



Review paper

The supply and demand of infrastructure robustness, resilience and sustainability

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ABSTRACT

Economics and engineering manage the built infrastructure in a dynamic equilibrium of supply and demand. The potentially contradictory constraints of the two domains within their governing dimensions of money and energy are explored. An analogy is drawn between mechanical and infrastructure network stability. The recently popularized terms of robustness, resilience and sustainability are defined as critical descriptors of the process and product to be jointly optimized by the two fields.

1 Introduction

In the present view, economics and engineering manage infrastructure processes and products as supply / demand (S/D) relationships in terms of money and energy. Figs. 1 – a) and b) illustrate the contrasting priorities governing engineering and economics. Energy is viewed as a rigid constraint, whereas money is regarded as a negotiable restraint. Thus, infrastructure management must reconcile the supply of and demand for structural performance under physically rigid constraints dimensioned in energy and economically negotiable restraints, negotiated in a dynamic mix of ultimately monetized economic and political priorities.

Both engineering and economics balance supply (R) and demand (Q), however their respective restraints and constraints can appear diverging. Engineered products must supply performance exceeding service demands by prescribed and uniformly accepted factors (such as  $\gamma$  and  $\phi$  in Fig. 1. a) over an intended useful life. In contrast, economic processes are planned over strategically and tactically varied time horizons. Except in the extreme high - and low - income areas, where social programs and philanthropy may reverse the governing pattern of Fig. 1. b), service demands exceed the supply by an indeterminate degree and motivate social progress. These diverging constraints and restraints are expressed in Eq. 1 – a, – b, as follows:

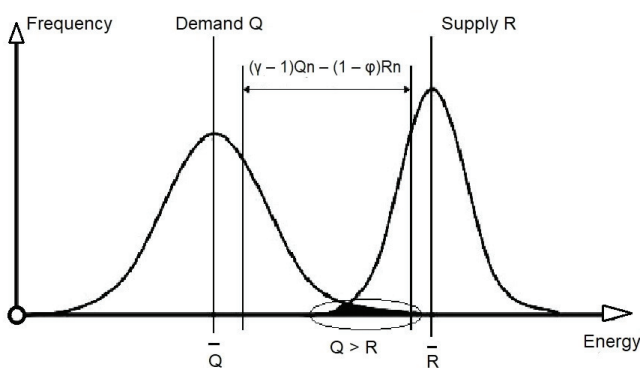
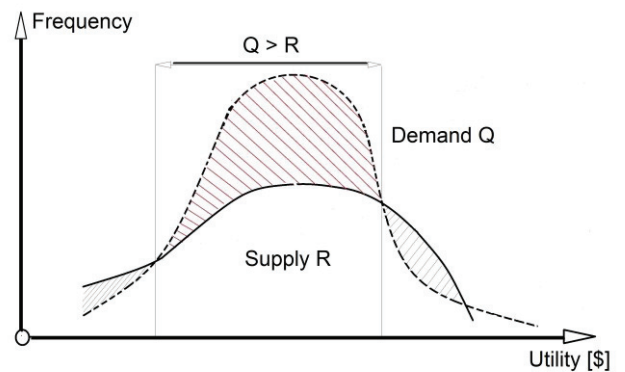


Figure 1. (a) The engineering constraint



(b) The economic restraint

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Engineering products:  $R > Q$  [Energy] (1–a)

Economic processes:  $R < Q$  [\$] (1–b)

Violating either condition of Eq. 1 amounts to failure. Less obvious, harder to address and more common are the failures of the two fields to reconcile the ostensibly contradictory constraints of their incongruent models and to render them compatible.

Engineering ensures a ‘stable’ equilibrium, such that  $R > Q$  in terms of energy. ‘Conservative’ oversupplies are professionally established and legally enforceable. Economics negotiates a ‘dynamic’ equilibrium, such that  $R < Q$  to an indeterminate degree in terms of money. ‘Shortfalls’, even catastrophic ones, are customary. Subject to litigation can be shortages in engineering and excesses in economics. Engineering products are acquired ‘ground-up’ under natural constraints in response to top-down economic demands. Economic processes are transacted ‘top-down’ under fiscal restraints in response to ground-up social demand. Hence, economists tend to regard engineering products as ‘static’, whereas engineers tend to view economic processes as ‘unstable’. Few if any are expert in both domains. The proverbial ‘meeting in the middle’ implies unattainable perfection, occasionally promised in political campaigns. Hence, both engineers and economists regard with skepticism up-to-the-moment politics, a.k.a. ‘the art of the possible’. Although only implicit in Fig. 1, politics dominates infrastructure management in the (also implicit) domain of intelligence / information.

Bridges are critical links in the built infrastructure, supplying instructive examples of  $Q/R$  disparities dimensioned in money and energy. The present exercise expands from events in the bridge network of a major city to more general conclusions applicable to infrastructure management in general. The first step is to examine the engineering methods of assessing the supply of structural performance.

## 2 Bridge conditions

The Federal-Aid Highway Act of 1968 initiated modern vehicular bridge management in the United States, and by extension, worldwide. The National Bridge Inventory (NBI),

established by the Federal Highway Administration (FHWA) rapidly built a database of 230,000 bridges, eventually expanding it to nearly 650,000. A vehicular tunnel database was initiated in 2015. Integration of the railroad bridge database, exceeding 220,000 bridges is pending.

In its present form, NBI is equipped to support strategic lifecycle decisions on local and national levels. Originally however, its overwhelming priority was to identify and avert disasters, such as the collapse of the Silver Bridge at Point Pleasant in 1967. Tactically, potential hazards had to be promptly identified and mitigated. Strategically, realistic lifecycle bridge performance had to be modeled, anticipated, and optimized. To these ends, the Act [1] mandated biennial inspections of vehicular bridges. To serve both objectives, the visual biennial inspections had to supply actionable qualitative and quantitative assessments of bridge conditions.

The NBI compensates for the vagueness of the term ‘condition’ with a database of complementary qualitative and quantitative, descriptive and prescriptive bridge assessments. Local owners supplement NBI according to their specific needs. The resulting condition database supports bridge management decisions on both project and network levels. Milestones in that process were the introduction of the LRFD Bridge Design Specifications by the American Association of State Highway Transportation Officials [2] and the AASHTO Bridge Element Condition States, adopted in [3].

The biennial inspections update the NBI with two types of assessments: descriptive and prescriptive. The original 10 – level condition ratings were essentially descriptive. The 4 element level condition states which superseded them combine the descriptive opinions of qualified engineers with quantitative measurements and, at the lowest level 4, imply prescriptive recommendations. Prescriptive assessments recommend action. Such are the ‘flag’ reports of potential hazards according to New York State Department of Transportation (NYS DOT) defined in [4, 5]. Based on its bridge inventory, NYS DOT also recognizes a number of vulnerabilities, such as steel details, concrete details, seismic, hydraulic, collision, overload, and acts of destruction. The vulnerability of overload was withdrawn. The variety of the federal and NYS DOT bridge condition assessments is summarized in Table 1.

