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A HYBRID APPROACH TO MODELING THE FLOTATION PROCESS FROM THE "VELIKI KRIVELJ" PLANT***

Abstract

The purpose of the flotation process control is optimization the concentrate grade and recovery, while maximizing profits. Consequently, the research into modeling and control of this process has always been an important area in control engineering practice. This paper presents the results of development and validation the predictive models, based on the ANFIS hybrid system. Models predict the values of copper concentrate and tailings grade as well as copper recovery in the flotation plant "Veliki Krivelj". The copper content in the feed ore, collector consumption in the rough flotation stage and consumption of frother, were selected as the independent variables. Other technical and technological parameters, relevant for the process of flotation concentration were considered constant. The results of the models validation have showed that the models provide the good predictions of changes in the copper concentrate grade, while the predictions of changes in the copper recovery and tailings grade are somewhat poorer.

Keywords: flotation, model, ANFIS, copper, concentrate, tailings, recovery

INTRODUCTION

One of the approaches of flotation process modeling is by the classical mathematical methods including the empirical, probabilistic, kinetic, and population-balance based models [1–6]. However, taking into consideration the complexity of the flotation process, caused by the interaction of many micro processes on the boundary of three phases (solid, liquid and gaseous), the classical mathematical equations have not been enough effective so far.

Recently, the soft computing methods emerged as a perspective alternative to the classical modeling approach. These methods, unlike the conventional mathematical methods, exhibit a certain tolerance to the imprecision and uncertainty of the technological parameters in description of real systems. Therefore, they offer more flexible and more robust solutions to the problems of modeling the stochastic processes such as the froth flotation. From the standpoint of process technology, the most commonly used soft computing techniques are the artificial neural networks, fuzzy logic and support vector machines, while the genetic algorithms are mainly applied to optimize the flotation circuit configuration [1].

The adaptive Neuro-Fuzzy Inference System (ANFIS) is a combination of two soft computing methods – artificial neural networks and fuzzy logic. Fuzzy logic has

the ability to change the qualitative aspects of human knowledge and insights into the process of precise quantitative analysis. However, it does not have a defined method that can be used as a guide in the process of transformation the human thought into the rule base fuzzy inference system (FIS), and it also takes quite a long time to adjust the membership functions (MFs). The artificial neural network (ANN) has a higher capability in the learning process to adapt to its environment. Therefore, the ANN can be used to automatically adjust the MFs and reduce the rate of errors in determining the rules in fuzzy logic [7,8].

The ANFIS architecture is an adaptive network that uses a supervised learning on learning algorithm having a function similar to the model of Takagi - Sugeno fuzzy inference system [7]. The ANFIS architecture is shown in Fig.1. Assuming that there are two inputs x and y, and one output z, two rules in the Takagi - Sugeno model can be expressed as:

If x is A_1 and y is B_1 then $z_1 = f_1(x,y)$

If x is A_2 and y is B_2 then $z_2 = f_2(x,y)$

where: A_1, A_2 and B_1, B_2 are the membership functions of each input x and y.
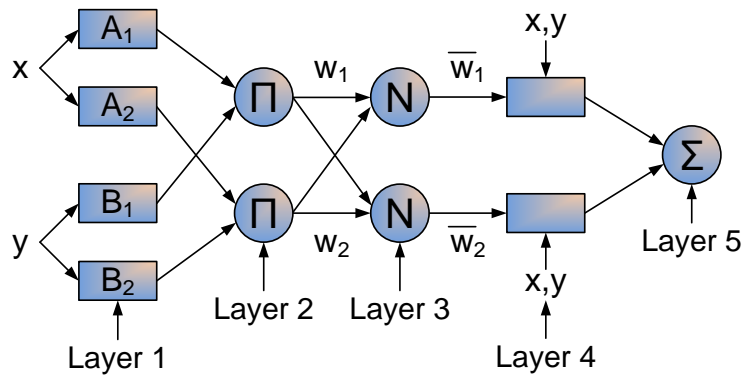

Figure 1 *ANFIS Architecture. Adapted from [7]*

The ANFIS architecture has five layers (see Fig. 1). In the layer 1, each node is adapted to a function parameter. The output from each node is a degree of membership value, given by the input of the membership functions. In the layer 2, each node is fixed or nonadaptive, and the circle node is labeled as Π. The output node is the result of multiplying of signal coming into the node and delivered to the next node. Every node in this layer represents the firing strength for each rule (wi). In the layer 3, each node is fixed or nonadaptive and the circle node is labeled as N. Each node is a calculation of the ratio between the i-th rules firing strength and sum of all rule firing strengths.

