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DEFORMABILITY PARAMETERS FOR FORMING THE STRESS - STRAIN STATE MODELS OF ROCK MASSIF**

Abstract

This paper describes the principle of development the structural models of rocks, based on which the geomechanical model rock massif is developed, to which the applied numerical methods of stress analysis can be applied in solving the practical problems in the field of underground and surface mining of mineral raw materials.

Keywords: deformability, stress-strain state, rock mass

INTRODUCTION

Stress-strain state of rock massif has great importance in designing and construction of structures and their stability. In construction the underground facilities, regardless of the cross-section of room, the rock massif is the main supporting element. Deformations of rock massif, around the underground facility, cause stresses in support structure or coverings, i.e. the both elements mutually affect each other. Due to a huge variety of the rock massif structure and its properties, it is very difficult to develop a model that can describe the stress-strain state of the rock massif. Such model should be as simple as it could be as it can be widely applied and the obtained results acceptable. The first step in the study of rock mechanics is to develop a structural model that reflects all characteristics of rock structures and their

physical-mechanical and deformation properties.

Walter Wittke developed in a detail this issue. Practically, he classified all rock types into granular structure, i.e. the rocks composed of individual elements of grain. Density was determined by packed grains or grains made of their aggregates with the specific disruption of the cracks. Rock is idealized as a quasi-plane and quasi-homogeneous element of rock massif. Depending on the shape and orientation of grains or aggregates in the rock, three main cases will be considered:

- 1 Rock with non-oriented granular structure, Figure 1a.
- 2 Rock with layered structure, Figure 1b.
- 3 Rock with schistose structure, Figure 1c.

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Figure1 Structural rock model

- a – non-oriented isotropic granular structure*
b - layered anisotropic structure
c - structure of anisotropic schistose

Based on the structural rock model, it is possible to make such geomechanical model of rock massif for the use of numerical methods of stress analysis in solving practical problems in the field of underground and open pit mining. These numerical methods are based on the assumption of elastic-viscous plastic deformation. Such model allows taking into account the mechanical-deformation anisotropy, caused by granular structure and surface texture cracks.

Suppose that in a part of the load that does not exceed the limit resistance to uniaxial pressure, the elastic deformations are exclusively developed independently on time and proportional to load, Figure 2. When the load reaches the limit value σ_{gr} , it causes the irreversible non-elastic deformations. The experiments proved that the irreversible deformations, and connected to this the redistribution of stress are highly depend on time. These deformations in geomechanical model are based on the viscous-plastic properties.

Figure 2 Deformability of isotropic rocks

- a - dependence of elastic and ideal viscous- plastic deformations ϵ_{ax} of stress σ_{ax}*
b - determination the elastic modulus and Poisson ratio ν for rocks
1 - elastic deformation
2 – viscous-plastic deformation

1 STRESS-DEFORMATION STATE OF ROCKS

1.1 Elastic deformation

Description of elastic deformations of rocks is derived from a model of their structure. In the rock with non-oriented granular structure in uniaxial or triaxial load which is less than limit strength, the elastic deformations appear independently of load direction. Deformable constants, modulus of elasticity E and the Poisson ratio ν are expressed as follows:

$$E = \frac{\sigma_z}{\varepsilon_z} [MPa]; \quad \nu = \frac{\varepsilon_x}{\varepsilon_z} = \frac{\varepsilon_y}{\varepsilon_z}$$

The shear modulus G depends on E and ν

$$G = \frac{E}{2(1+\nu)} [MPa] \quad (1)$$

Components of stress, strain and shear in the arbitrarily oriented Descartes coordinate system (x , y and z) are connected by the Hook law with the constants E and ν .

