PRIMENA NUMERIČKIH METODA U ANALIZI SPREGNUTIH KONSTRUKCIJA DRVO-BETON

APPLICATION OF NUMERICAL METHODS IN ANALYSIS OF TIMBER CONCRETE COMPOSITE SYSTEM

Dragan MANOJLOVIĆ
Tatjana KOČETOV MIŠULIĆ
Aleksandra RADUJKOVIĆ

ORIGINALNI NAUČNI RAD
ORIGINAL SCIENTIFIC PAPER
UDK:624.016.072.2
doi:10.5937/GRMK1804037M

1 UVOD

Kompozitni konstrukcijski sistemi podrazumevaju racionalno spajanje elemenata od odgovarajućih materijala, tako da se optimalno iskoriste njihova svojstva. Spregnute konstrukcije imaju najširu primenu u inženjerskim konstrukcijama velikog raspona [12], ali mogu se uspešno primeniti i u stambenim i poslovnim objektima. Adekvatnim spajanjem konstrukcijskih elemenata istih ili različitih fizičko-mehaničkih karakteristika u integralni poprečni presek, postiže se osnovni cilj postupka, tj. povećava se nosivost sistema u odnosu na pojedinačne elemente. U zavisnosti od primenjenih materijala, spregnute konstrukcije koje se često primenjuju u građevinarstvu uglavnom su tipa drvo–drvo, beton–beton, čelik–beton i drvo-beton.

Budući da su spregnute konstrukcije – napravljene od različitih materijala i različitim načinima spajanja – dostigle veoma visok stepen primene u građevinarstvu u poslednjih nekoliko decenija, neophodna je njihova preciznija analiza, kao i preciznije projektovanje. Poznato je da upotrebljeni tip sredstava za sprezanje najviše utiče na globalno ponašanje spregnutih konstrukcija. Stoga, od ključnog značaja je to kako da se uvede problem ponašanja veze između spregnutih materijala u analizi i proračunu.

Dragan Manojlović, MSc, dig, Departman za građevinarstvo i geodeziju, Fakultet tehničkih nauka, Univerzitet u Novom Sadu, e-mail: manojlovic.dragan@uns.ac.rs
Tatjana Kočetov-Mišulić, Dr, dig, Departman za građevinarstvo i geodeziju, Fakultet tehničkih nauka, Univerzitet u Novom Sadu, e-mail: tanya@uns.ac.rs
Aleksandra Radujković, Dr, dig, Departman za građevinarstvo i geodeziju, Fakultet tehničkih nauka, Univerzitet u Novom Sadu, e-mail: leksa@uns.ac.rs

1 INTRODUCTION

Composite construction systems consider the rational structural composition of the right materials at the right places in order to optimally exploit their properties. Composite structures have the widest application in large-span engineering constructions [12], but they can be applied successfully in residential and commercial buildings. By adequate coupling of the constructive elements of the same or different physical-mechanical characteristics into an integral cross-section, the basic goal of the procedure is achieved, i.e. the capacity of the system is increased in relation to the individual elements. Depending on the applied materials, composite structures that are often in use in the construction industry generally are timber-timber, concrete-concrete, steel-concrete and timber-concrete.

Since the composite structures, made by different materials and methods of joining, have reached very high level of application in the construction industry in last several decades, there is a demand for their more precise analysis and design. It is known that the type of used fasteners mostly influence the overall behaviour of the coupled structures. Therefore, it is of crucial importance how to introduce the problem of the connection behaviour between coupled materials into the analysis and design.

Dragan Manojlovic, MSc, civ.eng., Department of Civil Engineering and Geodesy, Faculty of Technical Sciences, University of Novi Sad, e-mail: manojlovic.dragan@uns.ac.rs
Tatjana Kocetov-Misulic, PhD, civ.eng., Department of Civil Engineering and Geodesy, Faculty of Technical Sciences, University of Novi Sad, e-mail: tanya@uns.ac.rs
Aleksandra Radujkovic, PhD, civ.eng., Department of Civil Engineering and Geodesy, Faculty of Technical Sciences, University of Novi Sad, e-mail: leksa@uns.ac.rs

Spajanje konstitutivnih elemenata može se postići na različite načine, pri čemu jedan od najčešćih postupaka jeste upotreba diskretno postavljenih sredstava za sprezanje (npr. mehanička spojna sredstva, sidra). Spojna sredstva treba da obezbede vezu dva različita materijala, prenoseći smičuće sile između dva elementa, s ciljem obezbeđenja kompozitnog sadejstva konstrukcije. S obzirom na to što je primena štapastih spojnih sredstava najčešća u spregnutim konstrukcijama drvo–beton (SDB), a pošto ponašanje celokupne konstrukcije zavisi od njihovog ponašanja, interes istraživača i projektanata, kao i brojnih studija i istraživačkih radova, odnosi se na ove tipove sredstava za sprezanje. Upotreba mehaničkih spojnih sredstava za sprezanje dva različita materijala, kao što su drvo i beton, ukazuje na to da je ponašanje SDB konstrukcija veoma složeno, budući da spojna sredstva dozvoljavaju izvesno klizanje u spoju koje dovodi do delimične interakcije (elastično sprezanje). Prema tome, analiza i proračun SDB konstrukcija zahteva da se imaju u vidu klizanja u spoju između elemenata.

Razmatrajući jednodimenzionalni problem, prve teorije elastičnog sprezanja kod greda izloženih statičkom dejstvu razvili su Newmark (1943,1951), Granholm (1949), Pleshov (1952) i Goodman (1967). Teoriju elastičnog sprezanja primenili su Girhammar i Gopu (1991) u analizi stubova s klizanjem u spoju, izloženih odgovarajućem slučaju aksijalnog opterećenja, koja je kasnije proširena i generalizovana u njihovom daljem radu. Na osnovu prethodnih istraživanja i analiza, oni su prikazali tačan analitički postupak statičke analize elastično spregnutih nosača s klizanjem u spoju [7], a u narednim radovima [8], [9], [10] predložili su tačne analitičke i pojednostavljene metode za analizu elastično spregnutih sistema s primenom na grede i stubove. U Srbiji, u oblasti sprezanja drvo–beton, teorijske osnove za analizu delimično spregnutih sistema uz eksperimentalne rezultate dao je B. Stevanović (1994) [17], kao i Lj. Kozarić [11] i R. Cvetković [3].

Teorija parcijalnog (elastičnog) sprezanja zasniva se na odgovarajućim pretpostavkama teorije elastičnosti i uzima u obzir klizanje spoja/veze pri njihovom proračunu. Analitički proračun elastičnog sprezanja podrazumeva rešavanje diferencijalnih jednačina, gde se rešenja u zatvorenom obliku mogu formulisati samo za pojedine (jednostavnije) slučajeve konturnih uslova i opterećenja.

U EN1995 [5] usvojen je pojednostavljeni manuelni postupak proračuna („γ-metod”), koji se u praksi široko primenjuje. Ovaj metod prvobitno je primenio Mohler (1956), razmatrajući problem klizanja u spoju između spregnutih elemenata (drvo–drvo), s mehaničkim spojnim sredstvima, ali uz odgovarajuće modifikacije, ovaj postupak se može primeniti i na druge tipove spregnutih konstrukcija, kao što su konstrukcije tipa drvo–beton. „Gama” metod razvijen je za statički sistem proste grede izložene sinusoidnom opterećenju $q=q_0\cdot\sin(\pi\cdot x/L)$. U ovom slučaju postoji jednostavno rešenje u zatvorenom obliku, koje se može primeniti i na druge vrste opterećenja, a zbog malog odstupanja od tačnog analitičkog rešenja diferencijalne jednačine. Ova metoda zasniva se na efektivnoj krutosti spregnutog sistema i teoriji elastičnog sprezanja, imajući u vidu konzervativni efekat raspodele sila unutar nosača, i skoro u potpunosti pokriva sve parametre koji utiču na

Coupling of the constitutive elements can be achieved in different ways, where one of the most common procedure is the use of number of individual shear connectors (mechanical fasteners, anchors,...). Shear connectors should ensure bond of two different materials, transferring the shear forces between two elements, enabling the composite action of the structure. The interest of researchers and constructors as well as numerous studies and research works refer to these types of fasteners since the application of dowel type connectors is the most common in timber-concrete composite structures (TCC), and additionally the behaviour of the overall construction depends on their behaviour. The use of mechanical fasteners for coupling two different materials such as timber and concrete shows that the behaviour of the TCC system is very complex, since the fasteners allow certain interlayer slip that leads to partially interaction (elastic composite action). Therefore, the analysis and design of TCC structures requires consideration of the interlayer slip between the sub-elements.

Considering one-dimensional problem, the first theories for partial composite action for beams subjected to static loads were developed by Newmark (1943,1951), Granholm (1949), Pleshov (1952) and Goodman (1967). The application of partial composite action theory was performed by Girhammar and Gopu (1991) in analysis of columns with interlayer slip subjected to one particular axial loading case which was extended and generalized in their further work. Based on previous research and analysis, they presented an exact static analysis of partial composite structures with interlayer slip [7] and afterwards in papers [8] , [9] , [10] they proposed an exact and simplified methods for analysis of the partial composite structures applied to the beams and columns. In Serbia, in the field of timber-concrete composites, the theoretical basis for analysis of partially composite system was given by B.Stevanović (1994) [17] and later on by Lj.Kozaric [11] and R.Cvetkovic [3], which was followed by experimental data.

The theory of partial (elastic) composite action is based on the corresponding assumptions of the theory of elasticity and takes into account the interlayer slip in the connection at their calculation. The exact calculation of the partial composite action implies solving differential equations where closed form solutions can be formulated only for some particular (simple) cases of boundary and loads conditions.

In EN1995 [5] the simplified manual design procedure ("γ-method") widespread in practice is adopted. This method was originally applied by Mohler (1956), considering the problem of interlayer slip between composite members (timber-timber) coupled with mechanical fasteners, but, with appropriate modifications, this procedure can be applied to the other types of composite constructions such as timber-concrete system. "Gamma" method was developed in the case of simply supported beam subjected to sinusoidal load $q=q_0\cdot\sin(\pi\cdot x/L)$. In this case, there is a simple closed-form solution, that could be applied to the other types of loads as well, due to a slight deviation from the exact analytical solution of the differential equation. This method is based on the effective stiffness of the composite system and on the theory of elastic ponašanje SDB konstrukcija.

Takođe, za proračun spregnutih sistema, moguće je primeniti aproksimativne metode zasnovane na diferencijalnoj [14] ili varijacionoj formulaciji [13].

Diferencijalna formulacija zasniva se na izvođenju diferencijalnih jednačina koje opisuju problem u određenom domenu, gde rešenje zavisi od graničnih uslova. U rešavanju problema, potrebno je naći nepoznatu funkciju koja će zadovoljiti diferencijalnu jednačinu, kao i granične uslove. Rešavanjem izvedenih diferencijalnih jednačina, dobija se analitičko rešenje problema, gde se rešenja u zatvorenom obliku mogu dobiti samo za ograničen broj jednostavnih proračunskih modela. Ako je proračunski model kompleksan, tada se najčešće primenjuju aproksimativne metode, pogodne za dobijanje prihvatljivog rešenja. Metode reziduuma u takvim slučajevima jesu pogodan način za formulisanje numeričkog rešenja.

