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LEVERED BETA: INFLUENCE OF DEBT AND CORPORATE INCOME TAX ON COMPANY SYSTEMATIC RISK. DISCUSSION ON THE CORRECT ALGEBRAIC EQUATION

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Abstract:

The increase in the share of debt in the capital structure is accompanied by an increase in the required return on equity because companies are exposed to higher financial risk. The beta coefficient of debt-financed companies differs from the beta coefficient of companies that are financed exclusively with equity. Namely, the beta coefficient, under the influence of financial leverage, tends to increase with the growth of indebtedness, that is the systematic risk measured by the coefficient beta of debt-financed companies is higher than the systematic risk of non-leveraged companies, due to financial risk. Because interest payments on the debt of leveraged companies are excluded as an expense from the tax base, corporate income tax reduces the beta coefficient of a company with debt, compared to the beta coefficient of the same company when income tax is abstracted. The higher the corporate income tax, that will be the lower the beta coefficient of companies that have debt in the capital structure. There are several algebraic equations, by different authors, for the beta coefficient of leveraged companies. The algebraic equation for the beta coefficient of leveraged companies, which is derived in this paper, was obtained using the net operating income approach.

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INTRODUCTION

The topic of this paper is the influence of financial leverage and corporate income tax on systematic risk measured by the beta coefficient. ''*The beta coefficient measures the sensitivity of the return of particular security concerning systematic or market factor*'' (Bodie, Kane, and Marcus, 2009, p. 184) or ''*beta is the measure of the sensitivity of a security's return to market movements, i.e. it is a measure of how sensitive a particular stock is to market movements*'' (Thompson, 2000, p. 249). It is investigated how the level of borrowing, measured by the ratio of market values of debt and equity, affects the systematic risk of companies. It also investigated how corporate income tax affects systematic risk.

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Exact algebraic equations are derived between the beta coefficients, when financial leverage is used, when financial leverage is not used, and with and without a corporate income tax. Also, equations for the required rate of return for companies that borrow with and without the influence of corporate income tax are derived. Therefore, the purpose of this paper is to show not only the way in which borrowing and corporate income tax affect systematic risk, but also to compare the above-mentioned equations derived by different authors.

In this paper, the following hypotheses will be shown to be valid:

H1: The company's indebtedness, measured by the debt-to-equity ratio, increases its exposure to systematic risk.

H2: Corporate income tax reduces exposure to systematic risk.

The goal of this paper is to derive an algebraically correct equation for the beta coefficient of leveraged companies, which would significantly improve the methodological approach to measuring systematic risk when companies approach borrowing to finance their own activities. In the empirical part of the paper, all equations for the beta coefficient of leveraged companies will be tested on real financial data of companies of global car manufacturers, which are taken from the web address: http:/pages.stern.nyu.edu/~adamodaran/.

This paper is composed of three parts. In the first part under the title Methodology, the equation for the required rate of return (k_e) and the beta coefficient of leveraged companies (β_e) was derived. In the next chapter, alternative equations of different authors are listed with a comparative analysis of theoretical and empirical approaches. In the last chapter under the title Conclusions, confirmations of the working hypotheses, which had to be proven, were presented. The empirical part of the paper is based on a sample of six globally integrated car manufacturing companies whose data on the movement of beta coefficients were available, dating from 2012 and 2013. However, the derived equations are valid for any company, regardless of the size of the sample and regardless of which industry they belong to, as well as regardless of the time of data collection.

LITERATURE REVIEW

The capital asset pricing model is used to determine the relationship between the expected return on security and the systematic risk. Given that the behavior of an investor who has an aversion to risk is observed, an equilibrium relationship between risk and expected return is implicitly introduced, for each security (Sharpe, Alexander and Bailey, 1995, p. 173-175, and p. 262-271). At equilibrium, each security is expected to achieve a return that is proportional to the magnitude of the systematic risk, that a risk that cannot be eliminated by diversification. Since linear regression is performed in the model, the obtained characteristic line, as stated by Van Horne (2001, p. 66-67), represents the line between the additional return of individual security and the additional return of the market portfolio, that the expected functional relationship between two sets of additional returns. During the regression and withdrawal of the ''*security characteristic line*'', a third quantity will appear, which represents an unsystematic part of the risk, and is manifested as the scattering of points from the characteristic line (Bodie, Kane and Marcus, 2009, p.185).

Since the coefficient 'α' represents, as part of the systematic risk, a segment of the characteristic line on the vertical axis (ordinates), it can practically have a value greater than zero, less than zero, and equal to zero. In fact, theoretically, in the state of the equilibrium, coefficient 'α' of individual security should have a value of zero. If the additional portfolio return would be equal to '0', the coefficient 'α' would be the additional return of a particular security. The second coefficient for measuring systematic risk is 'β', which represents the slope of the characteristic line. The coefficient 'β' describes the functional relationship between the additional return of individual security and the expected additional return of the market portfolio, that is it represents the systematic risk that results from the change in the prices of securities in the portfolio. Thus, the higher the slope of the characteristic direction of a particular security, which is defined as the coefficient 'β', the higher the systematic risk of that security (Van Horne, 2001).

