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CIRCULAR HOLES AS SOURCES OF STRESS CONCENTRATION IN THE PARTS OF MACHINES AND DEVICES USED IN MINING

Abstract

This work discusses the impact of circular forms on stress distribution in the parts of machines and devices used in mining, since it represents a typical source of stress concentration. The stress distribution around the hole, as a source of stress concentration, is significantly affected both by the hole size and type of material and its shape and position relative to the direction of external load effect. The aim of this work is to analyze the impact of circular holes, as a source of stress concentration, on the stress distribution since a large number of parts of which the machines and devices used in mining are made, is impaired by the stress. The work discusses only the parts made of isotropic material, because the practice has shown that most of the parts of which machines and devices used in mining are assembled, are made of isotropic materials. To obtain the results of stresses distribution in a plane isotropic field weakened by the holes, analytical and numerical methods were used.

Keywords: *circular hole, stress distribution, stress concentration, parts, machines and devices*

1 INTRODUCTION

The intention of each country is, as far as possible, to develop and expand its raw material base. Mining industry has a task of supplying an economy with the raw material in various forms. The supply of these raw materials is realized both by underground and surface mining exploitation the deposits of raw materials. Underground mining of mineral deposits involves obtaining the mineral raw material by the underground operation using the appropriate machines and devices and appropriate excavation method. Machines and devices in the underground mining are used for excavation, loading, transport, supporting elements of excavated area and its filling, and for making the underground chambers. Surface mining of deposits is significantly different from the underground, both in technological process of mining deposits, and in the application of

appropriate machinery. Surface mining represents a set of all works from the ground level to the unearthing of deposits for useful mineral raw materials, and a facility that is thus formed is called an open pit mine. Surface mining involves two basic groups of works: works on stripping waste rock (tailings) and works on useful mineral raw materials. Work on the overburden consist of the excavation, transportation and disposal of overburden tailing mass that cover, i.e. prevent free access and safe exploitation of useful raw materials. Works on useful raw material consist of the excavation, transport, transfer or storage of useful mineral resources. For all these works, it is necessary to use the appropriate machinery, machines and equipment [1].

Mining machines and devices have the built-propulsion systems, power transmis-

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sions, working bodies, control devices and brake systems which are composed of a large number of mechanical parts that make up a functional unit. In most cases these parts are complex geometric shapes. Bearing in mind the fact that each of these parts may have at least one zone in which there is a discontinuity, and then the issue covered by the title of this work becomes even more relevant.

In engineering practice, the knowledge of stress distribution and deformations is of great importance in the areas of geometric discontinuity the elements of machine design. Tests have shown that the stresses on the change of cross-sectional areas of machine parts increase, and all the more so if the change of cross-section is more intense. Such an occurrence of increasing stress on positions the cross-section change is called the stress concentration. Notches, grooves, passes from one width to another or from one diameter to another, holes and openings are typical sources of stress concentration.

The basic and most important task in design of complex mechanical structures, or parts of it, is the proper consideration of all parameters that influence the stress state. This approach involves defining the critical zones within the structure or its parts. These are, among others, the source zones of stress concentration around the holes of different shapes and positions. Solving problems in this case comes down to resolving differential equations, ordinary and partial, while satisfying the appropriate boundary and initial conditions [2-5].

For a more detailed calculation the stress, in order to take into account all influential factors in the field of machine elements weakened by the openings numerical methods, were used for research in this work. To simulate the real operating conditions (load, boundary conditions), the numerical methods were also used. Given the nature of the problem that is being addressed in this paper, finite element method (FEM) was chosen and two-dimensional (plane) finite elements were used [6-7]. As

numerical methods involve the use of computers and related software packages for generating a finite element mesh ANSYS software package was used herein [14].

A large number of structures, in order to ensure the conditions and meet the demands of strength, were made of isotropic materials. Isotropy is a feature of some bodies in different directions to show the same physical and mechanical properties, for example elasticity, stress, strain, etc. Many construction materials from which the parts of machines and equipment in mining are made of have the isotropic characteristics.

When it comes to research the phenomenon of stress concentration around a circular hole in the majority of cases, a thin plate weakened by a circular hole loaded with a certain load is analyzed [8-13]. The problem narrows down to the exact determination of stress distribution around the hole, as well as determination the stress concentration factor.

In this work, a special attention will be paid to the study of distribution the stresses in areas weakened by the circular hole, made of isotropic material and exposed to the static load.

2 DETERMINATION THE STRESS IN PANEL USING THE THEORY OF ELASTICITY EQUATIONS

In practical problems, it is often the case that a part of structure is burdened with external load so that only stresses occur in it and then it is spoken about the flat (planar) strain, or only deformation in two mutually perpendicular directions when it comes to flat (planar) deformation. Solving problems in this case comes down to resolving differential equations, ordinary and partial, while satisfying appropriate boundary and initial conditions [4, 5].

