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SEQUENTIAL DESIGN OF CONTROLLER FOR GRINDING CIRCUIT

Abstract

Multivariable controller design for grinding circuit is the subject of this work. Model of grinding circuit in Majdanpek Copper Mine is subjected to the newly proposed sequential design method, based on the Gauss-Jordan factorization and Nyquist-like techniques. Due to the nature of the applied design method, generalization to any other grinding circuit is straightforward. An extensive simulation analysis proved the effectiveness of proposed design in reference tracking, even in the case of some loop failures.

Keywords: grinding process, sequential design, Nyquist array

INTRODUCTION

Grinding of the ore to a fine product is the unit operation of crucial importance in the metallurgical extraction process. Its importance arises from the fact that it is the most expensive operation itself, but also, operation with the great influence to downstream extraction process. The ore exploitation with lower contain of useful component, as in the case of Serbian copper mines, make this importance even more significant.

The first major effort to automatic process control of grinding circuits made use of conventional PID-type controllers in many different single-loop control schemes. As a grinding process is not a single-variable control problem but is a multivariable system, conventional Inverse Nyquist Array (INA) method was favorite engineering tool for development a multivariable controllers for this kind of processes in last decades (Koudstaal et al., 1981, Hulbert et al., 1990). The direct extension of this approach, by introduction the model uncertainty and defines robust INA methodology, was presented by Ivezić and Petrović (2003). Craig and MacLoad (1995, 1996) applied μ -controller

synthesis and analysis methodology to design the robust controllers of grinding process. Duarte et al. (1999) and Pomerleau et al. (2000) made comparisons of different multivariable control strategies in grinding control.

For a number of reasons multi input - multi output (MIMO) control system, as for grinding circuits, could be designed, tuned and commissioned in sequential order, closing one feedback loop at the time. First, this sequential approach seems as an extension of well-known single input - single output (SISO) design methods into the area of generally much more complex MIMO systems. Second, a good sequential stability interpretation offers a possibility of obtaining a stable system at each stage of the design. A cautious use of this feature provides stability of the system even in the case of loop failure. Also, using of sequential design eliminate often a tedious work in direct decoupling of system but without loss of design quality and with faster solution obtain.

Basic concepts of the applied sequential method originate from Mayne (1979), as

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THE PLANT

suming that a pre-compensator is a full transfer function matrix, designed one column at each stage. Although such an approach simplifies a multivariable design into a series of simple multi input - single output design procedures, its implementation usually yields unnecessarily high order controllers within the pre-compensator. An improvement of the original sequential method was proposed in Bryant (1985), introducing Gauss elimination (GE) operations on the system return difference matrix. It was shown that the process of closing a system in sequential order could be formalized by GE, this way also avoiding afore mentioned controller order excess problem. Recently (Bryant and Yeung, 1996), additional method improvement is proposed the Gauss-Jordan (GJ) elimination instead of GE. Although both approaches offer a good sequential stability interpretation, using GJ elimination as a sequential transformation provides the full input - output transfer function of the partially closed system at each stage of design, whereas GE approach only provides transfer function associated with open-loop section of the system.

A grinding circuit in the Majdanpek Copper Mine is shown in Figure 1. This circuit consists of two primary rod mills and two secondary ball mills closed by hydro-cyclones. Also, in the secondary loop are two unit flotation cells that prevent the over grinding of coarse, floatable material of high density.

The levels in each of sumps and the flow rates and density of the streams leaving sumps are measured. Thus, the discharge stream from the secondary mills can be characterized from balances round the unit-feed sump and the primary mills, and the feed stream to the secondary mills can be characterized from balances round the cyclones and flotation-feed sump. The level in each sump was controlled by separate system, which allowed them to act as a buffer against fluctuations in the incoming flow while the outgoing flow was kept under tight control. On the overflow line from the cyclone a particle size monitor (P.S.M.) measures the particle size distribution and the mass percentage of the solids in this product stream. Only one point on the particle size distribution is measured, namely the percentage of the solids smaller than 75 μm , and no indication of the particle size spread is given.