$$\left. \begin{aligned} \sigma_x &= \frac{E}{1-\nu-2\nu^2} [(1-\nu)\varepsilon_x + \nu\varepsilon_y + \nu\varepsilon_z] \\ \sigma_y &= \frac{E}{1-\nu-2\nu^2} [\nu\varepsilon_x + (1-\nu)\varepsilon_y + \nu\varepsilon_z] \\ \sigma_z &= \frac{E}{1-\nu-2\nu^2} [\nu\varepsilon_x + \nu\varepsilon_y + (1-\nu)\varepsilon_z] \\ \tau_{xy} &= \frac{E}{2(1+\nu)} \cdot \gamma_{xy} \\ \tau_{yz} &= \frac{E}{2(1+\nu)} \cdot \gamma_{yz} \\ \tau_{zx} &= \frac{E}{2(1+\nu)} \cdot \gamma_{zx} \end{aligned} \right\} \quad (2)$$

The rocks with layered structures, isotropic elastic properties of its deformations, in the best case, are unacceptable. The experiments have confirmed that the uniaxial compressive strength of rocks, with a layered structure in the direction normal to the plane of isotropy, is much greater than the strength in the direction of parallel planar isotropy. Stress and strain state in the plane of stratification has an isotropic character. This description of a methodology of behavior in the zone of elastic deformation consists in application the five independent elasticity constants. It can be said that the stress-strain state is characterized by transverse isotropy [1]. Independent constants are E_1 and E_2 , normal and parallel to the plane of isotropy, ν_1 and ν_2 , and shear modulus G_2 for the shear stress in the plane of isotropy.

Let us consider the unit volume of rock with the Descartes system of coordinates (x' , y' and z'), wherein the axis z' matches with direction of the greatest deformability and x' and y' axes lie in the plane of isotropy, Figure 3.

From the conditions, Figure 3, the elastic constants are obtained from stress and linear and shear deformations of the unit volume, Figure 4. The values of the G_1 and ν_3 dependent on E_1 and E_2 , ν_1 and ν_2 .

Figure 3 Deformation anisotropy of rocks with a layered structure

a - horizontal plane of isotropy, b - vertical plane of isotropy

1 - plane of isotropy

2 - stratification

It is important to say that, according to the transverse isotropy, the theory of elasticity does not specify maximum value of the Poisson ratio as in the case of full isotropy, where $\nu < 0.5$. From considered strain energy, the limitation of inequalities follows:

$$2\nu_2 \cdot \nu_3 < 1 - \nu_1 \quad (3)$$

$$\nu_2 \leq \frac{E_2}{2E_1} \quad (4)$$

Based on the laboratory studies of deformation properties of rocks, ν_1 and ν_2 are always less than 0.5, and only ν_3 for transversely isotropic rocks in some cases may be higher than 0.5.

Figure 4, from *a* to *i*, shows determining the elastic constants of rock layered structure of transversely isotropic models.

a, b):

$$E_1 = \frac{\sigma_{x'}}{\varepsilon_{x'}} = \frac{\sigma_{y'}}{\varepsilon_{y'}};$$

$$c) E_2 = \frac{\sigma'_{z'}}{\varepsilon'_{z'}}$$

d, e i f):

$$G_1 = \frac{\tau_{x'y'}}{\gamma_{x'y'}} = \frac{E_1}{2(1 + \nu_1)};$$

$$G_2 = \frac{\tau_{z'y'}}{\gamma_{z'y'}} = \frac{\tau_{x'y'}}{\gamma_{z'x'}}$$

g, h i i):

$$\nu_1 = \frac{\varepsilon_{y'}}{\varepsilon_{x'}};$$

$$\nu_2 = \frac{\varepsilon_{x'}}{\varepsilon_{z'}} = -\frac{\varepsilon_{y'}}{\varepsilon_{z'}};$$

$$\nu_3 = -\frac{\varepsilon_{z'}}{\varepsilon_{y'}} = \frac{E_1 \nu_2}{E_2}$$

Figure 4 Determination the elastic constants for rocks in the case of transverse complexation for isotropic layered structure

