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Technical paper

Numerical assessment of shear force redistribution in beam-slab systems due to incompatible kinematic assumptions

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ABSTRACT

In finite element analysis, shear force in a Timoshenko beam can be grossly underestimated if the beam is coupled with a thin (Kirchhoff) plate in the finite element mesh. Although common knowledge in the field of structural mechanics, this error is rarely warned about and frequently overlooked in practice, since most of the commercial software packages do not prevent the user from combining finite elements with incompatible kinematic assumptions. Moreover, thin plates are commonly offered as the default option, so the mistake needs to be actively avoided. This article demonstrates and investigates the issue through numerical and analytical parametric studies. Theoretical background is provided to explain the observed behavior. The error is confirmed to be strongly dependent on model geometry and mesh refinement. As a solution, it is recommended to always combine beam and plate theories with mutually compatible kinematic assumptions: Euler-Bernoulli with Kirchhoff and Timoshenko with Mindlin.

1 Introduction

In present-day engineering practice, designed or assessed structures are routinely represented by elaborate, three-dimensional computational models. These models, in majority of cases, assume elastic behavior of the materials and idealized boundary conditions. As Powell [1] points out, a model that looks like the actual structure when rendered, does not necessarily have the same behavior. In fact, even the most basic use of linear-elastic finite element analysis in civil engineering carries numerous pitfalls and requires basic theoretical knowledge and constant critical assessment of the results.

Floors, bridge decks, retaining structures, as well as other planar structural systems that resist out-of-plane loads in bending and shear, can be designed as beam-and-slab systems (Fig. 1). In structural analysis, each component of such a system is normally modeled with an element of an analogous type – beams are modeled as one-dimensional (line) elements, while the slabs are represented by two-dimensional (planar) elements of the computational model. Such approach is quite convenient, as it captures load paths and shear lag without additional effort and manual input. Still, every step of the modeling process has to be approached in a deliberate manner.

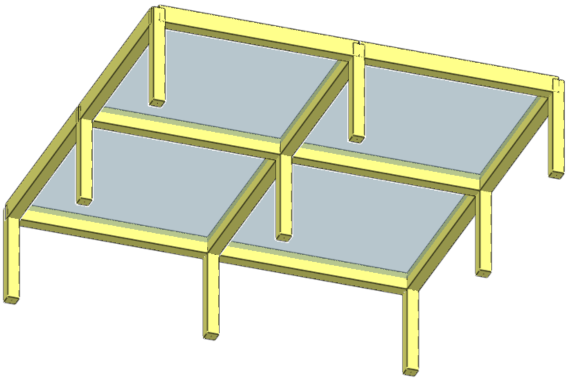

Figure 1. Floor slab with beams – Example of a computational model

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1.1 Description of the problem

The following observation was made: In beam-and-slab systems, modeled with beam finite elements of Timoshenko formulation [2] and thin plate elements (Kirchhoff theory [3]), shear forces in the beams tend to be incorrectly computed, the values being underestimated in all cases. The difference between the total and beam shear force is carried by the thin plate, which manifests through large values of transverse forces in the plate itself. The error can remain unnoticed for two reasons:

- Displacements and bending moments do not exhibit erroneous values as the shear forces do. Usually, response of a numerical model is critically assessed through these two parameters, as their distributions and values can be judged from intuition and experience. An error in shear force intensity is less likely to be noticed.
- Slabs supported by beams are normally not designed for shear, so plate shear forces can remain unexamined.

The computational error is eliminated if Bernoulli beams are used with thin plates, or Timoshenko beams with thick (Mindlin) plates.

1.2 Implementation of beam and plate theories in structural analysis software

Most of the commercial software tools for structural analysis implement the beam theory of Timoshenko for one-dimensional elements. This is the case with Radimpex Tower [4] and CSI SAP2000 [5], which were used for numerical analysis presented in Section 2. Although these programs do not offer Bernoulli beam as an option, its behavior can be emulated by artificially increasing shear area or appropriate stiffness modifiers of a Timoshenko beam.

Both mentioned programs offer two basic plate formulations: thin and thick, where thin refers to the Kirchhoff-Love theory of thin plates, and thick to the Mindlin theory, which includes the effects of transverse shear deformations [3]. Thin plates are offered as the default option in both programs. Autodesk Robot [6], another well-established program for analysis and design, offers thick plate formulation as the only option.

SAP2000 Basic Analysis Reference Manual [7] and CSI Analysis Reference Manual for SAP2000, ETABS and SAFE [8] explicitly recommend thick plate formulation for general use, except for special cases, where pure bending is sufficient to represent the structural behavior and/or a distorted finite element mesh is used. Problem of shear-locking of thick plates is briefly discussed, but there is no mention of the issue investigated in this article.

On the contrary, the Radimpex Tower 8 Instruction Manual [9] recommends thin plates for general use, while the thick plate formulation is suggested only for special cases such as thick foundation slabs.

None of the cited software manuals provide guidance on how to combine different types of beam and plate elements in the same model, nor do the programs impose any restrictions.

