MINIMUM COVARIANCE DETERMINANT-BASED BOOTSTRAPPING FOR APPRAISING AIR PASSENGER ARRIVAL DATA

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Air travel management is a case-special process since it includes different types of uncertainties such as ungovernable passenger mobility, variable costs as well as extraordinary restrictions like the Covid-19 pandemic. Therefore, the use of robust and reproducible statistical evaluations under uncertainty is required. The cornerstone of this study is the adaptation of bootstrapping and the robust Minimum Covariance Determinant (MCD)-based parameter estimation under a heterogeneous process. In addition, the study includes a novel bootstrapping regression implementation. The methodological developments have been tested by Serbia’s air transport data. The results showed that combining robust estimator and bootstrapping provides some advantages for determining outliers and also making advanced diagnostics. Thus, a state-of-the-art approach based on accuracy, reproducibility, and transparency has been introduced and its usability in the air travel mobility process has been exhibited.

Keywords: Air travel; Robust estimator; Bootstrapping; Correlation.

INTRODUCTION

As a rapid and safe transportation system, the airline includes many complex dynamics affected by different internal (personal management, equipment, organization) and external (climate, cost, distance) sources (Bogetic et al. 2014; Tomic et al. 2012). Air transportation deals with socio-economics, competitiveness, welfare, and also associated base sectors such as tourism and international trade. One of the key management terms is knowledge management for passenger satisfaction (Kavalić et al. 2022). The growth of air transportation positively affects the progress of various industries especially tourism (Oktal & Garcia, 2017). The United Nations World Trade Organization (UNWTO) describes tourism as a social, cultural, and economic phenomenon entailing the movement of people to countries or places outside their usual environment for personal or business/professional objectives (UNWTO, 2022).

Recently, an increasing dependency on air transport by the accommodation sector has created a relationship between air transport and tourism more complex trade (Dileep & Kurien 2022). In addition, both international and domestic tourist destinations now increasingly rely on air transport for tourism flows. The parallel lines between air transport and tourism-based passenger mobility were identified by Lohmann and Duval (2014) as common motivation, objective function, and shared emphasis on sustainability.

The investigations for analysing the relationships between air passenger dynamics and different sectors like tourism from a statistical point of view have gained popularity over the last two decades due to the requirements for economical optimization, system modelling, and investment planning. In one of the pioneer studies, Medeiros et al. (2008) discussed the air passenger arrival contribution to international tourism in Mallorca (Spain), and a neural network model was developed. In this way, the level of international demand was appraised computationally. In another study, some econometric models were suggested for evaluating air passenger traffic flows and related tourism demand (Fildes et al. 2011). To establish decision rules for long-term forecasting of air passengers, Sharma et al. (2018) suggested a
hybrid rough set model. In a similar novel work, for the purpose of projecting air passenger demand, the top 20 busiest airports' passenger counts were employed in a global vector autoregressive technique (Gunter & Zekan, 2021). In a recent study, it was discussed the effect of the Covid-19 pandemic on air passenger dynamics in the EU countries by using a time series analysis (Barczak et al. 2022).

Although many investigations in literature focused on the dynamics of the air transportation system and passenger demand topics, there are novel studies performed on simulation and robust estimation in the topic “airline and tourism” such as Assaf (2011) and Van De Vijver et al. (2011). However, the statistical analyses were performed only via conventional estimators and these modelling tools also do not consider critical issues such as the existence of outliers, the heterogeneous property of air passenger dynamics, and also the nonlinear structure of trend (location and scatter). Therefore, a highly robust estimator of location and scatter as well as a fast algorithm is required (Hubert et al. 2018).

To fill this gap in the literature, this study suggests a new algorithm by considering an appraisal based on complexity reduction and robustness. The theoretical cornerstone of the suggested model is to adapt bootstrapping and the robust Minimum Covariance Determinant (MCD)-based parameter estimation. In this way, a simulation-based robust correlation can be calculated. In addition, the study also conducts a novel bootstrapping regression application. Finally, revealing a state-of-the-art approach based on accuracy, reproducibility, and transparency is the main result of this study. The methodology has been tested using Serbia air passenger records and the merits of the hybrid statistical approach have been underlined.