Table 1. Bridge assessments

Assessment	Type	Source	Description
Element condition ratings	Descriptive	[1]	9 (New) – 0 (Imminent failure)
Bridge serviceability appraisal	Descriptive	[1]	9 (Superior to design criteria) – 0 (Closed)
Maintenance ratings	Prescriptive	[6]	9 (No repairs needed) – 1 (Closed)
Sufficiency ratings	Computed by weighted formula	[1]	$0 < S1 + S2 + S3 - S4 < 100\%$ , where: S1 – Structural adequacy & safety (< 55%) S2 – Serviceability & Obsolescence (< 30%) S3 – Essentiality for public use (< 15%) S4 – Special reductions (< 13%)
Load ratings	Computed analytically	[1, 2]	Inventory & Operating ratings
Element condition states	Descriptive Prescriptive	[3]	4 (Good), 3 (Fair), 2 (Poor), 1 (Severe)
Element condition ratings	Descriptive	[4]	7 (New) – 3 (Not functioning as designed) – 1 (Totally deteriorated or failed)
Potential hazards (Flags)	Prescriptive	[4]	Structural (PIA, Red, Yellow), Safety
Vulnerabilities	Descriptive Prescriptive	[5]	Hydraulic, Steel, Concrete details, Collision, Seismic, Destruction, Overload (withdrawn)

In another significant development, advanced technologies are offering a variety of non-destructive testing and evaluation (NDT & E) techniques [6], allowing for a quantification of previously purely qualitative assessments.

The qualitative condition ratings and quantitative diagnostics describe ‘as is’ conditions on the ‘project’ or ‘ground-up’ level. Also ground-up (a.k.a. hands-on), the prescriptive flag reports identify potential hazards, requiring a timely resolution. Load ratings, flag resolutions, and vulnerabilities are determined at the ‘top-down’ network level.

Serviceability combines ground-up findings and top-down determinations. The bridge management database integrates the overlapping complementary assessments in a ‘bilateral’ flow between the project and network levels. As in a redundant mechanical structure, the strengths of one block of information compensate for the weaknesses of another, enabling the redistribution in the event of partial failure. In subsequent sections, serviceability is qualified and to a degree, quantified, in terms of robustness, resilience and sustainability.

Up to 2015 bridge inspections according to the several updates of [4] included the following significant features:

- Inspection team leaders are professional engineers licensed in N.Y. All inspectors pass a state course;
- Fracture-critical elements are inspected hands-on and certified by the team leader;
- All bridge elements were rated in all spans on a scale from 7 (new) to 1 (failed), 3 signifying ‘not functioning as designed’;
- Potential hazards are designated as flags and processed in advance of the inspection reports.

In 2016 [5] adopted the four element condition states recommended by [2], superseding the seven condition rating levels of [4]. The other features pertain.

In their incongruous dimensions, the various assessments supply a multi-faceted view of the infrastructure and of each other. The ‘bridge condition’ and ‘sufficiency’ ratings of the 790 vehicular and pedestrian bridges of New York City, enumerated in Table 2, are plotted in Fig. 2 for 2008.

Table 2. New York City bridges & tunnels

Type	Quantity
East River Crossings	4
Moveable	25
Waterway	51
Arterial	208
Off – system (Local)	389
Pedestrian	107
Tunnels	6
Total	790

The two sets of ratings plotted in Fig. 2 are obtained by different weighted average formulae and hence, are fundamentally qualitative. The former is based on the NYS DOT descriptive condition ratings (7 – 1) according to [4]. The latter is based on the FHWA ratings (9 – 0) [1], comprising assessments of importance, serviceability, and obsolescence. The data points of both sets are generated deterministically, however in their continually expanding aggregate they offer abundant material for statistical, frequentist, and other probabilistic interpretations. Consistently with the basic management commitment to safety, structural conditions rated  $\leq 3$ , i.e., not functioning as designed according to [4] are few. In contrast, the FHWA sufficiency ratings  $< 50\%$  according to [1] are numerous. The conspicuous outliers in both graphs reflect rehabilitations. There are no outright structural failures, but quite a few serviceability ones. If the two sets of data points were reduced to average patterns over time, the ‘structural condition’ graph would be concave, tending asymptotically towards an average rating of 4, whereas the ‘sufficiency rating’ one would be convex, declining to 0 at about 85 years (essentially consistent with the 75 years useful life recommended by [2] and earlier editions. As postulated in Fig. 1 and Eq. 1, structural safety meets the demand, but serviceability is undersupplied. Both sets rate performance, however the ‘condition rating’ assesses structural integrity in engineering terms, whereas the ‘sufficiency’ rating reflects user’s satisfaction, hence containing economic considerations. Reversing the trends would be unsafe in the former case and possibly unaffordable in the latter.

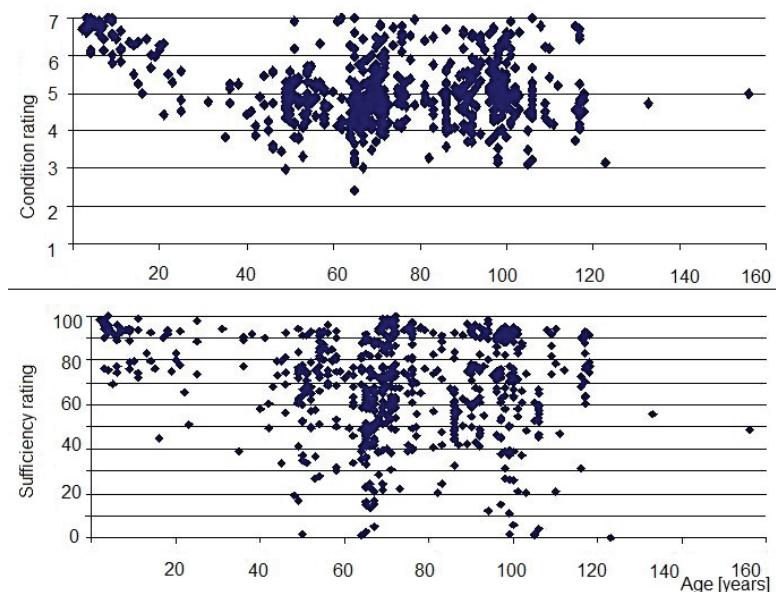


Figure 2. Structural condition and Sufficiency ratings for the NYC bridges (circa 2008)

The relationship of condition and load ratings similarly confirms the safe operation of the network. Qualitative visual inspections are the first to rate bridges unsafe, thus requiring AASHTO load ratings to determine whether the structure has quantifiably acceptable load-bearing capacity. The latter can be of levels I, II, and III, and may include proof loading. It is not uncommon for load ratings to find bridges deemed in fair to poor structural condition still fit to carry design loads. Once again, the reverse would have amounted to a misplaced relationship of the qualitative and quantitative assessments.