U varijacionoj formulaciji problema, potrebno je naći nepoznatu funkciju ili više funkcija koje zadovoljavaju uslov stacionarnosti funkcionala, gde u ovom slučaju nepoznata funkcija mora da zadovolji odgovarajuće dodatne uslove koji nisu implicitno sadržani u funkcionalu. Da bi se primenila varijaciona formulacija, neophodno je da za razmatrani problem postoji funkcional.

Na osnovu diferencijalne i varijacione formulacije problema, razvijene su brojne metode i postupci za određivanje približnih rešenja, pri čemu je od metoda reziduuma najzastupljenija Galerkinova metoda, dok je od varijacionih to Ritz-ova metoda.

Metod konačnih elemenata (MKE) jeste jedan od najčešće korišćenih numeričkih metoda u strukturalnoj analizi, pri čemu se formulacija konačnog elementa zasniva na rešenju diferencijalnih jednačina metodama reziduuma ili korišćenjem varijacione formulacije. MKE zasnovana na Galerkinovoj metodi (ili drugim metodama reziduuma) može se primeniti na mnogo širi skup diferencijalnih jednačina, jer nije potrebno imati odgovarajuću varijacionu formu, kao što je slučaj kada se koristi MKE bazirana na Raileigh-Ritz-ovoj metodi [1]. Na osnovu prethodnog izlaganja, može se zaključiti da je primena pojednostavljenih i/ili aproksimativnih numeričkih metoda za analizu i proračun SDB konstrukcija dobrodošla i preporučena. Upravo iz tog razloga, približne metode zasnovane na diferencijalnoj ili varijacionalnoj formulaciji [16] imaju široku primenu, jer mogu biti implementirane u programe za strukturalnu analizu, kako bi se obezbedio poseban alat inženjerima za proračun elastično spregnutih konstrukcija.

U radu je prikazana Galerkinova metoda u analizi SDB konstrukcija [14]. Analiziran je izbor probnih funkcija koje opisuju problem elastičnog sprezanja, kao i njihov uticaj na konačne rezultate. Za poređenje dobijenih rezultata, sprovedene su i analize prema analitičkim rešenju [17] i „gama” postupku [5]. Na osnovu predloženih numeričkih modela, model koji najbolje opisuje problem elastičnog sprezanja izabran je za dodatnu komparativnu analizu sa eksperimentalnim podacima [18]. Pored toga, predstavljena je i upotreba Ritz-ove metode u analizi SDB konstrukcija. Dobijeni rezultati prema Ritz-ovoj metodi, s različitim probnim funkcijama, analizirani su i upoređeni sa analitičkim rešenjem i „gama” postupkom. Sve analize su sprovedene upotrebom programa MATLAB [15].

coupling, taking into account the conservative effect of the distribution of forces within the girders, and so far most fully covers all the parameters that affect the behaviour of TCC.

Also, for the calculation of composite systems, it is possible to apply approximate methods based on the differential [14] or the variation formulation [13].

The differential formulation is based on the derivation of differential equations that describe the problem in a particular domain, where the solution depends on the boundary conditions. In solving the problem, it is necessary to find unknown function that satisfies differential equation as well as the boundary conditions. By solving the derived differential equations, an analytical solution of the problem arises, where the closed-form solution can be obtained only for a limited number of simpler design models. If the design model is complex, then the approximate methods are most commonly used and suitable for obtaining an acceptable solution. Residue methods are in such cases a convenient way to formulate a numerical solution.

In the variational formulation of the problem, it is necessary to find unknown function or several functions that satisfy the requirement of functional stationarity, where the unknown function must also satisfy the corresponding additional conditions that are not implicitly contained in the functional. In order to apply the variational formulation, it is necessary that functional exists for considered problem.

Numerous methods and procedures for determination of approximate solutions have been developed based on the differential and variational formulation of the problem. The Galerkin method is the most frequently applied one from the residue methods, while the Ritz method is most often used for variational formulation.

Finite element method (FEM) is one of the most used numerical methods in structural analysis where the final element formulations is based on the solution of differential equations by residual methods or using the variation formulation.FEM based on the Galerkin method (or other weighted residual methods) can be applied to a much broader set of differential equations because it is not necessary to have a proper variational form as it is the case when using Rayleigh-Ritz based FEM [1]. Based on the previous exposition, it can be concluded that the application of simplified and/or approximate numerical methods for the analysis and design of TCC structures is welcome and recommended. Therefore, the approximate methods based on differential or variational formulation [16] are widely used, because they can be implemented in structural analysis software in order to provide a specific tool for engineers for designing partial composite structures.

This paper presents the Galerkin method in the analysis of the TCC system [14]. The selection of trial functions that describe the problem of elastic composite action as well as their influence on the final results was analyzed. For comparison of the obtained results, analysis were performed according to analytical solution [17] and the "gamma" method [5]. On the basis of the proposed numerical models, a model that best describes the problem of elastic coupling was chosen for further comparative analysis with the experimental data [18]. In addition, the use of the Ritz method was also presented in the analysis of the TCC system. The obtained results according to the Ritz method with different trial functions were analyzed and compared to the analytical and the "gamma" method solutions. All analysis were performed using MATLAB software [15].

2 FORMULACIJA JEDNAČINA SDB SISTEMA

Za proračun spregnutih nosača od drveta i betona, gde se koriste mehanička spojna sredstva, primenjuje se teorija elastičnog sprezanja [17], [11].

Osnovne pretpostavke teorije elastičnosti koje se uvode jesu sledeće:

- drvo i beton su izotropni, elastični materijali – važi Hukov zakon;
- važi Bernulijeva hipoteza, odnosno ravni preseci i posle deformacije ostaju ravni i upravni na deformisanu osu preseka;
- spojna sredstva postavljena su na određenom razmaku i mogu se smatrati ekvivalentnom kontinualnom vezom s konstantnom elastičnosti spoja duž celog nosača;
- poprečni preseci betona i drveta konstantni su duž raspona;
- drvo i beton imaju jednake ugibe u svakoj tački spoja;
- aksijalna sila deluje u težištu betonskog preseka.

Pri savijanju SDB nosača, nastaje pomeranje (klizanje *v*) u spojnoj ravni dva materijala. Klizanje elemenata sprečeno je spojnim sredstvima, što uzrokuje pojavu sile klizanja (smičuće sile u kontaktnoj ravni) T_s koja izaziva silu pritiska N_1 i momenat savijanja M_1 – u gornjem, a silu zatezanja N_2 i momenat savijanja M_2 – u donjem elementu nosača, slika 1 (gde su sa *A* i *I* obeležene geometrijske karakteristike poprečnih preseka gornjeg i donjeg elementa, a sa *E* moduli elastičnosti primenjenih materijala). Intenziteti sila zavise od krutosti i deformabilnosti spojnog sredstva, odnosno njegovog modula pomerljivosti *K* [2].

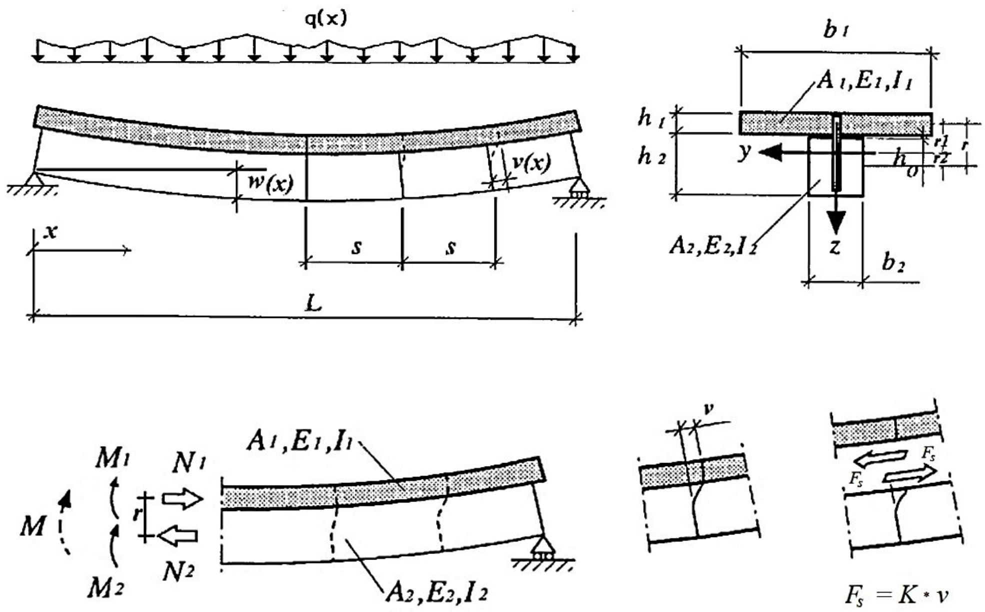
2 GOVERNING EQUATION OF TCC SYSTEM

The theory of elastic coupling [17], [11] is used for the calculation of TCC structures, where mechanical fasteners are used.

The basic assumptions of the theory of elasticity that are introduced are as follows:

- timber and concrete are isotropic, elastic materials and Hook's law applies,
- Bernoulli's hypothesis is valid, i.e. plane sections initially perpendicular to the midsurface will remain plane and perpendicular on deformed axis,
- coupling means are set at certain distances and can be considered as equivalent continuous connection with the constant elasticity along the beam,
- cross sections of concrete and timber are constant along the span,
- concrete and timber have equal deflections at each point of the connection,
- axial force acts at the centre of gravity (centroid) of the concrete section.

In TCC structure, one element slips (*v*)over the other along TC interface in the case of bending. Sliding of elements is prevented by the coupling means with appearance of interlayer slip (shear in contact interface) force T_s with compression force N_1 and the bending moment M_1 in the upper and the tensile force N_2 and the bending moment M_2 in the lower element of the structure, Figure 1 (where notation *A* and *I* represent geometrical properties of cross-sections of upper and lower element, while *E* represent the modulus of elasticity of applied materials). The intensities of forces depend on the stiffness and deformability of the coupling means and its slip modulus *K* [2].



Slika 1. Klizanje u kontaktnoj ravni SDB nosača [2]
Figure 1. Interlayer slip of TCC beam [2]

Kada se razmatra spregnuta greda drvo–beton, statičkog sistema proste grede, opterećena ravnomerno raspodeljenim opterećenjem *q(x)*, bez spoljašnje aksijalne sile, problem elastičnog sprezanja može se predstaviti diferencijalnom jednačinom drugog reda u aksijalne sile u betonu:

The problem of partial composite action could be represented with differential equation of the second order in the function of the axial force in concrete while observing the composite timber-concrete simply supported beam system with uniformly distributed load *q(x)* without an external axial force:

$$N_1''(x) - \alpha^2 N_1(x) = \beta M(x) \tag{1}$$

gde je:

where are:

$$\alpha^2 = k \cdot \left(\frac{1}{A_1 E_1} + \frac{1}{A_2 E_2} + \frac{r^2}{(EI)_0} \right) \tag{2}$$

$$\beta = \frac{k \cdot r}{(EI)_0} \tag{3}$$

M(x) – moment kruto spregnutog preseka ($k \to \infty$);
k – krutost spoja („raspodeljen” modul pomerljivosti) [N/m^2], *k=K/s;*
K – modul pomerljivosti spojnog sredstva [N/m], određen ispitivanjima;
s – rastojanje spojnih sredstava za sprezanje;
r – rastojanje između težišta betonskog i drvenog dela preseka.

