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MODELING THE BATCH POWER OF MILL**

Abstract

This paper presents a new method of modeling the batch power of mill on the basis of dimensional analysis and criterion equations. The grinding batch is represented by the grinding bodies, material and medium that is used to provide the flow of raw material through the mill, and most commonly water or air. The rate of energy consumption in the mill is regulated by the charge density and it presents the charge density. The feed power is dependent on the type of media milling body and prior to the treatment of mineral raw materials. Approximate power mill defined engine power is the power of batch mill highest charge density. In the work is varied density in batch laboratory mills and defined a model of batch power mill which we used in the adaptation of the mill plants in new industrial plant.The present model of the feed mill power is checked on industrial exploitation of quartz raw materials in Lukic polje near Milici. The process of mechano-chemical treatment defines as venture milling with longer residence time of material in the mill and small batch densities, all with the aim to reduce the force with which the batch effect on grain mineral resources. Radicality of fragmentation varies with the density of charge so that the densities obtained with smaller sized features special milling products with a narrower range of narrow size class and thus increased the specific surface area and reactivity.

Keywords: *batch strength, charge density, mechano-chemical treatment, reactivity*

INTRODUCTION

Modeling the batch strength of mill, shown in the work was done on the basis of theoretical considerations and dimensional analysis using the criterion equations. The paper uses a laboratory model and dimensional analysis, based on the similarity of dynamic invariants developed a model of mechano-chemical treatment in an industrial mill. The paper describes the process flow of physical modeling and determination of engine power for lower density batch mill $\rho_{\check{s}}$, given that the grinding is done with silex balls and not with the metal balls. Thus developed model provides answers to the question of the necessary batch strength of quartz sand grinding at the converted facility for so-called unconventional conditions of grinding or mechano-chemical treatment.

THEORETICAL CONSIDERATIONS

Theoretical considerations are related to the model of flow in the mill as a streaming media through the pipe with the certain characteristics. The required parameters that describe this process are:

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- Hydrodynamic *size:* υ , p , g
- Fluid properties: ρ , μ , σ

wherein:

υ - speed, $L \cdot t^{-1}$;
p - pressure of fluid, $M \cdot L^{-1} \cdot t^{-2}$;
g - gravitational acceleration, $L \cdot t^{-2}$;
ρ - density, $M \cdot L^{-3}$;
μ - dynamic viscosity, $M \cdot L^{-1} \cdot t^{-1}$;
σ - surface tension, $M \cdot t^{-2}$,
N - power, $M \cdot L^{2} \cdot t^{-3}$,
L - length, L,
M - mass, M,
t - time, t,

of these values is performed by a mechanical force acting, F_i force of inertia, F_g strength by weight, F_p pressure force, F_{tr} friction force, F_σ force of surface tension, F_q heat diffusion forces, F_m force mass diffusion. Putting the ratio of these two forces, the invariants or similarity criteria are obtained [1]. Figure 1 shows a cylindrical ball mill which can be seen as a pipe for streaming media with the characteristic sizes and parameters of grinding process and mechanochemical treatment.

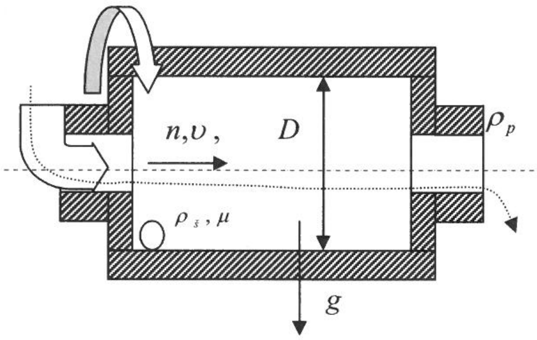

Figure 1 *Tube for media streaming, animation of mill*

Buckingham's theorem similarities

According to the Buckingham π theorem, each equation containing n related physical quantities (υ , ρ , μ etc.), between which m sizes are independent dimensions (M, L, t), may be converted to an equation which has n to m dimensionless criteria and simplex, composed of those values. As the criterion for simplex or taken tag P , then the above theorem can be written:

$$f(P_1, P_2, P_3, ...) = 0 \quad (1)$$

$$P_1 = f(P_2, P_3, ...) \quad (2)$$

This theorem is of great importance in the experimental and theoretical work. The relationship could be found between the dimensionless expression, and thereby the number of unknown reduced number of basic units of measure at least 3 conditions that significantly simplifies the experimentation and finding legality of interrelated physical size [1,2]. Criteria similarities are encountered practically in solving any problems from chemical engineering, particularly in problems magnification (scale-up). From the Annunciation-Stokes's equations of motion of viscous fluids, the following is got:

$$F_i = -F_g - F_p + F_{tr} \quad (3)$$

Or the balance of the forces of inertia, forces of weight, thrust force and friction force.

If the left side of the equation is shared with some members of the right side of the equation, the following is got:

$$\frac{\rho \cdot \frac{\upsilon^2}{l}}{\rho \cdot g} = \left[\frac{\upsilon^2}{l \cdot g} \right] = Fr \qquad (4)$$

$$\frac{\rho \cdot \frac{\upsilon^2}{l}}{\frac{p}{l}} = \left[\frac{\rho \cdot \upsilon^2}{p} \right] = Eu^{-1} \qquad (5)$$

$$\frac{\rho \frac{\upsilon^2}{l}}{\mu \cdot \frac{\upsilon}{l^2}} = \left[\frac{l \cdot \upsilon \cdot \rho}{\mu} \right] = \mathrm{Re} \qquad (6)$$

As the raw material in mill moves and mixes, the substantial inertial force of the gravitational force and frictional force are important. Gravity criterion, criterion of strength and flow criteria therefore should not be ignored. According to the above, the following is got:

$$E_U = f(\mathrm{Re}, Fr). \qquad (7)$$

MATERIALS AND METHODS

The experiment in which the grinding is performed has the characteristics of fluid flow through the tube. Fluid flow through tube is supported by mixing that is achieved by milling bodies because the mantle of mill is moved. In many experimental investigations, it was observed that the grinding process and mechano-chemical treatment will most affect the following parameters: the density of batch, $\rho_{\check{s}}$, speed of mill mantle *n*, diameter of mill mantle *D*, viscosity μ and, gravitational acceleration, *g*. All these parameters are included in a model of grinding development using specific criteria (Euler, Reynolds and Frude). Test conditions set so that we are in the first series of experiments, consisting of three experiments, experiments carried out in a single device when the criterion Frude was immutable, and when it is changed during the mechano-chemical treatment, or the flow rate of raw material through the mill, which is such a change had implications Reynolds criteria. This experiment would not have been feasible if we changed the type of grinding the batch so silex alum and steel grinding body were used. The measured density and viscosity of the pulp were constant for the first series of experiments.