The capital asset pricing model assumes that any risk that is not systematic will be eliminated by diversification, that is in the case of an efficient capital market, the relevant risk component of individual security becomes its systematic risk. Therefore, the expected return ($\hat{R}_j$) of an individual security is related to the degree of its systematic risk by the following equation (Sharpe, Alexander, and Bailey, 1995):

$$\hat{R}_j - R_f = (\hat{R}_m - R_f)\beta_j \tag{1.1}$$

where is R_f riskfree rate, $\hat{R}_m$ expected rate of return on the market portfolio, β_j direction coefficient for the j-th security. Thus, the expected return on the security is equal to the return on the risk-free investment increased by the risk premium.

Korteweg (2004, p. 2) said: "*Modigliani-Miller introduced the assumption that the expected return on capital increases with the amount of debt in the company's capital structure*" (proposition II *). "*This is one of the main principles of modern corporate finance*".

Modigliani and Miller (1958, p. 268-269) derived two basic propositions concerning the valuation of securities in companies with different capital structures:

Proposition I: The market value of the firm 'j' is:

$$V_j = S_j + D_j = X_j / p_k$$

where S_j is the market value of its common shares, and D_j is the market value of the debts of the company, X_j is the expected return on the assets owned by the company. "*That is, the market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate p_k*" (Modigliani and Miller, 1958, p. 268).

Proposition II*: "*Concerning the rate of return on common stock in companies whose capital structure includes some debt: the expected rate of return or yield, 'i', on the stock of any company 'j' is a linear function of leverage as follows*" (Modigliani and Miller, 1958, p. 271):

$$i_j = p_k + (p_k - r) D_j / S_j$$

Hamada (1972, p. 435) claimed: "*Both in the pricing mode and the Modigliani-Miller theory, borrowing, while maintaining a fixed amount of equity, increases the risk to investor. Therefore, the covariance of the asset's rate of return with the market portfolio's rate of return should be greater for the stock of a firm with a higher debt-equity ratio than for the stock of another firm with a lower debt-equity ratio*".

Harris and Pringle (1985, p. 237) claimed: "A number of different approaches have been developed to deal with investments with risk and financing characteristics different from those of a firm's 'average risk' project. For example, two methods of dealing simultaneously with risk and capital structure are adjusted present value (APV) suggested by Myers (1974) and adjusted discount rates along the lines of Modigliani and Miller (MM) or Miles and Ezzell (ME)".

Miles and Ezzell (1980, p. 719) claimed: "*In perfect capital markets, all the effects of the financing decision pertain to the tax shield created by debt financing. Thus, as originally shown by Modigliani and Miller, the value of a project's levered cash flow stream equals the market value the stream would have if it were unlevered plus the market value of the stream of tax savings on interest payments associated with the debt employed to finance the project*".

Taggart (1989, p. 1) said: "*The interaction between financing and investment is a classic problem in the valuation of firms and assets. If financing affects value, then an accurate estimate of value must take the financing mix into account. Recognition of this problem has in turn spawned a variety of methods for estimating asset value and the cost of capital, most of them focusing on the tax effects of financing*".

METHODOLOGY

The issue of capital structure is of key importance. Namely, the question can be asked, whether the company can influence the value of capital, by changing the combinations of financing. To answer this question, we perform an analysis of what happens to the value of a company if the debt-to-equity ratio changes.

Derivation of the equation for the beta coefficient with leverage

Assumptions of the net operating income approach (Van Horne, 2001, p. 253-254):

The ratio of debt to equity in total capital changes with the issue of bonds used to repurchase common shares, that is with the issue of common shares to repay the debt. This means that changes in the capital structure are happening instantly. We also assume that there are no transaction costs and that the entire net profit is paid in the form of dividends to the owners of common shares and that no growth is expected for the expected operating income. We assume yet that there are no costs of bankruptcy and corporate income taxation. The following three rates will be used:

k_i = F/B (annual interest costs/market value of debt outstanding)

where k_i is the yield on the company's debt, assuming that debt is perpetual.

k_e = E/S (earnings available to common stockholders/market value of stock outstanding)

where k_e represents the required rate of return for the investors in a company whose dividends are paid in full.

k_o = O/V (net operating earnings/the total market value of the company)

where V = B+S and where ko represents an overall capitalization rate of the company, i.e. the weighted average cost of capital.

EBIT = NOI and the equations apply:

EBIT -F = EBT EBT(1-T) = E_t

where E_t is the net profit available to stockholders and EBIT is earnings before interest and corporate income tax is paid. Suppose that the corporate income tax rate is T = 0, it follows that EBT = $E_{tf,}$ where E_{tf} is the earnings available to stockholders without taxation, and EBT is earnings before taxes, i.e. it follows:

$EBIT = F+E_{tf}$

where EBIT = O is net operating income, and from that follows:

$k_o = (F+E_{tf})/(B+S)$ respectively

$$k_o= k_i[B/(B+S)]+k_e[S/(B+S)] \tag{1.2}$$

that is, by rearrangement, the equation is:

$$k_e= k_o+(B/S)(k_o -k_i) \tag{1.3}$$

It can be seen that the derived equation (1.3) is identical to the equation derived by Modigliani and Miller (1958, p. 271, proposition II *):

$r_e(\text{levered}) = r_e(\text{unlevered})+(D/E)\ (r_e\ (\text{unlevered}) -r_d)$

where r_e is the required rate of return on equity, r_d is the required rate of return on borrowing, D/E is the ratio of debt to equity.