1.3 Research significance

For typical model geometries and levels of mesh refinement, underestimation of shear force in beams may amount to 30–50%, as is demonstrated in the following example (Section 2). If left undiscovered, the error carries over into the capacity check stage, which may result in severely underdesigned transverse reinforcement in concrete beams or web plates in steel beams. Considering the non-ductile nature of shear failure, it is clear that an error of this type may lead to detrimental consequences on safety of designed structures.

The modeling approach, where Timoshenko beams with unmodified geometric properties are coupled with thin plates, was encountered on numerous occasions in the authors' design and consulting practice and seems to be widely adopted in structural analysis of buildings and certain types of underground structures.

Although it may be considered trivial as a matter of structural mechanics, this phenomenon is poorly documented and generally disregarded in practice. Authors of this article were unable to find a textbook, paper, technical note or a software manual that contains an explicit warning about the described computational error and its consequences.

1.4 Aim, scope and methodology of the investigation

This article aims to provide an illustrative and systematic description of the computational error, caused by combination of shear-deformable beams with shear-rigid plates in finite element models, as well as practical guidance on how to avoid this error. No scientific contribution to the field of structural mechanics is intended. Rather, an important but neglected technical issue is reviewed using fundamental knowledge, accessible to practicing design engineers.

The problem was showcased and investigated by means of finite element method and fundamental beam theories:

- Numerical parametric study was performed on a structural model, to demonstrate the error on a practical example and to observe its general tendencies.
- The problem was simplified by establishing an analogy between thin plate theory and classical beam theory. Analytical parametric investigation, based on beam theories of Euler-Bernoulli and Timoshenko, was conducted to demonstrate the consequences of mutual kinematic incompatibility between connected finite elements. Behavior observed in numerical analysis of the beam-and-slab structure was replicated, confirming the given theoretical explanation of the computational error.

2 Numerical investigation

Analysis was performed using SAP2000 v14 and Radimpex Tower v8.5, and the results were cross-validated. Diagrams of deflections, forces and moments included in the following section were generated using Radimpex Tower.

2.1 Structural layout

A beam-plate structure with plausible proportions and loading was modeled to demonstrate the phenomenon (Fig. 2). Beam eccentricity, normally present in floor structures, does not affect the investigated issue, so it was not modeled for overall simplicity. Shape of the beam section (T-section may be used for bending design) also does not affect the behavior, so a simple rectangular section was adopted.

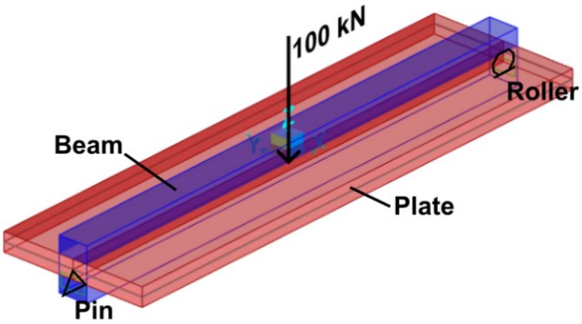

Figure 2. Beam-plate model (rendered view from SAP2000)

Table 1. General properties of the models

Beam cross-section (w/h)	40/70 cm
Plate cross-section (w/h)	200/25 cm
Span	6 m
Boundary conditions	Pin and roller
Loading	100 kN at midspan
Modulus of elasticity	34 GPa

Three variants of the described model were analyzed, using different beam and plate formulations (Table 2). T-K (Timoshenko-Kirchhoff) model is the one expected to produce the previously described computational error.

Table 2. Beam and plate formulations of the three analyzed models

Model	**Beam**	**Plate**
B-K	Euler-Bernoulli	Thin (Kirchhoff)
T-K	Timoshenko	Thin (Kirchhoff)
T-M	Timoshenko	Thick (Mindlin)
Note: Euler-Bernoulli beams were emulated in the software by increasing the shear area of Timoshenko beams to $1 \cdot 10^4$ m^2		

2.2 Reference solution

In order to critically assess the numerical results in the following parametric study, an analytical solution was derived. The entire beam-plate model was treated as a Bernoulli beam shown in Fig. 3. The total moment of inertia was calculated as the sum of the moments of inertia of the beam and plate cross-sections, shown respectively as blue and red rectangles in Fig. 3a. Shear force was calculated from static equilibrium, and the deflection was determined using the analytical formula for a simply supported Bernoulli beam with a point load. Moment and shear force are distributed between the beam and the plate in proportion to their respective bending stiffnesses. Deflection (w), total and beam bending moments ($M_{3,total}$ and $M_{3,beam}$), as well as total and beam shear forces ($V_{2,total}$ and $V_{2,beam}$) are listed in Table 3.