In another series of experiments, which also consists of three experiments, the conditions set by Frude criterion were investigated s changing and this was achieved using mills of various sizes. The Reynolds criterion in this series of experiments was maintained so as constant used milling different types of bodies and they have had a different feedstock residence time in the mill, and wherein the density and viscosity of the pulp have a constant value. This was achieved when the mill with the largest dimensions of the used silex balls, then at the mill medium dimensions of alumino used balls, and at the end of the mill with the smallest dimensions of the used steel balls.

Density and viscosity of the pulp within a single batch is maintained constant, and vary between batches, so that the two series were actually derived values of the two densities and viscosity of two values.

Dimensional Analysis and Criterion Equations

The formation of dimensionless numbers for a particular problem is most easily achieved using dimensional matrix. Dimensional matrix consists of a square and the remaining matrix. Rows of the matrix form the basis of size, and it will form a rank r matrix. The columns of the matrix represent the physical size or influential parameters. Sizes of squares elementary matrices appear in all the dimensionless numbers, while each element of the residual matrix appears in only one dimensionless number. For this reason, the remain-ning matrix should be comprised of the most important variables.

Table 1 *Basic dimensional matrix*

	ρ	D	n	N	μ	g
Mass, M	1.	0	0	1.	1.	0
Length, L	-3	1.	0	2	-1	1.
Time, t	0	0	-1	-3	-1	-2
	Basic matrix			Remaining matrix		

Rearrange matrix (linear transformation) is done by the core matrix becomes a common matrix. After the creation of a common matrix, dimensionless numbers arise in the following way. Each element of the remaining matrix, which is in the numerator is divided by the square matrix of the parameters that were graded under the number of the remaining elements of the matrix as shown in the example. Dimensional matrix for this case has the form.

Table 2 *Renovated dimensional matrix*

	ρ	D	n	N	μ	g
M	1	0	0	1	1	0
3M+L	0	1	0	5	2	1
-t	0	0	1	3	1	2
	Basic matrix			Remaining matrix		

$$\frac{N}{\rho^1 \cdot n^3 \cdot D^2} = \frac{N}{\rho \cdot n^3 \cdot D^5} \equiv Np$$

criterion power mill

Strength criterion and sometimes referred to as modified Euler expression (Euler) (Eu_M) because:

$$\left[\frac{\rho \cdot \upsilon^2}{p}\right] = Eu^{-1}$$

And

$$Eu_M = Eu^{-1} \equiv \left[\frac{\rho \cdot \upsilon^2 \cdot \frac{l^3}{t}}{p \cdot \frac{l^3}{t}}\right] \equiv \qquad (8)$$

$$\equiv \frac{\rho \cdot n^3 \cdot D^5}{N} \equiv Np^{-1}$$

wherein:

$Q = \frac{l^3}{t}$ - flow of pulp

$$\frac{\mu}{\rho^1 \cdot n^1 \cdot D^2} = \frac{\mu}{\rho \cdot n \cdot D^2} \equiv \mathrm{Re}^{-1}$$

- dimensionless Reynolds number for mixing process

$$\frac{g}{\rho^0 \cdot D^1 \cdot n^2} = \frac{g}{D \cdot n^2} \equiv Fr^{-1} \text{ -}$$

Froude's dimensionless number for mixing

Under the certain experimental conditions in the laboratory, it was attempted to reach a solution of the equation 7 and to find the coefficient k and exponents *a* and *b*

Its new analytical form has the appearance shown by equation 9

$$Eu_M = k \cdot \mathrm{Re}^a \cdot Fr^b \qquad (9)$$

By logarithms the criterion equation 9, the following is got:

$$\log Eu_M = \log k + a \cdot \log \mathrm{Re} + b \cdot \log Fr \quad (10)$$

It is necessary to determine experimentally the function Eu_M and one of the criteria, but the second criterion is kept constant in this series of experiments.

If the equation 9

$$k \cdot Fr^b = konst. = B \quad (11)$$

Obtained criterion equations:

$$\log Eu_M = a \cdot \log \mathrm{Re} + \log B \quad (12)$$

Thus the form:

$$y = a \cdot x + b$$

Dynamic viscosity μ and pulp density ρ_p in a laboratory mill can be adjusted so that their ratio is constant. Measuring viscosity is on the Brookfield viscometer. The equipment with rotating cylinders was used which allows determination of the viscosity over a wide range of consistency. The pulp density is measured in the mining pycnometer (to be used in flotation), and a scale to measure the density. Thanks to the use of different types of batch milling bodies (steel, alumino silicate spheres) the grinding times were different but the fineness of each of these experiments was the same.