Let us now examine what will happen to the rates of k_i, k_e, and k_o if the level of use of financial leverage is increased, which is measured by the increase in the B/S ratio. Using the net operating income approach with the example overtaken, for reasons of simplification of the presentation, we obtain the following results.

Example 1.

Let the company be debt-free and the expected value of NOI per year is $1000, and the overall capitalization rate ko is 15%. We express the value of the company as follows:

NOI	1000
k_o	0,15
$V=B+S=NOI/k_o$	6667
B	0
S	6667

Source: *Author according to Van Horne, J.C. (2001). Financial management anf policy (12th ed.), p. 255. Prentice Hall.*

E_{tf} = NOI -interest(F) =1000 -0 = 1000 $k_e = E_{tf}/S$ =1000/6667= 0,15 = 15%

The level of use of financial leverage, in this case, is B/S = 0

Example 2.

Let the company be now with \$1000 of debt with an interest rate of 10%, and the expected value of NOI per year is \$1000, and the overall capitalization rate k_o is 15%. We express the value of the company as follows:

NOI	1000
k_o	0,15
V=B+S	6667
B	1000
S	5667

Source: *Van Horne, J.C. (2001). Financial management anf policy (12th ed.), p. 255. Prentice Hall.*

E_{tf} = NOI -interest(F) =1000 -100 = 900 $k_e = E_{tf}/S$ = 900/5667 = 0,1588 =15,88%

The level of use of financial leverage, if the level of debt increases to \$1000 is 1000/5667 = 0,1765.

Example 3.

Let the company be now with \$3000 of debt with an interest rate of 10%, and the expected value of NOI per year is \$1000, and the overall capitalization rate k_o is 15%. We express the value of the company as follows:

NOI	1000
k_o	0,15
V=B+S	6667
B	3000
S	3667

Source: *Van Horne, J.C. (2001). Financial management anf policy (12th ed.), p. 255. Prentice Hall.*

E_{tf} = NOI -interest(F) =1000 -300 =700

$k_e = E_{tf}/S$= 700/3667 = 0,1909 =19,09%

The level of use of financial leverage, if the level of debt increases to \$3000 is 3000/3667=0,8181.

The critical assumptions of all the previous examples are that the k_i and k_o rates are constant. We will now examine the case where the corporate income tax rate is higher than zero (T>0). Since the equations apply:

$k_o = O/V$
$k_i = F/B$
$k_e = E_t/S$
$O = E_t + TE_{tf} + F$

it follows:

$k_o = (E_t+TE_{tf}+F)/(B+S)$ = (earnings available to stockholders after tax+the amount of corporate income tax+interest on debt)/(the total market value of the company)

whence it follows that it is:

$$k_o(B+S) = k_eS+TE_{tf}+k_iB \tag{1.4.1}$$

since equality applies E_{tf} = EBT, that is:

$$E_{tf} = NOI\text{ -interest} = O - k_iB = k_o(B+S) - k_iB \tag{1.4.2}$$

when we include equation (1.4.2) in equation (1.4.1) it follows that is:

$k_o(B+S) = k_eS+T[k_o(B+S)-k_iB]+k_iB$

respectively

$k_o = k_e[S/(B+S)][1/(1-T)]+k_i[B/(B+S)]$

from which after a series of transformations follows the equation:

$$k_e = (1 -T)[k_o+(B/S)(k_o -k_i)] \tag{1.5}$$

where equation (1.5) represents the correction of the required rate of return from equation (1.3) for the influence of corporate income tax. There is a clear difference between the derived equation (1.5) and the equation derived by Modigliani and Miller (1963, p. 439):

$r_e = r_o+(D/E)(r_o -r_d)(1 -T_c)$

where r_e is the required rate of return on equity with leverage, r_o is the cost of equity without leverage (D/E = 0), r_d is the required rate of return on borrowing, and T_c is the rate of corporate income tax.

Now we can use the equations (Ruback, 2002, p. 89) and (Fernandez, 2003, p. 4):

$$k_e = R_f+\beta_eP_m \tag{1.6}$$

$$k_o = R_f+\beta_oP_m \tag{1.7}$$

$$k_i = R_f+\beta_iP_m \tag{1.8}$$

where R_f is riskfree rate, $P_m=E(R_m)-R_f$, is market risk premium. We can calculate the beta coefficients as:

$\beta_e = (k_e -R_f)/P_m$ $\beta_o = (k_o -R_f)/P_m$ $\beta_i = (k_i -R_f)/P_m$

where β_e is the beta coefficient of leveraged equity, β_o is the beta coefficient of non-leveraged capital, β_i is the beta coefficient of debt.