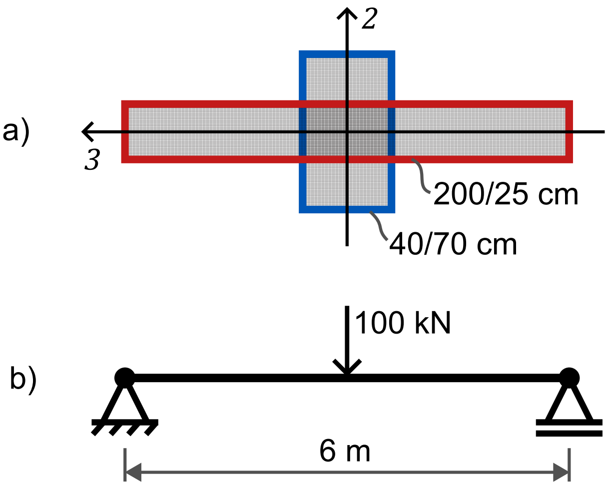

Figure 3. Beam model for the analytical reference solution: a) Cross-section; b) Structural system

Table 3. Analytical reference solution

Response parameter	**Value**
w	0.94 mm
max $M_{3,total}$	150 kNm
$V_{2,total}$	50 kN
max $M_{3,beam}$	122.2 kNm
$V_{2,beam}$	40.7 kN

Response of the numerical model B-K (Bernoulli-Kirchhoff) is expected to closely match the reference solution, while the T-M (Timoshenko-Mindlin) model should deflect slightly more, due to its deformability in shear.

It is worth noting that contribution of the slab to shear resistance would normally be neglected in traditional manual calculations, so the total force of 50 kN would be taken as the design value for the beam.

2.3 General behavior and mesh sensitivity

Results of numerical analysis of three models (Table 2) of the same structure (Fig. 2), with varying mesh refinement, are presented hereafter. Fig. 4 shows diagrams of vertical displacement (w), bending moment (M_3) and shear force (V_2) in the beam.

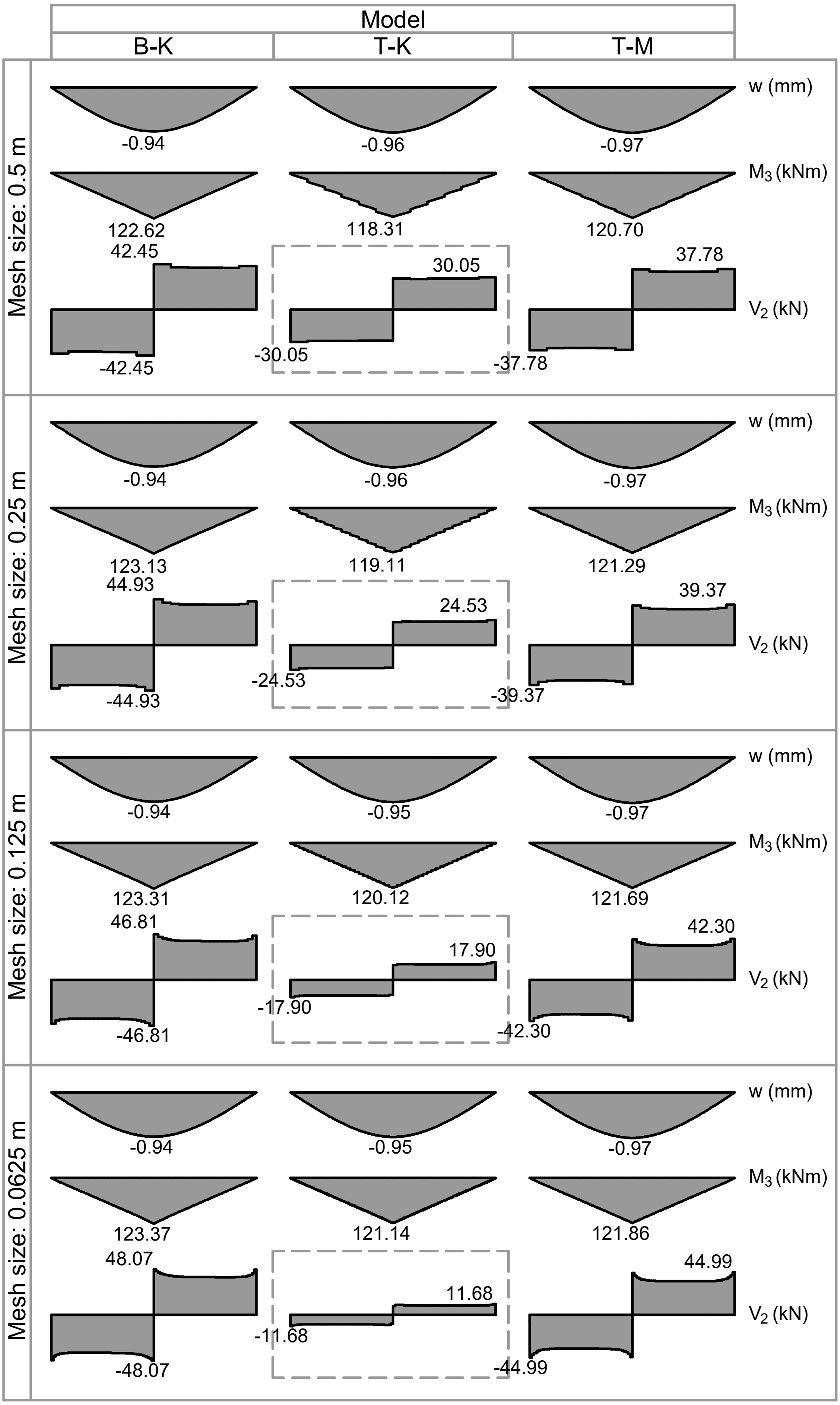

Figure 4. Results of numerical analysis for different mesh sizes: Deflection (w), moment (M_3) and shear force (V_2) diagrams of the beam (diagrams generated using Radimpex Tower)

Shear force $V_{z,x}$ in the plate, for 0.25 m finite element size, is shown in Fig. 5.