Change the Reynolds Re-Criterion, which is required for the formation of models, is achieved by changing the residence time in the mill, the raw material, the table 3. Change the value of Euler's criterion Eu_M is calculated according to equation 8, and the analytical expressions of equality and shows that all parameters are known ($N, \rho_{up}, n^3 i \ D^5$). Force N measure with the electric meter, or device that is connected to the engine laboratory mill. The density of the charge $\rho_š$ is a linear function of the density of the pulp ρ_p where the coefficient of direction and the intercept on the ordinate are dependent on the type of body that is milling ρ_{vk} and ρ_{sk} as can be seen from Equation 13: [3, 4, 5]

$$\rho_š = \rho_{vk} + 1{,}15 \cdot (1 - \frac{\rho_{vk}}{\rho_{sk}}) \cdot \rho_p \left[kg \cdot m^{-3}\right] \quad (13)$$

Where:

ρ_{vk} - *density of balls in bulk density in kg/m ^{3},*

ρ_{sk} - *density of the material from which the ball is made, in kg / m ^{3},*

ρ_p - *pulp density in kg / m ^{3}.*

The material from which it was made, ball, cast

Fe - ρ_{sk} = 7800 kgm^{-3},

Al_2O_3 - ρ_{sk} = *4800* kgm^{-3}

Silicate - ρ_{sk} = 2600 kgm^{-3} .

Density of balls in bulk density,

Fe cast - ρ_{vk} = 4100 to 4200 kgm^{-3} ,

Al_2O_3-pressed ρ_{vk} =2500 to 2700 kgm^{-3}

Silicate - ρ_{vk} = 1800 to1900 kgm^{-3}.

Frude constancy criteria in this series is achieved by the use of a mill of the same size for all of the individual experiments. When the data in the table shows the diagram it is evident that the points that represent the coordinates of individual experiments are approximately straight line. Figure 2 shows that the $tg\alpha = a =$ 0,6259, Or a the exponent of criteria Re, and cut-outs on the axes $\log Eu_M$ represents the value of $\log B$, and is $\log B = -1{,}8261$ and then B = 0,014924507.

In order to determine the coefficient of k from equation 9 is required to carry out another series of experiments in which to set the conditions of the experiment that the Reynolds number is constant. Thus, after the first series of experiments, the model is performed a second series of experiments, in which the criterion value is maintained constant Re, and from equation 9:

$$k \cdot \mathrm{Re}^{a} = konst. = C \qquad (14)$$

It follows from this criterion equation:

$$\log Eu_M = b \cdot \log Fr + \log C \qquad (15)$$

So, again, the straight line equation as a functional dependence Eu_M of changes Fr .

In order to remain constant the Reynolds criterion, it is necessary to be changed during the treatment so that the greater the mill is longer retention time of the raw material in the mill, the smaller shorter retention time of raw materials. The above experimental conditions were achieved by reducing the speed of medium and large, when the mill is decreased, and the efficiency of grinding by increasing the number of mill rotation when higher efficiency is achieved by grinding. To achieve the above conditions milling time except change the number of rotation of the mill was also necessary that the largest mill uses less density fed-batch (batch of sileks balls) in the medium mill, medium density fed-batch (batch of alumino-Ball), and the smallest mill, the largest batch density (batch of steel balls). Finally, in order to maintain the constant Reynolds criterion, it was necessary to perform several preliminary experiments to determine the pulp density

Table 3 *Experimental data for the power criterion when the Frude number is not changed*

Viscosity of pulp	Grinding time	Diameter of sheath	Speed pulp	Pulp density	Reynolds number		Speed	Frude number		Measured power mill	Milling body types	Density of charged balls	Material density	Charge density	Euler number Eu_M	
					Re	*LogRe*		*Fr*	*logFr*						Eu_M	$logEu_M$
μ, Pas	t, s	D, m	υ, ms $^{-1}$	*kgm* $^{-3}$			ns $^{-1}$			N, W		*kgm* $^{-3}$	*kgm* $^{-3}$	*kgm* $^{-3}$		
0.416	600	0.305	0.000510	1226	0.458	-0.3387	1.04	0.0329	-1.482	760	Si	1800	2600	2234	0.008726	-2.05917
0.416	400	0.305	0.000763	1226	0.686	-0.1637	1.04	0.0329	-1.482	760	Al	2600	4800	3246	0.012681	-1.89684
0.416	180	0.305	0.001694	1226	1.523	0.1826	1.04	0.0329	-1.482	760	Fe	4200	7800	4851	0.018949	-1.72241

Table 4 *Experimental data for the power criterion when the Reynolds number is not changed*

Viscosity of pulp	Grinding time	Diameter of sheath	Speed pulp	Pulp density	Reynolds number		Speed	Frude number		Measured power mill	Milling body types	Density of charged balls	Material density	Charge density	Euler number Eu_M	
					Re	*LogRe*		*Fr*	*logFr*						Eu_M	$logEu_M$
μ, Pas	t, s	D, m	υ, ms $^{-1}$	*kgm* $^{-3}$			n,s^{-1}			N, W		*kgm*$^{-3}$	*kgm*$^{-3}$	*kgm*$^{-3}$		
0.17	417	0.150	0.00036	1066	0.338612	-0.4703	1.6	0.0384	-1.415668	70	Fe	4200	7800	4766	0.02117649	-1.67415
0.17	898	0.220	0.000245	1066	0.337985	-0.4711	0.8	0.0141	-1.851397	250	Al	2600	4800	3162	0.00333724	-2.47661
0.17	1713	0.305	0.000178	1066	0.34043	-0.4679	0.5	0.0076	-2.117760	760	Si	1800	2600	2177	0.00094513	-3.02451