Takođe, problem SDB nosača može se izraziti putem diferencijalne jednačine četvrtog reda u funkciji vertikalnog pomeranja:

M(x)- the moment of the fully composite section ($k \to \infty$),
k - the slip modulus per-unit length (“smeared” slip modulus)[N/m^2], *k=K/s,*
K-the slip modulus [N/m], determined by testing,
s-the spacing between connections,
r - the distance between centroid of flange and web elements.

In addition, the problem of the TCC beam could be expressed through the differential equation of the fourth order in function of vertical displacement:

$$w''''(x) - \alpha^2 \cdot w''(x) = \frac{\alpha^2 \cdot M(x)}{(EI)_\infty} - \frac{M''(x)}{(EI)_0} \tag{4}$$

gde je:

where is:

$$(EI)_\infty = \frac{\alpha^2 \cdot (EI)_0}{\alpha^2 - \beta \cdot r} = E_1 I_1 + E_2 I_2 + \frac{r^2 \cdot E_1 A_1 \cdot E_2 A_2}{E_1 A_1 + E_2 A_2} \tag{5}$$

(EI)$_0$ i *(EI)$_\infty$* predstavljaju savojnu krutost za nespregnutu ($k \to 0$) i kruto spregnutu ($k \to \infty$) gredu, respektivno.

Rešavanje diferencijalnih jednačina (1 ili 4) predstavlja složen zadatak, i to naročito za različite slučajeve opterećenja i/ili granične uslove. U literaturi, za različite slučajeve opterećenja i uslove oslanjanja, mogu se pronaći analitička rešenja. U radu [17], analitička rešenja za aksijalnu silu *N*, silu klizanja u spoju T_s i vertikalno pomeranje *w*, za statički sistem proste grede i kontinualno opterećenje, data su jednačinama (6–8).

(EI)$_0$ and *(EI)$_\infty$* are the bending stiffness of non-composite ($k \to 0$) and fully composite ($k \to \infty$) beam, respectively.

Solving the differential equations (1 or 4) is a complex task, especially for different load cases and/or boundary conditions. In the literature, analytical solutions for different load cases and support conditions could be found. According to [17], analytical solutions for axial force *N*, interlayer slip force T_s and vertical displacement *w*, for a simply supported beam system and continuous load, are given by the equations (6-8).

Aksijalna sila

Axial force

$$N(x) = \frac{\beta}{\alpha^2} M(x) \left[1 - 2 \frac{\cosh \alpha \frac{l}{2} - \cosh \alpha (\frac{l}{2} - x)}{x(l-x)\alpha^2 \cosh \alpha \frac{l}{2}} \right] \tag{6}$$

Sila klizanja

Slip force

$$T_s(x) = \frac{\beta}{\alpha^2} T(x) \left[1 - \frac{\sinh \alpha (\frac{l}{2} - x)}{(\frac{l}{2} - x)\alpha \cosh \alpha \frac{l}{2}} \right] \tag{7}$$

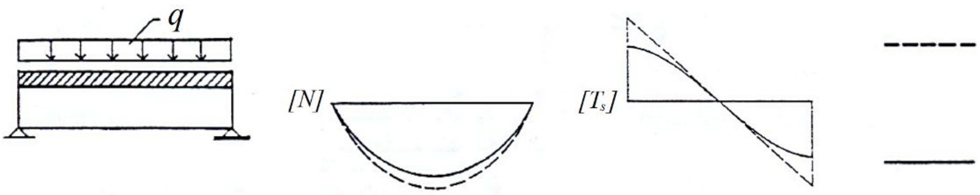
Vertikalno pomeranje

Vertical displacement

$$w(x) = w_\infty(x) + \frac{q \cdot \beta \cdot r}{\alpha^6 \cdot (EI)_0} \cdot f(x)$$
$$w_\infty(x) = \frac{q \cdot L^4}{24 \cdot (EI)_\infty} \cdot \left(\frac{x}{L}\right) \cdot \left(1 - 2 \cdot \left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right)^3\right) \tag{8}$$
$$f(x) = \left(\frac{\cosh\left(\alpha \cdot \left(x - \frac{L}{2}\right)\right)}{\cosh\left(\alpha \cdot \frac{L}{2}\right)} + \frac{x \cdot L \cdot \alpha^2}{2} - \frac{x^2 \cdot \alpha^2}{2} - 1\right)$$

Prikaz kvalitativne promene aksijalne sile N to jest sile klizanja u spoju T_s SDB nosača, dat je na slici 2.

A demonstration of the qualitative change of the axial force N i.e. slip force T_s in TCC system is shown in Figure 2.



Slika 2. Prikaz kvalitativne promene normalne tj. sile klizanja u SDB sistemu [17]
Figure 2. Demonstration of qualitative change of the axial i.e. slip forces in TCC [17]

3 APROKSIMATIVNE METODE

Problemi teorije elastičnosti opisani su pomoću diferencijalne formulacije (diferencijalne jednačine i odgovarajućih graničnih uslova) ili u varijacionoj formulaciji u obliku funkcionala. Iako rešenja ovih problema u matematičkom smislu egzistiraju kao jednoznačna, nalaženje analitičkih rešenja predstavlja zahtevan i često nerešiv zadatak. Stoga, približne metode često se koriste prilikom određivanja rešenja za ove probleme. Posebno značajne jesu one metode gde se kao polazna osnova koristi pretpostavka o rešenju u obliku aproksimativnih ili probnih funkcija, pri čemu je jedna od najčešće primenjivanih metoda reziduuma Galerkinova metoda, dok je od varijacionih metoda to najčešće Ritz-ova metoda.

Nepoznata funkcija *u(x)* diferencijalne jednačine problema aproksimira se približnim rešenjem $\bar{u}(x)$, izraz (9), koje se može predstaviti kao suppozicija proizvoda poznatih baznih funkcija Φ_m i nepoznatih koeficijenta c_m.

3 APROXIMATE METHODS

The problems of the theory of elasticity are described by means of the differential formulation (differential equations and corresponding boundary conditions) or in the variational formulation in the form of the functional. Although the solutions to these problems in the mathematical sense exist as unambiguous, finding analytical solutions is a delicate and often unsolvable task. Therefore, the approximate methods are often used to find solutions to these problems. Of particular interest are those methods in which the assumption of a solution in the form of approximate or trial function is used as the baseline, wherein one of the most commonly applied weight residual methods is Galerkin method, and commonly applied variational method is Ritz method.

The unknown function *u(x)* of problem's differential equation has to be approximated by the approximate solution $\bar{u}(x)$, expression (9), that could be represented as a superposition of products of known basis functions Φ_m and unknown coefficients c_m.

$$\bar{u}(x) = \sum_{m=1}^{n} c_m \cdot \Phi_m(x_m) \tag{9}$$

gde je:

Φ_m – skup izabranih linearno nezavisnih funkcija $\Phi_m(x_m)$;

c_m – nepoznati parametri, konstante ili funkcije, koje treba odrediti.

Najčešći oblici probnih funkcija jesu polinomi ili trigonometrijske funkcije. Funkcije Φ_m unapred se usvajaju, imajući u vidu granične uslove po

where are:

Φ_m - set of chosen linearly independent functions $\Phi_m(x_m)$,

c_m- unknown parameters, constants or functions to be determined.

The most common trial functions are polynomials or trigonometric functions. Functions Φ_m are adopted in advance by taking into account of essential boundary pomeranjima. Kada je reč o drugim uslovima, izbor funkcija Φ_m uglavnom je proizvoljan, ali kvalitet rešenja umnogome zavisi baš od izbora funkcija Φ_m. Poželjno je da funkcije Φ_m zadovoljavaju i granične uslove po silama, te da njihov oblik kvalitativno odgovara tačnom analitičkom rešenju. Dakle, kvalitativno poznavanje prirode rešenja veoma je korisno da bi se izbegao pogrešan izbor funkcija koje po svom obliku predstavljaju grubo odstupanje od analitičkog rešenja.

Galerkinova metoda ima širu primenu od Ritz-ove metode, jer se može primeniti pri rešavanju onih problema za koje funkcional ne postoji. U mehanici deformabilnih tela, ove dve metode su ekvivalentne, jer daju rezultate iste tačnosti. Izborom istih probnih funkcija u Ritz-ovoj i Galerkin-ovoj metodi, dobijaju se isti koeficijenti c_m (ista rešenja).

conditions. As regards other conditions, the choice of functions Φ_m is generally arbitrary, but the quality of the solution largely depends on the choice of functions Φ_m. It is desirable that the functions Φ_m also satisfies natural boundary conditions, and thattheir shape qualitatively corresponds to the exact analytical solution. Therefore, qualitative knowledge of the nature of the solution is very useful in order to avoid the wrong choice of functions that in their form represent a rough deviation from the analytical solution.

Galerkin method has wider application than Ritz's because it can solve even those problems in which the functional does not exist. In the mechanics of deformable bodies, these two methods are equivalent, as they give results of the same accuracy. By choosing the same trial functions in Ritz and Galerkin method, the same coefficients of c_m (i.e. same solutions) will be obtained.

3.1 Metoda reziduuma

Neka je posmatrani fizički problem, u domenu Ω, koji može da bude 1D do 3D, definisan diferencijalnom jednačinom:

3.1 Weighted residual method

A physical problem is observed in the domain Ω, which can be 1D to 3D, defined with a differential equation:

$$L(u) - \boldsymbol{f}_\Omega = 0 \tag{10}$$

gde je:

L – odgovarajući linearni diferencijalni operator;

$u(x)$ – nepoznata funkcija problema, koja zavisi od koordinate x unutar prostora Ω, pri čemu funkcija $u(x)$ zadovoljava date granične uslove na granicama domena Ω;

f_Ω – vektor slobodnih članova u jednačini u domenu Ω.

Nepoznata funkcija problema $u(x)$ aproksimira se s približnom funkcijom $\bar{u}(x)$, izraz (9), koja zadovoljava granične uslove po pomeranjima (esencijalne granične uslove), ali ne mora da zadovoljava i uslove po silama (prirodne granične uslove). Kako je $\bar{u}(x)$ približno rešenje jednačine (10), dobija se ostatak ili reziduum:

where are:

L - corresponding linear differential operator,

$u(x)$ - unknown function of the problem, that depends on the coordinate x within the domainΩ, where the function$u(x)$ satisfy the given boundary conditions at the boundaries of the domain Ω,

f_Ω – given force term in domainΩ.

