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Application of VaR (Value at Risk) method on Belgrade Stock Exchange (BSE) optimal portfolio

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Abstract

The main objective of this study is to determine the adequacy of the measurement of market risks of financial institutions in Serbia by the method of Value at Risk (VaR). For investors, in the current global financial crisis, it is particularly important to accurately measure and allocate risk and efficiently manage their portfolio. Possibility of application of VaR methodology, which is basically designed and developed for liquid and developed markets, should be tested on the emerging markets, which are characterized by volatility, illiquidity and shallowness of the market. Value of VaR in this study was calculated using historical and parametric methods and backtesting analysis was used to verify the adequacy of the application of VaR models. Backtesting VaR model performance analysis was conducted to compare the ex-ante VaR estimate to the ex-post returns. The empirical results show that parameter exponentially weighted moving average model gives lower values at risk in both cases (95% and 99%) due to the fact that this method assigns weights to more recent returns while our portfolio is exposed to a lower volatility in recent time. Based on the results of Kupiec's and Christoffersen's test, it was observed that VaR estimates obtained by both, parametric and historical simulation, give a good prediction of market risk, at 95% and 99% confidence level.

KEYWORDS: market risk, optimal portfolio, Value at Risk (VaR), backtesting, EWMA, GARCH models, historical simulation

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Introduction

Banks, investment funds and other financial institutions often use the concept of value at risk (VaR) as a measure of market risk. Value at risk is the assessment of the maximum loss in value of portfolio over a given time horizon at a given confidence level. Based on the VaR financial institutions are able to determine the level of capital that provides cover losses and ensure the financial position of extreme market movements (Mladenović, Miletić,Miletić, 2012).

In the global financial crisis conditions for investors are extremely important to accurately measure and allocate risk as well as to efficiently manage their portfolio. The influence of extreme events on the trends in financial markets in emerging countries is even more pronounced, since it is a market characterized by lower levels of liquidity and significantly smaller market capitalization. Financial markets in emerging countries are usually characterized by a number of reforms and greater likelihood of internal and external shocks such as inflation, a sudden depreciation of national currencies, changes in credit ratings, risk premium change, etc. As this market is characterized by a greater influence of internal trade and consequently a higher degree of volatility than the markets of developed countries, the distribution of returns is significantly more distorted than normal, which makes evaluation of VaR with standard methods that assume a normal distribution of returns more difficult. Application of VaR methodology, which is basically designed and developed for liquid and developed markets, is necessary to test on emerging markets that are characterized by extreme volatility, illiquidity and the shallowness of the market. Implementation of the VaR methodology in the investment process is directly related to the selection of appropriate method of estimation. In selecting the appropriate method of key importance is that it accurately determines the likelihood of losses (Mladenović, Miletić,Miletić 2012).

There is now a huge and increasing literature on value-at-risk. Some selected papers are reviewed here. Almost all researchers are unanimous that there is no single approach or a VaR model that is optimal in all markets and in all situations. According to previous published studies, models of VaR models based on moving averages give a good prediction of market risk, and that results vary depending on the loss function that was used, the chosen level of confidence VaR, the period for which the survey was conducted (turbulent or normal), used model for assessing the VAR and etc.

For example Degiannakis (2004) conclude that different technique of volatility are applied with different goals and objectives, and that the modeling of time varying volatility is necessary for estimating the VaR. Linsmeier and Pearson (1996) conclude that there is no simple answer to the question which VaR model gives better estimates of market risk given the fact that volatility is not constant but varies over time. Different statistical characteristics such as volatility clustering, flattening and asymmetry may affect the calculation and selection of appropriate model of VaR. Although most commonly used method of Risk Metrics assumes normality of distribution of returns, numerous empirical studies show that the distribution of returns is not normal. Thus, value of VaR obtained assuming normal distribution underestimate the true value of market risk. (Duffie et al., 1997).

Mladenović, Miletić and Miletić (2012) considers adequacy of VaR models in selected emerging economies with the daily returns of Bulgarian (SOFIX), Croatian (CROBEX), Czech (PX50), Hungarian (BUX), Romanian (BET) and Serbian (BELEX15) stock exchange indices before and during the financial turmoil. Authors conclude that GARCH type models with t error distribution give better 5% and 1% VaR estimation in comparison to normal error GARCH type models. Authors emphasize that GARCH type models for most confidence levels are not outperformed by EVT approach and estimations derived from POT.

Miletić and Korenak (2013) analyse the effectivenes of GARCH models in estimating Value-at-Risk for MSCI World Index, one of the most widely known benchmark for global stock funds, before and during the financial crisis. Daily returns of stock market index MSCI is analysed during the period Jun 3, 2002 to March 22, 2013 in respect. Authors aaplied symmetric GARCH and asymmetric GARCH models, as VaR forecast models. The performance of the VaR is assessed by Kupiec test unconditional coverage which represent the most famous test in this group. Results of backtesting show that assesed Value-at-Risk for EGARCH model is adequate for both confidence level according to Kupiec test for pre crisis period. On the other hand, EGARCH (2,1) model used for calculating VaR with 99% confidence level acording to Kupiec test seems to be adequate if we assume both normal and Student's t distribution of returns. At the same time, EGARCH (2,1) model did not pass Kupec test at 95% confidence level with assumption that residuals follow normal and Student's t distribution. Since, Basel Committee prescribes testing VaR model adequacy at 99% confidence level, at these confidence level our results show that VaR calculation based on EGARCH model is adequate measure of downside risk.