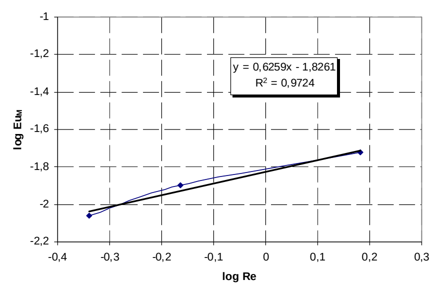

Figure 2 *Function* $\log Eu_M$ *and* $\log \mathrm{Re}$

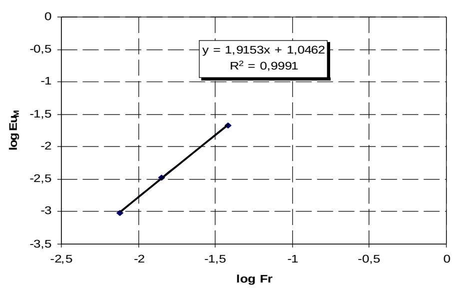

Figure 3 *Function* $\log Eu_M$ *and* $\log Fr$

Table 4 presents the experimental data for the second series of experiments, when the data in the table shows in graphical form to an almost straight line, Figure 3. Analogous conclusion of the earlier shows that $tg\beta = b = 1{,}9153$ and the intercept on the ordinate $\log Eu_M$ value $\log C = 1{,}0462$ and then $C = 11{,}12243817$ as the two series of experiments determined criteria exponent $\mathrm{Re}(a)$ and exponent of criteria $Fr(b)$, Constant k can be calculated from:

$$k = \frac{B}{Fr^b} = \frac{0.015}{Fr^{1.92}} = 10.27 \text{, apropos}$$

$$k = \frac{C}{\mathrm{Re}^a} = \frac{11.12}{\mathrm{Re}^{0.63}} = 21.91 \qquad (16)$$

To afford the median:

$$k_{sr} = \frac{\sum_1^2 k}{2} = \frac{k_1 + k_2}{2} = \frac{10.27 + 21.91}{2} = 16.1 \qquad (17)$$

Given that the system is interrogated for this constant k and exponents a and b constant values can be obtained for the experimental combination of criteria Re *and* Fr calculated value Eu_M.

$$Eu_M = k \cdot \mathrm{Re}^a \cdot Fr^b = 16.1 \cdot 0{,}338612^{0.63} \cdot 0.032989^{1.92} = 0.011869 \qquad (18)$$

Given that: $\frac{\rho_{up} \cdot n^3 \cdot D^5}{N} = Eu_M$ and $Eu_M = k \cdot \mathrm{Re}^a \cdot Fr^b$ it is seen that the use of laboratory tests occurred criterion equation batch power mill in treatment of silicate materials that can be used to check the mill batch strength in the accelerated conditions.

$$N = \frac{\rho_{š} \cdot n^3 \cdot D^5}{16.1 \cdot \mathrm{Re}^{0.63} \cdot Fr^{1.92}} \qquad (19)$$

Checking the Mill Batch Strength

Checking the mill batch strength adapted to the treatment of quartz sand in Lukic field near Milici was performed to check the batch loop power that has value $N = 280000$ W. $\rho_{š} = 2177\ kgm^{-3}$ - The density of the charge (ball + water + material), $D = 2{,}2\ m$ - Inner diameter of the mill, $n = 0.3\ s^{-1}$ - Speed of the mill,

$$N = \frac{\rho_{up} \cdot n^3 \cdot D^5}{16.1 \cdot \mathrm{Re}^{0.63} \cdot Fr^{1.92}} = \frac{2177 \cdot 0.3^3 \cdot 2.2^5}{0.012} = 255256\,W$$

As seen criterion equation model gives good results, because the calculated batch mill power less than the power the mill frame and engine power.

CONCLUSION

The importance of the present method of finding the criterion equation model batch mill power is great because the model can be used generally for all mineral materials [6,7]. Criterion equation model is possibly to apply to the industrial mills, because the laboratory conditions have been altered in all the relevant parameters that affect the process in the industry conditions.

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MODELOVANJE ŠARŽNE SNAGE MLINA**

Izvod

U radu je prikazana nova metoda modelovanja šaržne snage mlina na bazi dimenzione analize i kriterijumskih jednačina. Šaržu predstavljaju meljuća tela materijal i medijum koji se koristi ba bi se obezbedijo protok sirovine kroz mlin, a najčešće je to voda ili vazduh. Brzina trošenja energije u mlinu se reguliše gustinom šarže i predstavlja šaržnu snagu. Šaržna snaga zavisi od vrste meljućih tela i medija u kojem se vrši tretman mineralne sirovine. Okvirna snaga mlina definisana snagom motora predstavlja šaržnu snagu mlina za najveću gustinu šarže. U radu je varirana gustina šarže u laboratorijskim mlinovima i definisan model šaržne snage mlina kojeg smo primenili kod adaptacije mlinskog postrojenja u novi industrijski pogon. Prikazani model šaržne snage mlina proveren je u industrijskim uslovima ekspoloatacije kvarcne sirovine u Lukića polju kod Milića. Proces mehanohemijskog tretmana definše se kao poduhvat mlevenja sa dužim vremenom boravka materijala u mlinu i manjim šaržnim gustinama, a sve sa ciljem da se smanji sila kojom šarža deluje na zrna mineralne sirovine. Radikalnost usitnjavanja se menja sa promenom gustine šarže tako što se sa manjim gustinama dobijaju posebne karakteristike krupnoće proizvoda mlevenja sa užim dijapazonom uskih klasa krupnoće i time uvećanom specifičnom površinom i reaktivnošću.

Ključne reči: *šaržna snaga, gustina šarže, mehanohemijski tretman, reaktivnost*

UVOD

Modelovanje šaržne snage mlina prikazano u radu urađeno je na osnovu teorijskih razmatranja i pomoću dimenzione analize kriterijumskih jednačina. U radu je korišćen laboratorijski model i dimenziona analiza, a na osnovi invarijante dinamičke sličnosti razvien je model mehanohemijskog tretmana u industrijskom mlinu. U radu je prikazan tok procesa fizičkog modelovanja i određivanja snage motora za manju gustinu šarže mlina $\rho_{š}$ s obzirom da se mlevenje vrši sa sileks kuglama a ne sa metalnim kuglama. Ovako razvijen model daje odgovore na pitanje potrebne šaržne snage mlevenja kvarcnog peska u adaptiranom postrojenju za takozvane nekonvencionalne uslove mlevenja odnosno mehanohemijski tretman.