Figure 1 Grinding circuit in Majdanpek Copper Mine

The model of grinding circuit was obtained from its experimental identification around particular operating point, Grujic (1995). Transfer function of grinding circuit (consisting of the particle size of the product and the feed flow rate of the to the cyclone as outputs and the solid feed (together with its dilution) and the cyclone dilution as inputs) is given by Ivezić and Petrović (2003), as 2x2 transfer function matrix. That transfer function matrix is here somewhat extended

and modified for the reason of more adequate representing system dynamic but also for precise control. Namely, instead of total dilution as input variable, unit-feed sump dilution and cyclone-feed sump dilution are introduced as new input variables and density of the feed to the cyclone is introduced as new output variables. Also, sequence of input and output variables is rearranged so that model of grinding circuit is represented by 3x3 transfer function matrix:

$$P = \begin{pmatrix} \frac{36.49}{792s+1} & \frac{1.1405}{179s+1} & \frac{0.1866}{573s+1} \\ \frac{1676s+1}{78.9} & \frac{63.5s+1}{-4.51} & \frac{0.94}{683s+1} \\ \frac{-0.9362 \cdot e^{-350s}}{1164s+1} & \frac{(10.252s+0.003) \cdot e^{-200s}}{80218s^2+652s+1} & \frac{(132.7s+0.057) \cdot e^{-80s}}{36300s^2+358s+1} \end{pmatrix} \quad \begin{matrix} y = Pu \\ y = [y_1 \ y_2 \ y_3]^T \\ u = [u_1 \ u_2 \ u_3]^T \end{matrix} \quad (1)$$

u_1 - solid feed (together with its dilution)
 u_2 - unit-feed sump dilution
 u_3 - cyclone-feed sump dilution

y_1 - cyclone feed flow rate
 y_2 - cyclone feed density
 y_3 - particle-size measurement of the products

SEQUENTIAL METHOD BACKGROUND

The control objective of grinding circuit is to maintain the particle size of the product and flow rate and density of the feed to the cyclone close to set points $y_{ref} = [y_{1ref} \ y_{2ref} \ y_{3ref}]^T$.

The applied sequential design method is described in detail by Bryant and Yeung (1996). A closed loop system structure is shown in Figure 2, where $K(s)$ stands for controller (precompensator).

Figure 2 The considered system structure

It is assumed that $K(s)$ is a full transfer function matrix, rather than just diagonal or triangular. Anyhow, $K(s)$ can be written in the form

$$K(s) = K^{n+1}(s) \cdots K^2(s) K^1(s) \quad (2)$$

where $K^j(s)$, $j = 1, \dots, n+1$, are constructed from an identity matrix by replacing j -th column with a column of transfer functions. The condition of feedback loops for system in Figure 2 is described by real matrix F^i

$$F^i = \text{diag}\{f_1 \cdots f_i \ 0 \cdots 0\}, F^{n+1} = F = I \quad (3)$$

where $f_j = 1$, $j = 1, \dots, i$, stands for closed feedback loop. For precompensator structure (2) and a given plant transfer function matrix $P(s)$, the j -th column of the return difference matrix $R(s)$ of the system, defined as

$$R(s) = I + P(s)K(s)F \quad (4)$$

becomes a function of the j -th column of $K^j(s)$ only. Hence, the precompensator design is accomplished through a series of multi input - single output (MISO) designs, obtaining one $K^j(s)$ while simultaneously closing the associated feedback loop at each stage. The obvious advantage of this multi-stage sequential approach is a strong background in SISO design methods. On the other hand, a direct implementation yields precompensator of excessively high orders, and more seriously, the designer has no notion of the final form of $K(s)$.

In order to overcome these problems, Bryant and Yeung (1996) proposed use of sequential transformation, which consists of series of GJ operation on $R(s)$. These operations can be formalized as follows

$$N^{n+1}(s) \cdots N^2(s) N^1(s) R(s) = I \quad (5)$$

where $N^j(s)$ represents a row operation of the j -th stage of GJ elimination and is constructed from an identity matrix by replacing the j -th column with the transfer functions $n_{ij}^j(s)$, $i = 1, \dots, n+1$. If $R^{(j)}(s) = [r_{ik}^{(j)}(s)]$

denotes a return difference matrix after j stages of GJ elimination,

$$R^{(j)}(s) = S^j(s) R(s) \quad (6a)$$

$$S^j(s) = N^j(s) \cdots N^2(s) N^1(s) \quad (6b)$$

where $S(s)$ stands for sensitivity matrix

$$S(s) = S^{n+1}(s) = R^{-1}(s) \quad (7)$$

then $n_{ij}^j(s)$ is defined as

$$n_{ij}^j(s) = \begin{cases} -\frac{r_{ij}^{(j-1)}(s)}{r_{jj}^{(j-1)}(s)} & \text{if } i \neq j \\ \frac{1}{r_{jj}^{(j-1)}(s)} & \text{if } i = j \end{cases} \quad (8)$$