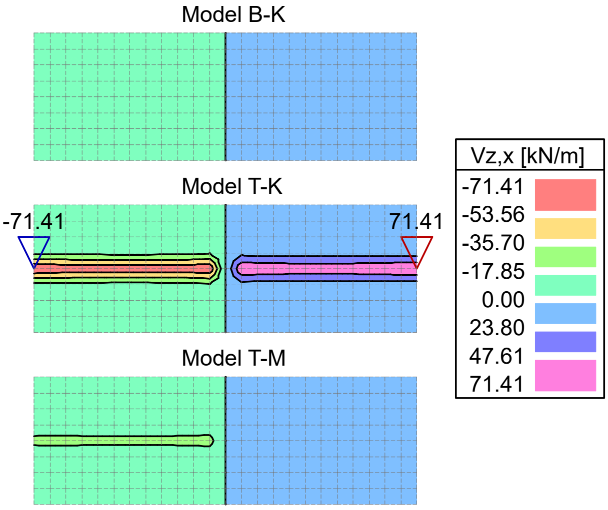

Figure 5. Transverse force $V_{z,x}$ in the slab for the mesh size of 0.25 m (contour plots generated using Radimpex Tower)

Note that, in the T-K model, reduction in the beam shear force V_2 is compensated by large values of $V_{z,x}$ in the plate, along the nodes that are shared with the beam.

Values of deflection (w), shear force (V_2) and midspan moment (M_3) of the beam, for each of the models, are shown as a function of mesh size in Fig. 6.

In Fig. 6b, mesh-dependent shear force peaks at the loading and support points were neglected, i.e. the lowest value across half of the span was taken from each set of results, as illustrated in Fig. 7.

It is evident, from the presented data, that results from the B-K and T-M models are numerically stable and consistent with the reference solution (Table 3). The B-K model exhibits slightly lower deflection than the T-M model, which is expected and in agreement with the reference solution (Table 3).

In the T-K model, strong mesh dependence of shear force redistribution between beam and plate can be observed (Fig. 4, middle column and Fig. 6b). As the mesh is refined, the beam carries a smaller, and the plate a larger portion of the total shear. For finite element size of 0.0625 m, the beam carries only one-fifth of the total shear force. Such behavior is, obviously, physically inadmissible.

Another artifact of the beam-plate incompatibility can be observed on the bending moment diagrams of the T-K model (Fig. 4, middle column). Moment concentrations, or ‘jumps’ in the diagram, are present at the nodes, where the beam is connected to the plate. Sign of the concentrated bending moments is consistent with the assumption about the imposed rotation compatibility between the beam and the plate, which is further discussed in Section 3.

2.4 Influence of plate thickness

As a second part of the parametric investigation, thickness of the plate in the model was varied from 5 to 30 cm, while all other quantities have remained constant. Finite element size is 0.125 m. Relation between plate thickness and beam response is shown in Fig. 8.

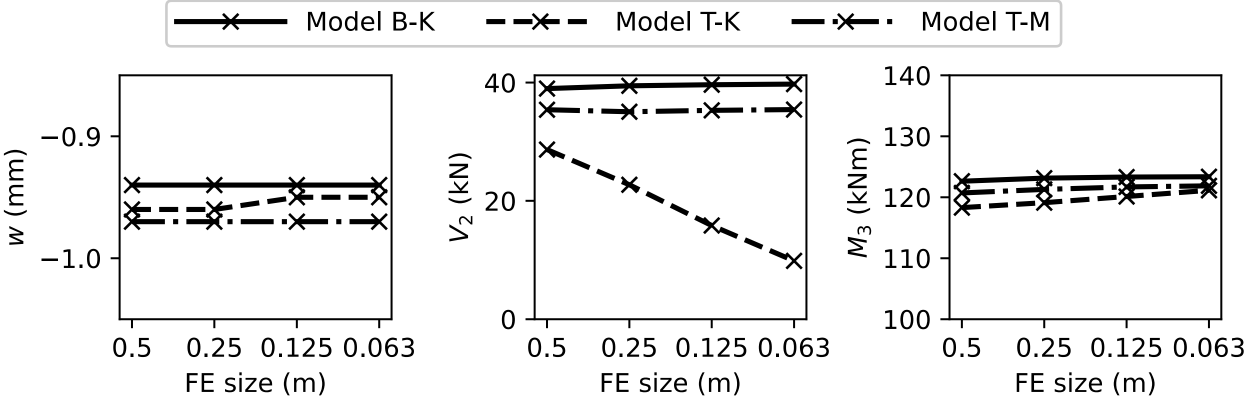

Figure 6. Mesh sensitivity of the solution: a) deflection; b) beam shear force; c) beam bending moment at midspan

