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## Optimalan izbor laptopa primenom DEA metode

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**Apstract:** Polaznu osnovu sistema za podršku odlučivanju predstavlja teorija odlučivanja. Ona nije zamena menadžmet disciplina već njihova podrška koja ima za cilj da unapredi funkcionisanje organizacije. U savremenim uslovima poslovanja sve je manje vremena za donošenje ispravnih odluka, njihovu analizu, praćenje efekata njihove implementacije, posebno na najvišim nivoima odlučivanja. Ideja rada je da se, relativno novom, metodom operacionih istraživanja napravi optimalan izbor laptopa. Alat korišćen u istraživanju je Analiza obavijanja podataka (DEA) i softver Criterium Decision Plus 3.04S. U prvoj fazi istraživanja je korišćena DEA metoda, a u drugoj fazi je softverom CDP sprovedeno rangiranje efikasnih alternativa. U istraživanju je tehnika rešavanja poslovnog problema, izbora optimalnog laptopa, u potpunosti opravdala upotrebu sistema za podršku odlučivanju.

**Ključne reči:** analiza obavijanja podataka, efikasnost, sistemi za podršku odlučivanju

## Optimal Choice of Laptop Applying Dea Method

**Abstract:** The starting point of decision support systems is the decision theory. It is not a substitute for management disciplines but their support, which aims to improve the functioning of the organization. In modern business conditions, there is less time for proper decision-making, their analysis, monitoring of the effects of their implementation, especially at the highest levels of decision-making. The idea of this study is to make the optimal choice of the laptop, using a relatively new method of operational research. The tool used in the research is the Data Envelopment Analysis (DEA) and the software Criterium Decision Plus 3.04S. In the first phase of research DEA method was used, and in the second phase the software CDP was used for ranking effective alternatives. Within the study, a technique for solving the business problem, selecting the optimal laptop, fully justified the use of decision support systems.

**Key words:** data envelopment analysis, efficiency, decision support systems

### 1. Introduction

In the last few decades, economic activity at the micro (companies, industries) and the macro level (regional, national level) are influenced by global trends which converge to improve the performance within which special attention is paid to assessing and improving the efficiency of production systems. Thanks primarily to its abilities to improve performance, in production and service activities, national economies of individual countries have achieved economic supremacy over the last three decades. One of the most important principles in any business is the principle of efficiency which consists in achieving the highest possible economic effects (outputs) with as little economic sacrifice (inputs). Efficiency can shortly be defined as the ability to achieve desired goals with minimum use of available resources. Unlike the effectiveness, which is the ability to identify and achieve real goals, or to "do the right things", efficiency consists of realizing these goals "the right way". Efficiency is a word of Latin origin (efficax), which means success. It shows the degree of effectiveness of production factors (resources involved) in the production of material goods and services. Efficiency is, in the simplest case, in organizations that use one input (costs, assets engaged, etc.) to produce a single output (gain, profit, revenue, etc.) defined as the ratio of output to input.

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For evaluating the success of organizations in practice, there are usually multiple inputs and outputs that are diverse in nature (financial, technical, technological, ecological, social, etc.) which have to be considered, and are expressed in different measuring units. It is therefore necessary to define a summary synthetic indicator of the efficiency, that consider all significant multiple results and all the resources that are used for their implementation. It is characterized by problems of aggregation of observed inputs/outputs in a virtual input/output. To determine the effectiveness of the organization, you should first solve the problem related to the expression of input and output data in the range of values that are comparable with each other, which is a problem of scaling. Another problem relates to the determination of the relative importance of individual input/output, and assigning weights or weighting. DEA method allows solving these problems.

## 2. DEA Method

Contemporary business trends in economy and management both in production and in service and non-profit sector, create tendency for more intensive use of quantitative methods in making decisions and analysis of the achieved results (Cvijanović, Dimitrijević, Grujić, 2002). That is a way the role and importance of the methods of operation researches is huge. The most significant and the most common method of operation researches is linear programming. It is essentially the mathematical analysis of the problem of optimum (Zorić, 2006). One of the methods of linear programming is the *data envelopment analysis*, that is, *DEA* method.