TEORIJSKA RAZMATRANJA

Teorijska razmatranja vezuju se za model strujanja u mlinu kao strujanje nekog medija kroz cev sa određenim ka-

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rakteristikama. Potrebni parametri koji opisuju ovaj proces su:

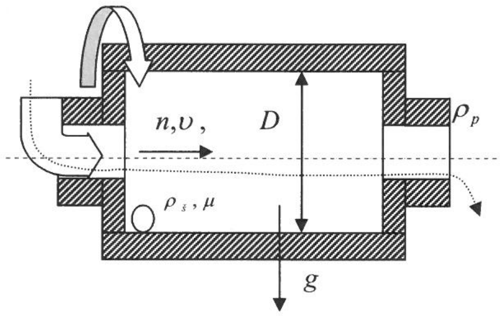
- hidrodinamičke veličine: υ, p, g
- svojstva fluida: ρ, μ, σ

gde je:

υ - brzina, $L \cdot t^{-1}$;
p - pritisak fluida, $M \cdot L^{-1} \cdot t^{-2}$;
g - gravitaciono ubrzanje, $L \cdot t^{-2}$;
ρ - gustina, $M \cdot L^{-3}$;
μ - dinamički viskozitet, $M \cdot L^{-1} \cdot t^{-1}$;
σ - površinski napon, $M \cdot t^{-2}$;
N - snaga, $M \cdot L^{2} \cdot t^{-3}$;
L - dužina, L;
M - masa, M;
- t - vreme, t.

Iz ovih veličina izvode se delujuće mehaničke sile, F_i sila inercije, F_g sila težine, F_p sila pritiska, F_{tr} sila trenja, F_σ sila površinskog napona, F_q sila difuzije toplote, F_m sila difuzije mase.

Stavljajući u količnički odnos po dve od navedenih sila, dobijamo invarijante ili kriterijume sličnosti [1]. Na slici 1 je prikazan cilindrični mlin sa kuglama koji se može posmatrati kao cev za strujanje medija sa karakterističnim veličinama odnosno parametrima procesa mlevenja i mehanohemijskog tretmana.



Sl. 1. *Cev za strujanje medija, animacija mlina*

Bakingem-ova teorema sličnosti

Prema Buckinghamovom π teoremi svaka jednačina koja sadrži n povezanih fizičkih veličina (υ, ρ, μ itd.), između kojih m veličine imaju nezavisne dimenzije (M, L, t), može biti prevedena u jednačinu koja ima n do m bezdimenzionih kriterijuma i simpleksa, sastavljenih iz tih veličina. Pošto je za simpleks ili kriterijum uzeta oznaka P, onda se gornja teorema može napisati:

$$f(P_1, P_2, P_3, \ldots) = 0 \qquad (1)$$

odnosno:

$$P_1 = f(P_2, P_3, \ldots) \qquad (2)$$

Ova teorema ima veliki značaj u eksperimentalnom i teorijskom radu. Nalazimo vezu između bezdimenzionih izraza, a pri tome je broj nepoznatih sveden na broj osnovnih jedinica mere najmanje 3 što veoma pojednostavljuje uslove eksperimentisanja i nalaženje zakonitosti o međusobnom odnosu fizičkih veličina [1,2]. Kriterijumi sličnosti se susreću praktično kod rešavanja svakog problema iz hemijskog inženjerstva, a posebno kod problema uvećanja (scale-up). Iz Navje-Štoks-ove jednačine kretanja viskozne tečnosti dobijamo da je:

$$F_i = -F_g - F_p + F_{tr} \qquad (3)$$

Odnosno ravnotežu sila inercije, sila težine, sila pritiska i sila trenja.

Ako se leva strana jednačine deli sa pojedinim članovima desne strane jednačine dobijamo:

$$\frac{\rho \cdot \frac{\upsilon^2}{l}}{\rho \cdot g} = \left[\frac{\upsilon^2}{l \cdot g}\right] = Fr \qquad (4)$$

$$\frac{\rho \cdot \frac{\upsilon^2}{l}}{\frac{p}{l}} = \left[\frac{\rho \cdot \upsilon^2}{p}\right] = Eu^{-1} \qquad (5)$$

$$\frac{\rho \frac{\upsilon^2}{l}}{\mu \cdot \frac{\upsilon}{l^2}} = \left[\frac{l \cdot \upsilon \cdot \rho}{\mu}\right] = \mathrm{Re} \qquad (6)$$

S obzirom da se u mlinu sirovina kreće i meša značajne su inercione sile gravitacione sile i sile trenja. Gravitacioni kriterijum, kriterijum snage i kriterijum strujanja stoga ne smeju biti zanemareni. Prema prethodnom imamo:

$$E_U = f(\mathrm{Re}, Fr). \qquad (7)$$

MATERIJAL I METODE

Eksperiment u kojem se vrši mlevenje ima karakteristike strujanja fluida kroz cev. Strujanje fluida kroz cev potpomognuto je mešanjem koje se ostvaruje pomoću meljućih tela jer se plašt mlina obrće. U mnogim eksperimentalnim istraživanjima primećeno je da na proces mlevenja i mehanohemijskog tretmana najviše utiču sledeći parametri: gustina šarže, $\rho_š$, broj obrtaja plašta mlina, *n*, prečmik plašta mlina, *D*, viskozitet μ i gravitaciono ubrzanje, *g*. Sve navedene parametre uključili smo u razvoj modela mlevenja pomoću određenih kriterijuma (Ojlera, Rejnoldsa i Frudea). Uslove ispitivanja podesili smo tako da smo u prvoj seriji opita, koja se sastoji od tri opita, eksperimente vršili u jednom istom uređaju kada je kriterijum Frude bio nepromenljiv, a pri tome se menjalo vreme mehanohemijskog tretmana, odnosno brzina proticanja sirovine kroz mlin, što je kao implikaciju imalo promenu Rejnoldsovog kriterijuma. Ovakav eksperiment ne bi bio izvodljiv ukoliko ne bismo menjali vrstu meljuće šarže pa smo koristili sileks, alumo i čelična meljuća tela. Izmerene gustine i viskoziteti pulpe su bili konstantni za prvu seriju opita.