*a, b and c - modulus of elasticity and corresponding strains
d, e, and f - shear modulus
g, h and i – Poisson coefficient*

The expression for determining the maximum value of independent shear modulus I. Kilj, proposes the following expression:

$$G_2 \leq \frac{E_2}{\left\{2 \left[v_2 (1 + v_1) + \sqrt{\left(\frac{E_2}{E_1} - v_2^2 \right) (1 - v_1^2)} \right] \right\}} \quad (5)$$

Since the shear modulus G_2 is very difficult to determine "in-situ", various authors have suggested that this independent

module can be determined approximately over other constants of elasticity.

L. Barden suggests the following relation [4]:

$$G_2 \cong \frac{E_2}{1 + \frac{E_2}{E_1} + 2v_2} \quad (6)$$

A less value can be obtained from the following relation:

$$G_2 \cong \frac{E_2}{2(1 + v_2)} \quad (7)$$

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PARAMETRI DEFORMABILNOSTI ZA FORMIRANJE MODELA NAPONSKO-DEFORMACIJSKOG STANJA STENSKOG MASIVA**

Izvod

U radu je opisan princip razrade strukturnog modela stena, na osnovu kojeg se razrađuje geomehnički model stenskog masiva, na koji se mogu primeniti numeričke metode naponske analize pri rešavanju praktičnih zadataka u oblasti podzemne i površinske eksploatacije mineralnih sirovina.

Ključne reči: deformabilnost, naponsko – deformaciono stanje, stenski masiv

UVOD

Naponsko-deformacijsko stanje stena stenskog masiva ima veliki značaj pri projektovanju i izgradnji objekata i njihove stabilnosti. Pri izradi podzemnih prostorija, bez razlike na poprečni presek prostorije, stenski masiv je osnovni noseći elemenat. Deformacije stenskog masiva oko podzemne prostorije izazivaju napone u konstrukciji podgrade, odnosno obloge, tj. oba elementa uzajamno utiču jedan na drugi. Obzirom na veliku raznolikost građe stenskog masiva i njegovih svojstava, vrlo je teško razraditi model koji može opisati naponsko-deformacijsko stanje stenskog masiva. Takav model mora biti jednostavan kako bi se mogao široko primenjivati, a dobijeni rezultati biti prihvatljivi. Prvi korak u istraživanju mehanike stena je razrada

strukturnog modela koji odražava sve osobenosti građe stena i njihova fizičko-mehanička i deformaciona svojstva.

Ta pitanja je detaljno razradio Walter Wittke. Praktično sve stene je svrstao u zrnastu strukturu, tj. stene sastavljene od pojedinih elemenata zrna. Gustina je određena upakovanim zrnima ili sastavljenih od njihovih agregata određene narušenosti pukotinama. Stena se idealizuje u vidu kvaziravnijskog i kvazijednorodnog elementa stenskog masiva. U zavisnosti od oblika i orijentacije zrna ili agregata u steni ovde ćemo razmatrati tri osnovna slučaja:

1. Stena sa neorijentisanom zrnastom strukturom, sl. 1.a.
2. Stena sa slojevitom strukturom, sl. 1.b.
3. Stena sa škriljavom strukturom, sl. 1.c.

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Sl. 1. Strukturni model stene

*a – neorijentisana izotropna zrnasta struktura,
b – slojevita anizotropna struktura,
c – škriljava anizotropna struktura*

Na osnovu strukturnog stenskog modela moguće je izraditi takav geomehnički model stenskog masiva na koji se mogu primeniti numeričke metode naponske analize pri rešavanju praktičnih zadataka u oblasti podzemne i površinske eksploatacije. Te numeričke metode se zasnivaju na pretpostavci o elasto-viskozno plastičnim deformacijama. Takav model omogućuje da se uzme u obzir mehaničko-deformaciona anizotropija uslovljena zrnastom strukturom i strukturom površina pukotina.