The remainder of the article is organized as follows. Section 2 introduces the methodological ground used in this study. Section 3 presents the numerical experiments and the results along with a brief discussion. Section 4 summarizes the findings of the investigation.

METHODOLOGY

Robust Distance Measure by Minimum Covariance Determinant (MCD)

If a multivariate data set is available, by using a \( n \times p \) data matrix \( X = (x_1, x_2, ..., x_n)^T \) with \( x_i = (x_{i1}, x_{i2}, ..., x_{ip})^T \), a \( p \)-dimensional tolerance ellipse can be structured based on Mahalanobis distance measure:

\[
MD_{(x)} = \sqrt{(x - \bar{x})^T S^{-1} (x - \bar{x})} \quad (1)
\]

In Eq. (1), \( MD \) and \( S \) correspond to the \( \alpha \)-quantile of the chi-squared distribution \( \sqrt{\chi_{p,0.975}^2} \), and the sample covariance matrix, respectively. Because the tolerance ellipse designed via MD aims to contain all measurements and also a sensitive outlier, the MCD estimator was suggested as a strong robust estimator of multivariate location and scatter based on robust distance (MCDRD) (Rousseeuw & Van Driessen 1999):

\[
MCDRD_{(x)} = \sqrt{(x - \mu_{MCD})^T \Sigma_{MCD}^{-1} (x - \mu_{MCD})} \quad (2)
\]

In Eq(2), \( \mu_{MCD} \) and \( \Sigma_{MCD} \) represent the estimate of location and covariance estimate, respectively. The robust distance expressed produces a much smaller ellipse compared with the MD and it includes the regular measurements. Thus, the MCD is projected for elliptically symmetric unimodal distributions. As recorded in Cator and Lopuhaa (2010), the distributional properties of the MCD estimators provide some tools to perform an effective robust statistical evaluation. These estimators have the same high breakdown point as the minimum volume ellipsoid estimators. To reach a high level of robustness the shape parameters \( \mu_{MCD} \) and \( \Sigma_{MCD} \) can be calculated by reweighting as follows (Hubert & Debruyne 2010).

\[
\hat{\mu}_{MCD} = \frac{\sum_{i=1}^{n} w(d_i^2) x_i}{\sum_{i=1}^{n} w(d_i^2)} \quad (3)
\]

\[
\hat{\Sigma}_{MCD} = c_1 \frac{n}{n-1} \sum_{i=1}^{n} W(d_i^2) (x_i - \hat{\mu}_{MCD}) (x_i - \hat{\mu}_{MCD})^T \quad (4)
\]

where \( d_i = \sqrt{(x - \mu_0)^T \Sigma_0^{-1} (x - \mu_0)} \). \( W \) and \( c_1 \) denote suitable weight function and a consistency factor, respectively. By using the robust covariance matrix, a robust correlation matrix between the variables \( X_l \) and \( X_j \) can be provided by

\[
r_{lj} = \frac{s_{lj}}{\sqrt{s_{ll}s_{jj}}} \quad (5)
\]

Where \( s_{lj} \) denotes the MCD covariance estimate.
Bootstrapping for parameter estimation and regression

As a nonparametric statistical technique, bootstrapping is applied for statistical inference based on the statistical distributions of the sample data. The main superiority of the bootstrap is its applicability with limited data and no requirement for distributional assumptions. In addition, this approach is relatively simple and general to handle complex relationships. The bootstrapping simulation can be considered for two purposes: estimating standard errors and optimizing regression coefficients.