The unknown function of the problem $u(x)$ is approximated with the approximate function $\bar{u}(x)$, equation (9), which satisfies the boundary conditions upon the displacements (essential conditions), but does not have to satisfy the conditions by forces (natural conditions). As $\bar{u}(x)$ is approximate solution of the equation (10), the residue or residuum is obtained:

$$L(\overline{\boldsymbol{u}}) - \boldsymbol{f}_\Omega = \boldsymbol{R}(\overline{\boldsymbol{u}}) \neq 0 \tag{11}$$

Kako je jednačina (10) sistem jednačina, odnosno matrična jednačina, ostatak jeste $R(\bar{u})$ vektor. Naravno, kada bi $\bar{u}(x)$ bilo tačno analitičko rešenje, onda bi vektor ostatka $R(\bar{u})$ bio jednak nultom vektoru. Ideja metode jeste da se vektor ostatka svede na nulti vektor „u prosečnom smislu". Stoga, uvode se linearno nezavisne težinske funkcije $W(\bar{u})$, uz uslov da integral skalarnog proizvoda vektora težinskih funkcija i vektora ostatka unutar domena Ω bude jednak nuli:

Since equation (10) is a system of equations i.e. a matrix equation, than the residue $R(\bar{u})$ is a vector. Of course, if $\bar{u}(x)$ would be the exact solution, then the residue vector $R(\bar{u})$ would be equal to the zero vector. The idea behind the method is to reduce the residue vector to the zero vector "in the average sense". Because of that, linearly independent weight functions $W(\bar{u})$ are introduced with the condition that the integral of the scalar product of the weight function vector and the residual vector within the domain Ω is equal to zero:

$$\int_\Omega \boldsymbol{W}^T(\overline{\boldsymbol{u}}) \cdot \boldsymbol{R}(\overline{\boldsymbol{u}}) \cdot d\Omega = \int_\Omega \boldsymbol{W}^T(\overline{\boldsymbol{u}}) \cdot (L(\overline{\boldsymbol{u}}) - \boldsymbol{f}_\Omega) \cdot d\Omega = 0 \tag{12}$$

Skalarni proizvod dva vektora jednak je nuli ukoliko su ti vektori međusobno ortogonalni. Prema tome, integralna jednačina (12) predstavlja uslov ortogonalnosti vektora ostatka na izabrani vektor težinskih funkcija.

Metode reziduuma sastoje se u nalaženju funkcija $\bar{u}(x)$ za koje će integralna jednačina (12) biti zadovo-

The scalar product of the two vectors is equal to zero if these vectors are mutually orthogonal. Accordingly, the integral equation (12) is a condition of the orthogonality of the residual vector to the selected vector of weight functions.

Residue methods consist of finding functions $\bar{u}(x)$ for which the integral equation (12) will be satisfied. If the ljena. Ako je jednačina (12) zadovoljena za bilo koji vektor težinskih funkcija, onda će se vektor ostatka približavati nultom vektoru.

Na taj način, približno rešenje $\bar{u}(x)$ aproksimira tačno rešenje $u(x)$. Sva rešenja $\bar{u}(x)$ koja zadovoljavaju (10) moraju da zadovoljavaju i (12), bez obzira na izbor težinskih funkcija. Dimenzija vektora težinskih funkcija odgovara broju nepoznatih koeficijenata c_m razmatranog problema.

Kao jedna od osnovnih varijanti metode reziduuma, koja usvaja težinske funkcije kao bazne funkcije Φ_m kojima je aproksimirano traženo rešenje, jeste Galerkinova metoda [16].

Na osnovu diferencijalnih jednačina elastičnog sprezanja (1 ili 4) i uslova (12), moguće je definisati sledeće relacije za određivanje problema SDB nosača u funkciji aksijalne sile u betonu ili u funkciji pomeranja za slučaj SDB grede opterećene kontinualnim opterećenjem q, prema sledećim izrazima:

equation (12) is satisfied for any weight functions vector, then the residue vector will approach the zero vector.

In this way, the approximate solution $\bar{u}(x)$ approximates the exact solution $u(x)$. All solutions $\bar{u}(x)$ that satisfy (10) must satisfy (12) regardless of weight functions' choice. The dimension of the weight functions vector corresponds to the number of unknown coefficients c_m of the considered problem.

As one of the basic variants of the residual method, which adopts weight functions as basis functions Φ_m for which the required solution is approximated, is Galerkin's method [16].

Based on the differential equations of the elastic coupling (1 or 4) and the condition (12), it is possible to define the following relations for determining the problem of the TCC girder trough the axial force in the concrete or trough displacements for the case of a TCC beam loaded with continuous load q, according to fallowing expressions:

$$\int_0^L \Phi_m(x) \cdot [N_1''(x) - \alpha^2 N_1(x) - \beta M(x)] \cdot dx = 0 \quad , \quad m = 1,2,\dots,n \tag{13}$$

$$\int_0^L \Phi_m(x) \cdot \left[\bar{w}''''(x) - \alpha^2 \bar{w}''(x) - \frac{\alpha^2 M(x)}{(EI)_\infty} + \frac{M''(x)}{(EI)_0}\right] \cdot dx = 0 \quad , \quad m = 1,2,\dots,n \tag{14}$$

Integralna formulacija koja u sebi implicitno sadrži diferencijalnu jednačinu problema, naziva se slaba formulacija (13 ili 14) koja izražava uslove i relacije koje moraju biti zadovoljene u prosečnom ili integralnom smislu.

Kako je diferencijalna jednačina problema parnog reda ($2r = 4$), parcijalnom integracijom izraza (12) red izvoda r u probnim funkcijama moguće je smanjiti sa $r = 4$ na $r = 2$. Parcijalnom integracijom izraza (13 ili 14) postiže se da odabrane probne funkcije moraju zadovoljavati samo granične uslove po pomeranjima, koji moraju biti zadovoljeni izborom samih probnih funkcija, dok su uslovi po silama već uključeni u formulaciju problema parcijalnom integracijom.

Rešavanjem integrala, dobija se sistem od n jednačina po nepoznatim koeficijentima c_m i približno rešenje za traženu funkciju $u(x)$ može se dobiti određivanjem koeficijenata c_m.

An integral formulation that implicitly contains a differential equation of the problem is called a weak formulation (13 or 14) that expresses the conditions and relations that must be satisfied in the average, or in an integral sense.

Since the differential equation of the problem is of even order ($2r = 4$), it is possible to reduce the required order of derivation in the trial functions by partial integration of the expression (12) from $r = 4$ to $r = 2$. By partial integration of expressions (13 or 14) is achieving that selected trial functions must satisfy only the essential conditions, that have to be satisfied by the selection of trial functions themselves, while the force conditions are already included into the formulation of the partial integration problem.

By solving the integrals, a system of n equations by unknown coefficients $c_m$is obtained andan approximate solution for the required function $u(x)$ could be derived by determination of coefficients c_m.

3.2 Varijaciona metoda

Kako se Ritz-ova metoda zasniva na varijacionoj formulaciji, potrebno je zadovoljiti uslov stacionarnosti funkcionala koji opisuje razmatrani problem. Za rešavanje problema u mehanici deformabilnih tela, funkcional je jednak ukupnoj potencijalnoj energiji, a stacionarna vrednost odgovara njenoj minimalnoj vrednosti. U *Teoriji konstrukcija*, ovaj metod je najpoznatiji varijacioni postupak. Razlog jeste to što postoji funkcional u obliku potencijalne energije [13].

Kada da se posmatra jednodimenzionalni linijski problem s domenom definisanosti $x Î [x_1,x_2]$, funkcional (potencijalna energija) izražava se putem integrala $I(u)$ u celom domenu:

3.2 Variational method

As the Ritz method is based on a variational formulation, it is necessary to satisfy the requirement of extremum of a functional that describes the problem under consideration. To solve problems in the mechanics of deformable bodies, the functional is equal to the total potential energy, and the stationary value corresponds to its minimum value. In the theory of structures this method is the most famous variation procedure. The reason for that is that there is a functional in the form of potential energy [13]. In case of one-dimensional beam problem with the defined domain $x Î [x_1,x_2]$, functional (potential energy) is expressed as an integral $I(u)$over the entire domain:

$$I(u) = \int_{x1}^{x2} \Pi\left(x, u(x), \frac{du(x)}{dx}, \frac{d^2u(x)}{dx}, \cdots\right) dx \tag{15}$$

gde je:

Π (...) – funkcional funkcija *u(x)*, *du(x)/dx*, $d^2u(x)/dx$,...

Uslov stacionarnosti funkcionala prikazuje se uslovom da je prva varijacija funkcionala jednaka nuli:

where is:

Π (...) - represents the functional of functions*u(x)*, *du(x)/dx*, $d^2u(x)/dx$,...

Extremum of a functional is represented by requirements that the first variation of the functional be zero:

$$\delta\Pi = 0 \tag{16}$$

ili zapisano u razvijenom obliku:

or shown in the developed form:

$$\delta\Pi = \frac{\partial\Pi}{\partial c_1}\delta c_1 + \frac{\partial\Pi}{\partial c_2}\delta c_2 + \ldots\ldots\ldots\ldots + \frac{\partial\Pi}{\partial c_n}\delta c_n = 0 \tag{17}$$

Kako su koeficijenti $c_1,c_2,...,c_n$ međusobno nezavisni parametri, onda se *δΠ*=0 svodi na sledeći uslov:

Since $c_1,c_2,...,c_n$ are mutually independent parameters, then *δΠ*=0 is represented by following condition:

$$\frac{\partial\Pi}{\partial c_m} = 0 \qquad (m = 1,2,\ldots,n) \tag{18}$$

što predstavlja sistem algebarskih jednačina po nepoznatim koeficijentima c_m.

Na osnovu diferencijalne formulacije problema elastičnog sprezanja, moguće je definisati funkcional na osnovu opštih varijacionih principa [8]. Pošto se najčešće koriste mehanička spojna sredstva za SDB nosače, usled spoljašnjeg opterećenja javljaju se izvesna pomeranja (tj. klizanja u spoju) na kontaktu između drveta i betona. Pored rada unutrašnjih sila (M_1, N_1, M_2 i N_2 where $N=N_1=-N_2$), potrebno je uzeti u obzir i deformacioni rad usled klizanja u spoju. Funkcional, ili ukupna potencijalna energija spregnutog sistema, u slučaju proste grede na koju deluje raspodeljeno opterećenje *q(x)*, može se prikazati u sledećem obliku [19]:

which represents a system of algebraic equations with unknown coefficients c_m.

Based on the differential formulation of the partially composite problem, it is possible to define a functional according to variation principles [8]. As the mechanical fasteners are commonly used for coupling in TCC, a certain displacements (i.e. an interlayer slip) occur on the TC interface due to the external load. Besides the strain energy due to internal forces (M_1, N_1, M_2 i N_2 where $N=N_1=-N_2$), it is also necessary to take into account the strain energy due to interlayer slip. Functional, or total potential energy of the composite system, in the case of simply supported beam with uniform distributed load *q(x)*, can be shown in the following form [19]:

$$I = W_i - W_e \tag{19}$$

$$W_i = \frac{1}{2}\int_0^l \frac{M_1{}^2(x)}{E_1I_1}dx + \frac{1}{2}\int_0^l \frac{M_2{}^2(x)}{E_2I_2}dx + \frac{1}{2}\int_0^l \frac{N^2(x)}{EA^*}dx + \frac{1}{2}\int_0^l \frac{(N'(x))^2}{k}dx \tag{20}$$

$$W_e = \int_0^l w(x)\cdot q(x)\cdot dx \tag{21}$$

gde je:

where is:

$$EA^* = \frac{E_1A_1\cdot E_2A_2}{E_1A_1 + E_2A_2} \tag{22}$$

W_i – potencijalna energija deformacije;

W_e – potencijal sila.

Kako moment savijanja *M(x)* možemo izraziti preko pomeranja *w(x)*, koristeći uslov jednakih rotacija spregnutih elemenata (drveta i betona), uvodeći odnos *g(x)*, izraz (20) prikazujemo u sledećem obliku:

W_i- strain energy due to internal forces,

W_e -potential energy due to external forces.