Pretpostavimo da se u delu opterećenja koje ne prevazilazi graničnu otpornost na jednoosni pritisak, razvijaju isključivo elastične deformacije nezavisne od vremena i proporcionalne su opterećenju, sl. 2. Kad opterećenje dostigne graničnu vrednost σ_{gr} , tada nastaju neelastične nepovratne deformacije. Eksperimenti potvrđuju da nepovratne deformacije i povezano sa tim preraspodela napona, u vrlo visokom stepenu zavise od vremena. Te deformacije u geomehničkom modelu zasnivaju se na viskozno-plastičnim svojstvima.

Sl. 2. Deformabilnost izotropnih stena

*a – zavisnost elastičnih i idealno viskozno-plastičnih deformacija ε_{ax} od napona σ_{ax}
b – određivanje modula elastičnosti i koeficijenta Poissona u stena
1 – elastična deformacija,
2 – viskozno-plastična deformacija*

1. NAPONSKO-DEFORMACIJSKO STANJE STENA

1.1. Elastične deformacije

Opis elastičnih deformacija stena proizilazi iz modela njihove strukture. Stena sa neorijentisanom zrnastom strukturom pri jednoosnom ili troosnom opterećenju, koje je manje od granične čvrstoće, nastaju elastične deformacije nezavisno od pravca dejstva opterećenja. Deformabilne konstante, modul elastičnosti E i koeficijent Poisson-a ν se izražavaju na sledeći način:

$$E = \frac{\sigma_z}{\varepsilon_z} [MPa]; \quad \nu = \frac{\varepsilon_x}{\varepsilon_z} = \frac{\varepsilon_y}{\varepsilon_z}$$

Modul smicanja G zavisi od E i ν :

$$G = \frac{E}{2(1+\nu)} [MPa] \quad (1)$$

Komponente napona, deformacija i smicanja u proizvoljno orijentisanom Dekartovom koordinatnom sistemu (x , y i z) povezani su po Huk-ovom zakonu konstantama E i ν :

$$\left. \begin{aligned} \sigma_x &= \frac{E}{1-\nu-2\nu^2} [(1-\nu)\varepsilon_x + \nu\varepsilon_y + \nu\varepsilon_z] \\ \sigma_y &= \frac{E}{1-\nu-2\nu^2} [\nu\varepsilon_x + (1-\nu)\varepsilon_y + \nu\varepsilon_z] \\ \sigma_z &= \frac{E}{1-\nu-2\nu^2} [\nu\varepsilon_x + \nu\varepsilon_y + (1-\nu)\varepsilon_z] \\ \tau_{xy} &= \frac{E}{2(1+\nu)} \cdot \gamma_{xy} \\ \tau_{yz} &= \frac{E}{2(1+\nu)} \cdot \gamma_{yz} \\ \tau_{zx} &= \frac{E}{2(1+\nu)} \cdot \gamma_{zx} \end{aligned} \right\} \quad (2)$$

Stene sa građom slojevite strukture, izotropne elastične karakteristike njenih deformacija u najboljem slučaju su nedopustive. Eksperimenti potvrđuju da jednoosna otpornost na pritisak stena sa slojevitom strukturom u pravcu normalnom na ravan izotropije znatno je veća od čvrstoće u pravcu paralelno ravno izotropije. Naponsko deformacijsko stanje u ravni slojevitosti poseduje izotropni karakter. Navedena metodologija opisa ponašanja u zoni elastičnih deformacija sastoji se u primeni pet nezavisnih jedne od drugih konstanti elastičnosti. Tada kažemo da naponsko-deformacijsko stanje karakteriše transversalna izotropija [1]. Nezavisne konstante su E_1 i E_2 , normalno i paralelno ravni izotropije, ν_1 i ν_2 , i modul smicanja G_2 za napone smicanja u ravni izotropije.