To estimate the standard error of the sample mean, the following procedure can be employed (James et al., 2021):

- Decide on the number of bootstrap samples \( M \).
- Randomly draw \( n \) measurements \( \tilde{Y}_b^*, Y_{b1}, ..., Y_{bn} \) for each sample \( b = 1, ..., M \) with replacement and compute the bootstrap mean:

\[
\bar{Y}_b^* = \frac{\sum_{i=1}^{n} Y_{bi}}{n} \quad (6)
\]

- Estimate the standard deviation of the bootstrap means based on \( M \) bootstrap samples,

\[
SE^*(\bar{Y}^*) = \sqrt{\frac{\sum_{b=1}^{M} (\bar{Y}_b^* - \bar{Y}^*)^2}{M-1}} \quad (7)
\]

\[
\bar{Y}^* = \frac{\sum_{b=1}^{M} \bar{Y}_b^*}{M} \quad (8)
\]

In general, it is difficult to determine standard errors in an analytical way and in most cases, the requirement for robust standard errors has also emerged (Fox & Weisberg, 2019). In addition, the SE expression given in Eq. (7) needs a generalization due to deriving the empirical sampling distribution for an estimator \( \hat{\theta} \) of the parameter \( \theta \). To fulfill these two conditions and adapt the calculations to MCD-based estimation the following generalized equation can be structured:

\[
SE^*(\hat{\theta}^*)_{(MCD)} = \sqrt{\frac{\sum_{b=1}^{M} (\hat{\theta}_b^*(MCD) - \hat{\theta}^*(MCD))^2}{M-1}} \quad (9)
\]

To provide the SE for the parameter \( \theta \), the robust estimator (MCD) considers the estimates for normally distributed data based on RD given in Eq (2). Before reaching the final solution in Eq. (9), the MCD-based bootstrapping algorithm considers the correlation matrix and some adaptive parameters. The bootstrap parameter estimation can be used for measuring the gauge of the robustness and also estimating regression coefficients of the bootstrap regression solution.

The conventional bootstrap-based regression parameter estimation procedure given in (Fox, 2016) can be modified. In the adapted procedure, the general regression expression \( z_i^* = [Y_{i1}, X_{i1}, ..., X_{ik}] \) is considered and the measurements are resampled. By using the bootstrapping, \( M \) sets of regression coefficients \( \beta_b^* = [A_{b1}^*, B_{b1}^*, ..., B_{bk}^*] \) are provided. The parameter estimation procedure can be summarized as follows:

- Estimating regression coefficients \( (\beta_b) \), response \( \hat{Y}_b \) and residual \( E_i \).
- Selecting bootstrap of the errors \( e_b^* = [E_{b1}^*, E_{b2}^*, ..., E_{bn}^*] \) and obtaining bootstrapped targets \( y_b^* = [Y_{b1}^*, Y_{b2}^*, ..., Y_{bn}^*] \).
- Using bootstrapped targets and fixed X-values, get re-sampled regression coefficients \( \beta_b^* = [A_{b1}^*, B_{b1}^*, ..., B_{bk}^*] \).

It should be underlined that the regression model procedure is bootstrapped by handling the indicator variables as (Fox, 2016):

- random and selecting bootstrap samples directly from the measurements \( z_i^* \).
- fixed and resampling from the residuals \( E_i \) of the modelled regression. The number of replications is one of the critical parameters in bootstrapping applications. James et al. (2021) suggest 1000 replications as general to provide accurate results.

IMPLEMENTATION

Travelers can be classified into two major groups in general: visitors (travel for leisure, business, etc.) and non-visitors. Tourists can be recruited for the first group with the excursionists (same-day visitor). UNWTO expresses a visitor that is a person who travels with a non-remunerative purpose to a foreign country for a period not exceeding one year. Similarly, a domestic tourist is described as a visitor whose maximum duration may be up to six months and the place being visited has to be inside the country of origin (Dileep, 2019). Due to the close connection among
transportation, tourism, and travel management, the case study aims attention at the air passenger (visitor) dynamics and mobility which includes a considerable amount of heterogeneity and uncertainty resources from personal and external management characteristics.