In this study we compared the assessment of VaR obtained by historical simulation method and parametric methods. Before we approached the calculation of the value of VaR we structured optimal portfolio of financial institution using Markowitz's modern portfolio theory, which is the most widely used and accepted model for determining the basic policy of investment in securities, portfolio composing, and mitigation of risks.

The main objective of this study was to determine the adequacy of different types of VaR methods for measuring market risks in Serbian financial market, where the value of VaR is calculated using historical and parametric methods as well as backtesting analysis used to verify the adequacy of these models.

The paper is structured as follows. In the second chapter two approaches often used to measure VaR are reviewed. In the third chapter methodology of backtesting is reviewed. Results of empirical analysis as well as results of backtesting are presented in the fourth chapter. Concluding remarks are given in the fourth chapter.

1. Methodological framework

1.1. Defining the concept of value at risk (VaR)

VaR is a measure that gives the maximum loss that can be realized from certain investments over a given time horizon (usually 1 day or 10 days), with a certain probability (most of this chapter is based on Jorion, 2001). Mathematically, VaR for the period of the *k* day in day *t* can be represented as follows:

$$P(P_t - P_{t-k} \leq VaR(t,k,\alpha)) = \alpha, \qquad (1)$$

where P_t is the price of a particular type of financial asset, and α represent a given level of probability.

VaR can be expressed in terms of a percentile of the return distributions. Specifically, if q_α is the α th percentile of the continuously compound return, VaR is calculated as follows:

$$VaR(t,k,\alpha) = \left(e^{q_\alpha} - 1\right)P_{t-k} \qquad (2)$$

Previous equation implies that a good estimate of VaR can only be produced with accurate forecast of the percentiles, q_α, which is obtained on the corresponding volatility modeling. Therefore, below we discuss the value of VaR for a series of returns.

Define a one-day return on day t as:

$$r_t = log(P_t) - log(P_{t-1}) \qquad (3)$$

For the time series of return rt, VaR can be expressed as:

$$P(r_t < VaR_t \mid I_{t-1}) = \alpha \qquad (4)$$

From this equation it follows that finding the VaR values is the same as finding a 100α% conditional quantiles. Formally, it is possible to develop models for the stock returns r_t as follows:

$$r_t = \mu_t + \epsilon_t,\ \epsilon_t = \sigma_t\eta_t,\ \mu_t = \mu(I_{t-1}|\theta),\ \sigma_t^2 = \sigma^2(I_{t-1}|\theta) \qquad (5)$$

where I_{t-1} is a set of information available at time *t-1*, and where μ and σ are functions of a certain dimensional vector of parameter values θ. In this model ϵ_t is innovation, σ_t is the unobserved volatility, and η_t is martingale difference sequence satisfying:

$$E(\eta_t \mid I_{t-1},\theta) = 0,\ V(\eta_t \mid I_{t-1},\theta) = 1 \qquad (6)$$

As a consequence, we have:

$$E(r_t \mid I_{t-1},\theta) = \eta_t,\ V(r_t \mid I_{t-1},\theta) = \sigma_t^2,\ \epsilon_t \mid I_{t-1} \sim D(O,\sigma_t^2) \qquad (7)$$

where $D(O,\sigma_t^2)$ represents the conditional distribution with zero mean value and variance σ_t^2.

If the return can be modeled by a parametric distribution, VaR can be derived from the distributional parameters. Unconditioned parametric models were determined with $\mu_t = \mu$ and $\sigma_t = \sigma$. Therefore we assume that returns are independent and equally distributed with a given density function:

$$f_r(x) = \frac{1}{\sigma} f_{r^*}\left(\frac{x-\mu}{\sigma}\right) \qquad (8)$$

where f_r is density function of distribution of r_t and f_{r^*} being density function of the standardized distribution of r_t.

Below we present the most commonly used parametric models that enables VaR estimates, the exponentially weighted moving average model (EWMA) and the conditional volatility models (GARCH type models).

1.2. The exponentially weighted moving average model (EWMA)

Since the JP Morgan 1994th RiskMetrics model was developed to measure VaR, VaR calculated in this way becomes a benchmark measure of market risk in practice. The starting assumption of RiskMetrics model is that returns of a certain type of financial assets have a conditional normal distribution with arithmetic mean zero and variance, expressed as the value of the exponential weighted moving average historical rate of squared values of return. However, in practice it was confirmed that the distribution of returns of financial assets generally deviates from the normal, i.e. has heavier tails, so the assessments of VaR obtained by this model are biased. Second, in many empirical studies (see for example Ding et al., 1993; So,2000) it was observed that returns of different types of financial assets are characterized by long memory, which is reflected in the assessment and prediction of market volatility.