As the bending moment *M(x)* can be expressed by deflection *w(x)*, using the condition of equal rotations of the composite members (timber and concrete), introducing the relation *g(x)*, the expression (20) is represented in the following form:

$$W_i = \frac{(EI)_0}{2}\int_0^l [w''(x)]^2 dx + \frac{(E_2I_2)^2}{EA^*}\int_0^l \left[\frac{w''(x)}{g(x)}\right]^2 dx + \frac{1}{2\cdot k}\int_0^l \left[\left(-\frac{E_2I_2\cdot w''(x)}{g(x)}\right)'\right]^2 dx \tag{23}$$

gde je: where:

$$g(x) = \frac{M_2(x)}{N(x)} = \frac{E_2I_2}{r}\cdot\left(\frac{1}{EA^*} - \frac{N''(x)}{N(x)}\right) \tag{24}$$

$$N(x) = \frac{M_2(x)}{g(x)} = -\frac{E_2I_2\cdot w''(x)}{g(x)} \tag{25}$$

Uvedeni odnos *g(x)* izveden je iz uslova kompatibilnosti pomeranja na spoju dva elementa, koji može da se zapiše u sledećem obliku:

The introduced relation *g(x)* was derived from the compatibility of displacements at the interface of the two subelements, that could be written in the following form:

$$\frac{N(x)}{EA^*} - \frac{M_2(x)\cdot r}{E_2I_2} = \frac{N''(x)}{k} \tag{26}$$

Poznajući rad unutrašnjih sila W_i, određen je funkcional za elastično spregnuti SDB nosač. Kako se u izrazu *g(x)* javlja normalna sila *N(x)*, za rešavanje problema, pored pretpostavljanja probne funkcije za pomeranje *w(x),* potrebno je pretpostaviti i probnu funkciju za normalnu silu *N(x)*. Primenom varijacionih principa na funkcional, Ritz-ovom metodom, možemo rešiti problem elastičnog sprezanja, odnosno odrediti pomeranje nosača i unutrašnje sile u spregnutom nosaču.

Knowing the strain energy due to internal forces W_i, functional for a partial TCC system is determined. As in the expression *g(x)* the normal force *N(x)* appears, for solving the problem, beside assumed trial function of displacement *w(x),* it is also necessary to assume the trial function for *N(x)*. By applying variation principles to a functional, with Ritz method, the problem of partially composite system can be solved, which means to determine displacement and internal forces in the composite members.

4 APROKSIMACIJA REŠENJA – PROBNE FUNKCIJE

Pogodne, a samim tim i najčešće, probne funkcije su polinomi ili trigonometrijske funkcije. Probne funkcije treba da zadovolje sledeće uslove:

- da su neprekidne i do potrebnog reda diferencijabilne;
- pored esencijalnih graničnih uslova, treba da zadovolje prirodne granične uslove;
- treba da oblikom kvalitativno odgovaraju tačnom rešenju;
- da budu potpune, npr. u slučaju polinoma određenog stepena, takođe treba da budu uključeni i svi članovi nižeg stepena.

Na osnovu dobro poznatih rešenja iz literature, jednačine (6) i (8), kao i kvalitativnog poznavanja oblika rešenja (slika 2), u ovom radu za probne funkcije izabrane su tri funkcije (hiperbolična, sinusna i funkcija oblika polinoma).

Usvojene probne funkcije opisuju zakon promene aksijalne sile *N*, sile klizanja u spoju T_s *(N')* i pomeranje *w* duž spregnutog nosača i kvalitativno odgovaraju analitičkim rešenjima (slika 2). Pomoću izraza (27, 28 i 29), date su odabrane probne funkcije za *N* i/ili *w*, dok je na slici 3 prikazan oblik funkcije i njen prvi izvod duž nosača.

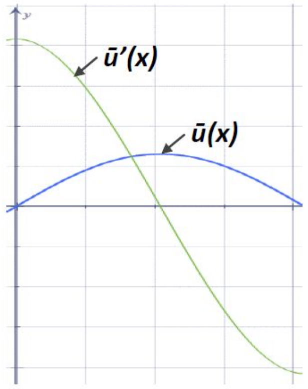
4 APPROXIMATION OF THE SOLUTION – TRIAL FUNCTIONS

Suitable, and therefore, most often, trial functions are polynomials or trigonometric functions. Trial functions should satisfy the following conditions:

- to be continuous and differentiable till the necessary order,
- in addition to essential, they also have to satisfy natural boundary conditions,
- to correspond qualitatively by the form to the analytical solution,
- to be complete, e.g. in the case of polynomials of a certain degree, all members of the lower degrees should also be included.

Based on the well-known solutions from the literature, equations (6) and (8), as well as on the qualitative flow of the solution (Fig. 2), three functions (hyperbolic, sinusoidal and polynomial functions) were selected for trial functions in this paper.

The adopted trial functions describe the law of the change of the axial force *N*, slip forces T_s *(N')* and displacement *w* along the composite girder and qualitatively correspond to the solutions (Fig. 2). By means of expressions (27, 28 and 29), selected trials for *N*and/or*w* are given, while on Fig. 3 the shape of the function and its first derivative along the beam are shown.



Slika 3. Probna funkcija i njen prvi izvod
Figure 3. Trial function and its first derivative

1. Hiperbolična funkcija / *Hyperbolic function*:

$$\bar{u}(x) = c_1 \cdot \Phi_1(x) = c_1 \cdot \left(\frac{\cosh\left(x - \frac{L}{2}\right)}{\cosh\left(\frac{L}{2}\right)} + \frac{x \cdot L}{2} - \frac{x^2}{2} - 1 \right) \tag{27}$$

2. Sinusna funkcija / *Sinusoidal function*:

$$\bar{u}(x) = c_1 \cdot \Phi_1(x) = c_1 \cdot \sin\left(\frac{\pi \cdot x}{L}\right) \tag{28}$$

3. Polinom / *Polynomial function*:

$$\bar{u}(x) = c_1 \cdot \Phi_1(x) = c_1 \cdot \left(\frac{x}{L}\right) \cdot \left(1 - 2 \cdot \left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right)^3\right) \tag{29}$$

U Galerkinovoj metodi, jednu od tri predložene funkcije treba usvojiti za probnu funkciju (za *N* ili *w*) prema odabranoj integralnoj formulaciji (jednačine 13 ili 14). U Ritz-ovoj metodi, potrebno je usvojiti dve probne funkcije (za *N* i *w*).

In Galerkin method, one of three suggested trial functions has to be adopted (for *N* or *w*) according to chosen integral formulation (Eq 13 or 14). In Ritz method it is necessary to adopt two trial functions(for *N* and *w*).

5 NUMERIČKE ANALIZE I POTVRDA REZULTATA

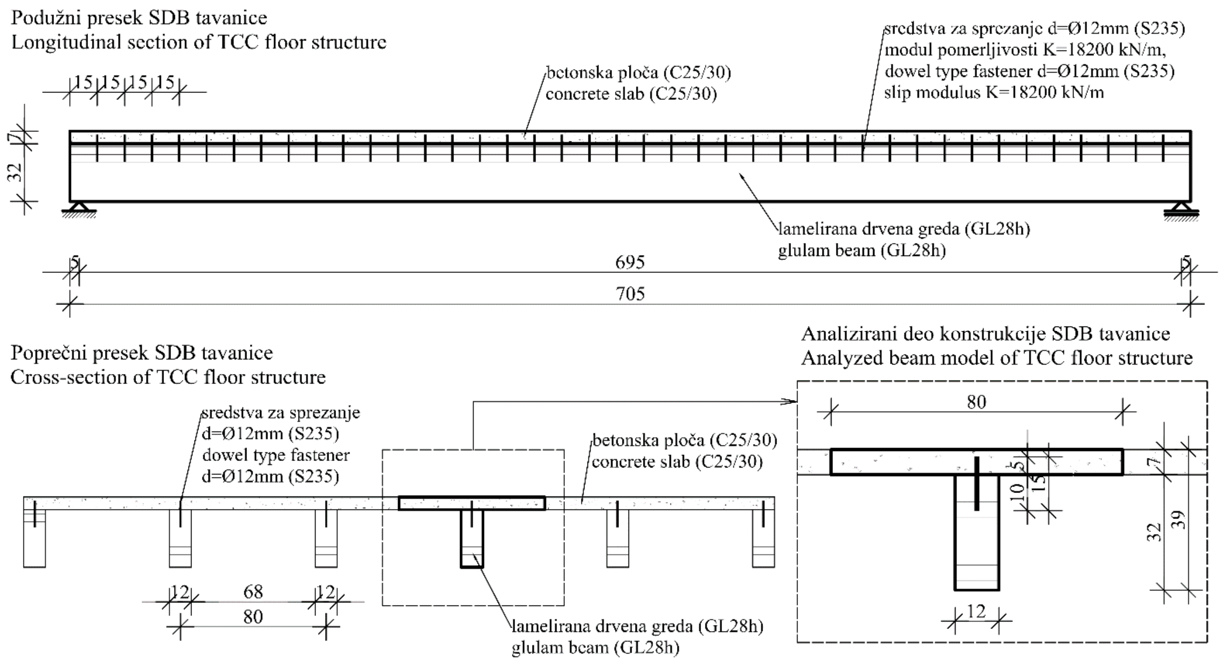
5.1 Opis analiziranog modela

Razmatrana je SDB konstrukcija tavanice za numeričku analizu metodama Galerkina i Ritz-a. Raspored elemenata i spojnih sredstava, kao i njihove dimenzije i svojstva primenjenih materijala prema evropskim standardima, prikazani su na slici 4.

5 NUMERICAL ANALYSIS AND VERIFICATION OF THE RESULTATS

5.1 Description of analyzed structural model

The TCC floor structure is considered for the numerical analysis by Galerkin and Ritz method. The disposition of the elements and fasteners, as well as their dimensions and properties of the applied materials according to European standards, are shown in Fig. 4.



Slika 4. Analizirani model međuspratne SDB konstrukcije [13]
Figure 4. Analyzed model of the TCC floor structure [13]

Konstrukcija tavanice sastoji se od lameliranih lepljenih drvenih (LLD) greda koje su – zajedno s betonskom pločom – spregnute vertikalno postavljenim štapastim spojnim sredstvima. U ovom radu, modul pomerljivosti K određen je Gelfijevim modelom [6]. Tavanica je opterećena sopstvenom težinom elemenata konstrukcije g, dodatnim stalnim opterećenjem d_g, kao i korisnim opeterećenjem p. Smatra se da će LLD grede biti poduprte u fazi izlivanja i očvršćavanja betonske ploče, te će spregnuti presek primati korisno i ukupno stalno opterećenje. Moguće je analizirati izdvojen deo spregnute tavanice (LLD greda s betonskom pločom efektivne širine), jer se smatra da u analiziranoj SDB tavanici betonska ploča nosi u jednom pravcu, a da su LLD grede statičkog sistema proste grede opterećene ravnomerno raspodeljenim opterećenjem. Sprovedena je numerička analiza SDB nosača prema Galerkinovoj i Ritz-ovoj metodi primenom programa MATLAB 2014 [15] u kome su napisani potprogrami/kodovi za njihov proračun. Takođe, pojednostavljenim „*γ* postupkom" izvršena je analiza kako bi se odredile referentne vrednosti predložene Evrokodom.

5.2 Numerička analiza primenom Galerkinove metode

Za potrebe numeričke analize, definisane su dve grupe modela na osnovu izbora probnih funkcija i integralnih formulacija (jednačine 13 ili 14) za aksijalnu silu $N(x)$ ili ugib $w(x)$: Modeli grupe A (N-HIP, N-SIN, N-POL) i Modeli grupe B (w-HIP, w-SIN, w-POL).