The average bridge life of 80 to 100 years suggested by the descriptive condition ratings is deceptive. For a realistic assessment, they must be combined with the prescriptive flag reports. The low – rated bridges of ages 40 to 45 years generate the most flags and govern the needs for repair and reconstruction. So long as deterioration is not delayed by other means, new bridges decline into this category while the current ones are rehabilitated. The NYS DOT flag protocol was designed as the first line of defense against the proliferating potentially hazardous bridge-related conditions and in the late 1980s became the critical descriptor of the state of the network. Flags are defined in [4, 5] as follows:

**Red Flag** - A structural flag that is used to report the failure or potential failure of a primary structural component that is likely to occur before the next scheduled biennial inspection.

**Yellow Flag** - A structural flag that is used to report a potentially hazardous structural condition which, if left unattended could become a clear and present danger before the next scheduled biennial inspection. This flag would also be used to report the actual or imminent failure of a non-critical structural component, where such failure may reduce the reserve capacity or redundancy of the bridge but would not result in a structural collapse.

**Safety Flag** - A flag that is used to report a condition presenting a clear and present danger to vehicular or pedestrian traffic but poses no danger of structural failure or collapse. Safety Flags can be issued on closed bridges whose condition presents a threat to vehicular or pedestrian traffic underneath or in their immediate vicinity.

**Prompt Interim Action (PIA)** – A flag demanding resolution by the responsible owner within 24 hours.

Defined as ‘potential hazards’, flags may or may not signify element or service failures. Their veracity and gravity

can vary widely. The bridge management action they invariably require is engineering review.

Applied jointly to the NYC bridge network of the considered period, the described structural assessments reassure tactically and disturb strategically. The ‘condition’ and ‘load’ ratings indicate acceptable bridge safety. However, failing ‘sufficiency ratings’ and accumulating ‘flags’ may foretell a pending crisis. In the more recent terminology, discussed in the subsequent sections, even though the individual structures appear on the average robust, the network’s resilience and overall sustainability may be approaching instability.

### 3 The NYC bridge network

During the last several decades, the vehicular bridges managed by NYC DOT have fluctuated around the numbers in Table 2. Without adjusting original dates of completion for rehabilitations, their average age circa 1990 was approximately 75 years. Another approximately 600 bridges on the arterial network in the five city boroughs are managed by NYS DOT. Their average age was approximately 40 years. Span numbers quantify bridge networks more meaningfully. NYC DOT manages approximately 5,000 spans.

Following the economic restrictions of the 1970s and early 1980s, the NY City bridge network suffered extreme neglect. By 1989 80 City bridges had been fully or partially closed and many were posted for restricted load. As intended, the proliferating flag incidence clearly signaled the unfolding network crisis. The flag history of the New York City bridges from their inception in 1982 to the ‘steady state’ reached circa 2006 is illustrated in Fig. 3 and discussed herein.

- The following five periods are discernible in Fig. 3:
- 1982–1987 Apparent equilibrium following initial adjustments (A – B)
- 1987–1992 Increase reaching annual factor of 2 (B – C)
- 1992–1996 Peaking approximately 24 times above the initial level (C – D)
- 1996–1999 Annual decrease by a factor of approximately 1.24 (D – E)
- 1999–2006 Apparent equilibrium at approximately 10 times the initial level (E –).

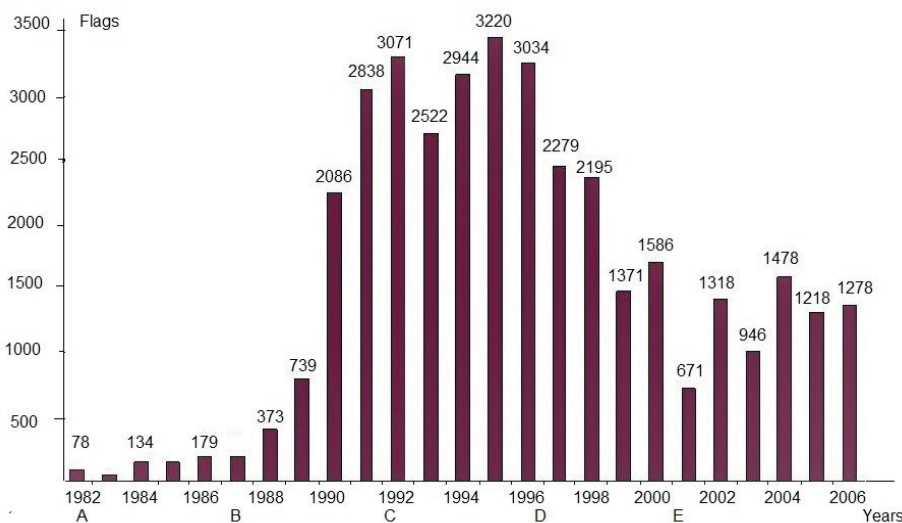


Figure 3. Flags on the New York City bridges, 1982 – 2006

Beyond 2006 the flag numbers have fluctuated about the number of 1200, suggesting a new equilibrium of service demand and bridge network performance supply.

During the years under consideration the direct costs of the (mostly temporary) repairs mitigating flagged conditions were averaging at approximately \$US 15 - 20K. The rough estimates of the notoriously intractable user costs due to traffic interruptions are invariably higher. The costs of the potential hazards escalating to actual accidents can be vaguely estimated, based on annual court case settlements in New York City. As a result, all levels of city management recognized the urgent need to address the looming crisis in bridge conditions.

Two events particularly impressed the public attention. In 1988 bridge inspectors found the deterioration of the Williamsburg Bridge, crossing East River since 1903, so advanced that its eight vehicular lanes and two subway tracks were temporarily closed. Following an in-depth inspection and analysis, [7] concluded that a rehabilitation, at a cost exceeding \$US 1 billion was feasible and urgent. On June 1, 1989, a piece of concrete spalled from the underside of the Franklin Delano Roosevelt (FDR) Drive on the Manhattan East Side at 19<sup>th</sup> St. and killed a motorist, as reported in [8].

A less visible, but no less significant consequence of the events was the re-establishment of the Bureau of Bridges (later Division) at the New York City Department of Transportation (NYC DOT). The financial crunches of the 20<sup>th</sup> century had reduced the powerful Bridge Commission of the early 1900s to a lesser department in various agencies for more 'general services'.

According to inspection reports nearly half of the City bridge decks were in conditions similar to FDR's. Under its constraining circumstances, the new City Bureau of Bridges had to obtain emergency funding and retain qualified in-house and contracted expertise. Hence, it needed a credible projection of the hazard mitigation needs. The Bridge Inspection & Management Unit established by the author undertook to model the flag expectations resulting from the next inspections. The first steps in that process for 1991 were reported in [9]. The projection for 1992 was reported in [10].

'Extreme events' of varying duration afflict the various aspects of social activities and assets at different frequencies. To varying degrees, they combine randomness and phenomenological causation. Thus, anticipating and managing any extreme event should benefit from both the random and causal features they may share. Traffic, climate and other factors cause structural deterioration by unrelated mechanisms, but in their domains correlate with population density, and human activities. Even without a full understanding of the underlying phenomena, network management plans for their identification and mitigation based on statistical data. During the ostensibly stable period A – B of Fig. 3 (1982 – 1987), however, frequentist reasoning alone misses the escalation during B – C (1987 – 1992). As [9] and [10] reported, causes for that development were discernible in the element 'condition' ratings.