**Data and structure identification**

The case study focused on understanding Serbia's air passenger mobility as one of the critical topics in air travel management. The air travel system in Serbia is structured by many parameters such as economic progress, international trade, new tourist destinations along with international relationships, growth of population with changes in demography, aviation technology, lifestyle, and income. Therefore, dynamic and ever-changing records encountered in similar countries can be mentioned.

The data includes the passenger (tourist) arrivals in Serbia from the records kept by The Tourism Organisation of Serbia (TOS) and the Statistical Office of the Republic of Serbia (PBC) such as (PBC 2020; TOS 2013). The data set consists of both domestic and foreign tourist records between 2003 and 2019. Due to the global pandemic coronavirus disease (COVID-19) and its travel restrictions (Aleksic et al 2022), the observations for the years 2020 and 2021 were omitted. The focal point of this implementation is to show the relationship between foreign and domestic mobility in Serbia. Table 1 summarizes the descriptive statistical properties of the data set. Figure 1 indicates the basic correlations and densities.

**Table 2: Descriptive statistics for passenger arrival data.**

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean (µ)</th>
<th>Standard Deviation (σ)</th>
<th>Coefficient of Variation CV = (σ / µ)*100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic</td>
<td>1165536</td>
<td>1843432</td>
<td>1535790</td>
<td>456146</td>
<td>29.7</td>
</tr>
<tr>
<td>Foreign</td>
<td>339283</td>
<td>1846551</td>
<td>900922</td>
<td>189067</td>
<td>21.0</td>
</tr>
</tbody>
</table>

**Application and results**

Before the MCD-based bootstrapping implementation, first, the relationship between conventional distance measure and MCD (robust distance) was shown. The data set contains the numbers of arriving passengers for 17 years. The scatter plot illustrated with the classical (Mahalanobis) and robust 97.5% tolerance ellipses in Figure 2 explores the domains. The MD identifies how far away the observation is from the centre of the cloud, relative to the size of the cloud.
Therefore, as seen in Figure 3, the MD-based tolerance ellipse encompassed all measurements. In addition, the MD produces a large area and none of the observations seems outlier. However, both Figures 2 and 3 illustrate that the RD-based tolerance ellipse has a much smaller structure and it encloses the regular data points. The MCD-based measure clearly specifies seven outliers.

Based on the detection and diagnosis capacity of MCD, a series of bootstrapping implementations were performed and the standard error for the correlation coefficient measured between the variables was estimated using case-resampling simulation instead of a formula. The following simulation procedure was followed by using the “robustbase” library in R (Hubert et al., 2012).

- determine the correlation via simulated data, provided by sampling $M$ cases with replacement from the $M$ cases of the measured data set used for input.
- repeat Stage 1 for a large number of bootstrap replications.
Following these two steps, the standard deviation of the bootstrapped values of the correlation coefficient estimated its standard error. Figure 4 illustrates the variation of the correlations provided as a result of bootstrapping with 1000 repetitions.

To explore the relationship between foreign and domestic passenger arrivals, first, the function structure was selected and then a bootstrapping regression algorithm was implemented. Figure 5 shows both linear and polynomial base regression models for simulations. As can be seen in Figure 5, the quadratic model has better performance for model fitting. In the bootstrap model, the following least-squares solution for the coefficients can be considered:

$$\beta_b^* = (X'X)^{-1}X'\hat{y}_b$$ for $b = 1, ..., M.$ \hspace{1cm} (10)

In other respects, the data set shows a tendency to a polynomial (quadratic) model below:

$$y_b = A_0 + B_1x + B_2x^2$$ \hspace{1cm} (11)

The functional (quadratic) regression model has been bootstrapped by using the independent variable foreign as fixed and resampling from the residuals $E_i$ of the fitted regression. For this application, 1000 replications were selected and the identically distributed residuals are considered. Table 2 summarizes the simulated regression model along with the base alternatives with the performances.
DISCUSSION

The data sets denote a long period and partial fluctuations resourced from internal and external factors encountered in Serbia. Table 1 gives a summary of the input data. The CV values calculated in Table 1 indicate that the variability levels for Foreign and Domestic arrivals are analogous. As a high-performance robust estimator, the MCD has a respected capacity to determine the outliers. Figures 2 and 3 indicate the masking effect: conventional (Mahalanobis) identification was affected by outliers and the diagnostic based on the MD can no longer detect the outlying observations. On the other hand, the MCD-based estimator has resistance against the outliers.