Sračunata su vertikalna pomeranja (w), momenti savijanja (M_1, M_2), aksijalne sile (N_1, N_2) i naponi (σ_1 i σ_2) u betonu / drvetu (gornje i donje vlakno) za presek u sredini raspona gednog nosača, kao i smičuće sile (F_s) u spojnim sredstvima i sile klizanja (T_s) na kontaktu betona i drveta nad osloncem. Rezultati numeričke analize modela iz grupe A i B upoređeni su s rezultatima analitičkog rešenja, a njihova procentualna odstupanja prikazana su na slikama 5, 6. Rezultati pojednostavljenog „*γ*-postupka" takođe su poređeni sa analitičkim rešenjem.

Sa slika 5 i 6, može se uočiti da rezultati numeričkih modela grupe A imaju manja odstupanja u odnosu na analitičko rešenje nego modeli grupe B.

Analiza rezultata dobijenih primenom različitih probnih funkcija za aproksimativno rešenje normalne sile *N,* pokazuje da se minimalno odstupanje javlja kada se usvoji hiperbolična funkcija, a maksimalno ako se usvoji sinusna funkcija. Modeli N-POL i N-SIN imaju znatna odstupanja kod napona ($\sigma_{1,b}$ and $\sigma_{2,t}$) i to čak do 21%, a manja odstupanja za sile klizanja i za smičuće sile (T_s and F_s) do 6,5%. Sve vrednosti su manje od onih dobijenih analitičkim rešenjem. Poredeći modele grupe A sa rezultatima „*γ*-metode", nijedan od ovih modela nema veća odstupanja u apsolutnom smislu, ali može se primetiti da modeli N-POL i N-SIN daju manje vrednosti.

Analiza rezultata dobijenih primenom različitih probnih funkcija za aproksimativno rešenje pomeranja *w,* pokazuje da se minimalno odstupanje javlja ukoliko se usvoji funkcija oblika polinoma, a maksimalno ako se usvoji hiperbolična funkcija. Modeli w-HIP i w-SIN imaju značajna odstupanja kod napona ($\sigma_{1,b}$ and $\sigma_{2,t}$) i to čak do 41%, a za sile klizanja i za smičuće sile (T_s and F_s) čak i do 28%. Model w-HIP pokazuje znatno manje

The floor structure consists of a glulam beams that are coupled with concrete slab by vertically arranged dowel type fasteners. In this paper, the slip modulus K is determined by the Gelfi model[6]. The floor structure is loaded by the self-weight of the structural elements g, by additional permanent load d_g, as well as by the imposed load *p.* It is considered that the timber glulam beams will be supported in the stage of pouring and hardening of the concrete slab, and the composite section will receive imposed and total permanent load. It is possible to analyze the part of the composite floor structure separately (glulam beam with the effective width of the concrete slab), because in analyzed TCC floor system all concrete slabs are one-way and glulam beams are simply supported with uniformly distributed load. Numerical analysis according to Galerkin and Ritz method of TCC structure was performed and several subprograms/codes are written in MATLAB 2014 [15]. The simplified "*γ*-procedure" was also performed in order to obtain the referent values suggested by Eurocode.

5.2 Numerical analysis by Galerkin method

For the purpose of numerical analysis, two groups of models are defined on the basis of selection of trial functions and integral formulation (eqs. 13 or 14) for axial force $N(x)$ ordeflection $w(x)$:Models of Group A (N-HIP, N-SIN, N-POL) and Models of Group B (w-HIP, w-SIN, w-POL).

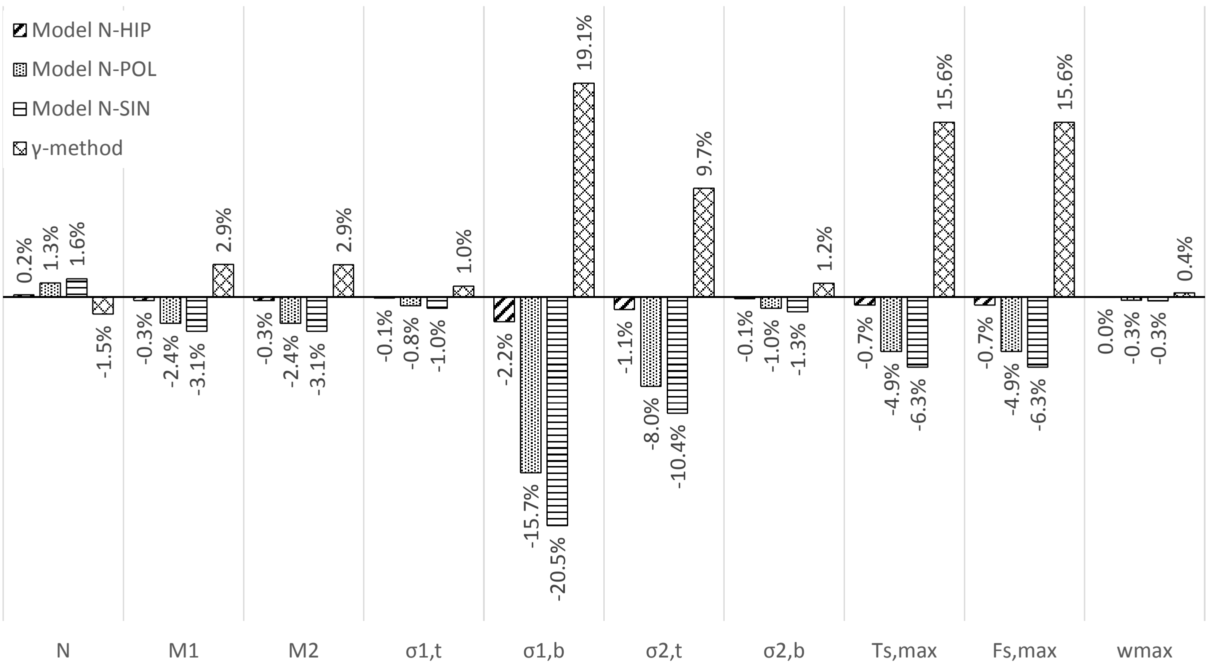
Vertical displacements (w), moments (M_1, M_2), axial forces (N_1, N_2) and stresses (σ_1 i σ_2) for the cross-section of concrete / timber element (top and bottom) in the middle of the beam span were calculated, as well as shear forces (F_s) in connectors and slip force (T_s) values at the concrete-timber contact in support zones. Results of performed numerical analysis for Models of group A and B were compared with analytical solution and their percentage deviations are shown at Figures 5,6. Results of simplified "*γ*-procedure" were also compared with analytical solution.

From Figs. 5 and 6, it can be noticed that numerical results of group A models have smaller differences in relation to the analytical solution than the models of group B.

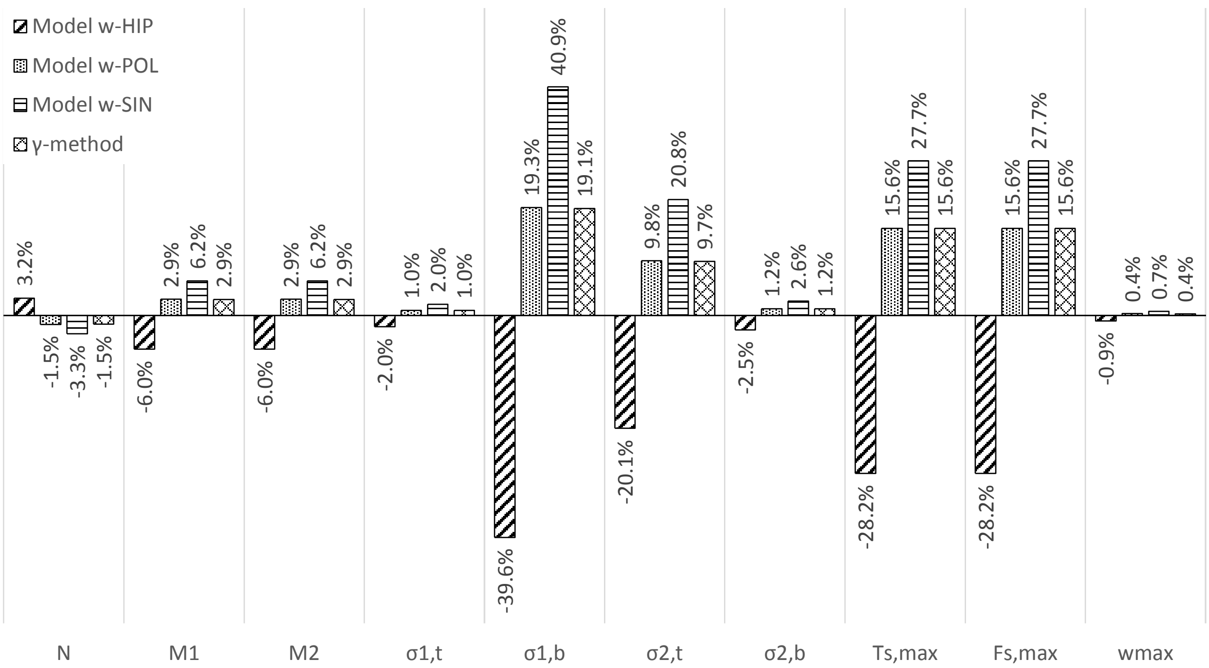
Analysis of results obtained by different trial functions for the approximate solution of the normal force N shows that a minimal deviation occurs when a hyperbolic function is adopted, and the maximum one if it is the sinusoidal function. The models N-POL and N-SIN have significant deviations in stresses ($\sigma_{1,b}$ and $\sigma_{2,t}$) even up to 21%, and minor deviations in the slip and shear forces (T_s and F_s) up to 6.5%. All values are smaller than those obtained by analytical solutions. Comparing the Group A models with results of the "*γ*" method, none of the models has greater deviations in absolute sense, but it can be noticed that the N-POL and N-SIN models give smaller values.

Analysis of results obtained by different trial functions for the approximate displacement w solution shows that a minimal deviation occurs if the polynomial function is adopted, and the maximum one if it is a hyperbolic function. The w-HIP and w-SIN models have significant deviations in stresses ($\sigma_{1,b}$ and $\sigma_{2,t}$) even up to 41%, and for slip and shear forces (T_s and F_s) even up to 28%. Model w-HIP shows significantly smaller values vrednosti u poređenju sa analitičkim rešenjem. Poredeći modele grupe B s rezultatima „γ-metode”, može se uočiti je da najbolje podudaranje sa „γ-metodom” ima model w-POL.

comparing to analytical. By comparing the models of group B with results of the "γ-method", it can be seen that the best match with the "γ-method" has the w-POL model.