The highly site / moment (i.e., space / time) – specific vehicular bridge network of New York City in the reviewed period qualify, retrospectively, as an extreme event gradually evolving from a 'potential' to an active crisis. A posteriori, the developments from 1987 to 2006 argue convincingly for prevention. However, the 'gestation' period between 1982 and 1987 could not have justified an emergency budget request, even though the five stages of the flag pattern are typical of most disaster scenarios and hence, could have been predictable.

#### 4 Supply & demand of services and expenditures: a stability analogy

Adopting Ernest Hemingway's words describing a character's bankruptcy in *The Sun Also Rises* (1926), crises occur first gradually, then suddenly. According to [12] a 'crisis' is "a state of rupture, negative and instantaneous, along a *trend* or 'tendency'". In [13] Parrochia traces the evolving view of 'crises' in all spheres of social activity from antiquity to the present. He views them as events, discontinuities, conflicts, and transactions.

Many phenomena qualifiable as critical or catastrophic display 5-stage patterns similar to those of Fig. 3. Such are the financial and political so-called crises and the health epidemics, including COVID-19 in the United States during 2020 – 21. Also similar are the phases of the 'Future Tech Hype Cycle', consisting of innovation, expectations, trough of disillusionment, enlightenment, and plateau of productivity, and the stages of grief, comprising denial, anger, bargaining, depression, and acceptance. Given any specifics, the cycles comprise an apparent equilibrium of supply and demand, imperceptibly degenerating from stable to neutral and to unstable, dynamic change perceived as collapse, peaking (or "hitting bottom"), and attenuation to a stable new equilibrium at elevated supply and demand. The moments when this scenario can be averted, for example by a smooth transition from the initial to the final equilibrium are of particular interest.

To that end, the rigorous definition of catastrophes in terms of energy instability supplies a generally applicable 'formal' analogy. Structural failures of strength are quantified by external demands exceeding the supply of material resistance (e.g.,  $Q > R$  in Eq. 1 – a). In contrast, instabilities are inherent in a structure's formal qualities. The described pattern is formally analogous to the 'snap-through' instability of Von Mises trusses and flat arches. Bažant and Cedolin [14] state: "The question of stability may be most effectively answered on the basis of the energy criterion of stability, which follows from the dynamic definition if the system is conservative." The authors present catastrophe theory as a "strictly qualitative viewpoint" analyzing the stability of conservative systems by energy methods as follows: "[Catastrophe theory] seeks to identify properties that are common to various catastrophes known in the fields of structural mechanics, astrophysics, atomic lattice theory, hydrodynamics, phase transitions, biological reactions, psychology of aggression, spacecraft control, population dynamics, prey-predator ecology, neural activity of brain, economics, etc. Simply, the theory deals with the basic mathematical aspects common to all these problems."

Both [13, 14] refer to René Thom's [15] demonstration that in a conservative system with one control parameter only one type of catastrophe is possible (the limit point or snap-through), with two independent control parameters, the fold and the cusp types of catastrophes are possible (asymmetric and symmetric bifurcation). For systems with three control parameters, five types of catastrophes become possible; and systems with up to four control parameters allow at most seven types of catastrophes. The 7 types of catastrophes are called 'elementary'.

Bažant and Cedolin [14] illustrate the snap-through of the von Mises truss as shown in Fig. 4. It is assumed that the bars will not buckle individually under the increasing load  $P$ . Rather, a 'global' instability occurs when the potential energy of the elastically deformed system reaches a bifurcation point. Equating to 0 the expression for the second derivative

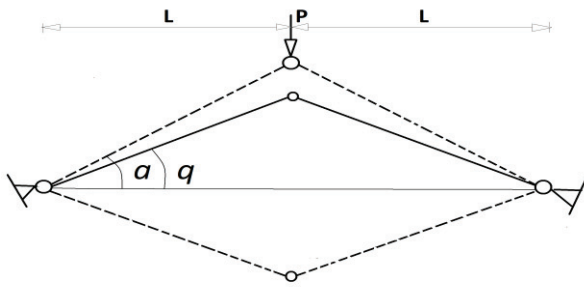
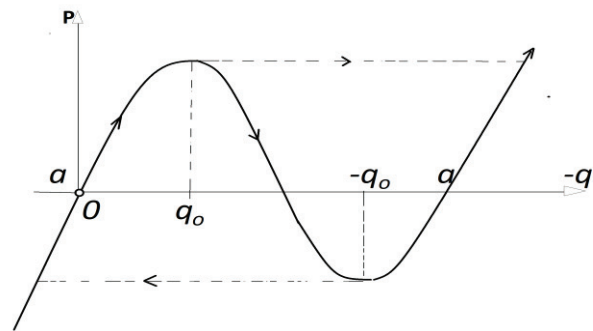


Figure 4 (a) Von Mises truss



(b) equilibrium path

of the system's potential energy obtains Eq. 2. The truss is stable for  $-q_0 \leq q \leq q_0$ .

$$\cos q_0 = (\cos a)^{1/3} \quad (2)$$

If energy and money were viewed as the two active parameters controlling the bridge network, each could cause its own type of instability. For four control parameters, for example if intelligence and information were regarded as additional parameters (e.g., representing political restraints), the possible types of catastrophes would increase to seven (Table 4.7.1., p. 300 [14]). Adding further indeterminacy, the inevitable 'passive' system imperfections strongly influence near-instability behavior.

The stability analogy reminds that, apart from failures quantifiable by demand exceeding the supply of strength, infrastructure assets and networks are vulnerable to those of qualitative form. It also cautions that the possible modes of system failure increase (more than linearly) with the number of 'control factors'. The analogy to mechanical instability advances the argument for anticipation and prevention, as opposed to relying on 'emergency response' at 'limit-points' or 'bifurcations'. However, the potential energy of conservative systems depends on measurable and calculable demands and supplies of applied and resisting energy. Hence, an infrastructure network, with its broadly estimated multi-parameter dynamic equilibrium of vaguely quantified and qualified supply and demand, cannot qualify as conservative. Once the stability analogy is not an exact predictor, it can be dismissed as one more doomsday warning. Aspiring Cassandras bear that curse since antiquity.

## 5 Robust, resilient and sustainable performance

In the terminology of stability theory, the consequences of extreme events escalating to 'national disasters' should qualify as catastrophes. The potentially catastrophic 'flag' history of Fig. 3 was contained both financially and mechanically without reaching disaster magnitude, but it gained 'emergency' status and absorbed substantial local and federal funding that could have served other purposes. Energy and money are the obvious control parameters traditionally quantifying infrastructure network performance. The stability analogy draws attention to the qualitative aspects of that performance. It raises the following questions: What critical parameters best reflect the potential instabilities of the infrastructure products and process, and what variables control them? The response demands terminology integrating the qualitative and quantitative features of energy and money.