This result is known as the normalized firing strength ($\overline{wi}$). In the layer 4, each node is an adaptive node to the output. In the layer 5, the single node is a fixed or non-adaptive node that computes the overall output as the summation of all incoming signals from the previous node [7].

EXPERIMENTAL

Experimental procedure was carried out in virtual conditions, using the MATLAB programming language. A total of three models (marked as ANF1, ANF2 and ANF3) have been developed using the ANFIS Editor Graphical User Interface.

The independent variables in each model were: the copper content in the feed ore (FCU), collector consumption in the rough flotation stage (PXR) and consumption of frother (FRT). The dependent variables were: the final copper concentrate grade (CCU) – for the ANF1 model; copper recovery of copper in the concentrate (RCU) – for the ANF2 model; and the final tailings grade (TCU) - for the ANF3 model. The basic structure of models is presented in Fig. 2.

Model development and testing was based on the real process data, collected from the flotation plant "Veliki Krivelj" during the multi-annual monitoring. The validation of the proposed models was performed in Microsoft Excel.
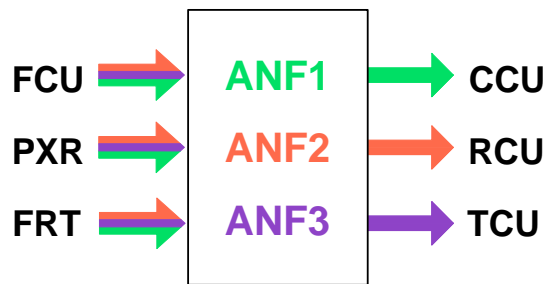
Figure 2 *Basic structure of models*

During the optimization of elementary conditions of the modeling process, the following two criteria were taken into consideration: (1) resulting surfaces should describe the real process in the best manner, and (2) the root mean square error of training should be minimal. In this regard, the expert analysis showed that the best results were achieved through the conditions presented in Table 1.

Table 1 *Optimal conditions for model development*

Model	Optimal membership function	Optimal output function	Learning algorithm	Number of epochs
ANF1	Gaussian	linear	Back propagation	100
ANF2	Bell	linear	Back propagation	400
ANF3	Gaussian	linear	Back propagation	400

For the neural network training, every second series of values of variables FCU, PXR, FRT and RCU (corresponds to the data of one shift) is chosen from the data base – "even cases". The structure of neural network, generated membership functions as well as generated surfaces are presented in Figures 3, 4 and 5, respectively.
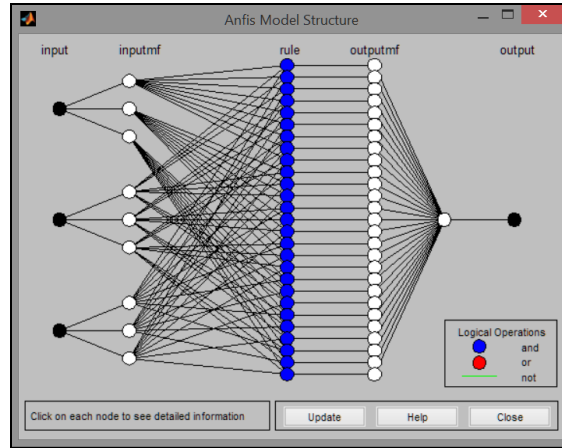
Figure 3 *Structure of the ANFIS neural network*