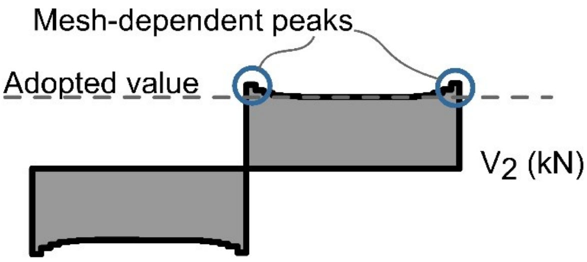

Figure 7. Reference value of shear force for model assessment

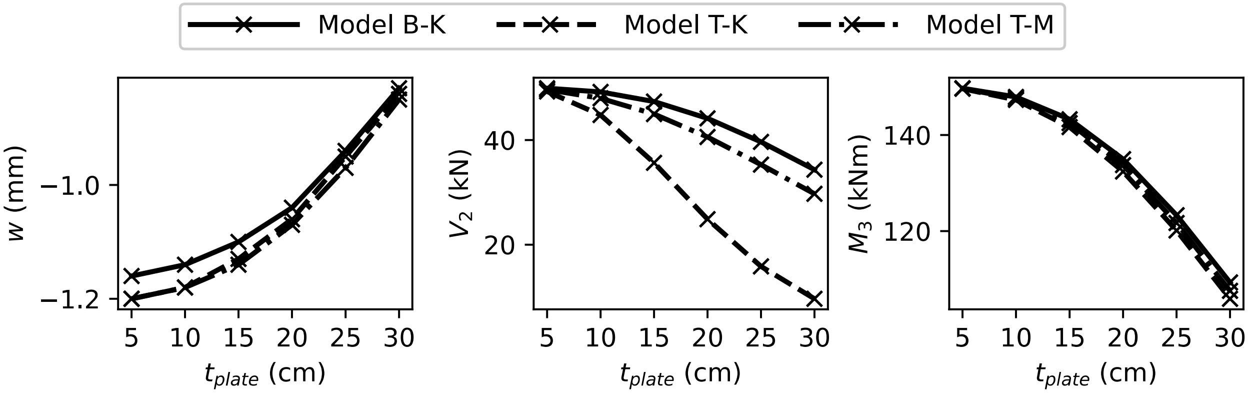

Figure 8. Influence of slab thickness on: a) deflection; b) beam shear force; c) beam bending moment

For the model T-K, but not for the other two models, it can be noticed that a relatively thicker plate leads to a disproportionately large error in shear force calculation. On the contrary, maximum values of deflection and bending moment seem to be accurate and consistent between the models.

3 Analytical investigation

While it is apparent that a combination of shear-deformable Timoshenko beams and shear-rigid Kirchhoff plates in a finite element mesh can lead to erroneous results, no direct explanation and description of the issue is available in literature. The following investigation aims to provide the theoretical background and demonstrate the problem in a more general and rigorous manner than the example given in Section 2.

To reduce the domain of the problem to the xz-plane, and simplify the analytical formulation, an equivalence can be established between the fundamental beam and plate theories. Both the Bernoulli beam theory and the Kirchhoff plate theory assume that the sections are plane and perpendicular to the deformed axis of the element (Fig. 9a). The same equivalence exists between the Timoshenko beam theory and the Mindlin plate theory, which take the shear deformation into account (Fig. 9b). The computational error due to mismatch of kinematic assumptions can also occur between two coupled beams, one of Bernoulli and the other of Timoshenko formulation (Fig. 10).

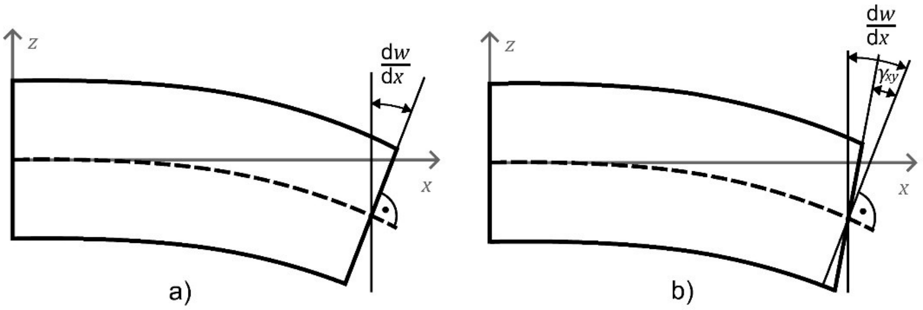

Figure 9. Deformation in the xz-plane of: a) Bernoulli beam or Kirchhoff plate; b) Timoshenko beam or Mindlin plate

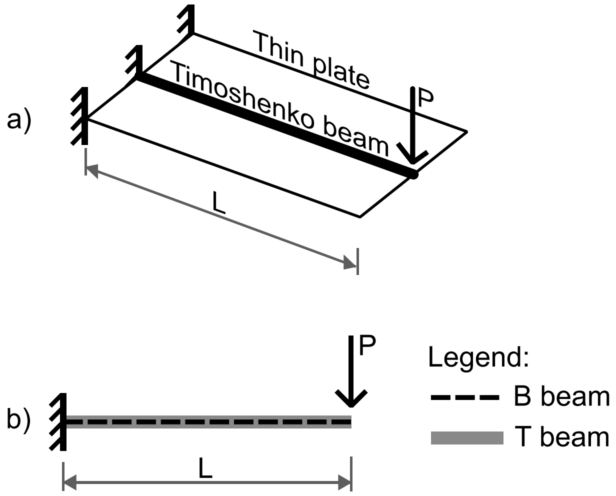

Figure 10. a) Beam-plate model loaded in shear and bending; b) Analogous beam model

Besides that, the conflict between a shear-rigid and a shear-deformable element occurs at every unrestrained node of the finite element model. Therefore, the error can be reproduced on a minimal model, consisting only of two nodes – one fixed and one free, and a pair of beam elements, one shear-rigid (Bernoulli) and the other shear-deformable (Timoshenko). A more complex model is an assembly of finite elements, and its overall behavior is governed by the element-level interactions. A computational error of this type, observed on the minimal example, is scalable to a more complex, discretized model.