U drugoj seriji opita, koja se takođe sastoji od tri opita, uslove ispitivanja podesili smo tako da Frudeov kriterijum bude promenljiv a to je bilo moguće postići upotrebom mlinova različitih veličina. Rejnoldsov kriterijum u ovoj seriji opita održavan je konstantantnim tako što su korišćene različite vrste meljućih tela i što smo imali različito vremena boravka sirovine u mlinu, a da pri tome gustine i viskoziteti pulpe imaju stalne vrednosti. Ovo je bilo moguće postići kada su u mlinu sa najvećim gabaritnim dimenzijama korišćene sileks kugle, zatim u mlinu sa srednjim gabaritnim dimenzijama korišćene alumo kugle, i na kraju u mlinu sa najmanjim gabaritnim dimenzijama korišćene čelične kugle.

Gustine i viskoziteti pulpe unutar pojedinačne serije se održavaju konstantnim, a između serija se razlikuju, tako da su u dve izvedene serije bile zapravo dve vrednosti gustine i dve vrednosti viskoziteta.

Dimenziona analiza i kriterijumske jednačine

Formiranje bezdimenzionih brojeva za određeni problem najlakše se postiže upotrebom dimenzionih matrica. Dimenziona matrica sastoji se od kvadratne i preostale matrice. Redovi matrica formiraju bazu dimenzija, i ona će formirati rang r matrice. Kolone matrice predstavljaju uticajne fizičke veličine ili parametre. Veličina kvadrata osnovne matrice pojavljuju se u svim bezdimenzionim brojevima, dok će se svaki elemenat preostale matrice pojavit samo u jednom bezdimenzionom broju. Iz ovog razloga preostala matrica bi trebalo da bude sastavljena od najvažnijih promenljivih veličina.

Tabela 1. *Osnovna dimenziona matrica*

	ρ	***D***	***n***	***N***	***μ***	***g***
Masa M	1	0	0	1	1	0
Dužina L	-3	1	0	2	-1	1
Vreme t	0	0	-1	-3	-1	-2
	Osnovna matrica			Preostala matrica		

Preuređivanje matrice (linearna transformacija) vrši se tako što jezgro matrice prelazi u zajedničku matricu. Nakon stvaranja zajedničke matrice bezdimenzioni brojevi nastaju na sledeći način. Svaki element preostale matrice koji stoji u brojiocu deli se sa parametrima kvadratne matrice koji su stepenovani brojem ispod elementa preostale matrice kao što je dato u primeru. Dimenziona matrica za naš slučaj ima oblik:

Tabela 2. *Preuređena dimenziona matrica*

	ρ	***D***	***n***	***N***	***μ***	***g***
M	1	0	0	1	1	0
3M+L	0	1	0	5	2	1
-t	0	0	1	3	1	2
	Osnovna matrica			Preostala matrica		

$$\frac{N}{\rho^1 \cdot n^3 \cdot D^2} = \frac{N}{\rho \cdot n^3 \cdot D^5} \equiv Np$$

- kriterijum snage mlina

Kriterijum snage se katkada naziva i modifikovani izraz Ojlera (Euler) (Eu_M) jer je:

$$\left[\frac{\rho \cdot \upsilon^2}{p}\right] = Eu^{-1}$$

a,

$$Eu_M = Eu^{-1} \equiv \left[\frac{\rho \cdot \upsilon^2 \cdot \frac{l^3}{t}}{p \cdot \frac{l^3}{t}}\right] \equiv$$

$$\equiv \frac{\rho \cdot n^3 \cdot D^5}{N} \equiv Np^{-1} \qquad (8)$$

gde je:

$Q = \frac{l^3}{t}$ - protok pulpe

$$\frac{\mu}{\rho^1 \cdot n^1 \cdot D^2} = \frac{\mu}{\rho \cdot n \cdot D^2} \equiv \mathrm{Re}^{-1}$$

- Rejnoldsov bezdimenzioni broj za proces mešanja

$$\frac{g}{\rho^0 \cdot D^1 \cdot n^2} = \frac{g}{D \cdot n^2} \equiv Fr^{-1}$$

- Froudeov bezdimenzioni broj za mešanje

Uz određene uslove eksperimenta u laboratorijskim uslovima pokušalo se je da se dođe do rešenja jednačine 7 i do pronalaženja koeficijenta k i eksponenata a i b .

Njen novi analitički oblik ima izgled prikazan jednačinom 9:

$$Eu_M = k \cdot \mathrm{Re}^a \cdot Fr^b \quad (9)$$

Logaritmujući kriterijumsku jednačinu 9 dobijamo:

$$\log Eu_M = \log k + a \cdot \log \mathrm{Re} + b \cdot \log Fr \quad (10)$$

Potrebno je eksperimentalno odrediti funkciju Eu_M i jedan od kriterijuma, s tim da se drugi kriterijum održava konstantnim u tom nizu eksperimenata.

Ako je iz jednačine 9:

$$k \cdot Fr^b = konst. = B \quad (11)$$

Dobija se kriterijumska jednačina:

$$\log Eu_M = a \cdot \log \mathrm{Re} + \log B \quad (12)$$

Dakle oblik:

$y = a \cdot x + b$.

Dinamički viskozitet μ i gustine pulpe ρ_p se u laboratorijskom mlinu mogu podesiti tako da njihov odnos bude konstanta. Merenje viskoziteta vršeno je viskozimetrom po Brukfildu. Korišćen je pribor sa rotirajućim cilindrima koji omogućava određivanje viskoznosti u širokom intervalu konzistencije. Gustina pulpe meri se rudarskim piknometrom (koji se upotrebljava u flotacijama) i vagom za merenje gustine. Zahvaljujući koriščenju različitih vrsta šarže meljućih tela (čelične, alumo i silikatne kugle) vremena mlevenja su se razlikovala ali je finoća mlevenja u svakom od pomenutih opita bila ista.