The structure of generated neural network is [3-9-27-1], and it is the same for each model. Number of nodes in the first hidden layer corresponds to the number of membership functions belonging to each input variable (9 in total), while the number of nodes in the second hidden layer corresponds to the number of fuzzy rules (27 in total), where the consequence of each rule is a linear function with different coefficients.
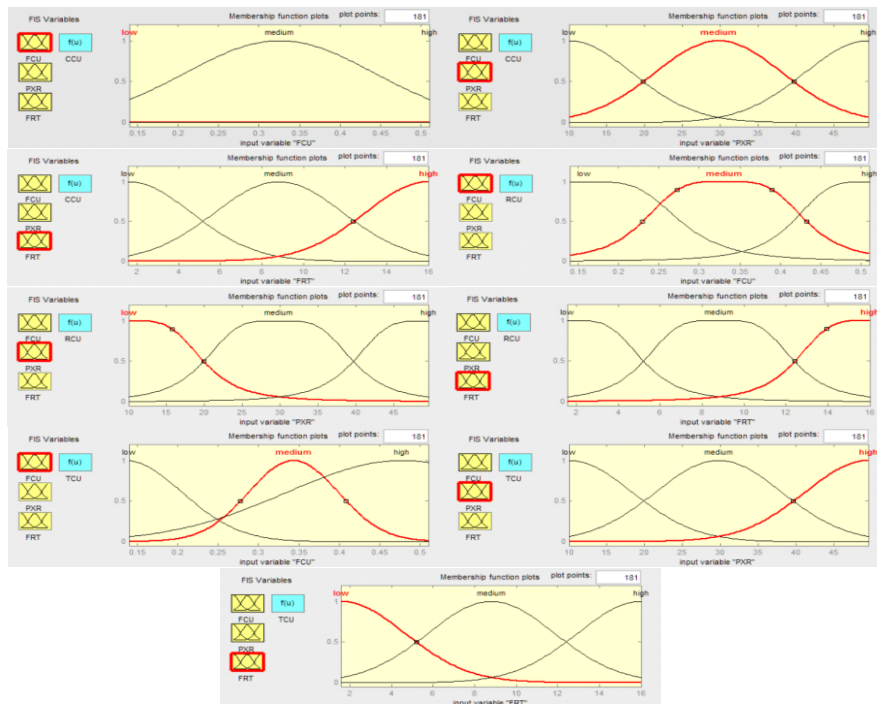
Figure 4 *Membership functions generated by the ANFIS models*
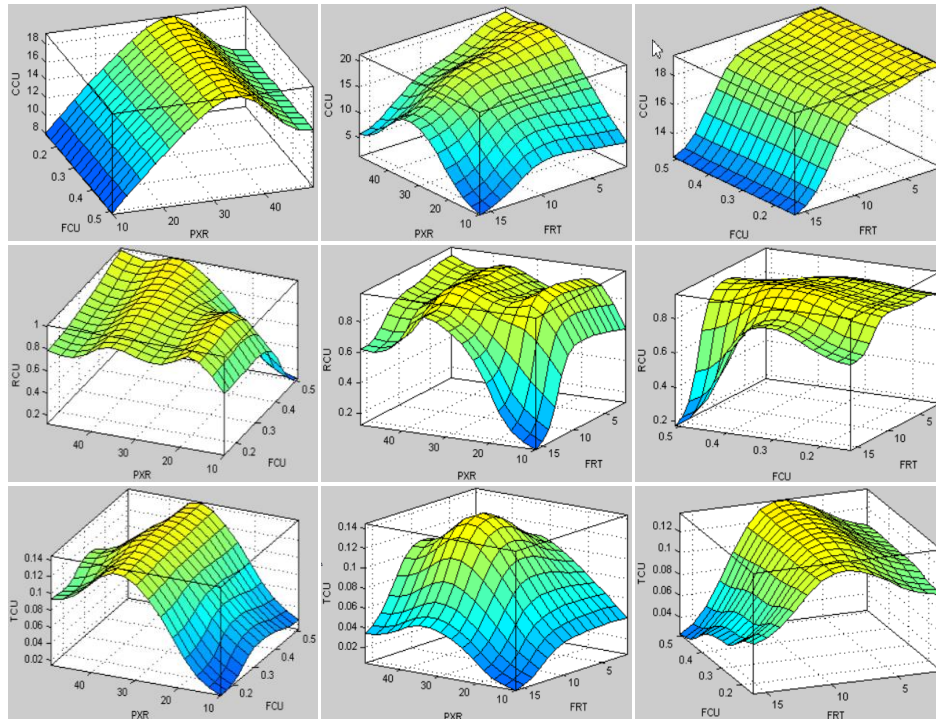
Figure 5 *Surfaces generated by the ANFIS models*