Combining equation (1.5) with equations (1.6), (1.7), and (1.8) it follows:

$R_f +P_m\beta_e = (1-T)[R_f +P_m\beta_o+(B/S)(R_f +P_m\beta_o -R_f -P_m\beta_i)]$

whence after a series of transformations the equation follows:

$$\beta_e= (1-T)[\beta_o+(B/S) (\beta_o -\beta_i)] -(T/P_m)R_f \tag{1.9}$$

which represents the beta coefficient with leverage, adjusted for the influence of corporate income tax and the level of indebtedness. From equation (1.9), for T = 0 follows equation:

$$\beta_e= \beta_o+(B/S)(\beta_o -\beta_i) \tag{1.10}$$

Equation (1.10) also follows from the combination of equation (1.3) and equations (1.6), (1.7), and (1.8):

$R_f +P_m \beta_e = R_f +P_m \beta_o+(B/S)(R_f +P_m \beta_o -R_f -P_m \beta_i)$

from which follows the equation:

$\beta_e= \beta_o+(B/S)(\beta_o -\beta_i)$

which represents a special case (for T=0) of the more general equation (1.9).

We can now test the obtained equations (1.5) and (1.9). Let the corporate income tax rate be T = 40%. The general assumption is R_f=3%, P_m=5%, k_o=15%, and k_i=10% (see, Table 1).

$\beta_o= (k_o -R_f)/P_m= (0.15 -0.03)/0.05 = 2.4$

$\beta_i= (k_i -R_f)/P_m= (0.1 -0.03)/0.05 = 1.4$

Table 1. Testing of equations (1.5) and (1.9)

1.	T=0	B/S=0/6667=0	→	β_e=2.4	k_e=0.15
2.	T=0	B/S=1000/5667=0.1765	→	β_e=2.576	k_e=0.1588
3.	T=0	B/S=3000/3667=0.8181	→	β_e=3.218	k_e=0.1909
4.	T=0.4	B/S=0/6667=0	→	β_e=1.2	k_e=0.09
5.	T=0.4	B/S=1000/5667=0.1765	→	β_e=1.306	k_e=0.0953
6.	T=0.4	B/S=3000/3667=0.8181	→	β_e=1.69	k_e=0.1145

Source: *Author*

Discussion: The net operating income approach implies that the overall capitalization rate, as well as the debt charges rate, remain constant regardless of the degree of use of financial leverage. In that case, the return rate on equity k_e and the coefficient β_e will grow linearly depending on the use of the degree of financial leverage, which is shown in cases 1, 2, and 3. However, the return rate on equity k_e and the coefficient β_e will decrease when the corporate income tax is included in the consideration, in proportion to the applied tax rate, which is shown through cases 4, 5, and 6. If we consider cases 1 and 4 for which B/S = 0, we can conclude that the income tax, which systematically affects all companies and capital structures, significantly reduces the systematic risk of non-leveraged companies. Suppose also an extreme theoretical case in which T=1 holds. From equation (1.5) it follows k_e=0, while from equation (1.6) it follows:

$\beta_e = -R_f /P_m$

Let us check whether the identical result follows from equation (1.9). Indeed, when we include the value of T=1 in equation (1.9), we obtain the value of the coefficient beta:

$\beta_e = -R_f/P_m$

which algebraically confirms the correctness of the derived equation. Thus, the coefficient β_e, for the extreme value T=1, is negative (if $R_f > 0$ and $P_m > 0$).

ALTERNATIVE EQUATIONS FOR THE BETA COEFFICIENT WITH LEVERAGE: A COMPARATIVE ANALYSIS

Theoretical results

For companies that maintain a fixed book-value leverage ratio, the equation is (Fernandez, 2003, p. 4):

$$\beta_l = \beta u+(\beta_u - \beta_d)D/E(1 - T) \tag{1.11}$$

According to Fernandez (2003), for companies that maintain a fixed market-value leverage ratio, the equation is (Miles and Ezzell, 1985, p. 1490-1491):

$$\beta_l = \beta_u+(D/E)(\beta_u - \beta_d)[1 - T\, k_d/(1+ k_d)] \tag{1.12}$$

According to Fernandez (2003), for companies with a preset level of debt in each period, the equation is (Modigliani and Miller, 1963):

$$\beta_l = \beta_u+[\beta_u - \beta_d](D - VTS)/E \tag{1.13}$$

where $VTS = D \bullet T \bullet k_d$ is value of the "tax shield" (Arzac and Glosten, 2005, p. 453); (Cooper and Nyborg, 2008, p. 368); (Fernandez, 2006, p. 4); (Fernandez, 2002, p. 5) or value of the "tax savings" (Besley and Brigham, 2015, p. 189).

With identical assumptions as for Table 1, we will test the previous equations (1.11), (1.12), and (1.13).

