LINEAR COMPACT LOCALIZED MODES IN FLUX-DRESSED TWO-DIMENSIONAL PLUS LATTICE

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It was studied a photonic linear two-dimensional plus lattice dressed by the artificial flux and explored the influence of the artificial gauge field on the energy band spectrum. The Aharonov-Bohm effect has caused the appearance of flat zones and for certain values of the flux, the lattice spectrum has been described by two momentum independent, fully degenerated flatbands and three dispersive bands. Three different types of fundamental non-orthogonal compactons have been obtained. The findings presented in this study could be tested in practice and they may be used for probing different artificial flatband systems, e. g. ultracold atoms in optical lattices, polariton condensates etc.

Keywords: photonic lattice, flat-band spectrum, compacton

Introduction

Flatband (FB) photonic systems have been in the spotlight of researchers since they represent an advantageous testbed for studying transport and localization properties at the linear level [1]. Although the FB systems were originally explored in condensed matter physics and afterwards realized on a variety of platforms (e.g. cold atom systems [2], quantum dot lattices [3]), photonic lattices have been established as the ideal ones, since working with them is very comfortable - they are easy to manipulate with and it is possible to directly observe the wave dynamics. FB photonic lattices represent arrays of coupled waveguides whose band structure has at least one entirely flat (i.e. dispersionless) band [4]. Due to their geometry, it is possible to design artificial gauge field effects which are equivalent to the magnetic field flux, i.e. the spin-orbit interaction in atomic systems [5].

One of the key features of FBs is the absence of dispersion, with fully degenerated energies, allowing the formation of a set of fully isolated localized structures – compactons, that are highly robust to environmental noise. These compact localized modes (CLMs) are eigenstates of FBs [6]. Transport of energy and mode propagation in these FB systems can be developed on the basis of the FB eigenmode, which is compact but not compulsorily orthogonal.

Here, we study a two-dimensional (2D) plus-like lattice [7], dressed by the artificial flux, which could be created by experimental techniques based on the coupled-spring resonators [4] and wave-guide networks [8]. This paper is a continuation of the previous study where this lattice geometry was proposed for the first time [7]. It was found that the energy spectrum of the corresponding linear lattice consisted of one fully degenerate FB placed between two inner and two external dispersive bands (DBs). The properties of linear and nonlinear compact modes were explored and it was shown that CLMs persist when nonlinearity is present in the system but they became unstable. Now, we go a step further and investigate the influence of the artificial gauge field on the linear, uniform plus-like lattice, by taking that a uniform flux threads each diamond plaquette. Since the flux-dressed lattice can host the Aharonov-Bohm (AB) effect which causes the appearance of flat zones in the corresponding energy spectrum [9], we expect the appearance of new FBs in this case. We find the energy band spectrum and corresponding CLMs.

The paper is organized in the following manner: After the Introduction, we present the mathematical model, the influence of the artificial flux on the energy spectrum of the lattice and finally the existence of CLMs is confirmed. A short summary is presented in Section V.

Model

The geometry of the uniform plus lattice under consideration is schematically presented in Fig. 1. The unit cell, marked with a dotted line, consists of five sites (*a, b, c, d, e*). Each of the four peripheral sites is linearly coupled with the central site as well as with surrounding elements of adjacent unit cells (coupling parameter *t*).

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The gauge field is introduced and it generates the artificial magnetic field inside the square plaquette (Fig. 1). The flux of this artificial field changes the coupling between sites of the diamond plaquette to $t\exp(\pm i\Phi/4)$, where t is the hopping parameter and ϕ is the artificial flux.