According to Henri Léon Lebesgue: "The definition of a new category requires the introduction of at least one new term." As a noun, 'sustainability' remains an abstract quality inviting well-intentioned attitudes and multiple descriptions, but no definition. As defined in Eq. 3 it could quantify the 'performance' of a bridge or a network. As adjective, it is restrained by the parameters of the qualitative politics, economics, environmental protection, and so on, all of which are ultimately monetized in quantifiable budgets. The *sustainability factor* proposed in Eq. 3 essentially measures the long-term affordability of an infrastructure network under the governing social restraints and natural constraints as follows:

$$\text{Sustainability factor} = \Sigma \text{ benefits} / \Sigma \text{ costs} \quad (3)$$

A sustainability factor has significance only relative to the estimated performances of other strategic alternatives (e.g., 'optimal', 'desired', 'prioritized', 'expected'), assessed under the same standards and conditions. The following properties distinguish the sustainability of a performance according to Eq. 3:

- Sustainable performance is not merely a reciprocal cost / benefit ratio. It implies, but is not limited to 'cost-effectiveness' and 'affordability' because neither 'benefits' nor costs are limited to direct immediate activities and services. For example, environmental considerations, still struggling for recognition, can influence both the benefits to the users and the operating costs in ways easier to qualify than quantify over diverging time-horizons. Beyond the known operating (a.k.a. 'direct') expenditures incurred by the responsible owner, the sustained costs include the 'user costs', perceived damages, the consumption and depletion of natural resources.

- Sustainable performance is both qualifiable and quantifiable because the network's optimal, prioritized, and otherwise restrained and constrained 'control parameters' are both calculated and negotiated in the political, economic, engineering and public domains. Hence, sustainability qualifies engineered performance, already quantified in energy and money. As a consequence, alternative solutions are evaluated for environmental, economic and political sustainability assuming (prematurely) that they are similarly feasible in the engineering domain.

- Sustainable performance pertains to *network lifecycles*, rather than to annual budgets and individual projects. As the infrastructure's lifecycle by far exceeds annual budget considerations, sustainability must be perpetually reassessed, optimized (or prioritized) and updated over specified periods. Bridges are sustainable within the transportation network which they re-define. The

networks are sustainable within the regional economy. The integration is organic.

By integrating the engineering, economic, political and public aspects of infrastructure performance, sustainability can reconcile the seemingly incongruent constraints and restraints of their domains. Conversely, the absence of such reconciliation can be shown as unsustainable. Tracing sustainability considerations in the UK since 1999, [16] concludes that “a bridge manager’s decision-making process will be much more complex when account has to be taken of sustainability. ... A procedure for assessing lifetime sustainability is needed to help the manager make consistent and good decisions.”

Consistently with Lebesgue’s postulate, the terms robustness and resilience have qualified, and to a degree quantified, ‘performance-based’ structural design within the energy and time constraints of engineering specifications prior to introducing ‘sustainability’. Robustness is defined as the ability of a structure or network with an impaired resistance to redistribute its supply to meet the load demands of ‘extreme events’ in constrained time. In the explicit forms of redundancy and ductility, robustness redistributes and sustains the load demands in the time and space of defined assets and events. Bruneau & Reinhorn [17] define resilience as “the ability to prepare and plan for, absorb, recover from, and more successfully adapt to adverse events”. Hence, resilience describes the capacity of the network to deliver services over extended lifecycles.

Networks consist of assets and assets are networks of elements. Hence, sustainability implies robustness and resilience on both the individual and group levels in both the mechanical and financial domains. The terms are not ratable according to any qualifying or quantifying scale so far. FHWA [18] has advanced bridge management towards the standardizing and codifying of their assessments. Figure 5, proposed in Yanev [19] illustrates the following realistic lifecycle of the engineered asset(s) in the plane of energy / robustness and time / resilience, under ‘normal’ demands and an extreme event.

Current design specifications prescribe bridge performance in terms of strength, stability, ductility,

redundancy, and criticality. A performance may deteriorate over time at a variable rate, depending on many external and intrinsic factors. Ensuing disruptive ‘extreme events’ can be external natural disasters, or internal structural non-performance. Robustness is the structural capacity to survive the energy onslaught of extreme events at discrete times with residual functionality. Resilience pertains to the process of network response, not only in terms of energy, but also in terms of money and extra-monetary considerations over extended, but nonetheless foreseeable periods. At both project and network levels, robustness and resilience imply redistributing a constrained supply of resistance in response to an expanded demand, as do structural redundancy and ductility. The structural condition ratings illustrated in Fig. 2 imply a moderate decline of robustness on the project level, but the sufficiency ratings indicate deficient serviceability and hence, a waning sustainability. Under the incongruent engineering constraints, economic and political restraints, post-event recovery may reach a sustainable level of structural robustness and network resilience or merely restore a state preceding the next crisis.

Assuming, as in Eq. 1, that engineering and economics are constrained and restrained in the 2-D space defined by the ‘control parameters’ of energy and money, sustainability adds the ‘third dimension’ of time-space necessary for evaluating and managing infrastructure network performance. Figure 6 illustrates the 3-D space in which engineering, economics and politics can jointly manage a sustainable infrastructure. Rotating the axes of energy, money and time-space in the respective planes of engineering / economics, economics / politics, and engineering / politics obtains the new axes of robustness, resilience and sustainability. By integrating energy, time-space and money, the new ‘control parameters’ of the social and physical performance restrain engineering, economics and politics into collaborating. As ‘control parameters’ intelligence / information would contribute further complexity beyond the present scope. The separate sets of intelligence / information inherent in the Energy, \$ and Time-Space dimensions adopted by engineering, economics and politics can account for their occasional contradictions.

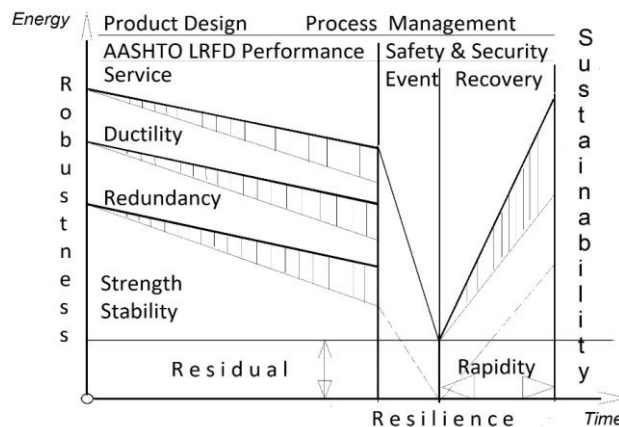


Figure 5. Robustness, resilience and sustainability of bridge products and network performance under routine and extreme service demands