The problem of parallel Bernoulli and Timoshenko beams is, to some extent and in a certain context, already covered in literature. Perelmuter and Slivker [10] discuss about the ‘paradox of coupling Bernoulli and Timoshenko bars in the same model’. The discussion concerns structural analysis of planar frames, where columns modeled as Bernoulli beams tend to carry a disproportionately large portion of the base shear, despite being more slender than strong columns of the same frame, intuitively modeled as Timoshenko beams. Analogy with the presently discussed issue is apparent, but the cited source is not concerned with beam-plate models. The following analytical study examines mesh dependence of the solution in a more elaborate manner, while relying only on fundamental theoretical concepts.

3.1 Parametric study

To reproduce the computational error within the analytical framework of fundamental beam theories, a minimal model of parallel shear-rigid and shear-deformable elements, loaded by a transverse force, was considered. There are two nodes in the model: node 1 and node 2. Node 1 is fixed, while node 2 is unrestrained. Two coincident beams, one of Bernoulli and one of Timoshenko formulation, span between the two nodes. Compatibility of displacements and rotations of the two beams is enforced at the nodes. Transverse force is applied at the node 2. The model is schematically represented in Fig. 11.

Rectangular cross section, 0.5 m by 0.5 m in size, was adopted for both beams in the initial configuration. The displayed results do not depend on the value of the modulus of elasticity E. Point load P is equal to 10 kN. Element length L is shown in every figure (Figs. 12–14). These particular values were chosen to facilitate clear and simple graphical representation of the results.

Three parameters of the response were monitored:

- Shear force in each of the beams, which is constant along their length (V);
- Bending moments at the support (M_1);
- Bending moments at the free end, which arise due to enforced compatibility of cross section angles of the two different beams (M_2).

Parametric analyses included variation of two parameters:

- Cantilever length (L);
- Ratio of cross-section heights of the two beams (H_B/H_T), controlled by variation of the Bernoulli beam cross-section height (H_B), while H_T remains constant.

In the following figures, values referring to Bernoulli and Timoshenko beams are labeled with letters B and T in the index.

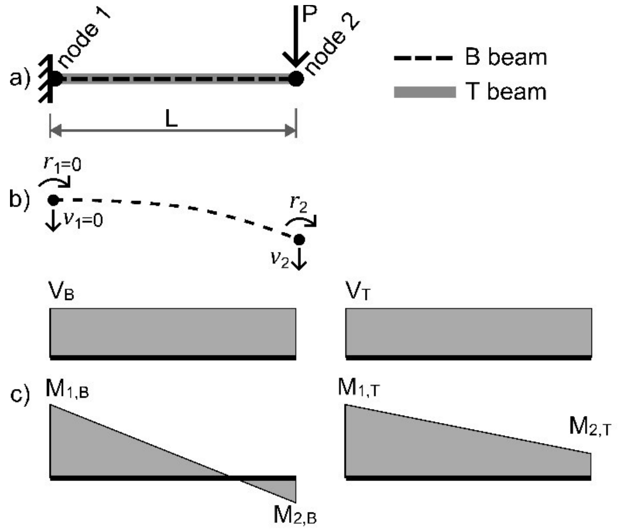

Figure 11. a) Structural system for analytical investigation; b) Degrees of freedom; c) Shear force and bending moment diagrams

A closed-form solution, based on conventional matrix analysis [11], was derived for the described elements and boundary conditions. Global stiffness matrix was assembled from stiffness matrices of Bernoulli and Timoshenko beams:

$$\boldsymbol{K}_B = \frac{EI_B}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{1}$$

$$\boldsymbol{K}_T = \frac{EI_T}{L^3(1+\Phi)}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & L^2(4+\Phi) & -6L & L^2(2-\Phi) \\ -12 & -6L & 12 & -6L \\ 6L & L^2(2-\Phi) & -6L & L^2(4+\Phi) \end{bmatrix} \tag{2}$$

Term Φ represents shear deformation parameter of the Timoshenko beam:

$$\Phi = \frac{12EI_T}{kGAL^2} \tag{3}$$

where k is the shear correction factor of the first-order shear deformation beam theory, which is, for rectangular sections, usually taken as 5/6 [12].

Vertical displacement and rotation of the free end (node 2), calculated from the equilibrium equations, can be written as:

$$v_2 = \frac{PL^3}{12K_1\left(1-\frac{3K_1}{4K_2}\right)} \tag{4}$$

$$r_2 = \frac{PL^2}{8K_2\left(1-\frac{3K_1}{4K_2}\right)} \tag{5}$$

where terms K_1 and K_2 are:

$$K_1 = EI_B + \frac{EI_T}{1+\Phi} \tag{6}$$

$$K_2 = EI_B + \frac{EI_T(4+\Phi)}{4(1+\Phi)} \tag{7}$$

I_B and I_T are moments of inertia of the cross sections of the Bernoulli and Timoshenko beams respectively.