RiskMetrics VaR model evaluation assumes a dynamic model of exponentially weighted moving average (EWMA) of the variance:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)(r_t - \mu)^2 \qquad (9)$$

In order to initialize a recursive equation of variance, the sampling variance is used:

$$\hat{\sigma}_t^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_t - \hat{\mu})^2 \qquad (10)$$

where, following RiskMetriks system, the value of the parameter λ is 0.94 for daily data and 0.97 for monthly data. The parameter λ is called a smoothing parameter which determines the exponentially declining weighting scheme of the observations. The smaller λ, the greater the weight is given to recent return data. Exponentially weighted moving average model can be represented as:

$$\sigma_t^2 = (1-\lambda)(r_{t-1}^2 + \lambda r_{t-2}^2 + \lambda r_{t-3}^2 + \cdots) \qquad (11)$$

If it is assumed that the conditional distribution of returns is normal with mean value zero and variance σ_t^2, then the one-day VaR on day *t* is obtained as follows:

$$VaR_t = \hat{\mu} + z_\alpha\hat{\sigma}_t = \hat{\mu} + \Phi^{-1}(\alpha)\hat{\sigma}_t \qquad (12)$$

wherez_α is 100α percent of *N(0,1),* respectively $\Phi^{-1}(\alpha)$ is the inverse distribution function of standardized normal random variable.

However, if returns are characterized by Student's t distribution with mean value zero, then the value of one-day VaR is calculated:

$$VaR_t = \hat{\mu} + st_{\alpha,v}\hat{\sigma}_t = \hat{\mu} + t_v^{-1}(\alpha)\,\hat{\sigma}_t\sqrt{\frac{v-2}{v}} \qquad (13)$$

where$st_{\alpha,v}$ being left quintile at α% and t being the distribution function for the Student's t distribution with the estimated number of degrees of freedom *v*.

1.3. GARCH type models

The GARCH type of models successfully captures several characteristics of financial time series, such as thick tailed returns and volatility clustering (Tsay, 2010). This type of models represents standard and very often used approach for getting VaR estimate. A general GARCH (p,q) model proposed by Bollerslev (1986) can be written in the following form:

$$y_t = \alpha_0 + \sum_{i=1}^{m} a_i y_{t-1} + \varepsilon_t - \sum_{j=1}^{n} b_j \varepsilon_{t-j} \quad (14)$$

$$\varepsilon_t = z_t \sigma_t, \qquad \varepsilon_t | I_{t-1} \sim N(0, \sigma_t^2),$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \quad (15)$$

$$\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0$$

The first equation actually describes the percentage level of return, $y_t = 100 * r_t$, which is presented in the form of autoregressive and moving average terms, i.e. ARMA(m,n) process. Error term ε_t in the first equation is a function of z_t, which is random component with the properties of white noise. The third equation describes the conditional variance of return y_t, which is function of ε_t^2 of *q* previous periods and conditional variance of *p* previous periods. The stationarity condition for GARCH (p,q) is $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$.

In many applications with high frequency financial data the estimate for $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j$ turns out to be very close to unity. This provides an empirical motivation for the so-called integrated GARCH(p,q), or IGARCH(p,q), model (see Bollerslev et al. (1994).In the IGARCH class of models the autoregressive polynomial in previous equation has a unit root, and consequently a shock to the conditional variance is persistent in the sense that it remains important for future forecasts of all horizons. A general IGARCH (p,q) process can be written in the following form:

$$\sigma_t^2 = \alpha_0 + A(L)\varepsilon_t^2 + B(L)\sigma_t^2, A(L) + B(L) = 1 \quad (16)$$

where A(L) and B(L) are lag operators.

In order to capture asymmetry Nelson (1991) proposed exponential GARCH process or EGARCH for the conditional variance:

$$\log(\sigma_t^2) = \alpha_0 \sum_{i=0}^{\infty} \pi_i \, g\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) \quad (17)$$

Asymmetric relation between returns and volatility change is given as function $g\left(\frac{\varepsilon_t}{\sigma_t}\right)$, which represent linear combination of $|\frac{\varepsilon_t}{\sigma_t}|$ and $\frac{\varepsilon_t}{\sigma_t}$:

$$g\left(\frac{\varepsilon_t}{\sigma_t}\right) = \theta\left(\left|\frac{\varepsilon_t}{\sigma_t}\right| - E\left|\frac{\varepsilon_t}{\sigma_t}\right|\right) + \gamma\left(\frac{\varepsilon_t}{\sigma_t}\right) \quad (18)$$

where ϑ and γ are constants.

By construction, equation is a zero mean process (bearing in mind that $z_t = \varepsilon_t/\sigma_t$). For 0 $< z_t < \infty$, g(z_t), is linear function with slope coefficient $\theta + \gamma$, while for $-\infty < z_t \leq 0$ it is linear function with slope coefficient $\gamma - \theta$. First part of equation, $\theta(|z_t| - E\,|z_t|)$, captures the size effect, while second part, $\gamma(z_t)$, captures the leverage effect.