Starting from the aforementioned equations of the theory of elasticity, the expressions are derived in literature [3, 8], which define the stress state as the uni-axial strained homogeneous isotropic plate

weakened by a circular hole in polar coordinates:

$$\sigma_\rho = \frac{p}{2} [(1 - \rho^2) + (1 - 4\rho^2 + 3\rho^4) \cos 2\theta] \quad (1)$$

$$\sigma_\theta = \frac{p}{2} [(1 + \rho^2) - (1 + 3\rho^4) \cos 2\theta] \quad (2)$$

$$\tau_{\rho\theta} = \frac{p}{2} (1 + 2\rho^2 - 3\rho^4) \sin 2\theta \quad (3)$$

where:

p – surface forces of plate stretching,

ρ – polar pull measured from the hole center,

θ – polar angle.

The expression for calculation the stress to the contour of circular hole is obtained if in (1), (2) and (3), $\rho = 1$:

$$\sigma_\theta = p(1 - 2 \cos 2\theta) \quad (4)$$

From (4), it is shown that maximum values of stress σ_θ are obtained at $\cos 2\theta = -1$, i.e. at $\theta = \pm \pi / 2$:

$$(\sigma_\theta)_{\max} = 3p \quad (5)$$

According to the literature [3, 8], the expressions to calculate the stresses $\sigma_{\max}$, $\sigma_{\min}$ and $\tau_{\max}$ will be determined by the following formulae:

$$\sigma_{\max} = \frac{\sigma_\rho + \sigma_\theta}{2} + \sqrt{\left(\frac{\sigma_\rho - \sigma_\theta}{2}\right)^2 + \tau_{\rho\theta}^2} \quad (6)$$

$$\sigma_{\min} = \frac{\sigma_\rho + \sigma_\theta}{2} - \sqrt{\left(\frac{\sigma_\rho - \sigma_\theta}{2}\right)^2 + \tau_{\rho\theta}^2} \quad (7)$$

$$\tau_{\max} = \pm \sqrt{\left(\frac{\sigma_\rho - \sigma_\theta}{2}\right)^2 + \tau_{\rho\theta}^2} \quad (8)$$

Based on the expressions (1-8), an algorithm is formed in the software package Microsoft Office Excel and the values of stresses $\sigma_{\max}$, $\sigma_{\min}$ and $\tau_{\max}$ are calculated. Figure 1 shows: a) lines of equal $\sigma_{\max}$ and $\sigma_{\min}$, b) lines of equal $\tau_{\max}$.

Figure 1 The uniaxial tensioning of homogeneous isotropic plate weakened by a circular hole: a) lines of equal $\sigma_{\max}$ and $\sigma_{\min}$, b) lines of equal $\tau_{\max}$

3 DETERMINATION THE STRESS ON PANEL USING THE FINITE ELEMENT METHOD

To determine the value of the stress and in order to take into account all the influential factors in the field of machine elements weakened by the circular opening, as well as to simulate real operating conditions (load, boundary conditions), numerical methods can be successfully used. Given the nature of the problem that is addressed in this work, the finite element method (FEM) was chosen. The model was discretized using 2D triangular solid finite elements. A detailed overview of equations is given in [7, 8], and here some of the most important relationships will only be given.

Components of surface forces are the vector F_n elements:

$$F_n = \begin{Bmatrix} F_{nx} \\ F_{ny} \end{Bmatrix} \quad (9)$$

At any point of observed body, the shift vector s with the displacement components u and v in the direction of coordinate axes x and y , is shown as:

$$s = \begin{Bmatrix} u \\ v \end{Bmatrix} \quad (10)$$

The links between the displacement vector s and the deformation vector ε can be represented by the Cauchy's revolutionary kinematic equations in the form:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \\ \varepsilon_{xy} &= \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \end{aligned} \quad (11)$$

According [7, 8], the relation between the deformation vector ε and displacement vector s can be displayed in the matrix form:

$$\varepsilon = d \cdot s \quad (12)$$

The matrix of differential operator d and its transposed matrix d^T have the forms that can be found in [7, 8].

Stress state in the given point of stressed body is determined by three component stresses as follows: two normal (σ_x and σ_y) and tangential ($\tau_{xy} = \tau_{yx}$) which operate at that point. Conditions of balance between the internal and external forces at the part of contour where the contour conditions are specify by the surface forces are given by the Cauchy's revolutionary equations (Cauchy's boundary conditions):

$$d_s^T \sigma = F_n \quad (13)$$

where: d_s^T - the transposed matrix of the matrix d_s [7, 8].