Section forces in Bernoulli and Timoshenko beams are calculated from the displacement field. Shear force, which is constant along both beams, is equal to:

$$V_B = \frac{EI_B}{L^3}(12v_2 - 6Lr_2) \tag{8}$$

$$V_T = \frac{EI_T}{L^3(1+\Phi)}(12v_2 - 6Lr_2) \tag{9}$$

Moment at the fixed end (node 1) is:

$$M_{1,B} = \frac{EI_B}{L^3}(-6Lv_2 + 2L^2r_2) \tag{10}$$

$$M_{1,T} = \frac{EI_T}{L^3(1+\Phi)}(-6Lv_2 + (2-\Phi)L^2r_2) \tag{11}$$

Free-end (node 2) moments, which are of equal intensity and opposite signs, are:

$$M_{2,B} = \frac{EI_B}{L^3}(6Lv_2 - 4L^2r_2) \tag{12}$$

$$M_{2,T} = \frac{EI_T}{L^3(1+\Phi)}(6Lv_2 - (4+\Phi)L^2r_2) \tag{13}$$

Free-end moments M_2 are caused by imposed compatibility of cross-section rotation angles between the two elements with different kinematic assumptions. For the given configuration, the Bernoulli beam exhibits larger section rotation at the free end than the Timoshenko beam (refer to the Fig. 9). When compatibility is enforced at the node 2, mutually balanced bending moments of opposite signs are introduced at the free end of the beams. The 'jumps' in the bending moment diagrams, observed in Section 2 in the T-K model (Fig. 4), have the same principal cause.

3.2 Mesh dependence of the error

Influence of length (L) of the beams, while other parameters remain unchanged, was analyzed. Dependence of shear forces (V), support moments (M_1) and free-end moments (M_2) on element length is shown in Fig. 12. The solution is obtained using the presented equations. This parametric analysis should be analogous to the mesh sensitivity study presented in Section 2.3.

It can be observed that, if the beams are slender enough, difference in shear behavior has negligible influence, as the response is dominated by bending, so the B and T beams share the load equally, i.e. in proportion to their respective geometric properties.

As the elements are shortened (which is analogous to mesh refinement), shear force in the Timoshenko beam approaches zero, while the Bernoulli beam carries the total shear force. This is consistent with observations from the numerical analysis (Section 2).

3.3 Influence of beam proportions on the error

To examine how different proportions of shear-rigid (Bernoulli) to shear-deformable (Timoshenko) elements influence the error in shear force calculation, a second parametric analysis was conducted, which should reflect the numerical results from the Section 2.4.