Zakoian (1990) proposed TGARCH (p,q) model as alternative to EGARCH process, where asymmetry of positive and negative innovations is incorporated in the model by using indicator function:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q}(\alpha_i \varepsilon_{t-i}^2) + \sum_{i=1}^{q}\gamma_i d\,(\varepsilon_{t-1} < 0)\varepsilon_{t-i}^2 + \sum_{j=1}^{p}(\beta_j \sigma_{t-j}^2), \quad (19)$$

where γ_i are parameters that have to be estimated, *d(·)* denotes the indicator function defined as:

$$d\,(\varepsilon_{t-1} < 0) = \begin{cases} 1, & \varepsilon_{t-1} < 0 \\ 0 & \varepsilon_{t-1} \geq 0 \end{cases} \quad (20)$$

TGARCH model allows good news, $(\varepsilon_{t-i} > 0)$, and bad news, $(\varepsilon_{t-1} < 0)$, to have differential effects on the conditional variance. For instance, in the case of TGARCH (1,1) process, good news ha san impact of α_1, while bad news has an impact of $\alpha_1 + \gamma_1$. For $\gamma_1 > 0$, the leverage effect exsists.

APARCH (p, q) process, proposed by Ding, Granger and Engle (1993), includes seven different GARCH type models (ARCH, GARCH, AGARCH, TGARCH, TARCH, NGARCH and Log-GARCH):

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^{q}\alpha_i(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{p}(\beta_j \sigma_{t-j}^\delta), \quad (21)$$

where $\alpha_0 > 0, \delta \geq 0, \beta_j \geq 0, j = 1, \cdots, p, \alpha_i \geq 0, -1 < \gamma_i < 1$, and $i = 1, \cdots, q$. Parameter δ in the equation denotes exponent of conditional standard deviation, while parameter γ describes asymmetry effect of good and bad news on conditional volatility. Positive value of γ means that negative shocks from previous period have higher impact on current level of volatility, and otherwise.

Based on estimated parameters of GARCH type process, it is possible to make forecast of $\hat{y}_t(k)$ and conditional volatility $\hat{\sigma}_t^2(k)$ for next *k* periods [for details see Mladenović et al. (2006)]. Forecasted value of return and the conditional volatility for the next period is obtained as follows:

$$\hat{y}_{n+1}(1) = \alpha_0 + \sum_{i=1}^{m} a_i y_{n+1-1} + \varepsilon_t - \sum_{j=1}^{n} b_j \varepsilon_{n+1-j} \quad (22)$$

$$\hat{\sigma}_n^2(1) = \alpha_0 + \sum_{i=1}^{q}\alpha_i \varepsilon_{n+1-i}^2 + \sum_{j=1}^{p}\beta_j \sigma_{n+1-j}^2, \quad (23)$$

If residuals z_t follow standardized normal distribution, VaR at 95% confidence level could be calculated as:

$$\hat{y}_t(1) - 1.65\hat{\sigma}_n(1),$$

while if residuals z_t follow standardized t^s distribution with v degrees of freedom, then VaR could be calculated as:

$$\hat{y}_n(1) - t^s\sqrt{\frac{v-2}{v}}\,\hat{\sigma}_n(1) \quad (24)$$

1.4 Historical Simulations Model

Historical model calculation of VaR does not use the assumption of certain types of distributions, but uses actual data from the past (Barone-Adesi, Giannopoulos 2001). The main advantage of historical simulation is non-parametrical or non-existence of assumptions regarding the distribution of portfolio returns as this model seems appropriate for inefficient markets. Historical simulation is based on the assumption that the returns are independently and identically distributed. IID assumption is based on the theory that the returns are periodically mutually uncorrelated which means that the return of one period does not depend on the return of the previous period. This assumption is consistent with the theory of market efficiency, where the present price of securities reflects all information relevant to the price of those securities. If price changes depend only on the new information, which means that it can not be predicted and therefore it will be time uncorrelated (Neftci, 2004).

There are several ways in which VaR can be calculating using historical simulation of the basic principle. The first step in carrying out historical simulation is the collection of sufficient historical data on gains and losses or returns for portfolios that are going to be used in order to conduct historical simulations.

Considered portfolio which is made up of *N* securities, and for each security there are observations for each of the *n* periods (e.g. days) in the historical sample, it will have a simulating return in period *t*:

$$Returnt = \sum_{i=1}^{N} W_i \, R_{i,t} \quad (25)$$

where: W_i-share in assets currently invested in securities and, $R_{i,t}$-return on security *i* during the period *t*.

Previous formula gives the simulated historical series of returns for the portfolio and thus serves as a basis to calculate the VaR using historical simulation. The resulting series returns will differ from the actual return earned on the portfolio because the actual composition of the portfolio changes over time. Simulated historical returns represent returns that portfolio achieved if the investor has changed his portfolio at the end of each working day in a manner to ensure that each of securities always have the same relative share of the portfolio.

The obtained returns are applied to the histogram and from the histogram the value VaR for a desired level of risk is read. Depending on the desired level of risk, the *n*-th greatest loss is taken for the value of VaR with the predetermined probability.

In applying the historical method to calculate the VaR in emerging markets and economies that are in transition, a significant limitation is the length of time-series data that is available. This problem is particularly acute in countries with a short history of the market economy, where securities are not listed on stock exchange long enough to be able to calculate the VaR for long periods of time.

Empirical studies have shown that series of portfolio shares on the capital market are characterized by the existence of heteroscedasticity and autocorrelation between returns (Radivojević et al., 2010). The assumption that returns are independently and identically distributed is unrealistic due to the fact that volatility varies depending on time and that the time periods are grouped by high and low volatility. Because of these shortcomings of the standard historical simulation approach developed are weighted models that in different ways process the returns in order to remove autoregression and serial correlation between variables and transferred them into independent and equally distributed returns. The most famous modifications of historical simulation include time weighted model and volatility weighted model.