Promena Rejnoldsovog Re - kriterijuma, koja je potrebna za formiranje modela, postiže se menjanjem vremena boravka sirovine u mlinu, tabela 3. Promena vrednosti Ojlerovog kriterijuma Eu_M računa se prema jednačini 8, a iz analitičkog izraza te jednakosti se vidi da su poznati svi parametri ($N, \rho_{up}, n^3 i\ D^5$). Snagu N merimo pomoću električnog brojila, odnosno uređaja na koji je priključen motor laboratorijskog mlina. Gustina šarže $\rho_š$ je linearna funkcija gustine pulpe ρ_p gde koeficijent pravca i odsečak na ordinati zavise od vrste meljućih tela odnosno ρ_{vk} i ρ_{sk} kao što se vidi iz jednačine 13:[3, 4, 5]

$$\rho_š = \rho_{vk} + 1{,}15 \cdot (1 - \frac{\rho_{vk}}{\rho_{sk}}) \cdot \rho_p \ \frac{kg}{m^3} \quad (13)$$

gde je:

ρ_{vk} - *gustina kugli u nasutom stanju, u kg/m^3;*

ρ_{sk} - *gustina materijala od kog je sačinjena kugla, u kg/m^3;*

ρ_p - *gustina pulpe, u kg/m^3.*

Gustina materijala od kog je sačinjena kugla:

Fe livene - ρ_{sk} = 7800 kgm^{-3},

Al_2O_3 - ρ_{sk} = *4800* kgm^{-3},

Silikatne - ρ_{sk} = 2600 kgm^{-3}.

Gustina kugli u nasutom stanju:

Fe livene - ρ_{vk} = 4100 do 4200 kgm^{-3},

Al_2O_3 presovane - ρ_{vk} = 2500 do 2700 kgm^{-3},

Silikatne - ρ_{vk} = 1800 do 1900 kgm^{-3}.

Konstantnost Frudeovog kriterijuma u ovoj seriji postiže se upotrebom mlina istih dimenzija za sve pojedinačne opite. Kada se podaci iz tabele predstave dijagramom vidljivo je da se tačke koje predstavljaju koordinate pojedinih opita nalaze na približno pravoj liniji.

Na slici 2 je vidljivo da je $tg\alpha = a = 0{,}6259$, odnosno a je eksponent kriterijuma Re, a odsečak na osi $\log Eu_M$ predstavlja vrednost $\log B$, pa je $\log B = -1{,}8261$, a tada je B =0,014924507.

Da bi utvrdili koeficijent k iz jednačine 9 potrebno je da izvršimo još jednu seriju eksperimenata u kojoj će se podesiti uslovi eksperimenta da Rejnoldsov broj bude konstanta. Dakle, nakon prve serije eksperimenata na modelu se izvodi druga serija eksperimenata, u kojoj se održava konstantnim vrednost kriterijuma Re, odnosno iz jednačine 9:

$$k \cdot \mathrm{Re}^{a} = konst. = C \qquad (14)$$

Iz ovoga sledi kriterijumska jednačina:

$$\log Eu_M = b \cdot \log Fr + \log C \qquad (15)$$

Dakle opet jednačina prave linije kao funkcionalna zavisnost Eu_M od promene Fr.

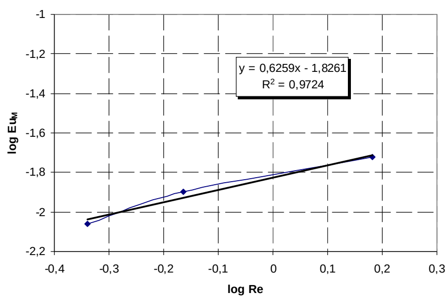
Da bi Rejnoldsov kriterijum ostao konstanta potrebno je bilo menjati vreme tretmana na način da u većem mlinu bude duže vreme zadržavanja sirovine, a u manjem mlinu kraće vreme zadržavanja sirovine. Navedene eksperimentalne uslove postigli smo sa smanjenjem broja obrtaja srednjeg i velikog mlina kada je efikasnost mlevenja manja i povećanjem broja obrtanja malog mlina kada je postignuta veća efikasnost mlevenja.

Tabela 3. *Eksperimentalni podaci za kriterijum snage kada se ne menja Frudeov broj*

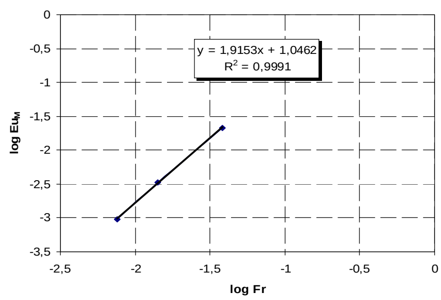
Viskozitet pulpe	Vreme mlevenja	Prečnik plašta	Brzina pulpe	Gustina pulpe	Rejnoldsov broj		Broj obrtaja	Frudeov broj		Merena snaga mlina	Vrsta meljućih tela	Gustina nasutih kugli	Gustina materijala	Gustina šarže	Ojlerov broj Eu_M	
					Re	*logRe*		*Fr*	*logFr*						Eu_M	$logEu_M$
μ,Pas	t, *s*	D,m	υ,ms^{-1}	*kgm*$^{-3}$			ns^{-1}			N,W		*kgm*$^{-3}$	*kgm*$^{-3}$	*kgm*$^{-3}$		
0,416	600	0,305	0,000510	1226	0,458	-0,3387	1,04	0,0329	-1,482	760	Si	1800	2600	2234	0,008726	-2,05917
0,416	400	0,305	0,000763	1226	0,686	-0,1637	1,04	0,0329	-1,482	760	Al	2600	4800	3246	0,012681	-1,89684
0,416	180	0,305	0,001694	1226	1,523	0,1826	1,04	0,0329	-1,482	760	Fe	4200	7800	4851	0,018949	-1,72241