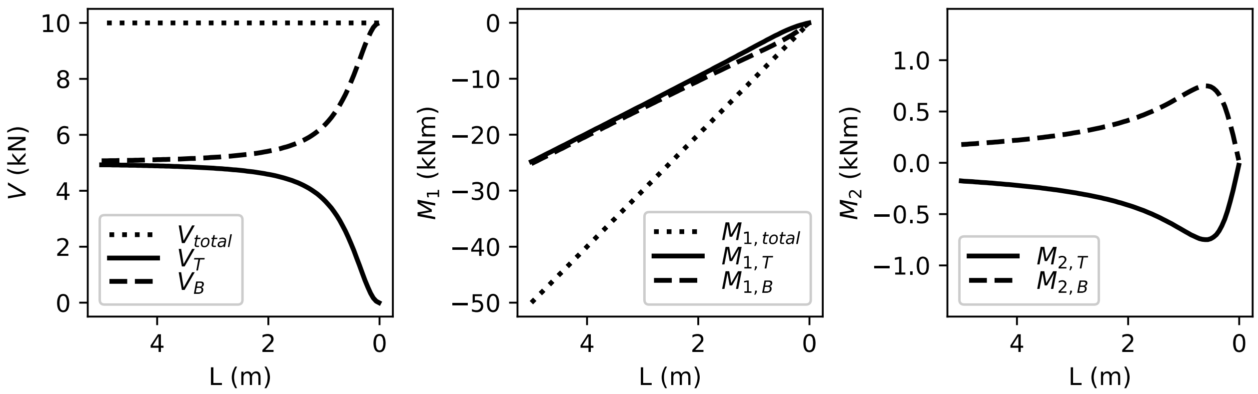

Figure 12. Influence of cantilever length on the response

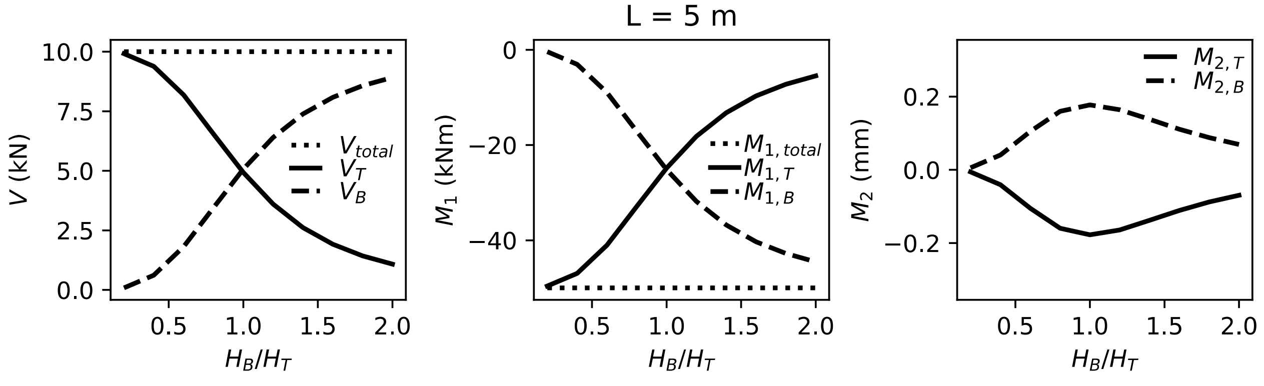

Figure 13. Cantilever length of 5 m: Influence of cross-section height proportions

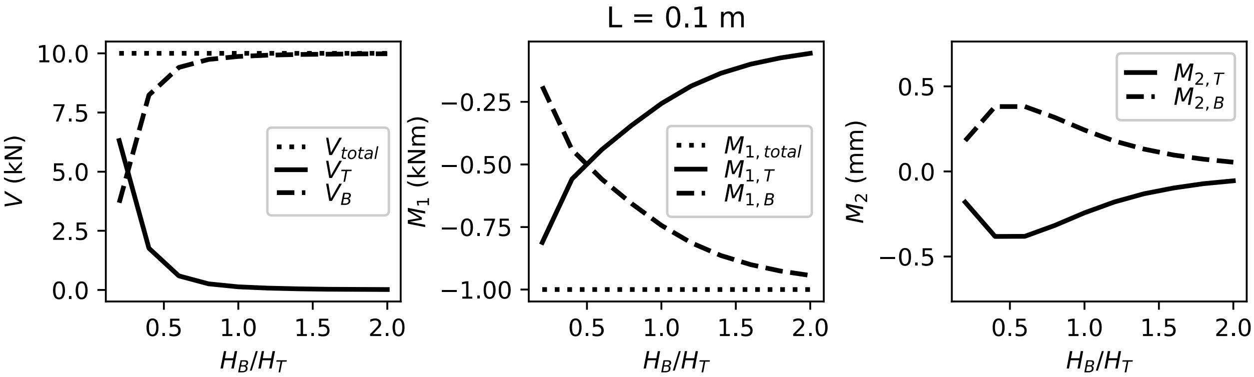

Figure 14. Cantilever length of 0.1 m: Influence of cross-section height proportions

For a relatively long cantilever (which would correspond to a coarse finite element mesh), redistribution of the shear force follows the relative change in bending stiffness, which is an admissible result for relatively slender beams.

However, if the beams (or the finite elements) are sufficiently short relative to their cross-section size, shear force in the Timoshenko beam quickly converges to zero as height of the Bernoulli beam is increased, while the Bernoulli beam carries the total shear by itself (Fig. 14). This makes the result size-dependent (or mesh-dependent in finite element analysis) and thus unreliable.

4 Conclusions

Finite element models, in which shear-rigid and shear-deformable line and/or planar elements are combined, always exhibit erroneous and mesh-dependent results in terms of shear forces. Models with Timoshenko beams and Kirchhoff plates are an example that is often encountered in practice. The difference in kinematic assumptions between the beam theory of Timoshenko and plate theory of Kirchhoff inherently leads to underestimated shear forces in beams and unrealistic shear force concentration in plates. Results obtained from such models are physically inadmissible and can lead to unsafe structural designs.

Magnitude of the error is dependent on geometric proportions of the finite elements, so it is less pronounced with coarse meshes and relatively thin plates. Refinement of the finite element mesh, which may be intuitively employed to improve accuracy of the results, actually worsens the computational error and leads to grossly unconservative values of shear forces in beams.

The authors conclude that there is no conceivable practical scenario where a combination of Timoshenko beams and Kirchhoff plates in a computational model of a beam-and-slab system would be necessary or desirable. In all cases, beam and plate theories with mutually consistent kinematic assumptions should be used (Bernoulli with Kirchhoff and Timoshenko with Mindlin). In such a way, the only gross error that could plausibly occur would be the underestimation of deflections due to inadequate application of a pure-bending (Bernoulli-Kirchhoff) model. Still, such an error would be rather predictable and much less critical than erroneous underestimation of shear forces in beams.

To prevent such errors, software tools for analysis and design could implement model checks and warnings, or automatic adjustment of beam properties, in order to apply consistent kinematic assumptions in the entire finite element mesh. Due to the different workflows that design engineers tend to use, the error should be avoided altogether, rather than mitigated in design modules of the software packages.

CRediT authorship statement:

Branislav Stupar: Conceptualization, Validation, Writing – Review & Editing, Investigation; **Jelena Jotanović**: Supervision, Writing – Review & Editing; **Nikola Tomić**: Visualization, Writing – Original Draft, Software, Investigation, Methodology, Data curation.

Declaration of competing interest:

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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