Slika 5. Procentualna odstupanja modela grupe A u odnosu na analitičko rešenje
Figure 5. Percentage deviations of models of group A in relation to the analytical solution



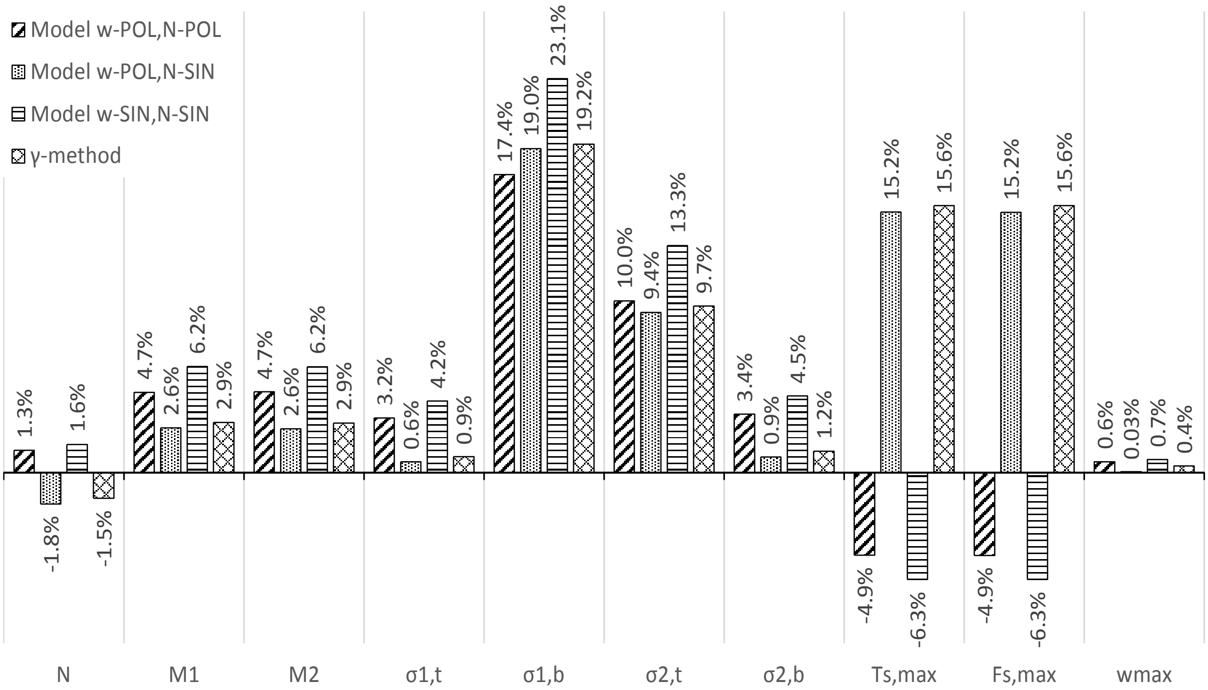
Slika 6. Procentualna odstupanja modela grupe B u odnosu na analitičko rešenje
Figure 6. Percentage deviations of models of group B in relation to the analytical solution

5.3 Numerička analiza primenom Ritz-ove metode

Za potrebe numeričke analize, definisana je grupa modela na osnovu istovremenog izbora dve probne funkcije za aksijalnu silu *N(x)* i ugib *w(x)*: Modeli grupe C (w-POL,N-POL; w-POL,N-SIN, w-SIN,N-SIN). Svi uticaji analizirani Galerkinovom metodom sračunati su i prema Ritz-u, a njihova procentualna odstupanja – u odnosu na analitičko rešenje – prikazana su na slici 7. Rezultati pojednostavljenog „*γ*-postupka" takođe su prikazani i upoređeni sa analitičkim rešenjem.

5.3 Numerical analysis by Ritz method

For the purpose of numerical analysis, the group of models is defined on the basis of simultaneous selection of two trial functions for axial force *N(x)* anddeflection *w(x)*: Models of Group C (w-POL,N-POL; w-POL,N-SIN; w-SIN,N-SIN). All effects analyzed by Galerkin method are calculated according Ritz as well and their percentage deviations compared to analytical solution are presented on Fig 7. Results of simplified "γ-procedure" were also compared with analytical solution.



Slika 7. Procentualna odstupanja modela grupe C u odnosu na analitičko rešenje
Figure 7. Percentage deviations of models of group C in relation to the analytical solution

Na slici 7, može se primetiti da minimalno odstupanje od analitičkog rešenja pokazuje varijantni model (w-POL,N-SIN), dok se maksimalna odstupanja javljaju kod varijantnog modela (w-SIN,N-SIN). Rezultati određeni varijantnim modelom (w-POL,N-SIN) jesu na strani sigurnosti, jer daju neznatno veće vrednosti (do 3%) za unutrašnje sile i pomeranja, dok odstupanja za normalne napone i sile klizanja / smičuće sile, dostižu vrednosti do 19% i 15% respektivno, u odnosu na analitičko rešenje. Razlog za takvo povećanje leži u činjenici da su normalni naponi i sile klizanja / smičuće sile izvedene veličine osnovnih nepoznatih *w(x)* i *N(x)*, pa su kumulativne greške veće. Očigledno je da izbor probnih funkcija za *w(x)* i *N(x)* ima značajan uticaj na konačni rezultat, kao i na izvedene statičke veličine. Takođe, može se primetiti da model (w-POL,N-SIN) ima najbolje poklapanje s pojednostavljenim „*γ*-metodom", predloženim u EN 1995. Iako varijantni model (w-POL,N-POL) daje manja odstupanja od varijantnog modela (w-SIN,N-SIN), poredeći ih sa analitičkim rešenjem, može se uočiti da dobijene vrednosti precenjuju ili potcenjuju analitičke, te ovi modeli nisu na strani sigurnosti.

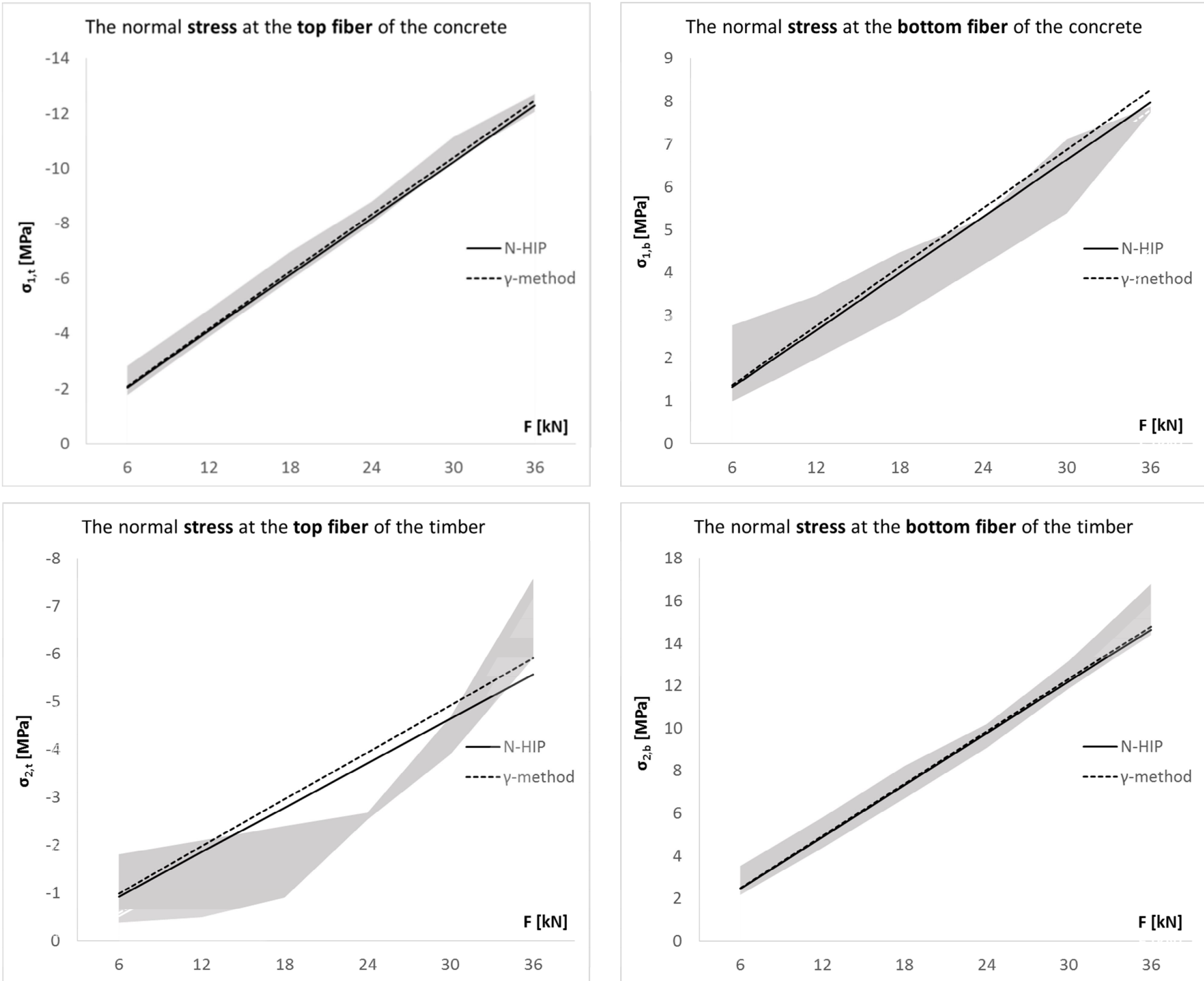
From Fig. 7, it can be noticed that the minimum deviation from the analytical solution shows the variant model (w-POL,N-SIN), while the max deviation occurs in variant model (w-SIN,N-SIN). The results obtained by variant model (w-POL,N-SIN) are on the safe side because they give a slightly higher values (up to 3%) for internal forces and displacements, while deviations for normal stresses and slip/ shear forces, arise up to 19% and 15% respectively, comparing to analytical solution. The reason for such increase lays in the fact that normal stresses and slip/ shear forces are derived values from baseline unknowns *w(x)* and *N(x),* so the cumulative errors are higher. It is obvious that the selection of trial functions for *w(x)* and *N(x)* has the significant impact on final result, as well as on derived statical values. It can be also noted that model (w-POL,N-SIN) has the best match with the approximate "γ-method" proposed in EN 1995. Although variant model (w-POL,N-POL) shows smaller deviations from variant model (w-SIN,N-SIN), comparing these two models with analytical solutions it can be seen that obtained values overestimate or underestimate analytical ones, but both models are not on the safe side.

5.4 Potvrda Galerkinove metode

Radi provere Galerkinove metode u analizi SDB konstrukcija, upoređene su dobijene numeričke vrednosti sa eksperimentalnim podacima. Kako se pokazalo da N-HIP model na najbolji način opisuje problem SDB, ovaj model izabran je za komparativnu analizu sa eksperimentalnim rezultatima SDB greda (EP1 and EP2) [18], gde su upotrebljena mehanička spojna sredstva. Dijagrami na slici 8 predstavljaju napone u poprečnom preseku konstitutivnih elemenata SDB greda za presek u sredini raspona, u odnosu na intenzitet sile tokom faza nanošenja opterećenja (F = 6, 12, 18, 24, 30 i 36 kN). Osenčene površine predstavljaju anvelopu eksperimentalno dobijenih rezultata za grede EP1 i EP2, dok pune i isprekidane linije predstavljaju rezultate numeričke analize pomoću N-HIP modela i „*γ*-metode”, respektivno. Razmatrajući napone ($\sigma_{1,b}$ and $\sigma_{2,t}$) na kontaktu dva materijala, znatno odstupanje od eksperimentalnih rezultata može se primetiti za napon $\sigma_{2,t}$, pri opterećenjima F = 18 i 24 kN. Primetno je odlično poklapanje numeričkih vrednosti sa eksperimentalnim rezultatima kod gornjeg i donjeg vlakna spregnutog poprečnog preseka, kao i donjeg vlakna u betonskoj ploči.