The general form of constitutive equations, i.e. the relationship between the matrix of the components of stress tensor and matrix components of strain tensor of elastic material is given by the formula:

$$\sigma = D \cdot \varepsilon, \quad (14)$$

where: D - is the matrix of material stiffness, which for homogeneous isotropic elastic materials can be expressed through the Young's modulus of elasticity E and Poisson's ratio μ hence the name for this matrix "the matrix of elastic constants, or matrix of the tensor of elasticity" [7, 8].

The basic equation of finite element, i.e. equilibrium equation, which gives the relationship between the nodal displacements and nodal forces, can be written in the form of:

$$F = K \cdot S \quad (15)$$

where:

K - finite element stiffness matrix.

The following section presents the results of stress distribution σ_{max} and σ_{min} in a plane isotropic field weakened by a circular opening. The results refer to the element of plate type, whose sizes are in this case $2\text{ m} \times 5\text{ m} \times 0.1\text{ m}$, and the same is weakened by the circular opening of diameter $d = 100\text{ mm}$. The load is uniaxial, in the direction

of y axis, and is $p = 1\text{ [N/m}^2\text{]}$. Steel is the material of which the plate is made with the elasticity modulus $E = 2.1 \times 10^5\text{ [MPa]}$ and Poisson's coefficient of $\mu = 0.33$. In the case 2D triangular solid finite elements are used.

Figure 2 shows distribution of the highest stresses σ_{max} .

Figure 2 Distribution of stress σ_{max} on the panel weakened by the circular hole of diameter $d = 100\text{ mm}$ and uniaxially strained along the y axis

It can be seen from Figure 2 that the greatest stress values are at the intersection of contour of hole and x-axis (red color of the stress field in Figure 2).

Maximum stress value in this case is $\sigma_{max} = 2,994\text{ [N/m}^2\text{]}$.

Figure 3 shows distribution of normal stress σ_{min} . It can be seen from Figure 3 that the highest stress values are at the intersection between the contours of hole and y-axis (blue stress field in Figure 3). Maximum value of this stress is $\sigma_{min} = -0.994333\text{ [N/m}^2\text{]}$.

Figure 3 Distribution of stress σ_{min} on the panel weakened by the circular hole of diameter $d = 100$ mm and uniaxially strained along the y axis

4 COMPARISONS OF ANALYTICALLY AND NUMERICALLY OBTAINED RESULTS

In order to verify the numerical results obtained by the finite element method (FEM), the results obtained analytically were compared. Comparison of the obtained values of normal stresses σ_{max} and σ_{min} for uniaxial strained homogeneous isotropic plate along the y axis, weakened by the

circular hole of diameter $d = 100$ mm was done.

Table 1 presents the highest values of components of the normal stress σ_{max} and σ_{min} obtained by finite element method (FEM) and by analytical manner.

Table 1 Values of stress σ_{max} and σ_{min}

Method	σ_{max}	σ_{min}
FEM	2,994	-0.994
ANAL	3,000	-1.000

Based on the results obtained by the finite element method (FEM) and analytical way, it can be seen that the highest values of

stress σ_{max} , as expected, are obtained on contour of hole and in the points of contour intersection of hole and x-axis. In those points,

the cracks will first occur during uniaxial strain of parts weakened by the circular hole, which can later lead to the breakage of parts, and even accidents on machines and equipment used in mining. The highest obtained values of stress σ_{min} are on contour of hole, the points of contour intersection of the hole and the y-axis, which should be taken into account in designing a uniaxial pressed parts weakened by a circular hole.

5 CONCLUSION

In this paper to determine the image of the stress state in mechanical parts of which machines and equipment used in mining are assembled analytical and numerical methods are used. Analytical problem solving, for more complex problems may be difficult or even impossible often due to bulky mathematical procedure and deadlines that are set in solving problems. Therefore, in this work, the analytical methods are used only for obtaining the results of stress distribution in simpler problems. Numerical method, i.e. finite element method, presented in this paper, has enabled to reach certain results which are certainly important for the future work in the field of research the stress concentration. The analysis of the results obtained by the finite element method and using the computer program ANSYS, confirms the usefulness and justifies the use for solution the similar problems in design.

These studies demonstrated the influence of circular holes on the stress state, particularly on a very contour of the hole. Based on the results, reached in this paper, it can be concluded that any change in shape causes a disruption in the stress state of strained element and stress concentration. Therefore, in designing the machinery and equipment used in mining, special attention should be paid to the construction of parts weakened

by the holes, prestressed both at the strain and pressing, as well as to the other types of strain.

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