Table 2. Testing of equation (1.11)

1.	T=0	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
2.	T=0	D/E=1000/5667=0.1765	→	β_l=2.576	k_e=0.1588
3.	T=0	D/E=3000/3667=0.8181	→	β_l=3.218	k_e=0.1909
4.	T=0.4	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
5.	T=0.4	D/E=1000/5667=0.1765	→	β_l=2.506	k_e=0.1553
6.	T=0.4	D/E=3000/3667=0.8181	→	β_l=2.89	k_e=0.1745

***Source:** Author*

Discussion: If we consider cases 1 and 4, in contrast to equation (1.9), in equation (1.11) the corporate income tax, which acts systematically on all companies, does not lower the beta coefficient without leverage (D/E = 0), and thus gives the wrong result, whose interpretation leads to the conclusion of underestimation of equity. In the extreme case, T=1 follows:

$\beta_l = \beta_u$

which is a result that is in contradiction with equations (1.5) and (1.6).

Table 3. Testing of equation (1.12)

1.	T=0	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
2.	T=0	D/E=1000/5667=0.1765	→	β_l=2.576	k_e=0.1588
3.	T=0	D/E=3000/3667=0.8181	→	β_l=3.218	k_e=0.1909
4.	T=0.4	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
5.	T=0.4	D/E=1000/5667=0.1765	→	β_l=2.57	k_e=0.1585
6.	T=0.4	D/E=3000/3667=0.8181	→	β_l=3.188	k_e=0.1894

***Source**: Author*

Discussion: As with the previous equation (1.11) and equation (1.12), the corporate income tax on condition (D/E=0) does not lower the beta coefficient of the non-leveraged company. So, this equation as a previous leads to the conclusion about the underestimation of equity. In the extreme case T=1 follows:

$\beta_l = \beta_u + (D/E)(\beta_u - \beta_d)[1 - k_d/(1+ k_d)]$

which is a result that is also in contradiction with equations (1.5) and (1.6).

Table 4. Testing of equation (1.13)

1.	T=0	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
2.	T=0	D/E=1000/5667=0.1765	→	β_l=2.576	k_e=0.1588
3.	T=0	D/E=3000/3667=0.8181	→	β_l=3.218	k_e=0.1909
4.	T=0.4	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
5.	T=0.4	D/E=1000/5667=0.1765	→	β_l=2.569	k_e=0.1584
6.	T=0.4	D/E=3000/3667=0.8181	→	β_l=3.185	k_e=0.1892

***Source:** Author*

Discussion: When considering cases 1 and 4, as in the previous two equations (1.11) and (1.12) and using equation (1.13), the corporate income tax under condition (D/E = 0) does not lower the beta coefficient and thus gives the wrong result. In the extreme case T=1 follows:

$\beta_l = \beta_u + [\beta_u - \beta_d](D/E - Dk_d/E)$

which is a result that is also in contradiction with equations (1.5) and (1.6). Given that the results obtained for the coefficient β_l, for all three alternative equations of adjusting the beta coefficient for the effect of financial leverage and corporate income tax, in cases 1, 2, and 3 are identical to the results obtained by equation (1.9), while the results in cases 4, 5, and 6 (when T > 0 and D/E ≥ 0) are significantly higher than the results obtained using equation (1.9), implies the conclusion that equity is significantly underestimated.

Practitioners use the equation (Bence, 2011, p. 12) and Fernandez (2003):

$$\beta_l = \beta_u(1 + D/E) \tag{1.14}$$

Table 5. Testing of equation (1.14)

1.	T=0	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
2.	T=0	D/E=1000/5667=0.1765	→	β_l=2.824	k_e=0.1712
3.	T=0	D/E=3000/3667=0.8181	→	β_l=4.363	k_e=0.2482

Source: Author

Discussion: The results obtained by equation (1.14) are significantly higher than the results obtained by equations (1.9), (1.11), (1.12) and (1.13) except in case 1 when (T = 0 and D/E = 0).

According to Fernandez (2003), the equation (1.15) was derived by (Harris and Pringle, 1985, p. 238):

$$\beta_l = \beta_u + (D/E)(\beta_u - \beta_d) \tag{1.15}$$

Table 6. Testing of equation (1.15)

1.	T=0	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
2.	T=0	D/E=1000/5667=0.1765	→	β_l=2.576	k_e=0.1588
3.	T=0	D/E=3000/3667=0.8181	→	β_l=3.218	k_e=0.1909

Source: Author

Discussion: Since equation (1.15) is a special case of the more general equation (1.9), the results are identical in cases (T = 0), as the results obtained by equations (1.9), (1.11), (1.12), and (1.13).

According to Fernandez (2003), the equation, which is valid for perpetual growth, was derived by (Myers, 1974, p.19-20):

$$\beta_l = \beta_u + (D/E)(\beta_u - \beta_d)[1 - T\, k_d/(k_d - g)] \tag{1.16}$$

whence with the assumption g = 5%, k_d = 10%, thus g < k_d, it follows:

Table 7. Testing of equation (1.16)

1.	T=0	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
2.	T=0	D/E=1000/5667=0.1765	→	β_l=2.576	k_e=0.1588
3.	T=0	D/E=3000/3667=0.8181	→	β_l=3.218	k_e=0.1909
4.	T=0.4	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
5.	T=0.4	D/E=1000/5667=0.1765	→	β_l=2.435	k_e=0.1518
6.	T=0.4	D/E=3000/3667=0.8181	→	β_l=2.564	k_e=0.1582

Source: *Author*

Discussion: And in equation (1.16) in cases 1 and 4, it happens that the corporate income tax does not affect the reduction of the beta coefficient without leverage, which gives the wrong result, which implies a conclusion about the underestimation of equity. When T = 1, follows:

$\beta_l = \beta_u + (D/E)(\beta_u - \beta_d)[1 - k_d/(k_d - g)]$

which is in contrast to equations (1.5) and (1.6).