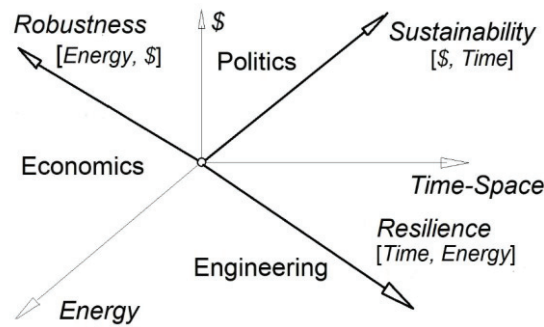


Figure 6. Control parameters of the infrastructure performance space: robustness, resilience and sustainability in engineering, economics and politics

### 6 Engineering and economic prioritization of maintenance and reconstruction

Bridge management operating options can be reduced most generally to maintenance and (re)construction. Under the disparate political, economic, engineering, and other restraints and constraints governing the process these options cannot be rigorously optimized. Most major failures are caused by more than one critical deficiency. Bridge-related hazards proliferate catastrophically due to deferred maintenance and delayed reconstruction, both of which are precipitated by shortages of money (and occasionally, information). Once arising however, they command funding allocation. Economic decisions take for granted and occasionally disregard the analysis of the engineering information during periods perceived as stable but rely on it incontestably in catastrophic extreme events. Between 1991 and 1998 the reported up to 3,200 annual flags could not have been addressed physically ground-up if top-down analysis had not reviewed and prioritized their urgency. However, the decline in structural condition ratings prior to 1987 and the corresponding mild increases in potential hazards were signaling the approaching instability. The following lifecycle model of bridge network supply of performance and demand for maintenance and rehabilitation illustrates the point.

If the condition of a bridge network  $R$  with total deck area  $A$  were in a 'steady state' from one year to the next, with ratings distributed close to uniformly along the scale, Eq. 4 should describe the equilibrium between their deterioration rate  $r$  and the quantity  $A_{rec}$  entering reconstruction annually. The ratio  $A / A_{rec}$  expresses the benefit / cost sustained by both the community in terms of service reduction and by the responsible owner in terms of construction costs. The improvements due to repairs without closures, discussed in Example 18 [9], are ignored herein.

$$A / A_{rec} = \Delta R_{rec} / r + n \tag{4}$$

where:

- $A$  is the deck area of the bridge network
- $A_{rec}$  - deck area entering reconstruction annually
- $R$  - average bridge condition rating on the NYS rating scale (7 - 1)
- $\Delta R_{rec}$  - average total change of  $R$  of  $A_{rec}$
- $r$  - annual rate of bridge deterioration ( $\partial R / \partial t$ )
- $n$  - average duration of reconstruction in years

Inspection records of the period suggested the following values:

$$A \approx 1,500,000 \text{ m}^2; r \approx 0.2 \text{ points}; \Delta R_{rec} \approx 4.5 \text{ points}; n \approx 3 \text{ years}$$

Substituting the preceding values in Eq. 4 obtains a condition rating equilibrium requiring  $A_{rec} \approx 0.04A = 58,824 \text{ m}^2$ . Reconstructing annually  $n(0.04A) = 0.12A \approx 170,000 \text{ m}^2$  is physically and economically unsustainable, hence tantamount to a network failure. Financially, at an average reconstruction cost of 10,000 \$US/m<sup>2</sup>, the demand for annual expenditures would amount to approximately \$US 600 million (1989) and the benefits would take  $n$  years to materialize. Inevitably, hazards must be mitigated as they arise. After the fatal accident at FDR on June 1, 1990, \$US 50 million were dedicated to addressing similarly rated bridge deck conditions, affecting half of the city bridges. Over 20 years, the four East River crossings were rehabilitated with partial traffic closures, each absorbing more than \$US 1 billion. By the year 2000 capital reconstructions reached the annual cost of \$US 600 million, effectively reducing annual flag numbers to the manageable 1200. In 2020 the annual budget of the agency approached \$US 1 billion, predominantly in reconstruction costs.

In Fig. 7 the potential hazard history of Fig. 3 is reduced to the polygon A - B - C - D - E. Since hazards and their mitigation expand the operating costs quantifiably while reducing the service both quantitatively and qualitatively, inverting their history can be regarded as reflective of the network's sustainability. Given sustainability's qualitative nature, it could be scaled to the anti-symmetric pattern illustrated by the dotted line of the polygon A' - B' - C' - D' - E'. Its path reflects the drop in the supply of services. Inverting the graph of Fig. 5 would produce a similar pattern.

The five stages described by the polygon A - B - C - D - E in Fig. 7, as well as the flag history in Fig. 3 can be considered as symptomatic of a catastrophic event. They include an apparent equilibrium of services and expenditures (A - B), expanding demands for corrective actions (B - C), a state of maximum demand (C - D), a decline in the demand (D - E), followed by a new equilibrium at a higher supply of services / demand for expenditures. A formal similarity is discernible between the potential energy of 'conservative' mechanical systems and the 'sustainability' of the bridge network described by the inverse polygon A' - B' - C' - D' - E' in Fig. 7.



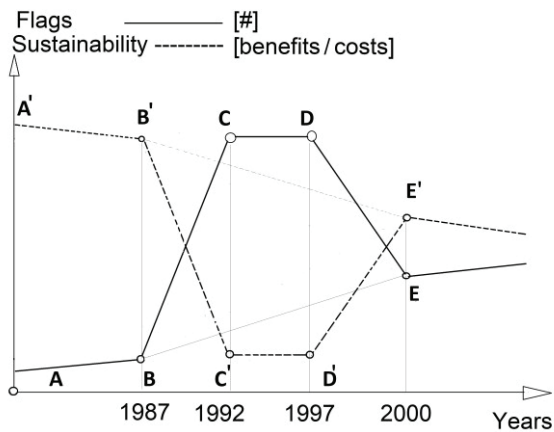


Figure 7. Potential hazards and respective sustainability of a bridge network (not to scale)

Since neither the quantifiable energy and money, nor the qualifiable intelligence and information of the network are conservative, infrastructure managers, their critics, and the media assess its condition in broadly varying terms, ranging from ‘challenging’ to ‘catastrophic’, with comparable conviction. Modifying the standards of the descriptive bridge condition and prescriptive potential hazard assessments in response to an infrastructure crisis would be a non-conservative equivalent of the fiscal manipulations attempting to stave off a financial collapse. Particularly salient are the following features of the Von Mises truss analogy:

- The potential energy of the conservative system equals the difference between the energy of the loads and that of the elastic structural deformation. By analogy, both increasing demand and reduced supply can cause instability. The established supply of and demand for network services and needs could be assumed as constant over relatively short periods, however in general, management should anticipate their growth.

- Under increasing load, single degree of freedom (SDOF) systems are stable while the slope of the potential energy is positive and unstable while it is negative. A declining system robustness (analogous to ‘stiffness’) is potentially unstable. The escalating demands related to potential hazards trigger the ‘extreme event’ of A’ – B’ – C’ – D’ – E’ in Fig. 7, however if they were avoided, the slope B’ – E’ still remains negative and hence, tends towards unsustainable.