As it can be seen from Fig. 5, a dependence of concentrate grade (CCU) on copper content in feed (FCU) and collector consumption in the rough flotation (PXR) is described pretty well by the presented surface. As in the real flotation process, with the increase of collector dosage, the concentrate grade primarily increases, and then decreases. However, it should be noted that at low collector consumptions, the concentrate grade is quite low, which does not correspond to real conditions. The other two surfaces that show the dependence of CCU variable on FCU, PXR, and FRT variables also describe the real flotation process pretty well. Namely, with increase in the frother dosage, there is a certain decrease in the concentrate grade. This phenomenon is related to ability of the frother to "pull" the tailings particles into the concentrate, if it is in surplus in the flotation pulp.

When it comes to the copper recovery (RCU), the presented surfaces do not describe a real dependences in the flotation process in the best manner. In the observed range of values, with an increase of the amount of collector and frother, copper recovery in concentrate should be constantly increased. In this case, the resulting surfaces have a "wavy" shape.

Finally, when it comes to the tailings grade (TCU), it can be concluded that the surfaces do not correspond to the real process. In the observed range of values, with an increase of the frother and collector dosage, the copper content in tailings should be decreased. In both cases, the TCU variable firstly increases to the certain limit, and then decreases.

RESULTS AND DISCUSSION

The evaluation of models was performed in the software package MATLAB – entering the real values of the process input variables from the industrial flotation plant "Veliki Krivelj" and generating correspo-nding outputs, predicted by the models. The evaluation of each model was made forming a matrix of three independent variables and generating the column matrix for the output variable, using the module Fuzzy Logic Toolbox.

Possibility of models to reliably predict flotation parameters, on the basis of given input parameters, was determined by the regression analysis in Microsoft Excel. The egression analysis showed the correlation between actual process values and values predicted by models.

The results of the regression analysis are presented in Tables 2–4.

Table 2 *Statistical correlation of real and predicted values*

Statistical parameter	Technological indicator of flotation process		
	CCU	RCU	TCU
Correlation coefficient R	0.98347	0.99608	0.91642
Coefficient of determination R^2	0.96721	0.99217	0.83982
Adjusted R^2	0.96669	0.99164	0.83930
Root mean square error	3.44739	0.079921	0.04786
Observations	1910	1910	1910

Table 3 *Statistical analysis of regression linear equations*

|	C	SE	t	p	L95	U95
CCU	0.95997	0.00405	237.30	0	0.95203	0.96790
RCU	1.06415	0.00216	491.78	0	1.05991	1.06840
TCU	2.46691	0.02466	100.04	0	2.41855	2.51527

Table 4 *Analysis of variance (ANOVA)*

		df	SS	MS	F	SF
CCU	Regression	1	669217.43	669217.43	56309.99	0
	Residual	1909	22687.56	11.88		
	Total	1910	691905.99			
RCU	Regression	1	1544.76	1544.76	241848.19	0
	Residual	1909	12.19	0.00639		
	Total	1910	1556.95			
TCU	Regression	1	22.9236	22.9236	10008.83	0
	Residual	1909	4.3722	0.00229		
	Total	1910	27.2958			

The regression analysis (Tables 2–4) indicates a strong link between the actual and predicted values of copper grade and recovery, given that the correlation coefficients are very high. In other words, the models ANF1 and ANF2 well follow the changes in real values of the observed parameters related to their rise or fall over time. Correlation coefficient of real and predicted tailings grade is somewhat lower.

