IK EQUATIONS & MODELING FOR SCARA ROBOTS

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Abstract — The paper presents the backward kinematics analysis, modeling equations for a 4 axis robot.

Index Terms — SCARA Robot, Computer Control

I. INTRODUCTION

Robotics is an interdisciplinary field that mixes various engineering disciplines such as electrical, electronics, instrumentation, mechanical, computer science, control engineering, mathematics, communications and many other fields into one. In this, work a unique 4 axes system is designed and fabricated with indigenous components with a brief kinematic analysis of the designed robot. The kinematically modeled designed robot is used for performing some PNP operations and was named as a Selective Compliance Assembly Robot Arm (SCARA). The primary motive behind the work was to develop a modular educational robotic system, the CRUST 2002 (Computerized Robotic Unit with Selective Tractability system) with the help of locally available components and sub-systems as shown in Fig. 1 and also to develop a user friendly GUI to control it [1].

Fig. 1. The designed SCARA robot

II. PHYSICAL STRUCTURE DESIGN

A four axis / four DOF designed SCARA robot arm as shown in Fig. 1. A SCARA robot is a 4 DOF stationary robot arm having base, elbow, vertical extension and tool roll and consisting of both rotary and prismatic joints [2]. There is no tool yaw and tool pitch (only tool roll) [1]. There are 4 joints, 4 axis (three major axes - base, elbow, vertical extension and one minor axis - tool roll). The 4 DOF's are given by Base, Elbow, Vertical Extension and Tool Roll, i.e., there are three rotary joints and one prismatic joint. Since $n = 4$; 16 KP's are to be obtained and 5 RHOFC's are to be attached to the various joints [3].

The vector of joint variables is a combination of θ and d , i.e., $q = \{ \theta, d \}^T$.

Vector of joint variables are $q = \{ \theta_1, \theta_2, d_3, \theta_4 \}^T$.

Vector of joint distances are

$$d = \{ d_1, 0, d_3, d_4 \}^T = \{ 877, 0, d_3, 200 \}^T \text{ mm.}$$

Vector of link lengths are

$$a = \{ a_1, a_2, 0, 0 \}^T = \{ 425, 375, 0, 0 \}^T \text{ mm.}$$

Vector of link twist angles are

$$\alpha = \{ \alpha_1, \alpha_2, 0, \alpha_4 \}^T = \{ \pm \pi, 0, 0, 0 \}^T.$$

All the 4 joint axes are vertical in nature (all the z - axes can be pointing down or up) as shown in Fig. 3. The first three (B, E, VE) axes are called

Fig. 3. Inverse Kinematic Model of the robot

III. DEVELOPMENT OF THE LCD AND KPT

The analysis of the robot is developed in 2 passes, viz., development

as the major axes and are used for positioning the wrist, while the last one, the minor axes (TR) is used to orient the tool in the direction of the object [4]. The 1st 3 major axes determines the shape and size of work envelope. It consists of a L shaped structure, to the end of which the second link is attached. There are two links a_1 and a_2 which move parallel to the work surface ; The vertical extension d_3 is variable and moves in a direction $\perp^r$ to the work surface ; the length of the gripper / tool / EE is d_4 [2].

IV. BLOCK DIAGRAM OF THE INVERSE KINEMATIC MODELING

of the link coordinate diagram as shown in Fig. 4 and development of the kinematic parameter table as shown in the Table 1 [10]. The link coordinate diagram is developed as shown in the Fig. 6 with the kinematic parameter table as shown in Table 1 [2].