- In a ‘snap through’ instability the two bars of the Von Mises truss do not buckle individually. The overall geometry of the 2-bar system becomes unstable due to ‘unsustainable’ elastic deformation. Thus, network instability can be due to the failing robustness of one critical link or to the decline in the overall resilience of the network (as in unsustainable traffic volume and structural safety demands).

The formal analogy between network sustainability and structural stability draws attention to the period denoted as A – B in Figs. 3 and 7, corresponding to 1982 – 1987 in Fig. 3,

as well as the period of steady decline preceding the ‘extreme event’ in Fig. 5. In terms of ‘snap-through’ instability, this is the period when potential energy is approaching instability due to ‘elastic’ deformation of the system. The bridge condition and sufficiency ratings of Fig. 2, and the declining robustness of Fig. 5 suggest that the resilience is approaching critical un-sustainability. Certain languages use the same word for ‘stability’ and ‘resilience’. Beyond point B in Fig. 7 the system is already in a catastrophic ‘extreme event’ when only emergency measures are appropriate. Also evident is that beyond a certain loss of robustness (Fig. 5) and sustainability (Fig. 7), full replacement becomes the only option.

Possible alternatives of operating costs for the considered period are plotted not to scale in Fig. 8. The corresponding numerical values in Table 3 are tentative and non-homogeneous because reconstruction is funded by federal, state and local sources, whereas maintenance was funded only locally at the time. The numbers include inflation. Nevertheless, they realistically quantify the monetary implications of the alternative strategies balancing reconstruction / maintenance, as well as the increase of total expenditures from initial to ultimate. The New York City Bureau of Bridges was founded in 1988 to a large extent in response to the looming bridge crisis. By then the ‘extreme event’ was in progress and the paths A – B (and A’ – B’) of Fig. 7 were physically unsustainable. Mitigating the hazards to the public was the emergency priority. By the year 1997 however, both options, denoted as 1 and 2 in Fig. 8, were viable. By 2000 the difference is distinct. Reconstruction and maintenance could continue along path 1 at the established ratio. Alternatively, preventive maintenance could be radically increased, reducing the demand for reconstruction over time, as in E’ and path 2. The Report [20] recommended dedicating 1% of the network’s replacement cost to annual maintenance, amounting to approximately \$US 100 million (2000). That maintenance should extend bridge life from 40 to 120 years, implying  $r \approx 0.067$ . The implied effectiveness of the investment in maintenance is notoriously prone to the ‘system imperfections’ of poor execution.

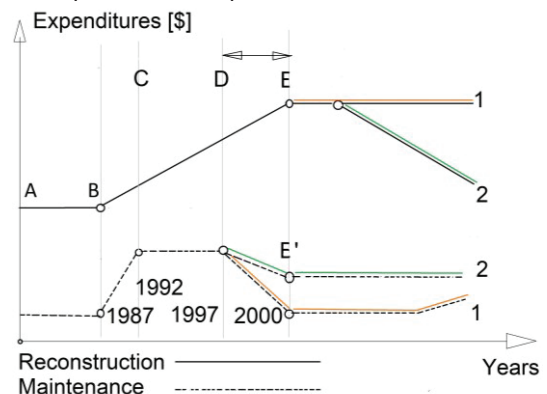


Figure 8. Alternative funding to the reconstruction and maintenance of a bridge network (not to scale)

Table 3. Hypothetical operating costs for the NYC bridge network (1987 – 2000) in million \$US

Year	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	Alt.
Reconstruction	90	100	110	120	130	140	160	180	200	250	300	400	500	600	600
Maintenance	10	20	50	50	60	60	60	60	60	60	50	50	50	50	100
Total	100	120	160	170	190	200	220	240	260	310	350	450	550	650	700

Given the high direct and user costs associated with reconstruction and hazard mitigation, the cumulative maintenance and reconstruction expenditures represented by path 2 offer superior long-term sustainability. For example, let more effective maintenance and reconstruction modify the terms in Eq. 4 as follows:

$$r \approx 0.1 \text{ points}; \Delta R_{rec} \approx 5; n \approx 2$$

Then  $A_{rec} / A \approx 0.02$ . Hence, the immediate investment in more effective maintenance reduces future reconstruction costs and improves community benefits two-fold. Improved long-term sustainability is popular during post-event recovery periods, however the demands of the same recovery preempt immediate investments in it. Moreover, costlier maintenance adds to the budget without reducing imminent reconstruction needs. Such is the \$US 50 million increase in the Alternative budgeting for the year 2000 (last column of Table 3). The customary administrative preference for reconstruction contracts over in-house maintenance similarly influences management choices. By 2008 NYC DOT raised bridge conditions above the NYS DOT [4] rating of 3 (not functioning as designed) primarily by intensifying reconstruction and struggles to maintain that dynamic equilibrium since.

## 7 Discussion

Using the 'stick-slip' terminology of mechanics, Umberto Eco [21] advises that "History is sticky and slippery. We must always keep in mind that tomorrow's catastrophes are secretly ripening today." In the present view, in order to be understood and managed, crises must be viewed and dimensioned integrally as current processes and products of past ones. Historically, infrastructure network management has advanced mostly after the 'disasters waiting to happen' happen. Possibly explicable in the cases of the relatively random natural disasters, that course appears irrational in 'extreme events' extended over time, as in the case of the predictable structural decline. Why infrastructure management fails to adopt stable strategies of prevention until the instabilities illustrated in Figs. 3 and 6 become unsustainable? An explanation is sought in the insufficiently scrutinized incongruence between the constraints and restraints of energy and money governing supply / demand in engineering and economics, illustrated in Fig. 1 and Eq. 1. If engineering, economics, politics, and popular sentiment fail to reconcile their different attitudes towards the common 'control parameters' of energy, money, and time, they shall continue to court catastrophes.

In an infrastructure network, a perfect balance of the physically constrained engineering quantities and socially restrained economic, political and other qualities is not only impossible, but unsustainable and unstable. Conservative mechanical structures can suffer from catastrophic instability unless their initial and deformed shapes are analyzed with respect to the potential energy of the acting loads. In a formal analogy, the services and performance of a transportation network, traditionally quantified in terms of money and energy can be also qualified in terms of a dynamic sustainability, expressed as a function of its robustness and resilience in space and time.