Furthermore, using (6) and (3), (4) can be rewritten as

$$S^j(s) P(s) K(s) = P^j(s) K(s) = R^{(j)}(s) - S^j(s) \quad (9)$$

showing that after j stages of GJ operations on $R(s)$, obtained transfer function $P^j(s)K(s)$ represents partially a closed system with first j feedback loops closed; here $P^j(s)$ is interpreted as a new open-loop system matrix with first j loops closed. It was already shown in Bryant (1985) that sequential loop closure can be formalized through a series of GE factorizations. However, in Bryant and Yeung (1996) it was provided that applying successive GJ operations on $R(s)$ yields not only the transfer function of partially closed system, but also provides that columns of precompensator $K(s)$ can be determined separately, one at each stage, even more directly than suggested by (2). In elements $r_{ij}^{(j-1)}(s)$ of $R^{(j-1)}(s)$, called sequential return difference functions, which is due to the nature GJ operations.

A sequential method via GJ factorization can now be outlined in the following procedure:

0. Let $K^0(s) = N^0(s) = S^0(s) = I$,

$$P^0(s) = Y^0(s) = P(s), \quad j = 1.$$

1. Prepare the Nyquist array

$$Y^{j-1}(s) = \begin{bmatrix} I - S_{11}^{j-1}(s) & P_{12}^{j-1}(s) \\ P_{21}^{j-1}(s) & P_{22}^{j-1}(s) \end{bmatrix}$$

partitioned into the blocks $Y_{11}^j(s)_{j \times j}$,

$Y_{12}^j(s)_{j \times (n+1-j)}$, $Y_{21}^j(s)_{(n+1-j) \times j}$ and

$Y_{22}^j(s)_{(n+1-j) \times (n+1-j)}$.

2. Examine the Nyquist array $Y^{j-1}(s)$

and choose precompensator column $k_{ij}(s), i = 1, \dots, n+1$. Then compute

$r_{ij}^{(j-1)}(s)$, plot its Nyquist contour and count the number of counter-clockwise encirclements about the origin, c_j . In order for closed loop system to be stable, it must be $\sum_{i=1}^{n+1} c_i = -p_0$, where p_0 is the number of unstable poles of the original $P(s)$. Repeat this step until the stability margin requirements are satisfied.

3. Update $K^j(s)$ by substituting the j -th column $K_{*j}^j(s)$ with $k_{ij}(s)$, when update $R^{(j-1)}(s)$ according to the newest column, $R_{*j}^{(j-1)}(s) = P^{j-1}(s)K_{*j}^j(s) + e_j$, e_j representing the j -th column of identity matrix.

4. Compute $N^j(s)$ and perform one stage of GJ factorisation by updating

$$R^{(j)} = N^j(s)R^{(j-1)}(s) \quad (10a)$$

$$S^j(s) = N^j(s)S^{j-1}(s) \quad (10b)$$

$$P^j(s) = S^j(s)P(s) = N^j(s)P^{j-1}(s) \quad (10c)$$

5. Compute a transfer function of partially closed system, $H^j(s) = P^j(s)K^j(s)$.

Submatrix $H_{11}^j(s)_{j \times j}$ should be close to

the unity matrix I_j , and submatrices

$$H_{21}^j(s)_{(n+1-j) \times j} \quad \text{and} \quad H_{12}^j(s)_{j \times (n+1-j)}$$

should be as small as possible over the operating bandwidth. If not, repeat the procedure from step 1.

6. Increase j by 1. If still $j \leq n+1$, continue with the procedure from step 1, else the design is completed.

Since the Nyquist approach is utilized when choosing $k_{ij}(s)$ in the stages 1 and 2 according to Bryant and Yeung (1996) this method is called the Basic Gauss-Jordan Nyquist design procedure.