Data envelopment analysis, or the DEA method is one of the new methods of operation researches and presents the method of linear programming for evaluating relative efficacy of organizational units which use multiple diverse inputs to create multiple diverse outputs. In the beginning the method was developed for evaluating the efficacy of non-profit organizations (schools, hospitals, cinemas, libraries) but in the later period the area of its usage has been greatly expanded. The development of the DEA method (Charnes, Cooper, Golany, Seiford, Stutz, 1985) was reflected in the development of family of basic DEA models and later in their numerous expansions, but also in the development of the specialized DEA software. Given that the procedure on how to apply DEA method and how to use more types of DEA model in practice by solving some problem has been developed, many authors for this method use the term „DEA methodology“.

*Data Envelopment Analysis – DEA* is developed by Charnes et al., 1985. In order to measure the efficiency of organizational units, primarily non-profit ones, they suggested that the optimization task of linear fractional programming should be solved for each *Decision Making Unit – DMU*. In the mentioned paper, they formulated appropriate tasks of linear programming which are known in literature as *CCR primary* and *CCR dual models*. If we endeavor to minimize the inputs in production of the given output it is an *input-oriented DEA model*, but if we endeavor to maximize outputs in existing level of input it is *output-oriented DEA model*. *Constant returns to scale* are defined so that proportional increase of input results with proportional increase of output of some of the DMU. Banker, Morey, 1986., gave the expansion of original CCR DEA models, by allowing the units to operate with *variable returns to scale*, which lead to creation of *BCC model*. In the meantime, DEA has been applied in different areas of work such as, forest holdings, agricultural goods, power plants, fast food kiosks, scientific research projects, banking subsidiaries, military institutions, social and health facilities, schools, faculties, economic areas (regions), and recently also for evaluating the efficiency of e-commerce.

The development of DEA method and its models, presented in the Table 1, was motivated, with the need to evaluate efficiency of the work of related organizational units, where the presence of large number of inputs and outputs makes the comparability more difficult. The initial idea was the principle of efficiency which was introduced by Farrell for the manufacturing organizations, and according to him, the goals should be achieved with minimal efforts, that is, with minimal usage of funds. As in the evaluation of the success of organizations, especially non-profit ones, various inputs or outputs usually should be considered, which by their nature are mostly diverse and are expressed in different units of measurement, it is necessary to solve the problem of aggregation of observed inputs/outputs into one *virtual* input/output. First of all, it is necessary to express input and output data in the range of values which are mutually comparable, that is, it is necessary to solve the problem of scaling so that the efficiency can be expressed with number between 0 and 1, and each organizational unit has the freedom to determine the value of the weighting coefficient in the manner which is the most convenient for it, that is, to maximize its efficiency, with limitation that the weights must be positive values and that the quotient of virtual output and virtual input of each unit can not have the value higher than 1. Then, it is necessary to solve the problem of weighting, that is, to determine relative importance of individual inputs and outputs (to assign weighting coefficients to inputs and outputs). In such circumstances, the creators of the DEA method, defined the *efficiency* of the observed unit as:

$$h = \frac{\text{the sum of weighted output}}{\text{the sum of weighted input}} \quad (1)$$

The main characteristic of the DEA method is that the efficiency measure gives the *relative measure*. In other words, each of the units (entity) is evaluated, based on the inputs and/or outputs data, as relatively efficient or inefficient in regard to other units (entities) which are included in the analysis. Thus, DEA checks whether each of the entities is on the efficiency limit. It is important to emphasize that the *identification* of inefficient units are strong, while, the identification of efficient units is weak, because it can happen that certain units are efficient only because they included favorable weights into the measurement of its efficiency. The efficiency limit presents, in economic terms, empirically derived maximum of outputs which each unit can achieve with available inputs. On that occasion the efficiency limit behaves as an envelope for inefficient units. The envelope is the curve which in every point touches one of the curves which belong to the same family.