According to Fernandez (2003), Modigliani & Miller (1963) set the equation for perpetual growth:

$$\beta_l = \beta_u + (D/E)[\beta_u - \beta_d + (T\, k_d/P_m) - VTS(k_u - g)/(D\, P_m)] \tag{1.17}$$

whence, assuming g = 5%, it follows:

Table 8. Testing of equation (1.17)

1.	T=0	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
2.	T=0	D/E=1000/5667=0.1765	→	β_l=2.576	k_e=0.1588
3.	T=0	D/E=3000/3667=0.8181	→	β_l=3.218	k_e=0.1909
4.	T=0.4	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
5.	T=0.4	D/E=1000/5667=0.1765	→	β_l=2.704	k_e=0.1652
6.	T=0.4	D/E=3000/3667=0.8181	→	β_l=3.807	k_e=0.2204

Source: *Author*

Discussion: Also, equation (1.17) in cases 1 and 4 does not lower the beta coefficient, which gives an erroneous result and leads to the conclusion that equity is underestimated. When T=1, follows:

$\beta_l=\beta_u+(D/E)[\beta_u - \beta_d + (k_d/P_m) - k_d(k_u - g)/P_m]$

which is in contrast to equations (1.5) and (1.6).

According to Taggart (1991, p. 11), equation (1.18) was derived by (Hamada, 1972, p. 437):

$$\beta_l = \beta_u[1+(1-T)D/E] \tag{1.18}$$

Table 9. Testing of equation (1.18)

1.	T=0	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
2.	T=0	D/E=1000/5667=0.1765	→	β_l=2.824	k_e=0.1712
3.	T=0	D/E=3000/3667=0.8181	→	β_l=4.363	k_e=0.2482
4.	T=0.4	D/E=0/6667=0	→	β_l=2.4	k_e=0.15
5.	T=0.4	D/E=1000/5667=0.1765	→	β_l=2.654	k_e=0.1627
6.	T=0.4	D/E=3000/3667=0.8181	→	β_l=3.578	k_e=0.2089

Source: *Author*

Discussion: Equation (1.18) also in cases 1 and 4, does not lower the beta coefficient. In the extreme case T=1, follows $\beta_l = \beta_u$, which, as in equation (1.11), is in contrast to equations (1.5) and (1.6). In all cases of the previous six equations, the coefficient β_l is significantly higher (for values T > 0 and D/E ≥ 0) than when equation (1.9) is applied, which implies the conclusion that equity is significantly underestimated in these cases as well.

Extension in the case of perpetual growth of dividends at a constant rate

We start from the equation of the model of perpetual dividend growth at a constant rate 'g' (Van Horne, 2001); (Besley and Brigham, 2015, p. 115):

$$k_e - g = D_t + 1/P_t \tag{1.19}$$

where $k_e > g$. If the previous equation is multiplied and divided by 'N', the number of shares outstanding, it becomes equivalent to the expression:

$k_e - g = E_t/S$ (earnings available to common stockholders/market value of stock outstanding) it follows:

$k_o(B+S) = (k_e - g)S + T(k_o(B+S) - k_iB) + k_iB$

whence after a series of transformations follows the expression:

$$k_e = g + (1-T)[k_o + (B/S)(k_o - k_i)] \tag{1.20}$$

If we now use equations (1.6), (1.7), and (1.8), the previous expression is transformed into:

$R_f + P_m\beta_e = g + (1-T)(R_f + P_m\beta_o + (B/S)(R_f + P_m\beta_o - R_f - P_m\beta_i)$

whence after a series of transformations the expression is obtained:

$$\beta_e = (1-T)[\beta_o + (B/S)(\beta_o - \beta_i)] + [(g - TR_f)/P_m] \tag{1.21}$$

Equation (1.21) represents the beta coefficient of leveraged companies whose dividends have perpetual growth at a constant rate 'g'. The necessary condition is that the rate 'g' be lower than the required rate of return on equity k_e.