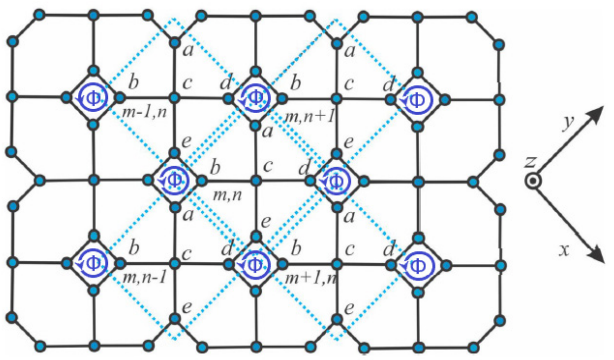

Figure 1. Schematic of 2D plus-like lattice with artificial flux. The unit cell is encircled by a dotted line.

The light propagation through the linear flux-dressed plus lattice in the tight binding approximation can be described by a set of 5*MN* coupled differential equations:

$$\partial_z a_{m,n} + t\left(b_{m,n+1}e^{i\phi/4} + d_{m-1,n}e^{-i\phi/4}\right) + tc_{m,n} = 0$$
$$\partial_z b_{m,n} + t\left(a_{m,n-1}e^{-i\phi/4} + e_{m-1,n}e^{i\phi/4}\right) + tc_{m,n} = 0$$
$$\partial_z c_{m,n} + t\left(a_{m,n} + b_{m,n} + d_{m,n} + e_{m,n}\right) = 0$$
$$\partial_z d_{m,n} + t\left(a_{m+1,n}e^{i\phi/4} + e_{m,n+1}e^{-i\phi/4}\right) + tc_{m,n} = 0$$
$$\partial_z e_{m,n} + t\left(b_{m+1,n}e^{-i\phi/4} + d_{m,n-1}e^{i\phi/4}\right) + tc_{m,n} = 0$$

where z is the normalized propagation axis, $a_{m,n}$, $b_{m,n}$, $c_{m,n}$, $d_{m,n}$, and $e_{m,n}$ are localized mode's amplitudes located at the *a, b, c, d* and e sites of the (*m, n*) unit cell (*m*=1, ...*M*; *n*=1,...*N*). Here, *M* and *N* denote the total number of unit cells in the *x* and y direction, respectively. The strength of the hopping parameter is scaled to *t*=1.

The (linear) Hamiltonian of the flux-dressed plus lattice in the reciprocal lattice space can be expressed in a matrix form:

$$\begin{pmatrix} 0 & te^{ik_y}e^{i\phi/4} & t & te^{-ik_x}e^{-i\phi/4} & 0 \\ te^{-ik_y}e^{-i\phi/4} & 0 & t & 0 & te^{-ik_x}e^{i\phi/4} \\ t & t & 0 & t & t \\ te^{ik_x}e^{i\phi/4} & 0 & t & 0 & te^{ik_y}e^{-i\phi/4} \\ 0 & te^{ik_x}e^{-i\phi/4} & t & te^{-ik_y}e^{i\phi/4} & 0 \end{pmatrix}$$

where $\vec{k} = (k_x, k_y)$ is a 2D Bloch vector. When the eigenvalue problem of linear Hamiltonian is solved, we obtain the corresponding eigenvalue bands $\beta(k_x, k_y)$, β being the corresponding eigenenergy which plays the role of propagation constant.

The energy spectrum

As was already mentioned in the Introduction, FB photonic lattices have at least one entirely FB in the photonic band structure. A FB is an entirely dispersionless energy band that spans the entire Brillouin spectrum [10]. Corresponding eigenstates are known as CLMs and they are constructed as superposition of degenerate Bloch waves. The amplitudes of eigenstates vanish except at a limited number of unit cells. In this way, the waves stay localized and they may be robust against the eventual disorder of the system [11]. FBs can be classified into three types: 'symmetry-protected' FBs, 'accidental' FBs and 'topologically protected' FBs [4]. The last type is robust under the perturbation of coupling parameters, whereas the 'accidental' FBs are formed by fine-tuning system parameters.