Time weighted model was developed by Boudoukh, Richardson and Whitlaw (1998). This model of historical simulation by observed returns from recent past gives relatively high weights that decline exponentially over time, and their sum is 1.

$$w_{t-i-1} = \lambda w_{t-i}$$

$$\sum_{i=1}^{N} w_{t-i} = 1 \quad (26)$$

where; w_i portfolio returns in period i and λ is the decay factor.

Exponential weighting is done in such a way that the exponential decay factor λ assigns a value between 0 and 1, and w (1) is the weight of recent historical portfolio return. Observation that precedes the new observation will get the latest factor w (2), which is w (2) = λ x w (1). Third return will get the weight λ ^ 2 x w (1), and so on until the nth number of observations. Once the weights are assigned to the observed returns the value of VaR is calculated using the empirical distribution of returns adjusted for the assigned weights (Boudoukh, Richardson,Whitlaw, 1998).

Volatility weighted model is developed by Hull and White (1998). The basic idea was to adjust the changes of volatility in historical returns which occurred in the recent past. To predict the VaR for day t the most important return is used $r_{i,t-1}$, and conditional volatility $\sigma_{t,i}$, obtained through EWMA or GARCH method. The obtained amount of anticipated volatility at time t, $\sigma_{t,i,,}$ represents a multiplier with which historical returns $r_{i,t}$are multiplied with at time t and weighted with the necessary volatility analyzed by EWMA or GARCH method in time t (Hull,White 1998) .

$$r_{i,t}^{*} = \sigma_{t,i} \times \frac{r_{i,t}}{\sigma_{t,i}} \quad (27)$$

where: $r_{i,t}^{*}$ is weighted volatility return.

Weighting the returns in this way historical losses are increasing or decreasing depending on the current market volatility. In order to assemble a histogram of historical return volatility weights are used instead of the actual returns. In order to account for the accumulation of volatility in forecasting future volatility is useful to use the EWMA or GARCH models, given that both models consider the current variance as a function of the previous square variance of past returns.

2. Backtesting

Backtesting represents a statistical procedure by which losses and gains are systematically compared to the appropriate valuation of VaR. In the backtesting process it can be statistically examined if the frequency exceptions, during the selected time interval, are in accordance with the chosen confidence level. These types of tests are known as tests of unconditional coverage. The most famous test in this group is the Kupiec test.

In theory, however, a good VaR model not only shows the correct amount of exceptions, but exceptions that have been evenly distributed over time, i.e. that are independent from each other. Grouping of exemptions indicate that the model does not register changes in the market volatility and correlation in the correct manner. Conditional coverage test, therefore, examines conditionality and changes in data over time (Jorion 2001). The most commonly used test of this group is the Christoffersen independence test.

2.1. The Kupiec test

The Kupiec test, known as the proportions of failures test (POF), measures whether the number of exemptions is consistent with a given confidence level (Jorion 2001). If the null hypothesis is true, then the number of exemptions follows the binomial distribution. Therefore, to implement the POF test it is necessary to know the number of observations (n), the number of exceptions (x) and confidence level.

The null hypothesis of the POF test is:

$$H_0: p = \hat{p} = \frac{x}{T}$$

The basic idea is to determine if the observed excess rate $\hat{p}$ is significantly different from p, excess rate determined by the given confidence level. According to Kupiec (1995) POF test is best implemented as a likelihood-ratio test (LR). The statistical test has the following form:

$$LR_{POF} = -2\ln\left(\frac{(1-p)^{T-x}p^{x}}{\left[1-\left(\frac{x}{T}\right)\right]^{T-x}\left(\frac{x}{T}\right)^{x}}\right) \tag{28}$$

If the null hypothesis is correct, LR_{POF} statistics in asymptotic conditions has χ^2 distribution with a single degree of freedom. If the value of LR_{POF} statistics exceeds the critical value of χ^2 distribution, the null hypothesis is rejected and the model is considered to be imprecise.

2.2. The Christoffersen independence test

Christoffersen (1998) uses the same idea of the credibility test as Kupiec, but extends the test by introducing separate statistical values for independent exceptions. In addition, this test observes if the probability of exceptions on any day depends on the outcome of the previous day.

Let n_{ij} be defined as the number of days when an outcome j occurs assuming that event i occurred on the previous day. Besides that, let π_i represents probability of observing an exception conditional on state i on the previous day (Nieppola, 2009):

$$\pi_0 = \frac{n_{01}}{n_{00}+n_{01}}, \pi_1 = \frac{n_{11}}{n_{10}+n_{11}} \text{ i } \pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}} \tag{29}$$

If the model is correct, the exception that occurs today should not depend on exception that occurred on the previous day. In other words, if the null hypothesis is true, probabilities π_0 and π_1 should be equal.

Independence test of the exceptions is best implemented as a likelihood-ratio test (LR). The statistical test has the following form:

$$LR_{ind} = -2\ln\left(\frac{(1-\pi)^{n_{00}+n_{01}}\pi^{n_{10}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{n_{11}}}\right) \tag{30}$$

LR_{ind} also has asymptotically χ^2 distribution with one degree of freedom. If the value LR_{ind} is lower that the appropriate critical value of the χ^2 distribution, the model passes the test. On the contrary, the value of LR_{ind} statistic greater than the critical value implies that the model is rejected.