As seen in Figures 6 and 7, the MCD has better parallelism and accuracy and can also be employed to flag the outliers. In particular, Fig 7 illustrates the comparative evaluation based on Eigenvalues. It should be noticed that the robust distance measure in Eq. (2) has no sensitivity to the masking effect and to perform reliable data analysis (location and scatter) in an uncertain environment including fluctuations, the robust MCD is preferred.

The studies on correlation trend models indicated that this indicator has many fluctuations. Therefore, instead of a conventional measure, the robust measure would be preferable for a data diagnostic. It should be highlighted that although Figure 5 and Table 2 give us a general perspective, there is a big difference between the functional correlations in Figure 5 and the MCD-based bootstrapped correlations in Figure 4. If we zoom in on the data set, we can identify two different periods for foreign arrivals. The simulations presented in Figure 4 clearly exhibited this two-piece analysis.
requirement. Even though domestic arrivals have been just steady between the years 2003-2015 and a relatively constant trend has been recorded between domestic and foreign arrivals, a dramatic increase in foreign arrivals and relatively different trends have been recorded in recent years. Therefore, a two-piece regression model would be more realistic. The difference between these two periods is presented in Figure 8.

In the last step of the implementation, the coefficients used in the regression model have been optimized by bootstrapping. Because the number of data is limited and a requirement on exhibiting the improvement provided by bootstrapping is tested, all the measurements in the data set were employed in the regression analyses. Figure 9 depicts the variabilities and also densities of the coefficients. As seen in Figure 8, the simulations produced nearly normal distributions for all parameters. It should be noticed that this study did not prefer scaling and no data transformation has been employed in the simulation studies.

The results showed that nonlinear robust bootstrapping reveals exact findings to understand the relationship between foreign and domestic arrivals in Serbia. A negative correlation and functional relationship have been recorded. One of the important findings explored by the MCD diagnosis is the trend of outliers recorded for the last 4-5 years before the Covid-19 pandemic. A strong increase and the accompanying linear trend show the improved international relationships (tourism, trade, meeting, etc) for Serbia. However, future investigation is required with more data and a long period to exhibit the relationships among the sub-dynamics.

![Figure 9: Histograms of bootstrapped regression parameters.](image-url)
CONCLUSIONS

Air travel management is a case-special process since it includes different types of uncertainties such as qualitative (uncontrollable personal and passenger behaviour) and quantitative (fluctuated records, variable costs, international trends, etc.) factors as well as ungovernable parameters like the Covid-19 pandemic. Hence, sustainable management requires strong numerical tools such as robust statistical analyses. The MCD has been used to analyse the mobility of air passengers because conventional estimators do not take into account the key difficulties. It is a very robust estimator of location and dispersion as well as a fast method. The experimental studies were performed based on three methodological grounds: illustrating the superiority of the suggested robust estimator compared with the conventional ones, improving the accuracy of the correlation coefficient using the MCD-based bootstrapping, and regression parameter optimization via bootstrapping regression. The algorithmic procedures were examined by using Serbia air passenger arrival data and some useful results have been provided. The suggested approach is not sensitive to outliers and more accurate and explainable outcomes can be provided. The correlations and coefficients for entire and two-piece structures have been produced and discussed.

REFERENCES


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Minimum covariance determinant-based bootstrapping for appraising air passenger arrival data


**“BOOTSTRAPING” METODA BAZIRANA NA MINIMALNOJ KOVARIJANSNOJ DETERMINANTI ZA PROCENU PODATAKA O DOLASCIMA PUTNIKA U VAZDUŠNOM SAOBRAĆAJU**


**Ključne reči:** Putovanje avionom; Robust estimator; Bootstrapping; Korelacija.