5.4 Verification of Galerkin method

In order to verify the application of Galerkin's method in TCC system, the comparison of numerical obtained values and experimental data was done. As it was shown that the N-HIP modeldescribes the problem of TCC partial composite action on the best way, this model was applied for comparative analysis with experimental test results of TCC (EP1 and EP2) beams [18], where mechanical fasteners were used. The diagrams in Figure 8 show the stresses in cross-section of constitutive elements of TCC beams in the middle of the span, in relation to force intensity throughout experimental loading phases (F = 6, 12, 18, 24, 30 and 36 kN). The shaded surfaces represent the envelopes of the results obtained from experiments for beams EP1 and EP2, while the full and dashed lines represent the results of the numerical analysis by N-HIP model and by "*γ*-method" respectively. Considering the stresses ($\sigma_{1,b}$ and $\sigma_{2,t}$) on the contact of two materials, deviations from the experimental results can be noticed, significantly for stress$\sigma_{2,t}$ at loads F = 18 and 24 kN. A good match with experimental results on the top and bottom sides of the cross-section as well as in bottom of the concrete slab is obvious.



Slika 8. Uporedna analiza rezultata dobijenih N-HIP modelom i eksperimentalnih podataka
Figure 8. Comparative analysis of numerical model N-HIP and experimental data

6 ZAKLJUČAK

Na osnovu predstavljenih analiza primenom Galerkinove i Ritz-ove metode, može se zaključiti da izbor probnih funkcija u formulaciji problema ima najveći uticaj na konačne vrednosti dobijenih i prikazanih rezultata. U Galerkinovoj metodi, značajan uticaj ima i sam izbor osnovnih nepoznatih (*w* i *N*). Takođe, kvalitativno poznavanje prirode rešenja može značajno doprineti smanjenju odstupanja rezultata (greške) od analitičkog rešenja. U prikazanoj analizi primenom Galerkinove metode, predloženi N-HIP model smatra se najpogodnijim za rešavanje problema elastičnog sprezanja. Kada se primenjuje varijaciona formulacija, funkcional može biti definisan putem jedne promenljive ili više njih (sila/pomeranje), dok će nepoznate koje su izabrane za osnovne biti određene s većom tačnošću od ostalih izvedenih veličina.

Kako su metode reziduuma ili varijacione metode prilično uobičajeni oblik formulacije u MKE, predstavljene metode Galerkina i Ritz-a mogu se uspešno primeniti prilikom definisanja spregnutog KE za elastično spregnute konstrukcije drvo–beton, a time se dobija efikasan inženjerski alat u praksi. Uvođenjem Evrokoda za spregnute sisteme drvo–beton [4], očekuje se preciznije definisanje osnovnog ulaznog parametra za različita sredstva spajanja – modula pomerljivosti spojnog sredstva *K*, a time i realniji odgovor spregnutih nosača u numeričkim analizama.

ZAHVALNOST

Ovaj rad proistekao je iz istraživačkog projekta TR 36043 „Razvoj i primena sveobuhvatnog pristupa u projektovanju novih i proceni sigurnosti postojećih konstrukcija u cilju smanjenja seizmičkog rizika u Srbiji", koji je finansiralo Ministarstvo nauke Republike Srbije.

6 CONCLUSION

Based on the presented analysis using Galerkin's method and Ritz method, it can be concluded that the selection of trial functions in problem formulation has the major influence on the final effect values of obtained and presented results. An important influence is also the choice of baseline unknowns (*w* and *N*) for Galerkin's method. Also, the qualitative knowledge of solution nature can significantly contribute tothe reduction of errors in obtained results related to analytical solution. In presented analysis by Galerkin's method the proposed N-HIP model qualifies as the most appropriate in order to solve the problem of partial coupling. When using a variational formulation, functional could be defined trough one or more unknowns (forces/displacements), while the unknowns that are chosen as basic will be determined with more accuracy than the derived ones.

As the weighted residual method or the variation formulation in the FEM is quite usual form, presented the Galerkin and Ritz methods could be successfully applied when defining a composite FE for partially timber-concrete composite systems, thereby enabling an efficient engineering tool in practice. With introduction of Eurocode for timber-concrete composite structures [4], it is expected that more precise definition of basic input parameter - slip modulus for different types of fasteners *K*, will contribute to the more realistic response of composite beams in numerical analysis.

ACKNOWLEDGMENTS

The work has been done within the scientific research project TR 36043 "Development and application of a comprehensive approach to the design of new and safety assessment of existing structures for seismic risk reduction in Serbia", which is funded by the Ministry of Science of Serbia.

7 LITERATURA
REFERENCES

[1] Cassel K. W. (2013): Variational Methods with Applications in Science and Engineering University Press, Cambridge.

[2] Ceccotti A. (2002): *Composite concrete–timber structures*, Progress Structural Engineering and Material (4), pp. 264–275.

[3] Cvetković R. (2016): *Mehaničko ponašanje spregnutih konstrukcija tipa drvo-beton*, Doktorska disertacija, Građevinsko-arhitektonski fakultet, Univerzitet u Nišu, Niš, 204 str.

[4] Dias A., Schänzlin J., Dietsch P. (eds.) (2018): Design of timber-concrete composite structures: A state-of-the-art report by COST FP1402/WG 4, Shaker Verlag Aachen.

[5] EN 1995-1. (2008): Eurocode 5: Design of timber structures – Part 1–1: General – Common rules and rules for buildings, CEN, Brussels.

[6] Gelfi P., Giuriani E., Marini A. (2002): *Stud Shear Connection Design for Composite Concrete Slab and Wood Beams*, Journal of Structural Engineering, pp. 1544–1550.

[7] Girhammar U.A., Gopu V.K.A. (1993): *Composite beam-columns with interlayer slip – Exact analysis*, ASCE Journal of Structural Engineering, 119(4), pp. 1265–1282.

[8] Girhammar U.A., Pan D. (2007): *Exact static analysis of partially composite beams and beam-columns*, International Journal of Mechanical Sciences, 49, pp. 239–255.

[9] Girhammar U.A. (2008): *Composite beam-columns with interlayer slip – Approximate analysis*, International Journal of Mechanical Sciences, 50(12), pp. 1636–1649.

[10] Girhammar U.A. (2009): *A simplified analysis method for composite beams with interlayer slip*, International Journal of Mechanical Sciences, 51(7), pp. 515–530.

[11] Kozarić Lj. (2016): *Vibracije izazvane ljudskim delovanjem kod spregnutih međuspratnih konstrukcija tipa drvo-laki beton*, Doktorska disertacija, Građevinski fakultet Subotica, Univerzitet u Novom Sadu, Subotica, 112 str.

[12] Manojlović, D., Kočetov Mišulić, T. (2016): Analiza i modeliranje spregnutih sistema drvo–beton – primena na proračun mostovske konstrukcije prema EN, Građevinski materijali i konstrukcije, 59(4), pp. 47–74.
[13] Manojlović D., Radujković A., Kočetov Mišulić T. (2017): *Application of Ritz method in analysis of timber-concrete composite system*, 17. International Symposium MASE, Ohrid, Faculty of Civil Engineering, pp. 721–728.
[14] Manojlović D., Kočetov Mišulić T., Radujković A. (2018): *Weighted residual method in analysis of timber concrete composite system*, Zbornik radova sa nacionalnog kongresa, 15. Kongres Društva građevinskih konstruktera Srbije, Zlatibor, pp. 573-582.
[15] MATLAB R2014a (2014): MathWorks Inc. The Language of Technical Computing.
[16] Reddy J. N. (2002): *Energy Principles and Variational Methods in Applied Mechanics*, 2nd Edition, John Wiley & Sons, Inc., New York, pp. 608.
[17] Stevanović B. (1994): *Analiza spregnutih nosača od drveta i betona*, Magistarska teza, Građevinski fakultet Univerziteta u Beogradu, Beograd, 119 str.
[18] Stevanović B. (2004): Eksperimentalna i teorijska analiza spregnutih nosača drvo-beton izvedenih mehaničkim spojnim sredstvima, Materijali i konstrukcije 47, str. 29–46.
[19] Szabó B. (2006): *Influence of shear connectors on the elastic behaviour of composite girders*, Doctoral dissertation, Helsinki University of Technology, Espoo, Finland, pp. 124.

REZIME

PRIMENA NUMERIČKIH METODA U ANALIZI SPREGNUTIH KONSTRUKCIJA DRVO-BETON

Dragan MANOJLOVIĆ
Tatjana KOČETOV MIŠULIĆ
Aleksandra RADUJKOVIĆ

Analiza i proračun spregnutih drvo–beton (SDB) konstrukcija, gde je veza konstitutivnih elemenata ostvarena mehaničkim spojnim sredstvima, predstavlja kompleksan zadatak zbog uzimanja u obzir pomerljivosti sredstava za sprezanje tj. klizanja na kontaktu dva materijala. Primena pojednostavljenih postupaka i metoda u analizi SDB nosača predstavlja pogodan i poželjan način proračuna, koji inženjerima u praksi omogućava efikasan alat. Široko rasprostranjen, pojednostavljen proračun tzv. γ-metod dat je u EN 1995. Metode zasnovane na diferencijalnoj ili varijacionoj formulaciji često su u upotrebi kada su u pitanju programi za strukturalnu analizu konstrukcija. U radu je prikazana Galerkin-ova i Ritz-ova metoda za analizu i proračun SDB nosača za slučaj proste grede izložene raspodeljenom opterećenju. Analiziran je izbor probnih funkcija koje opisuju problem elastičnog sprezanja, kao i njihov uticaj na konačne rezultate. Za potrebe numeričke analize, na osnovu predloženih numeričkih modela, napisani su kodovi u MATLAB-u. Model primenjen u analizi Galerkinovom metodom, koji najbolje opisuje problem elastičnog sprezanja, izabran je za komparativnu analizu sa eksperimentalnim podacima.

Ključne reči: Ritz-ova metoda, Galerkinova metoda, γ-metod, slaba formulacija, numerička analiza, MKE, sprezanje drvo–beton, elastično sprezanje, klizanje u spoju.

SUMMARY

APPLICATION OF NUMERICAL METHODS IN ANALYSIS OF TIMBER CONCRETE COMPOSITE SYSTEM

Dragan MANOJLOVIC
Tatjana KOCETOV MISULIC
Aleksandra RADUJKOVIC

The analysis and design of composite timber-concrete (TCC) structures, where the connection of the constituent elements is achieved by dowel type fasteners, is a complex task due to taking into account the slip of the coupling means, i.e. interlayer slip on the contact surface of two materials. The application of simplified procedures and methods in the analysis of the TCC system is a convenient and desirable way of design that enables efficient tool for engineering practice. Widespread simplified calculation procedure, so called "γ-method", is adopted in EN 1995. Methods based on differential or variational formulation are commonly applied when software for structural analysis are used. Galerkin's and Ritz's methods for analysis and design of TCC systems in the case of simply supported beam loaded with uniformly distributed load are shown in this paper. The selection of trial functions that describe the problem of elastic composite action as well as their influence on the final results were analyzed. For the purposes of numerical analysis, based on the proposed numerical models, several codes are written in MATLAB. The model applied in analysis by Galerkin's method, that best describes the problem of elastic coupling, was chosen for further comparative analysis with experimental data.

Key words: Ritz's method, Galerkin's method, γ-method, Weak form, Numerical analysis, FEM, Timber-concrete composite, Partial interaction, Interlayer slip.