3. Empirical Analysis

The first step of the empirical analysis examines the characteristics of stocks that are continuously traded on the BSE, and that constitute the index Belex-15. This study used a sample time series of daily rates of return of these stocks for the period of 18.10.2005 - 21.10.2011.

The next step in the empirical analysis is measuring of market risk by analytical and historical VaR model. Backtesting analysis based on Kupiec and Christoffersen test was conducted for comparison of VaR estimates with actual values of returns.

During the construction of the portfolio, we applied the Markowitz theory of optimal portfolio choice. Comparing the features and characteristics of the optimal portfolio and BELEX-15, including the measurement of market risk through VaR, we tested the adequacy of the application of modern portfolio theory. According to Markowitz the optimal portfolio theory stocks that have the most optimal combination of expected return and standard deviation should be chosen (higher expected return and a lower standard deviation, the greater the expected return and the same standard deviation or the same expected return and a lower standard deviation (Markowitz 1959) This means that in the optimal portfolio, stocks that have the lowest coefficient of variation should be chosen. By applying the mentioned criteria the optimal portfolio is made up of following stocks: Veterinarski zavod Subotica (VZAS), Tehnogas (TGAS), Imlek (IMLK), Aik banka (AIKB).

Comparing the statistical characteristics of the optimal portfolio under the assumption of equal participation of selected stocks in the portfolio with Belex-15 stock index, one can conclude the following:

- accumulation of volatility observed in the case of both series (which can be seen from the graph – Fig. 1);
- correlogram and Ljung-Box statistics for residuals and square values of residuals indicate the existence of autocorrelation of returns in the case of the two series, which implies that the value of return in one period depends on the actual return values in the previous period;
- distribution of rates of return of both series deviates from the normal, elongated, which means that it has heavier tails and more likely to exercise extreme return values (Fig. 2 and 3). Indication of kurtosis in the case of both analyzed a series of more than three, which shows that if the realized profit / loss, it is higher than in the case of stocks whose rates of return are normally distributed. Kurtosis is higher than normal, which is usual characteristics of financial series. This is indicated by the Jarque-Bera test which value suggests rejection of the null hypothesis, which assumes a normal distribution. The standard deviations of both series of returns are extremely high, especially when compared with an average rate of return.

The average rate of return of the optimal portfolio for the analysed period was positive, although very low, unlike Belex-15 index, which is the negative. Also, the return distribution of optimal portfolio is closer to the normal distribution, i.e. has a lower coefficient of skewness (closer to 0) and a lower coefficient of kurtosis (closer to 3). Therefore, all indicators with the exception suggest that standard deviation of the optimal portfolio has a better performance than the index Belex-15.

At the risk of 1% ARCH - LM test indicates the presence of ARCH effects for both series of returns (the results of this test are shown in Table 1).