Table1. The review of the basic DEA models

INPUT ORIENTED MODELS	
The weight problem (PIp)	The envelopment problem (DIp)
(Max) $h = \mu^T y_0 + u^*$ With limitations: $v^T x_0 = 1,$ $u^* e^T + \mu^T y - v^T x \leq 0,$ $\mu^T \geq 0,$ $v^T \geq 0,$ where: $u^* \begin{cases} \dots = 0 \dots \dots \dots \text{in} \dots PI_0 \\ \dots \leq 0 \dots \dots \dots \text{in} \dots PI_1 \\ \dots \geq 0 \dots \dots \dots \text{in} \dots PI_2 \\ \dots \text{unlimited} \dots \dots \dots \text{in} \dots PI_3 \end{cases}$	(Min) Z With limitations $y \cdot \lambda \geq y_0,$ $Z \cdot x_0 - x \cdot \lambda \geq 0,$ Z unlimited, $\lambda \geq 0.$  For DI0 : no additional limitation For DI1 : add $e^T \cdot \lambda \leq 1$ For DI2 : add $e^T \cdot \lambda \geq 1$ For DI3 : add $e^T \cdot \lambda = 1$
OUTPUT ORIENTED MODELS	
The weight problem (POp)	The envelopment problem (DOp)
(Min) $h = v^T y_0 + v^*$ With limitations: $\mu^T y_0 = 1,$ $v^* e^T - \mu^T y + v^T x \geq 0,$ $\mu^T \geq 0,$ $v^T \geq 0,$ where: $v^* \begin{cases} \dots = 0 \dots \dots \dots \text{in} \dots PO_0 \\ \dots \leq 0 \dots \dots \dots \text{in} \dots PO_1 \\ \dots \geq 0 \dots \dots \dots \text{in} \dots PO_2 \\ \dots \text{unlimited} \dots \dots \dots \text{in} \dots PO_3 \end{cases}$	(Max) $\phi$ With limitations $x \cdot \lambda \leq x_0,$ $\phi \cdot y_0 - y \cdot \lambda \leq 0,$ $\phi$ unlimited, $\lambda \geq 0.$  For DO0 : no additional limitation For DO1 : add $e^T \cdot \lambda \leq 1$ For DO2 : add $e^T \cdot \lambda \geq 1$ For DO3 : add $e^T \cdot \lambda = 1$

Source: (Grujić, 2001)

In order for the inefficient units to be on the efficiency limit and become efficient, they have to either reduce inputs or increase outputs. If the unit is possible to envelop, then it is relatively inefficient, and if it is not, it is included in forming of the efficiency limit. Depending on which DEA method is used, the efficiency limit is in the shape of *convex cone* or *convex layer* (cape). The content and level of inefficiency, for each of the inputs and outputs, is determined for each inefficient entity, based on its distance from the efficiency limit. The level of inefficiency is defined with comparison with one reference unit or with convex combination of other reference units placed on the efficiency limit.

Based on the above, DEA can be described as an analysis which is oriented towards data because it derives evaluations of performances and other conclusions directly from observed data with minimal assumptions. DEA method (Charnes et al., 1985) involves several different approaches and models which are mutually connected and which are used together for evaluation of the efficiency limit and evaluation of the relative efficiency of

observed DMU. DEAMethod provides informations which are significant for managing further work of efficient or inefficient units. For inefficient units, DEA finds ways for achieving the efficiency of those units. For the units which are already efficient, DEA provides informations on how to become more efficient. In order for DEA to be successfully used in the evaluation of performance, it is necessary for all the phases in the use of DEA to be entirely conducted. In conducting the study of efficiency by using DEA method, four phases can be singled out:

- Definition and choice of DMU, whose relative efficiency should be defined,
- Defining input and output factors which are relevant and convenient for evaluation of relative efficiency of the chosen DMU,
- Choosing the adequate DEA model and
- Solving DEA model, analysis and interpretation of the results.

### 3. Methodology and Results of Research

The intention of the research was primarily to identify the part of the product, a laptop, which should never be included when choosing the optimal solutions. It may seem strange, but when you make a division between what is good and what is not, the potentially great decision has already been made. This was the primary requirement, and the secondary was to find the solution among the acceptable/optimal ones, which is the best in given circumstances.