Since it is necessary to make a comparison between the predictive abilities of models with different scales, a root mean square error is not a suitable parameter. Therefore, a normalized root-mean-square error (NRMSE) is taken into consideration. Values are shown in Figure 6.
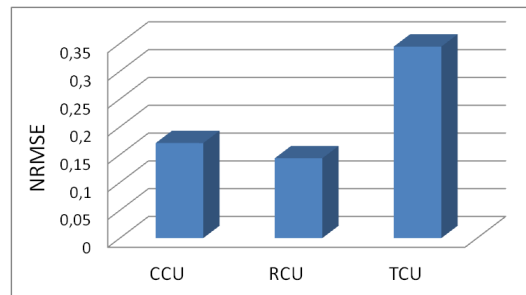
Figure 6 *Normalized root-mean-square error of prediction*

Normalized root-mean-square errors indicate that the ANF1 and ANF2 models can predict the concentrate grade and recovery values similarly good. On the other hand, the ANF3 model has significantly poorer predictive abilities.

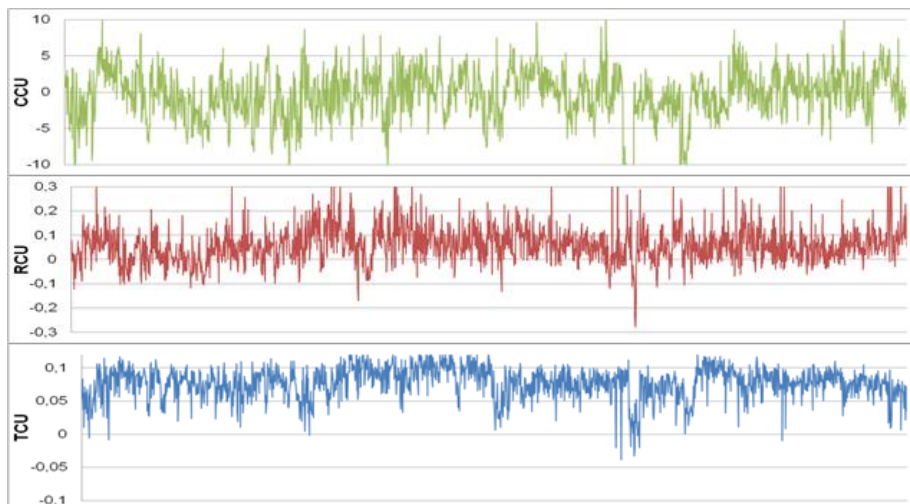Figure 7 shows the prediction errors of developed models. Error of prediction (ε), which also serves as one of the criteria for assessment the predictive properties of model, is calculated according to formula (1):

$$\varepsilon = y_{pr} - y_{re} \qquad (1)$$

where:

y_{pr} – predicted value of technological indicator (CCU, RCU, TCU)

y_{re} – real value of technological indicator (CCU, RCU, TCU)

Figure 7 *Prediction error of concentrate grade, recovery and tailings grade*

Despite that correlation coefficients are pretty high (Table 2), better insight into the predictive abilities of models can be achieved considering the prediction errors (Figure 7), as well as by the visual analysis of results in relation to the training and test data (Figures 8 – 10). It should be noted that the test data, similarly to the training data, corresponds to the values of variables from every second shift – "odd cases".
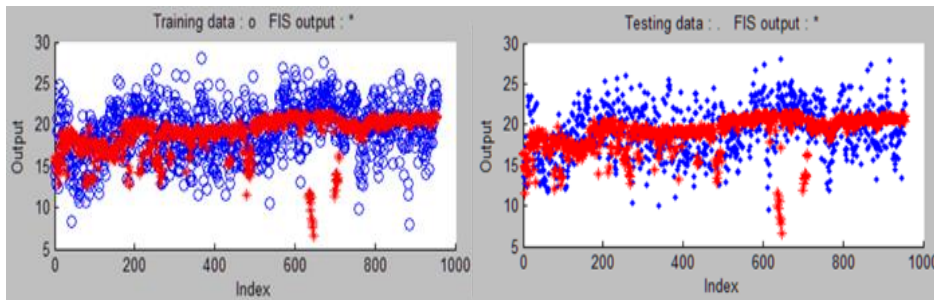
Figure 8 *Prediction of the concentrate grade in comparison to the training and test data*