Empirical results. Testing of equations for beta coefficient with leverage on real financial data of companies, global car manufacturers

Source of data: *http:/pages.stern.nyu.edu/~adamodaran/*

Daimler AG (2013)

$\beta_{levered}$=1.7646*, P_m= 5%; $R_{riskfree}$=3.04%; T=21%; k_e=11.86%(after tax); k_d=4.54% (before tax); k_d=3.59% (after tax); k_u=9.44%(before tax); k_u=7.46%(after tax); D/E=113.67%;

Daimler AG (2012)

$\beta_{levered}$=1.7754*, P_m= 5.80%; $R_{riskfree}$=1.76%; T=29.5%; k_e=12.06%(after tax); k_d=2.77%(before tax); k_d=1.95%(after tax); k_u=8.11%(before tax); k_u=5.72%(after tax); D/E=168.00%;

Bayerische Motoren Werke Aktiengesellschaft (2013)

$\beta_{levered}$=1,5723*; P_m=5%; $R_{riskfree}$=3,04%; T=21%; k_e=10,90%(after tax); k_d=4,04%(before tax); k_d=3,19%(after tax); k_u=8,342%(before tax); k_u=6,59%(after tax); D/E=126,5%;

Bayerische Motoren Werke Aktiengesellschaft (2012)

$\beta_{levered}$=1,5134*; P_m=5.80%; $R_{riskfree}$=1,76%; T=29.5%; k_e=10,54%; k_d=2,76%(before tax); k_d=1,95%(after tax); k_u=7,70%(before tax); k_u=5,43%(after tax); D/E=146,69%;

Audi AG (2013)

$\beta_{levered}$=0,8881*; P_m=5%; $R_{riskfree}$=3,04%; T=21%; $_{ke}$=7,48%(after tax); k_d=4,04%(before tax); k_d=3,19%(after tax); k_u=9,17%(before tax); k_u=7,24%(after tax); D/E=6,0%;

Audi AG (2012)

$\beta_{levered}$=0,7749*; P_m=5.80%; $R_{riskfree}$=1,76%; T=29.5%; k_e=6,25%; k_d=2,26%(before tax); k_d=1,59%(after tax); k_u=8.86%(before tax); k_u=6.25%(after tax); D/E=0,00%;

Fiat S.p.a (2013)

$\beta_{levered}$=3,3095*; P_m=7,85%; $R_{riskfree}$=3,04%; T=21%; k_e=29,02%(after tax); k_d=6,94%(before tax); k_d=5,48%(after tax); k_u=12,873%(before tax); k_u=10,17%(after tax); D/E=401,78%;

Fiat S.p.a (2012)

$\beta_{levered}$=3,7115*; P_m=8,43%; $R_{riskfree}$=1,76%; T=31.4%; k_e=33,05%(after tax); k_d=5,51%(before tax); k_d=3,78%(after tax); k_u=11,92%(before tax); k_u=8,18%(after tax); D/E=565,47%;

Peugeot S.A (2013)

$\beta_{levered}$ =9,4331*; P_m=5,60%; $R_{riskfree}$=3,04%; T=21%; k_e=55,87%(after tax); k_d=7,44%(before tax); k_d=5,88%(after tax); k_u=13,14%(before tax); k_u=10,38%(after tax); D/E=1009,61%;

Peugeot S.A (2012)

$\beta_{levered}$ =14,2156*; P_m=6,18%; $R_{riskfree}$=1,76%; T=33.3%; k_e=89,61%(after tax); k_d=3,51%(before tax); k_d=2,34%(after tax); k_u=10,64%(before tax); k_u=7,10%(after tax); D/E=1734,48%;

Renault Societe Anonym (2013)

$\beta_{levered}$=1,9334*; P_m=5,60%; $R_{riskfree}$=3,04%; T=21%; k_e=13,87%(after tax); k_d=5,44%(before tax); k_d=4,30%(after tax); k_u=9,50%(before tax); k_u=7,51%(after tax); D/E=197,73%;

Renault Societe Anonym (2012)

$\beta_{levered}$=2,2719*; P_m=6,18%; $R_{riskfree}$=1,76%; T=33.3%; k_e=15,80%(after tax); k_d=3,51%(before tax); k_d=2,34%(after tax); k_u=8,83%(before tax); k_u=5,89%(after tax); D/E=279,72%;

Table 10. Data testing for Daimler AG, BMW AG, Audi AG

	Daimler AG		**BMW AG**		**Audi AG**	
	2013	**2012**	**2013**	**2012**	**2013**	**2012**
Author	β_e=1,7640	β_e=1,7733	β_e=1,5718	β_e=1,5131	β_e=0,8895	β_e=0,7735
Fernandez	β_l=2,1601	β_l=2,1858	β_l=1,9224	β_l=1,9047	β_l=1,2746	β_l=1,2241
Miles & Ezzell	β_l=2,3815	β_l=2,6300	β_l=2,1424	β_l=2,2633	β_l=1,2871	β_l=1,2241
Modigliani & Miller	β_l=2,3811	β_l=2,6296	β_l=2,1421	β_l=2,2630	β_l=1,2870	β_l=1,2241
Hamada	β_l=2,4294	β_l=2,3919	β_l=2,1223	β_l=2,0830	β_l=1,2841	β_l=1,2241
Practitioners	β_l=1,8888	β_l=1,8446	β_l=1,6102	β_l=1,5615	β_l=0,8904	β_l=0,7741
Harris&Pringle	β_l=1,7646*	β_l=1,7750*	β_l=1,5722*	β_l=1,5131*	β_l=0,8881*	β_l=0,7741*

Source: *Author; * relevant data for $\beta_{levered}$ were calculated by equation (1.10), formulated by (Harris & Pringle, 1985)*