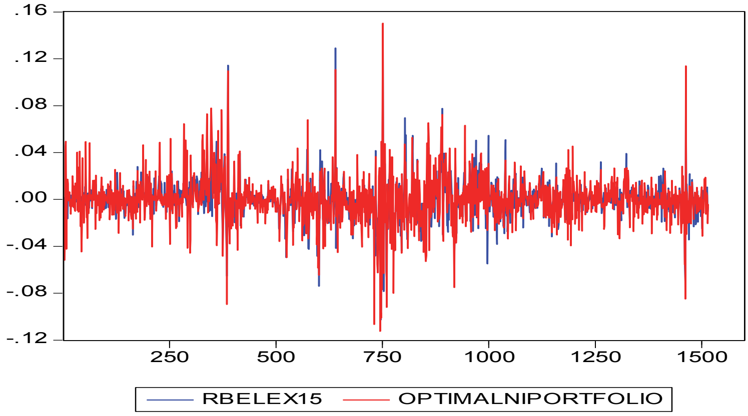

Fig.1: Rates of returns of the optimal portfolio and Belex-15 index

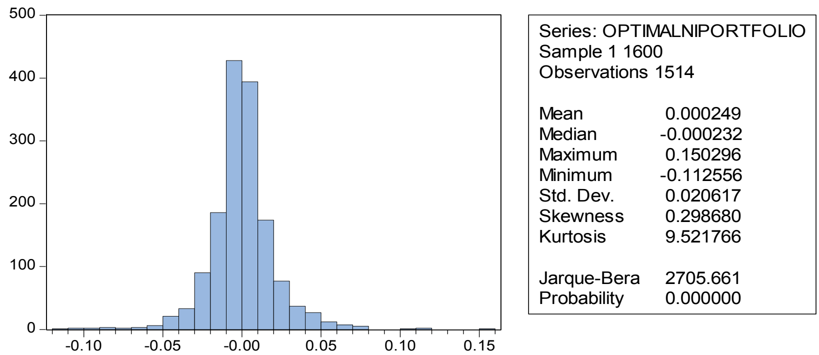

Fig. 2: Statistical characteristics of the optimal portfolio returns

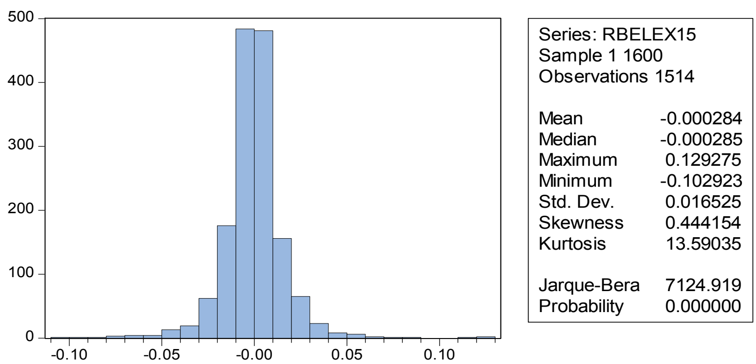

Fig. 3: Statistical characteristics of the Belex-15 index returns

Table 1: ARCH – LM return test of the optimal portfolio Belex 15 index

	OPTIMAL PORTFOLIO	BELEX15
F STATISTIC	197,8577 (0,000)	140,0955 (0,000)
$N*R^2$	175,1806 (0,000)	128,3781 (0,000)

Source:Author's calculations.

Since the optimal portfolio gives higher returns and is less likely to generate greater losses, we started measuring the market risk for the optimal portfolio using historical simulation and parametric VaR model that assumes a normal distribution, as well as using GARCH methodology, assuming that residuals followed by Student's t distribution.

Parametric VaR measurement method, which assumes a normal distribution of returns requires knowledge of the mean value, the returns and their variance and covariance matrix, which is estimated by two different approaches: with equal weights (EW) and the method of exponentially weighted moving average (EWMA).

The observed period of holding a portfolio is one-day and ten-day. For liquid markets adequate evaluation of the VaR is one day due to the constantly changing of the prices, while Basel I recommends ten day (Basel Committee on Banking Supervision, 1996). VaR values are given for two intervals of confidence - 95% and 99%.

The results of the analysis using parametric VaR can be interpreted as follows: in 95% of cases not more than 3.21% of the portfolio value will be lost through the method of equal weights. In 95% of cases not more than 2.18%, will be lost if method of exponentially weighted moving average is applied (Table 2).

We notice that the method of exponentially weighted moving average gives lower values of risk in both cases. The main reason is that the method of exponentially weighted moving average assigns more weights to recent returns as well as our portfolio is exposed to lower volatility in recent times. It should be noted that the value of VaR using parametric method was calculated for $\lambda = 0.94$.

Table 2:Value of the parametric model VaR of the optimal portfolio

	Equal weights method		The method of exponentially weighted moving average	
Confidence level	1 day	10 days	1 day	10 days
95%	3,21%	10,16%	2,18%	6,91%
99%	4,54%	14,37%	3,09%	9,77%

Source:Author's calculations.

In the next step, we used the GARCH methodology to assess the value of VaR. GARCH (1.1) model with assumption of a normal distribution and student t distribution has a satisfactory statistical properties. In the estimated model there is no presence of ARCH effects and autocorrelation in the squared values of residuals. Residuals of the estimated model, however still does not have a normal distribution, but the asymmetry and higher kurtosis of the distribution are less pronounced than in the portfolio. The presence of asymmetric effects is not observed. Series of returns of the optimal portfolio is best described by ARMA (2,1) specification, assuming normal distribution of residuals, or ARMA (1,1) specification assuming the student's t distribution.

Table 3: Parameter estimates of the GARCH model

	GARCH (1,1) Normal distribution	GARCH (1,1) *Student's t* distribution
Mean equation		
AR(1)	-0.608 (0.000)	0.503 (0.000)
AR(2)	0.176 (0.000)	
MA(1)	0.745 (0.000)	-0.396 (0.000)
Variance equation		
c	0.233 (0.000)	0.174 (0.000)
α	0.238 (0.000)	0.209 (0.000)
β	0.713 (0.000)	0.769 (0.000)
Specification tests		
Q2(30)	32.747 (0.48)	30.216 (0.654)
ARCH (10)	5.07(0.886)	6.09 (0.807)

Source: Author's calculations.

Based on estimated parameters by GARCH type models we make forecast of returns and conditional volatility to obtain VaR estimates.

Table 4:VaR estimates based on GARCH models

	GARCH (1,1) Normal distribution	GARCH (1,1) *Student'st* distribution
Forecast returns	0.542	-0.064
Forecast conditional variance	5.437	4.750
VaR (1, 0.95)	3.306	3.350
VaR (1, 0.99)	4.891	5.839

Source: Author's calculations.

The maximum daily loss obtained by GARCH type models at 95% confidence level is about 3.3%.

We calculated the value of VaR using historical simulation. Historical method does not assume any type of return distribution. The only assumption is that the returns are independent and identically distributed. The main advantage in comparison with the previous method is the ability to customize the extreme values of return. Returns are sorted in a rising way starting from the largest negative values. We used the empirical distribution as a proxy true distribution generating returns and, finding returns that provide return with cumulative distribution function equal to 1 - a confidence interval. In order to find the true value of VaR, we used linear interpolation. In doing so, we assume that the weights of all the rates of return are equal, but that recent returns have higher weights.