V. COMPUTATION OF THE JOINT VARIABLES

The output of the direct kinematics of the SCARA is [2]

$$T_{\text{Base}}^{\text{Tool}} = T_0^4 = \begin{bmatrix} C_{1-2-4} & S_{1-2-4} & 0 & a_1 C_1 + a_2 C_{1-2} \\ S_{1-2-4} & -C_{1-2-4} & 0 & a_1 S_1 + a_2 S_{1-2} \\ 0 & 0 & -1 & d_1 - q_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Tool Configuration Vector (TCV) is given by [2] [9]

$$w(q) = \begin{bmatrix} a_1 C_1 + a_2 C_{1-2} \\ a_1 S_1 + a_2 S_{1-2} \\ d_1 - q_3 - d_4 \\ \dots\dots\dots \\ 0 \\ 0 \\ -\exp\left(\frac{q_4}{\pi}\right) \end{bmatrix}$$

Note : To find out q_1, q_2, q_3, q_4 ; using various trigonometric identities
apply row operations used in [2], [11].
mathematics to the components of w

To Extract Elbow Joint Variable $q_2 = \theta_2$

$$\cos q_2 = \frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2} \quad q_2 = \pm \cos^{-1} \left[\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2} \right]$$

Eqⁿ 3.12:4

$$q_2 = \pm \arccos \left(\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2} \right) \quad \text{Eq}^n \text{ 3.12:5}$$

From the above equations, we see that the IK solution is not unique and hence we get two solutions. Hence, solution to IKP to the shoulder is not unique [10].

To Extract Base Joint Variable $q_1 = \theta_1$

$$w_1 = a_1 C_1 + a_2 C_{1-2}$$

$$w_2 = a_1 S_1 + a_2 S_{1-2}$$

Expand C_{12} and S_{12} using sum of sines and cosines; isolate C_1, S_1 write in matrix form, collect all C_1 terms and S_1 terms, find A^{-1} and $|A|$, solve for q_1 [2], [15].

$$w_1 = a_1 C_1 + a_2 (C_1 C_2 + S_1 S_2) = (a_1 + a_2 C_2) C_1 + (a_2 S_2) S_1$$

$$w_2 = a_1 S_1 + a_2 (S_1 C_2 - C_1 S_2) = (-a_2 S_2) C_1 + (a_1 + a_2 C_2) S_1$$

$$\tan q_1 = \frac{S_1}{C_1} = \frac{\sin q_1}{\cos q_1} = \left[\frac{(a_1 + a_2 C_2) w_2 + (a_2 S_2) w_1}{(a_1 + a_2 C_2) w_1 - (a_2 S_2) w_2} \right]$$

$$q_1 = \arctan 2 \{ (a_1 + a_2 C_2) w_2 + a_2 S_2 w_1, (a_1 + a_2 C_2) w_1 - a_2 S_2 w_2 \}$$

$$q_1 = \tan^{-1} \left\{ \frac{(a_1 + a_2 C_2) w_2 + (a_2 S_2) w_1}{(a_1 + a_2 C_2) w_1 - (a_2 S_2) w_2} \right\}$$

This solution given by this gives the values of the base angle q_1 over the complete range $(-\pi, +\pi)$; since, we

had used the arc tan 2 function. Hence, if we use the arc tan 2 function, we

can recover the base angles over the

To Extract Vertical Extension Joint Parameter, $q_3 = d_3$

From the 3rd component of TCV ;
w, we have

$$w_3 = d_1 - q_3 - d_4$$

The 3rd variable q_3 is extracted as
[13]

$$q_3 = d_1 - d_4 - w_3$$

To Extract Tool Roll Angle, $q_4 = \theta_4$ [2]

complete range, i.e., 360° [17], [18]

The tool roll angle is computed from the last component of the TCV, i.e., w_6 [14].

From TCV, we have ;

$w_4 = w_5 = 0$, i.e., there is no yaw and pitch.

Using the equation [15]

$$q_n = \pi \ln \frac{\sqrt{w_4^2 + w_5^2 + w_6^2}}{\sqrt{0^2 + 0^2 + w_6^2}} = \pi \ln$$

$$q_4 = \pi \ln |w_6|$$

VI. CONCLUSION

A four axes inverse kinematic analysis was performed for the designed robot and was successfully implemented in the laboratory. The robot was controlled using a GUI developed in visual basic language in various modes. A number of pick and place operations were successfully performed by the developed robot by using teaching mode, manual mode and programming modes and the inverse kinematic model.

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