Another classification of FBs is in use and it is related to the size of the CLM, U, which represents the number of unit cells occupied by the CLM. Now, symmetry-protected FBs are replaced with *U*=1 FBs, with CLM occupying only one unit cell, hence creating an orthogonal set [12]. Accidental FB corresponds to U ≥ 2, where CLM extends beyond one unit cell and creates a non-orthogonal set. Topologically protected FBs can be found in systems with a bipartite symmetry (e. g. Lieb lattice with two sublattices [4]). For the sake of completeness, we should add that FB can be classified as singular and nonsingular. The FB is considered singular if the singularity is present in the FB's Bloch functions, otherwise, the FB is nonsingular. The singularity is generated by the band crossing with another DB.

As it was shown in the preceding paper [7], in the absence of the flux, the energy spectrum of the uniform lattice has one fully degenerate FB that is centred at β=0 and placed between two inner and two outer, mirror symmetric DBs (Fig. 2a). The upper DB is connected to the FB at the centre of the Brillouin zone ($k_x = k_y = 0$). The lower DB is connected to the FB as well but at the borders of the Brillouin zone ($|k_x| = |k_y| = \pi$). Now, by introducing the flux, *Φ*, and by changing its value from [0, 8π], the energy spectrum is being affected, as it is illustrated in Figs. 2b, c and d. Due to the AB effect, flat zones appear in the corresponding energy spectrum, hence for ϕ=π, the lattice spectrum is described by two momentum independent, fully degenerated FBs (corresponding to), and three DBs. The upper FB is connected with two DBs at one point ($|k_x|$=-π, $|k_y|$=-π), whereas the lower FB is connected with two DBs at the centre of the Brillouin zone ($k_x = k_y = 0$). The same structure of bands is found for ϕ=3π (2 FBs and 3 DBs) except for the different geometry of DBs, as it can be seen from Figs. 2c and 2d. For ϕ=2π, the five eigenenergies of linear Hamiltonian form one FB at β=0 and four DBs, but in this case, the whole spectrum is symmetric with respect to FB, Fig.2b. Here, the only FB is also connected with 2 DBs, with both of them at four points: ($|k_x|$ =0, $|k_y|$ =π) and ($|k_x|$ =π, $|k_y|$ =0) of the Brillouin zone.

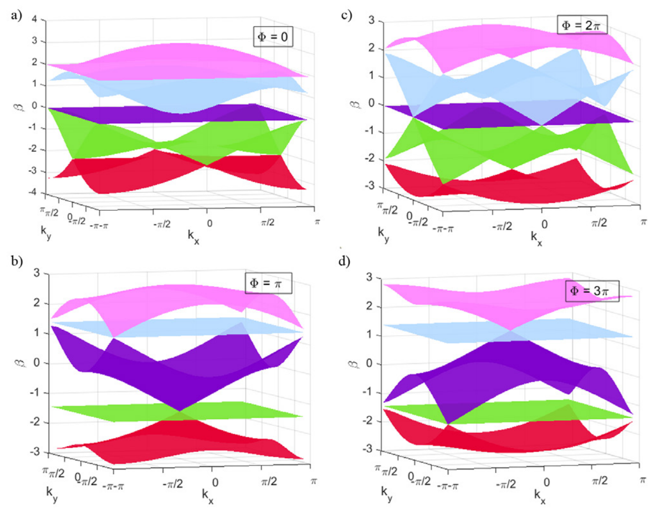

Figure 2. Energy bands of the linear plus lattice for different values of flux: a) Φ=0, b) Φ=π, c) Φ=2π and d) Φ=3π

Compact localized modes

In the case of the uniform flux-free plus lattice, the FB eigenbase is spanned by a set of corresponding compact, nonorthogonal, localized eigenstates -fundamental compactons [7]. They are a consequence of the destructive interference effect which is induced geometrically. Each compacton represents an eight-site structure that is shared by four unit cells, i.e. compactons are of the class *U*=4, as defined previously. Inside the central structure, the amplitude is zero, while two sites in each square plaquette have nonzero alternating equal amplitudes.