Razmotrimo jediničnu zapreminu stene sa Dekartovim sistemom koordinata (x' , y' i z'), pri čemu osa z' se poklapa sa pravcem najveće deformabilnosti, a ose x' i y' leže u ravni izotropije, sl. 3.

Iz uslova sl. 3 konstante elastičnosti se dobiju iz napona i linearnih i smičućih deformacija jedinične zapremine, sl. 4. Pri čemu su takođe veličine ν_3 i G_1 zavisne od E_1 i E_2 , ν_1 i ν_2 .

Sl. 3. Anizotropija deformabilnosti stena sa slojevitom strukturom
a – horizontalna ravan izotropije, b – vertikalna ravan izotropije
1 – ravan izotropije, 2 - slojevitost

Važno je reći, da pri transverzalnoj izotropiji, teorija elastičnosti ne određuje maksimalnu vrednost koeficijenta Poisson-a kao u slučaju pune izotropije, gde je $\nu < 0,5$. Iz razmatrane energije deformacija sledi samo ograničenje nejednačina:

$$2\nu_2 \cdot \nu_3 < 1 - \nu_1 \quad (3)$$

$$\nu_2 \leq \frac{E_2}{2E_1} \quad (4)$$

Na osnovu laboratorijskih istraživanja deformacionih svojstava stena ν_1 i ν_2 su uvek manji od 0,5, a samo ν_3 za transverzalno izotropne stene u pojedinim slučajevima može biti veći od 0,5.

Na sl. 4 od *a* do *i* prikazano je određivanje elastičnih konstanti stena slojevite strukture transverzalno izotropnog modela.

a, b):

$$E_1 = \frac{\sigma_{x'}}{\varepsilon_{x'}} = \frac{\sigma_{y'}}{\varepsilon_{y'}};$$

$$c) E_2 = \frac{\sigma'_z}{\varepsilon'_z}$$

d, e i f):

$$G_1 = \frac{\tau_{x'y'}}{\gamma_{x'y'}} = \frac{E_1}{2(1 + \nu_1)};$$

$$G_2 = \frac{\tau_{z'y'}}{\gamma_{z'y'}} = \frac{\tau_{x'y'}}{\gamma_{z'x'}}$$

g, h i i):

$$\nu_1 = \frac{\varepsilon_{y'}}{\varepsilon_{x'}};$$

$$\nu_2 = \frac{\varepsilon_{x'}}{\varepsilon_{z'}} = -\frac{\varepsilon_{y'}}{\varepsilon_{z'}};$$

$$\nu_3 = -\frac{\varepsilon_{z'}}{\varepsilon_{y'}} = \frac{E_1 \nu_2}{E_2}$$

Sl. 4. Određivanje elastičnih konstanti stena za slučaj transversalno izotropni model slojevite strukture

*a, b i c – modul elastičnosti i odgovarajuće deformacije,
d, e i f – moduli smicanja,
g, h i i – koeficijenti Poisson-a*

Izraz za određivanje maksimalne vrednosti nezavisnog modula smicanja I. Kilj, predlaže sledeći izraz:

$$G_2 \leq \frac{E_2}{\left\{ 2 \left[\nu_2 (1 + \nu_1) + \sqrt{\left(\frac{E_2}{E_1} - \nu_2^2 \right) (1 - \nu_1^2)} \right] \right\}} \quad (5)$$

Obzirom da je modul smicanja G_2 , vrlo teško odrediti „in-situ“ to razni autori su predložili za taj sam po sebi nezavisni modul

da se odredi približno preko drugih konstanti elastičnosti.

L. Barden predlaže sledeću zavisnost [4]:

$$G_2 \cong \frac{E_2}{1 + \frac{E_2}{E_1} + 2\nu_2} \quad (6)$$

Nešto manja vrednost se dobije po sledećoj zavisnosti:

$$G_2 \cong \frac{E_2}{2(1 + \nu_2)} \quad (7)$$

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