Table 11. Data testing for Fiat S.p.a, Peugeot S.A, Renault S.A

	Fiat S.p.a		**Peugeot S.A**		**Renault S.A**	
	2013	**2012**	**2013**	**2012**	**2013**	**2012**
Author	β_e=3,3081	β_e=3,7109	β_e=9,4295	β_e=14,2142	β_e=1,9298	β_e=2,2742
Fernandez	β_l=3,6526	β_l=4,1549	β_l=9,9223	β_l=14,7876	β_l=2,2861	β_l=2,7500
Miles & Ezzell	β_l=4,2491	β_l=5,4345	β_l=11,931	β_l=21,2269	β_l=2,5716	β_l=3,5246
Modigliani & Miller	β_l=4,2462	β_l=5,4306	β_l=11,9199	β_l=21,2189	β_l=2,5708	β_l=3,5237
Hamada	β_l=5,2300	β_l=5,8803	β_l=16,1889	β_l=18,0616	β_l=2,9556	β_l=3,2784
Practitioners	β_l=4,5577	β_l=5,0682	β_l=14,5437	β_l=15,8499	β_l=2,3765	β_l=2,5377
Harris&Pringle	β_l=3,3089*	β_l=3,7134*	β_l=9,4239*	β_l=14,2195*	β_l=1,9316*	β_l=2,2753*

Source: *Author; *relevant data for $\beta_{levered}$ were calculated by equation (1.10), formulated by (Harris & Pringle, 1985)*

Discussion: In all cases of testing of beta coefficient of leveraged company, equation (1.9) formulated by the author of this paper gives identical results as equation (1.15) (which is a special case of equation (1.9)) formulated by (Harris & Pringle, 1985). However, for the use of equation (1.15) in the analysis, the data for the interest rate of debt and the rate of total capitalization must be adjusted so that they have values after tax. All other equations give significantly higher results for the beta coefficient of leveraged companies, which leads to the conclusion that the equity of such companies is underestimated. Given that the level of indebtedness, measured by the D/E ratio, was significantly lower during (2013) compared to (2012), for all companies except for Audi AG (where the growth of the beta coefficient was calculated), this is also shown in the lower results for leveraged beta coefficient.

CONCLUSIONS

Thus, the general conclusion is that with increasing debt levels, that is higher debt-to-equity ratio, the company's systematic risk increases, which is shown by the analysis of the theoretical model and the analysis of real company data, which confirms hypothesis H1. However, when the corporate income tax rate is included, the situation changes, so that the beta coefficient decreases in proportion to the amount of the tax rate, which is also shown by the analysis, which confirms hypothesis H2. However, when using the equations of other authors presented in this paper, the decrease in the beta coefficient does not occur to the extent defined by equation (1.9), performed by the author of this paper, which may indicate that the returns on shares are too high, and thus the shares price is lower than it should be, which means that the use of such equations yield results that may lead to the conclusion that the company's shares are undervalued.

From the attached results of all equations in which the tax rate 'T' appears, it can be concluded that the corporate income tax reduces the systematic risk. That is, the beta coefficient of leveraged companies, in the β_l designation, is lower when the tax rate is higher than zero ($T > 0$), which is shown by the analysis of the theoretical model and the analysis of real data of companies of global car manufacturers.

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LEVERIDŽOVANA BETA: UTICAJ DUGA I POREZA NA DOBIT NA SISTEMATSKI RIZIK KOMPANIJE I RASPRAVA O PRAVILNOJ ALGEBARSKOJ JEDNAČINI

Rezime:

Porast učešća duga u strukturi kapitala je praćen porastom zahtevanog prinosa na sopstveni kapital jer su kompanije izložene većem finansijskom riziku. Beta koeficijent kompanija koje se finansiraju dugom, razlikuje se od beta koeficijenta kompanija koje se finansiraju isključivo sopstvenim kapitalom. Naime, beta koeficijent, pod uticajem finansijskog leveridža ima tendenciju da raste sa rastom nivoa zaduženosti, odnosno, sistematski rizik meren koeficijentom beta kompanija koje se finansiraju dugom, veći je od sistematskog rizika neleveridžovanih kompanija, zbog postojanja finansijskog rizika. Iz razloga što se plaćanja kamata na dug leveridžovanih kompanija izuzimaju kao trošak iz poreske osnove, porez na dobit smanjuje beta koeficijent kompanije sa dugom, u poređenju sa beta koeficijentom iste kompanije kada se porez na dobit apstrahuje. Što je veći porez na dobit to će biti niži beta koeficijent kompanija koje imaju dug u strukturi kapitala. Postoji niz algebarskih jednačina, različitih autora, koje dovode u vezu beta koeficijent leveridžovanih kompanija sa beta koeficijentom neleveridžovanih kompanija, beta koeficijentom duga i porezom na dobit kompanija. Algebarska jednačina za beta koeficijent leveridžovanih kompanija koja je izvedena u ovom radu, dobijena je korišćenjem pristupa neto operativnog prihoda.

Ključne reči:

sistematski rizik,
beta koeficijent,
finansijski leveridž,
poreski štit.

JEL klasifikacija:
G11, G32, C10.