo kvazistatičko rešenje, $\delta \rightarrow 0$ ($\delta = \omega_\sigma / \omega = \omega_\sigma T^D$), tako da su očekivani i dopunski neistraženi efekti koje RDA teorija daje u problemu dinamičke stabilnosti pod uticajem frekvencije ω_σ (frekvencija sile). RDA transformiše materijalno-nelinearan problem u linearno-dinamički problem, tako da su dobijena rešenja analitička.

Osim toga, RDA na efikasan način daje objašnjenja i u postkritičnom ponašanju analizirane grede, gde pokazuje da se greda izrađena od duktilnog materijala može lomiti od krtog do izrazito duktilnog ponašanja, zavisno od zadate početne imperfekcije. Iako je ovo odavno utvrđeno eksperimentalno, u ovom radu se to i teorijski potvrđuje. Kako je postkritično stanje u domenu mehanike oštećenja i nelinearne mehanike loma, RDA je upešna u poređenju s mehanikom oštećenja preko kritične varijable oštećenja.

Zahvalnost

Ovaj rad predstavlja deo istraživanja koja se sprovode u okviru istraživačkih projekata OI 174027 „Računarska mehanika u građevinskom inženjerstvu” i TR 36017 „Istraživanje mogućnosti primene otpadnih i recikliranih materijala u betonskim kompozitima, sa ocenom uticaja na životnu sredinu, u cilju promocije održivog građevinarstva u Srbiji”, uz podršku Ministarstva za nauku i tehnologiju Republike Srbije.

initial imperfections shows that the limit load capacity converging to the Euler-critical force. Convergence is slower for the larger the initial imperfections. According to the theory of elasticity the softening effects that observed experimentally under the load close to the critical load can not be obtained. However, the key disadvantage of this model is that in the full sense of the word elastic material does not exist, or it is only hypothetical thought.

The term limit load capacity under the assumption of ideal plasticity provides in the analyzed beam an upper limit load capacity. This theory describes softening effects, since it gives a lower limit load capacity at higher initial imperfections. The key disadvantage of this model is that it does not explain the critical buckling stresses.

The limit load capacity obtained by RDA is always located between the above-described theories. RDA theory describes the critical stresses for inelastic buckling. The reason is that RDA involves inelastic properties of materials in the analysis of buckling. In this paper the quasi-static solution is analyzed only, $\delta \rightarrow 0$ ($\delta = \omega_\sigma / \omega = \omega_\sigma T^D$), so that the expected additional unexplored effects that RDA theory provides in the problem of dynamic stability under the influence of frequency ω_σ (frequency of force). RDA transformed materially nonlinear problem into the linear dynamic problem so that the obtained analytical solutions.

Apart from this, RDA effectively provides explanations in post-critical behavior of the analyzed beam, which show that a beam made from ductile material can break from brittle to extremely ductile, depending on the initial imperfections. Although this a long time established experimentally in this paper is theoretically confirmed. Because that post-critical state is in the field of damage mechanics and nonlinear fracture mechanics, the RDA is successful compared with damage mechanics through the critical variables of damage.

Acknowledgements

The work presented in this paper is a part of the investigation conducted within the research projects OI 174027 "Computational Mechanics in Structural Engineering" and TR 36017 "Utilization of by-products and recycled waste materials in concrete composites for sustainable construction development in Serbia: investigation and environmental assessment of possible applications", supported by the Ministry of Science and Technology, Republic of Serbia. This support is gratefully acknowledged.

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REZIME

GRANIČNA NOSIVOST PRITISNUTE GREDE SA IMPERFEKCIJAMA

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Smilja ŽIVKOVIĆ

Ovaj rad predstavlja teorijsko istraživanje elastičnog i neelastičnog izvijanja tankozidne grede s početnim imperfekcijama. Glavni cilj rada jeste da pokaže da lom grede varira od krtog do izrazito duktilnog – u zavisnosti od imperfekcija. Elastično rešenje za procenu uticaja imperfekcija na graničnu nosivost u elastičnom području dobro je poznato. Međutim, elastično rešenje može biti primenjeno samo do linije granične nosivosti. Istraživanje granične nosivosti konstrukcije veoma je komplikovan problem materijalne nelinearnosti, jer ovaj problem mora da uključi plastični mehanizam loma. U ovom radu analizirana je granična nosivost grede primenom reološko-dinamičke analogije (RDA) [4]. Radi prezentovanja mogućnosti RDA, urađeno je poređenje s numeričkim i eksperimentalnim rezultatima jedne čelične grede iz literature [1] od Murray-a.

Ključne reči: Tankozidna greda, početne imperfekcije, plastični mehanizam loma, RDA, granična nosivost, postkritično ponašanje, varijabla oštećenja

SUMMARY

LIMIT LOAD CAPACITY OF COMPRESSED BEAM WITH IMPERFECTIONS

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This paper presents a theoretical investigation of elastic and inelastic buckling of thin-walled beam with initial imperfections. The main aim of this paper is to show that the fracture of beams varies from brittle to extremely ductile depending on the imperfections. Elastic solution for estimation of intial imperfections against the limit load capacity in the elastic range is well known. However, the elastic solution can be applied only to the line of limit load capacity. Examination of the limit load capacity of structure is very complicate problem of material non-linearity, because this problem must includes the plastic mechanism of failure. In this paper the limit load capacity of beam is analyzed using the reological-dynamical analogy (RDA) [4]. In order to demonstrate the ability of RDA, the comparison with experimental and numerical results of one steel beam from Ref [1] by Murray, is done.

Key words: Thin-walled beam, initial imperfections, plastic mechanism of failure, RDA, limit load capacity, post-critical bahaviour, damage variable