Figure 8 shows that model ANF1 generally predicts the concentrate grade in the range between 17 and 22% Cu. This predicttion can be considered as a quite correct, although, at first sight, it does not seem to be the case. The reason is a large dispersion of real data in the plant Veliki Krivelj. This is in line with the prediction error, which noticeably "oscillates" around the value zero, during the entire period of the plant operation (see Figure 7).
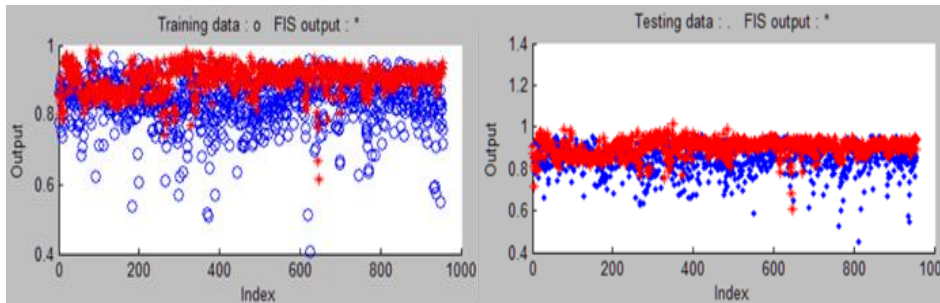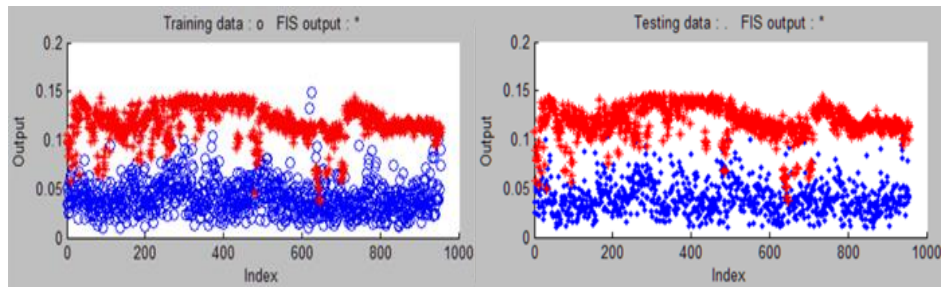

Figure 9 *Prediction of the copper recovery in comparison to the training and test data*

The recovery values obtained by the model ANF2 are generally higher than the real ones (Figure 9). This is also confirmed by the prediction error, which is mainly positive (Figure 7).

Figure 10 *Prediction of the tailings grade in comparison to the training and test data*

It can be seen from Figure 10 that the predicted values of copper content in the tailings are significantly higher than the real process data (when it comes to both training and test data), as indicated by a positive prediction error during the entire operating period of the plant (Figure 7). This error value often exceeds 0.1% Cu, so it is sugges-ted that such model cannot be considered adequate. This is a good example of how a high correlation coefficient means that the model well follows changes in the actual values of the observed parameters, but at the same time the predicted values do not meet the required criteria in terms of precision.

CONCLUSIONS

Modeling of flotation processes is not a simple task, mainly thanks to the complexity of the process, where the classical mathe-matical equations have not been effective so far. Recently, the soft computing methods have been emerged as a perspective alternative to the classical modeling approach. One of soft computing methods is the Adaptive Neuro-Fuzzy Inference System (ANFIS). The architecture of this system consists of five layers and integrate the principles of fuzzy logic and artificial neural networks.

For the purpose of this research, three ANFIS models have been developed. These models predict concentrate grade and reco-

very, as well as tailings grade in the flotation plant Veliki Krivelj. Predictive abilities are considered through several criteria.

According to the indicators of regression analysis (correlation coefficient and normali-zed root-mean-square error) model, which predicts the copper recovery, has the best predictive abilities. However, observing the prediction error, it seems that the model which predicts concentrate grade has the better predictive abilities. According to most criteria, the model which predicts the tailings grade cannot be said to be adequate.

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