Table 5: VaR estimates of the optimal portfolio based on historical simulation method

	The method of equal weights		The method of exponentially weighted moving average	
Confidence level	1 day	10 days	1 day	10 days
95%	2,71%	8,57%	2,39%	7,56%
99%	4,99%	15,80%	3,38%	10,69%

Source: Author's calculations.

Compared with the parametric method at the 95% confidence level, value of VaR under the assumption of equal weights is smaller by the method of historical simulation, while at the 99% confidence level value of VaR is higher.

The results obtained by the method of historical simulation assuming equal weights suggest that in 95% of cases maximum daily loss is about 2.71% of the portfolio, and in 99% of cases maximum daily loss is about 4.99% of portfolio value (Table 5). Historical simulation method under the assumption that recent returns have higher weights gives a lower VaR estimate in relation to the assumption of equal weights, but higher mark than the parametric VaR assuming that recent returns have higher weights.

Backtesting VaR model performance analysis is carried out with the aim of comparing ex-ante VaR estimate the ex-post returns. This analysis was performed for the last 200 observations returns.

Based on the results of Kupiec's and Christoffersen's test (Table 6), it was observed that VaR estimates obtained by both, parametric and historical simulation, give a good prediction of market risk, at 95% and 99% confidence level.

Table 6:Backtesting results of parametric VaR-a and historical simulation method

	Parametric method		Historical simulation method	
Test	EW	EWMA	EW	TW
Kupiec 95%	3.199	0	3.199	0.451
Kupiec 99%	0.619	3.208	0.619	0.438
Christoffersen 95%	0.26	1.06	0.26	0.67
Christoffersen 99%	0.01	0.26	0.01	0.09

Source: Author's calculations. Note: Critical values of the test for the 95% and 99% confidence level are amounted to 3.84 and 6.635, respectively.

Table 7: Backtesting results of a parametric VaR using GARCH models

Test	GARCH (1,1) Normal distribution	GARCH (1,1) Student's t distribution
Kupiec 95%	4.857*	4.857*
Kupiec 99%	0.619	4.020
Christoffersen 95%	0.16	0.16
Christoffersen 99%	0.01	0.01

Source: Author's calculations. Note: Critical values of the test for the 95% and 99% confidence level are amounted to 3.84 and 6.635, respectively.

Kupiec's test shows that the VaR score obtained by GARCH model is inadequate for the 95% confidence level and overestimate risk value, while with the 99% confidence level, this score gives good predictions of market risk. According to Christoffesren's test VaR estimates obtained by GARCH type models are satisfactory even at the 95% confidence level.

Conclusion

The main objective of this analysis was the measurement of market risk for stocks that constitute the index Belex-15 on the basis of which the hypothetical optimal portfolio of financial institutions is constructed. The optimal portfolio is constructed of the following stocks: Veterinarski zavod Subotica (VZAS), Tehnogas (TGAS), Imlek (IMLK), Aik banka (AIKB).

Comparing the statistical characteristics of the optimal portfolio under the assumption of equal participation of selected stocks in the portfolio with Belex-15 stock index we can observe common characteristics such as the accumulation of volatility and the existence of autocorrelation. The standard deviations of both series of returns are extremely high, especially when compared with an average rate of return. The average rate of return for the observed period in the optimal portfolio is positive, although very low, unlike Belex-15 index, which is the negative. Also, the optimal portfolio distribution of return has a lower coefficient of skewness (closer to 0) and a lower coefficient of kurtosis (closer to 3). Therefore, all indicators of the descriptive statistics with the exception of the standard deviation suggests that the optimal portfolio has a better performance than the index Belex-15. At the risk of 1% ARCH - LM test indicates the presence of ARCH effects for both series of returns.

Since the optimal portfolio gives higher returns and is less likely to generate greater losses, we started measuring the market risk applying historical simulation VaR method, parametric VaR assuming a normal distribution, as well as using GARCH methodology, assuming that residuals follow Student's t distribution.

The empirical results show that parameter exponentially weighted moving average model gives lower values at risk in both cases (95% and 99%) due to the fact that this method assigns weights to more recent returns while our portfolio is exposed to a lower volatility in recent times.

In the case of application of GARCH methodology in assessing the value of VaR, GARCH (1.1) model with assumption of a normal distribution and student t distribution have satisfactory statistical properties. Series of returns of the optimal portfolio is best described by ARMA (2,1) specification, assuming normal distribution of residuals, or ARMA (1,1) specification assuming the student's t distribution. Results show that with the significance level of 95% this model gives higher values of VaR than in the case of exponentially weighted average method.

Backtesting VaR model performance analysis is carried out with the aim of comparing ex-ante VaR estimate the ex-post returns. Based on the results of Kupiec's and Christoffersen's test, it was observed that in the case of VaR obtained using parametric and historical simulation model at the 95% and 99% confidence level the observed models give a good prediction of market risk.

The research results may have important implications for investors and risk managers who operate in the turbulent markets of developing countries. However, the main limitation of this study is that empirical research referred to only Serbian capital market so that its results cannot be generalized to other emerging markets.

In the future research we will try to overcome these limitations by using a larger sample of returns at the capital markets of new EU member states and accession countries to join the EU, which represent an attractive market for foreign investors.

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