CONTROLLER DESIGN

One of the most critical point in a grinding circuit is the cyclone underflow, since overloading at that point can lead to sanding-up of the secondary mills and to discharge of very coarse material to the product. Cyclone underflow is not measured direct in the Majdanpek Concentrator, but it is affected directly by flow and density of its feed. Therefore control of these two variables has priority so that loop closing procedure will start with one of these two loops.

According to steps 1 and 2 of the sequential procedure, the Nyquist array of $Y^0(s) = P(s)$ is examined. Its Nyquist plot is presented in Figure 3. It is evident that $Y_{21}^0(s)$ is relatively very large with respect to $Y_{11}^0(s)$, therefore the cross-couplings from loop-2 to loop-1 is high and therefore sequential design will start from loop 2. For further design is important to notify that $Y_{13}^0(s)$ and $Y_{23}^0(s)$ are relatively small with respect to $Y_{11}^0(s)$ and $Y_{22}^0(s)$, respectively.

Exploiting the last conclusions and using classical loop-shaping techniques, the second column of controller (precompensator) is chosen to be:

$$K_{*,2}^1(s) = \begin{bmatrix} \frac{1}{P_{21}(s)} \frac{k_1}{s(T_1s+1)} & \frac{1}{P_{22}(s)} \frac{k_1}{s(T_1s+1)} & 0 \end{bmatrix}^T \quad (11)$$

Figure 3 Nyquist plot of $Y^0(s)$

Figure 4 Sequential return difference functions

Time constant $T_1 = 1$ and gain $k_1 = 0.1$ are chosen according to step 2 requirement for number of encirclement sequential return difference function $r_{22}^{(0)}(s)$ about the origin. Since the open loop transfer function $P(s)$ is stable, the sum of the origin counter-clockwise encirclements c_j of the sequential return

difference function $r_{jj}^{(j-1)}(s)$ at each stage should be equal to zero, in order for a closed loop system to be stable. Figure 4 shows that this hold for $r_{22}^{(0)}(s)$. Physically, it means that cancel of loops 1 and 3 will not affected to regular feed density to cyclone. This ensure that sending-up of mills will be avoided

Figure 5 Nyquist plot of $h_{22}^1(s)$

Figure 6 Nyquist plots of $H_{21}^1(s)$ and $H_{12}^1(s)$

Figure 7 Nyquist plot of $Y^l(s)$

Figure 8 Nyquist plots of $H^2(s)$

Following further steps 3 to 5 of the procedure, a partially closed system with only the second feedback closed $H^1(s) = [h^1_{ik}(s)]$ is computed and reviewed. Thus Figure 5 shows that $H^1_{11}(s) = h^1_{22}(s)$ is close to unity over the operating bandwidth, while

$$H^1_{12}(s) = [h^1_{21}(s) \quad h^1_{23}(s)]$$

and

$$H^1_{21}(s) = [h^1_{12}(s) \quad h^1_{32}(s)]$$

submatrices are insignificant comparing to $h^1_{22}(s)$, Figure 6. This means that $K^1(s)$ design so far yields both a good loop-shaping and decoupling, the later providing that the following stages of the design will not affect significantly the results of this one.

The second stage of design starts with putting $j = 2$ and examining the Nyquist array of $Y^1(s)$, Figure 7, formed by the previously computed $S^1(s)$ and $P^1(s)$. It is obviously that $Y^1_{22}(s) = h^1_{22}(s)$ and off-diagonal elements of $Y^1_{11}(s)$ are significantly smaller of it. According to this fact, that is loop-1 is now insensitive to another, and using classical loop-shaping techniques, the first column of controller (precompensator) is chosen to be:

$$K^{2,*}_1(s) = \begin{bmatrix} \frac{1}{P_{11}(s)} \frac{0.005}{s(s+1)} & 0 & 0 \end{bmatrix}^T \quad (12)$$

Following further steps of the procedure, a partially closed system with the first and the second feedback closed $H^2(s) = [h^2_{ik}(s)]$ is computed and reviewed. Figure 8 shows that $h^2_{11}(s)$ is close to unity over the operating bandwidth, $h^2_{22}(s)$ is unchanged, as expected and off-diagonal elements of these two terms are significantly smaller. From the point of view of overall system stability it is worth nothing that, just like in the previous stage of design, sequential return difference function $r^{(1)}_{11}(s)$ has zero origin encirclements, $c_2 = 0$, Figure 4. Physically, it means that cyclone can work independently, regardless to loop-3 of system. This concludes the second stage.