Bažant and Cedolin [14] caution: "The study of structural stability is often confusing because the definition of structural stability itself is unstable. ... one definition of stability – the dynamic definition – is fundamental and applicable to all structural stability problems. Dynamic stability analysis is

essential for structures subjected to nonconservative loads, such as wind or pulsating forces. Structures loaded in this manner may falsely appear to be stable according to static analysis while in reality they fail through vibrations of ever-increasing amplitude or some other accelerated motion." Bažant and Cedolin [14] reiterate that in inelastic systems instability can occur below the critical loads but may follow stable paths. Since a transportation network is neither 'conservative' nor 'elastic', the stability analogy is purely formal but usefully underscores the following critical imperatives of infrastructure management:

- The assets must be managed as a process in time, as well as a network of products in space.
- A sustainable process (as a stable structure) will depend at minimum on product robustness and process resilience, which in turn can be quantified in the traditional control parameters of energy and money. Implicit but critical, intelligence and information add significantly to the modes of instability.
- Given the 'energy' and 'money' dissipation characterizing the supply and demand governing an infrastructure network, a 'horizontal' slope of the modeled parameter a fortiori corresponds to a potentially unstable equilibrium.
- At losing stability, the Von Mises truss 'snaps through' from one stable state to a geometrically opposite one. It is assumed that its bars will neither buckle in compression nor rupture in tension. In an infrastructure network such dynamic transitions could be compared to rapidly escalating demands for money and energy, possibly exceeding the economic and productive capacity of the system and causing local failures.

## 8 Conclusions and directions

The pursuit of sustainability advances by defining it. A World Summit on Sustainable Development was held in Johannesburg in 2002, following a related event in 1992 at Rio de Janeiro. In 2010 the US Report [22] focused on sustainable development of chemicals, transport, mining, waste management, and sustainable consumption and production. In 2011 the Office of Sustainable Development at the United Nations (UNSOD) established 17 goals (SDGs), emphasizing least developed countries (LDCs). The subject can advance from well-intentioned general directives to specific tasks if 'sustainability' and its constitutive 'robustness' and 'resilience' are defined in consistent and accepted qualified and quantified terms. To that purpose, the present view reduces the scope to the management of a local transportation infrastructure of a metropolis. Even on that scale, an equilibrium of supply and demand in the disparate terms of energy and money cannot be rigorously established. A general analogy between mechanical instabilities and crises in other socially critical domains however can be discerned and qualified in terms of sustainability, robustness and resilience.

Over the considered period the robustness and resilience of a bridge network slid from stable through neutral equilibrium to potential instability, whereas the established equilibrium suggested no potential instability until public safety demanded emergency funding. Below a certain qualitative level, declining bridge condition ratings trigger an increase in potentially hazardous conditions and hence, an economic 'bifurcation' quantifiable in money. Direct and user expenditures sustained a loss of quality of life, until a relatively manageable stable state was reached at higher annual direct costs.

The reported flag forecast, as well as most current models of bridge condition deterioration nationwide, were and remain based on the 10 and the 7 – level qualitative condition ratings of [1] and [4]. The recent transition to the 4 quantified element condition states of [3], all of which can coexist in the same element (of a span or the bridge), introduces a critical discontinuity in the invaluable NBI database. Changes in the NYS DOT flagging procedure have had a similar effect. According to [5] non-structural conditions are no longer ‘flagged’ and utilities are treated separately. The forecasting reported herein would have been impossible without the preceding decades of consistent qualitative assessments. Duplication and re-distribution of effort have ensured most engineering successes, whereas their elimination (advertised as streamlining by fiscal-oriented management) has caused many failures. A single perfect condition assessment system does not exist. Management, as all other engineering branches, becomes robust, resilient and ultimately, sustainable, by relying, as much as possible, on redundant and complementary strengths.

Given the heterogeneous, inherently discontinuous information, a rigorous, universally applicable algorithm could not have been developed then nor is available currently. In the words of Von Neumann and Morgenstern [11]: “Even in sciences which are far more advanced than economics, like physics, there is no universal system available at present.” The authors highly recommend quantification but acknowledge its limitations. Engineering management must maximize reliance on science, but, particularly under severe constraints, has to produce art, as the French term *ouvrages d’art* implies. As extensively quantifiable as decision support might be, managers contribute, if at all, by executing qualitative decisions. No generic algorithm could have supported a legitimate budget request. There is no substitute for qualitative managerial expertise and soundly motivated choice. The element level condition states adopted on the federal and state levels in [3, 5] supply a valuable quantifier but not a substitute for the qualitative assessments. The complex information and far-reaching implications contained in the flag history of Fig. 3 demonstrate that no single system of parameters can fully capture the diverse and incongruent supply and demand inequalities governing the engineering and economic management. Isolated violations of the constraints in Eq. 1 may not be critical, whereas approaching their breach on a network scale would guarantee a crisis. Therefore, it becomes imperative to examine the available engineering and economic indicators in order to discern a potential crisis while it can still be averted, and if necessary, identify new such indicators and ‘control parameters’.

Since the reported period FHWA has acknowledged recognition by several innovations. Bridge life-cycle performance has become a central design consideration. Rehabilitation, repair and maintenance activities were integrated in Bridge Preservation [6], eligible for federal funding on par with capital reconstruction. (The repainting of a major bridge can cost hundreds of millions of \$US and hence, qualifies as a capital project.) As [18] suggests, condition assessments remain a work in progress. Since instabilities in non-conservative systems are even harder to identify, they should be precluded by broad margins.

The robustness and resilience of transportation networks are gradually gaining forms allowing for their qualitative & quantitative assessments. By integrating engineering, economic, and environmental criteria, they lend a manageable meaning to the ‘third dimension’ of

sustainability, beyond elementary cost-effectiveness. Sustainability, quantified and qualified in engineering and economic terms, becomes indispensable for managing infrastructure performance. Sustainable lifecycle strategies anticipate and prevent relapses to the potentially unstable conditions of NYC bridges in 1987. Unmanageable project-level losses of robustness and resilience become unsustainable and hence irreversible on the network level. System ‘imperfections’ near the points of instability invalidate routine expectations. Consequently, infrastructure planning requires at a minimum a 20-year horizon in order to anticipate and avoid the ‘poli-crises’ currently discussed at international gatherings of economic experts.

Figures 3 and 7 imply that post-extreme event equilibrium is attained at higher costs and hence, diminished sustainability. This phenomenon corresponds to the endemic inflation, the ubiquitous entropy, and the traditional lament for the ‘good old days’ when ‘things were better’. The network sustainability improves by expanding benefits and reducing expenditures, again reducing to superior robustness and resilience of engineering products and economic process.

In 2021 \$US 1.3 trillion were allocated by Act of Congress to rebuilding the national ‘hard’ infrastructure towards an unquantifiable but presumably sustainable level. Emphasized are the mythical “shovel-ready” projects, which President Barak Obama has called “nonexistent”. There is no explicit mention of the ensuing perpetual maintenance costs, however every new construction must imply a financially sustainable commitment to maintain the product and the process in robustly and resiliently performant condition over the designed useful life. Since the transportation infrastructure is part of the general social fabric, along with many other domains, such as the energy, chemical, natural resources, waste disposal, and social services, optimization invariably yields to prioritization, and can easily degenerate into emergency management by triage. The deadlocked negotiations over spending and national debt limits between the Legislative and Executive Branches of U. S. Government in 2023 demonstrate the political precarity that engineering and economics management must be able to neutralize. A qualitative and quantitative reconciliation of the energy and money supply and demand between these two essential infrastructure management domains would ensure the resilient, robust and sustainable management of the ‘hard’ infrastructure that society depends on.

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