Now, if the artificial flux is 'turned on', we have the following situation, illustrated in Fig. 3. For Φ=2π, the fundamental compactons of the (only) zero energy FB are marked as C1. They have the same structure as the ones obtained for the flux-free lattice – they are of the class *U*=4, have the central zero amplitude, but the difference is that the two sites in each square plaquette have equal amplitudes. The fundamental compactons corresponding to two non-zero energy FBs ($\beta = \pm\sqrt{2}$), formed for Φ=π, are marked C2 and C3 (Fig.3). These compactons are class *U*=5, i.e. they occupy 5 unit cells. Except for the amplitude of the central site which is zero, all other 4 sites of the unit cell have nonzero amplitudes. Same compactons C2 and C3 correspond to the case of Φ=3π, but since the geometry of the band is 'reversed', the C2 type characterizes the FB $\beta = -\sqrt{2}$, whereas the C3 type compacton correlates with the FB $\beta = +\sqrt{2}$. As a reminder, all linear combinations of fundamental compactons are the FB modes solutions, as well.

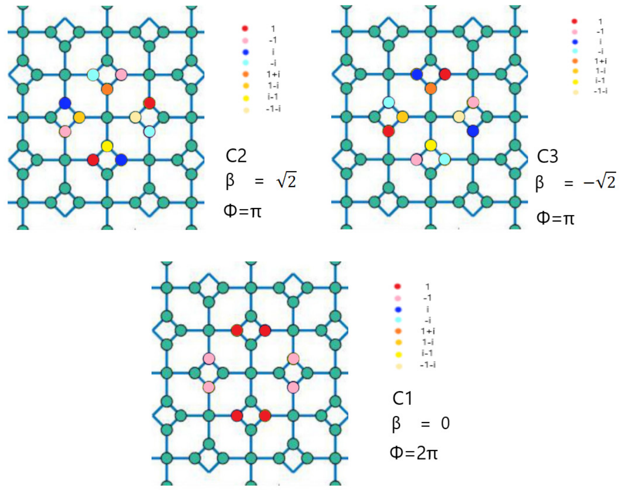

Figure 3. The linear compact mode amplitudes of FBs corresponding to different values of the artificial flux.

Conclusion

In this paper, we explored the influence of the artificial gauge field on the energy band spectrum in the linear case and corresponding compact localized modes. We have found that, as a consequence of the artificial flux, the lattice hosts the Aharonov-Bohm effect which causes the appearance of flat zones in the corresponding energy spectrum. Hence, for certain values of the flux, the lattice spectrum is described by two momentum independent, fully degenerated flatbands and three dispersive bands. Corresponding compact localized modes have been obtained. In the comparison with the flux-free case, now, 3 different types of fundamental non-orthogonal compactons were found.

In the follow-up of this work, we will investigate the dynamical properties of compact localized modes, and their stability in the presence of disorder and/or nonlinearity. Since the plus lattice can be realized experimentally, these findings could be tested in practice and they may have diverse practical applications, in settings like superconducting wire networks, topological lattices etc.

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Izvod

LINEARNE KOMPAKTNE LOKALIZOVANE MODE U DVODIMENZIONALNOJ, FLUKSOM TRETIRANOJ, PLUS REŠETKI

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U ovom radu proučavana je fotonska linearna dvodimenziona plus rešetka tretirana fluksom kalibracionog polja kao i uticaj veštačkog kalibracionog polja, na energetski spektar. Usled Aharonov-Bomovog efekta dolazi do pojave ravnih zona i za određene vrednosti fluksa energetski spektar rešetke može se opisati sa dve potpuno degenerisane ravne zone i tri disperzivne zone. Pronađena su tri različita tipa fundamentalnih ne-ortogonalnih kompaktona. Predstavljeni rezultati mogu se proveriti u praksi i imaju potencijal za primenu u različitim okruženjima, kao što su mreže superprovodnih žica, topološke rešetke itd.

Ključne reči: fotonske rešetke, ravne zone u spektru, kompaktoni.