The last stage of design starts with putting $j = 3$ and examining the Nyquist array of $Y^2(s)$, formed by the previously computed $S^2(s)$ and $P^2(s)$, Figure 9. Existing the time delay in row 3 and somewhat large element $Y^2_{31}(s)$ according to $Y^2_{33}(s)$ require introducing of PI and phase-lag control in the precompensator third column:

$$K_{*,3}^3(s) = \begin{bmatrix} \frac{1675s+1}{500s+1} \cdot \left(-0,019 - \frac{0,0002}{s} \right) & 0 & \left(1 + \frac{0,01}{s} \right) \end{bmatrix}^T \quad (13)$$

Final closed loop transfer function matrix $H^3(s) = H(s)$ has excellent frequency characteristics for all diagonal elements, together with strong decoupling of the off-diagonal elements, as shown in Figure 10. Precompensator choice defined by (11-13) gives zero origin encirclements of sequential return difference function $r_{jj}^{(j-1)}(s)$, $c_j = 0$, at each design stage, Figure 4. Together with the fact that the open loop

transfer function matrix $P(s)$ is stable, this guarantees stability for closed loop transfer function matrix $H(s)$. Thus, the design is completed with stability and performance objectives satisfied. Also, it is interesting to note that controller obtained by GJ sequential design is simple (all transfer functions of $K(s)$ are easy to implement) and that the controller is sparse matrix (four elements of nine of $K(s)$ are zero).

Figure 9 Nyquist plot of $Y^2(s)$

Figure 10 Nyquist plots of $H(s)$

THE TRANSIENT ANALYSIS

Extensive simulations have been performed to confirm the obtained results. Time responses of closed-loop system, using sequential controller (SC) and previously de-

signed, INA and decentralized (DC) controllers are compared in order to observe how the system tracks set points changes. A unit step is used as a test signal.

Figure 11 Time responses of plant with different controllers to unity step signal in input u_1

Figure 12 Time responses of plant with different controllers to unity step signal in input u_2

Figure 13 Time responses of plant with different controllers to unity step signal in input u_3

The most typical time responses of particle size of the product, cyclone feed flow rate and density on cyclone-feed sump dilution, solid feed (together with its dilution) and unit-feed sump dilution are shown in Figures 11-13. It is evident that characteristics of sequential controller lie somewhere between of characteristics of INA and DC controllers. Sequential controller has faster response compared to INA controller. The use of very complex structure of decoupling precompensator in INA controller design as a consequence has somewhat better decoupling characteristic. But simplicity of sequential controller, together with straight

forward procedure for its derivation overcomes that advantage of INA design. Moreover, time responses of plant with failure of loops 1 and 3 show that decoupling characteristic of sequential controller is superior, even to INA controller, Figure 14 (responses in output y_2 are similar as in case of all loops closed - Figure 12). It is because special case, analogous to the first stage of sequential design, was considered. DC controller is with the similar responses characteristics as sequential controller but with the worst decoupling of the system which is more evident in the case of loop failures.

Figure 14 Time responses of plant with failure of loops 1 and 3 to unity step signal in input u_2

In this paper the design of a multivariable controller for grinding process is presented. The applied design method is the Gauss-Jordan Nyquist sequential procedure. Adequate sequential controller for the grinding circuit in Majdanpek Copper Mine is obtained, its structure discussed and variable parameters tuned. The simulation analysis revealed satisfactory closed loop performance. Comparison with INA and decentralized controllers justified use of this design method. Procedure is more straightforward than in case of INA design but with quite well decoupling result. Also, sequential design procedure provides in some degree integrity of the system, that is stability robustness in the face of loop failures. In the case of decentralized controller use, interaction between the loops is still significant and so unacceptable.

Regardless to different structure of grinding circuits used in different metallurgical extraction processes, mathematical description of physical dependences and phenomena is similar. As the utilization of Nyquist array plots in controllers' design is recognizable for the engineers in this branch and the proposed sequential design is based to Nyquist approach, given arguments and explanations for use of sequential design method, define this method as powerful tool for controllers design for this class of processes.

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