Tabela 4. *Eksperimentalni podaci za kriterijum snage kada se ne menja Rejnoldsov broj*

Viskozitet pulpe	Vreme mlevenja	Prečnik plašta	Brzina pulpe	Gustina pulpe	Rejnoldsov broj		Broj obrtaja	Frudeov broj		Merena snaga mlina	Vrsta meljućih tela	Gustina nasutih kugli	Gustina materijala	Gustina šarže	Ojlerov broj Eu_M	
					Re	*logRe*		*Fr*	*logFr*						Eu_M	$logEu_M$
μ,Pas	t, *s*	D,m	υ,ms^{-1}	*kgm*$^{-3}$			n,s^{-1}			N,W		*kgm*$^{-3}$	*kgm*$^{-3}$	*kgm*$^{-3}$		
0,17	417	0,150	0,00036	1066	0,338612	-0,4703	1,6	0,0384	-1,415668	70	Fe	4200	7800	4766	0,02117649	-1,67415
0,17	898	0,220	0,000245	1066	0,337985	-0,4711	0,8	0,0141	-1,851397	250	Al	2600	4800	3162	0,00333724	-2,47661
0,17	1713	0,305	0,000178	1066	0,34043	-0,4679	0,5	0,0076	-2,117760	760	Si	1800	2600	2177	0,00094513	-3,02451



Sl. 2. *Funkcija* $\log Eu_M$ *i* $\log \mathrm{Re}$



Sl. 3. *Funkcija* $\log Eu_M$ *i* $\log Fr$

Da bi postigli pomenute uslove vremena mlevenja osim promene broja obrtanja mlina takođe je bilo potrebno da se u najvećem mlinu koristi manja šaržna gustina (šarža od sileks kugli), u srednjem mlinu srednja šaržna gustina (šarža od alumo kugli), a u najmanjem mlinu najveća šaržna gustina (šarža od čeličnih kugli). Na kraju, da bi Rejnoldsov kriterijum ostao konstantan bilo je potrebno izvršiti više preliminarnih opita da bi se utvrdila i gustina pulpe. U tabeli 4 su prikazani eksperimentalni podaci za drugu seriju opita, a kada se podaci iz tabele predstave u grafičkom obliku dobijamo skoro pravu liniju, slika 3. Analogno ranijim zaključivanjima vidi se da je $tg\beta = b = 1{,}9153$ i odsečak na ordinati $\log Eu_M$ je vrednost $\log C = 1{,}0462$, a tada je C=11,12243817. Pošto je u dva niza eksperimenata određen eksponent kriterijuma $\mathrm{Re}(a)$ i eksponent kriterijuma $Fr(b)$, konstanta k se može izračunati iz:

$$k = \frac{B}{Fr^b} = \frac{0{,}015}{Fr^{1{,}92}} = 10{,}27\,,$$

odnosno

$$k = \frac{C}{\mathrm{Re}^a} = \frac{11{,}12}{\mathrm{Re}^{0{,}63}} = 21{,}91 \qquad (16)$$

Pa se dobija srednja vrednost:

$$k_{sr} = \frac{\sum_1^2 k}{2} = \frac{k_1 + k_2}{2} = = \frac{10{,}27 + 21{,}91}{2} = 16{,}1 \qquad (17)$$

Uzevši da su za ovakav ispitivani sistem konstanta k i eksponenti a i b konstantne vrednosti može se za eksperimentalno dobijenu kombinaciju kriterijuma Re i Fr izračunati vrednost Eu_M.

$$Eu_M = k \cdot \mathrm{Re}^a \cdot Fr^b = = 16{,}1 \cdot 0{,}338612^{0{,}63} \cdot 0{,}032989^{1{,}92} = = 0{,}011869 \qquad (18)$$

Obzirom da je: $\frac{\rho_{up} \cdot n^3 \cdot D^5}{N} = Eu_M$ i $Eu_M = k \cdot \mathrm{Re}^a \cdot Fr^b$ vidimo da se primenom laboratorijskih ispitivanja došlo do kriterijumske jednačine šaržne snage mlina u tretmanu silikatne sirovine koji može biti upotrebljena za proveru šaržne snage mlina u uvećanim uslovima.

$$N = \frac{\rho_{š} \cdot n^3 \cdot D^5}{16{,}1 \cdot \mathrm{Re}^{0{,}63} \cdot Fr^{1{,}92}} \qquad (19)$$

Provera šaržne snage mlina

Provera šaržne snage mlina za adaptirane uslove tretmana kvarcnog peska u Lukića polju kod Milića vršena je da bi se proverila okvirna šaržna snaga koja ima vrednost $N = 280.000$ W.

$\rho_{š} = 2177\ kgm^{-3}$ - gustina šarže (kugle+voda+materijal)

$D = 2{,}2\,m$ - unutrašnji prečnik mlina,

$n = 0{,}3\ s^{-1}$ - broj obrtaja mlina,

$$N = \frac{\rho_{up} \cdot n^3 \cdot D^5}{16{,}1 \cdot \mathrm{Re}^{0{,}63} \cdot Fr^{1{,}92}} = = \frac{2177 \cdot 0{,}3^3 \cdot 2{,}2^5}{0{,}012} = 255.256\,W$$

Kao što se vidi kriterijumska jednačina modela daje dobre rezultate, jer je izračunata šaržna snaga mlina manja od okvirne snage mlina odnosno snage motora.

ZAKLJUČAK

Značaj prikazanog postupka iznalaženja kriterijumskih jednačina modela šaržne snage mlina je velik jer se model može koristiti uopšteno za sve mineralne sirovine [6,7]. Kriterijumske jednačina modela moguće je primeniti i na industrijskim mlinovima zato što su u laboratorijskim uslovima menjani svi relevantni parametri koji utiču na proces i u industrijskim uslovima.

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