For this purpose was used the DEA method. Its specificity is that it quickly detects the sources/DMU inefficiencies, which means that the identification of inefficient units (laptops) is strong while the identification of efficient units is weak. Once the inefficient units/alternatives are rejected, it is needed to find the best/optimal remaining alternative. According to the DEA method, all the remaining units are efficient, thus for ranking within the DEA methodology is used the Anderson-Peterson's model, while in the study the ranking of alternatives was carried out using the software Criterium Decision Plus, version 3.04S, by Info Harvest Company.

The research referred to the selection of the optimal laptop, within the same price range. The study involved the combination of three inputs and three outputs. On the input side there were analyzed:

- processor,
- graphics card and
- RAM memory.

On the output side there were analyzed:

- monitor resolution,
- battery life and
- price.

In Table 2 the laptop models were given with the aforementioned input/output values.

Table 2. Overview of input/output in choosing the laptop

No	MODEL	CPU	RAM	GPU	RESOL	BATT	PRICE
1	Lenovo IdeaPad 510-15	Intel Core I3 6100U, 2.30GHZ	8GB	NVIDIA Ge Force	Full HD	Li 2 cell	65.999,00 RSD
2	Dell Inspiron 3157	Intel Celeron N3050, 2.13GHZ	4GB	Intel HD 5500	HD	Li 3 cell	65.999,00 RSD
3	Asus K555 DGXO103T	AMD A10 8700P, 3.20GHZ	12GB	AMD Radeon R5	HD	Li 2 cell	66.999,00 RSD
4	HP ProBox 450G2	Intel Core I5 5200U, 2.70GHZ	4GB	Intel HD 5500	HD	Li 4 cell	66.999,00 RSD
5	Lenovo IdeaPad B50-80	Intel Core I5 5200U, 2.70GHZ	8GB	AMD Radeon R5	Full HD	Li 4 cell	66.999,00 RSD
6	Asus K555 LBXO532D	Intel Core I5 5200U, 2.70GHZ	6GB	NVIDIA Ge Force	HD	Li-Poly 2 cell	68.999,00 RSD
7	Lenovo E50-80	Intel Core I5 5200U, 2.70GHZ	8GB	AMD Radeon R5	Full HD	Li 4 cell	68.999,00 RSD
8	Lenovo IdeaPad YOGA500	Intel Core I3 5005U, 2.00GB	4GB	Intel HD 5500	HD	Li 4 cell	69.299,00 RSD
9	Dell Inspiron 5559	Intel Core I5 6200U, 2.80GHZ	8GB	AMD Radeon R5	HD	Li 4 cell	69.999,00 RSD

10	HP 250G4	Intel Core I5 6200U, 2.80GHZ	8GB	AMD Radeon R5	HD	Li 3 cell	69.999,00 RSD
11	Lenovo V310-15ISK	Intel Core I5 6200U, 2.80GHZ	8GB	AMD Radeon R5	HD	Li 4 cell	69.999,00 RSD

Source: (ComTrade SHOP, 2016)

Using DEA Solver, which is an addition to Excel, performed according to the efficiency rating of given laptop computers. The analysis involved the evaluation of efficiency of observed units, by the implementation of:

- input-oriented model with a constant return to scale,
- input-oriented model with a variable return to scale,
- output-oriented model with a constant return to scale and
- output-oriented model with a variable return to scale.

In order for the comparison to make sense, DEA requires the previous implementation during two steps. The first involves scaling. This means that the value of the inputs/outputs presented in Table 2, is necessary to be expressed within the range of values that are comparable to each other, so the efficiency estimate could be presented as a number between 0 and 1. Then, in the second step, it is necessary to implement weighting, i.e. to determine the relative importance of individual inputs and outputs. For this purpose Table 3 was created.

Table 3. Overview of the input/output on which scaling and weighting was conducted

Typ	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3
1	80	93	90	90	70	90
2	70	75	65	75	80	90
3	80	100	80	75	70	80
4	90	75	65	75	90	80
5	90	93	80	90	90	80
6	90	85	90	75	90	70
7	90	93	80	90	90	70
8	80	75	65	75	90	65
9	100	93	80	75	90	60
10	100	93	80	75	80	60
11	100	93	90	75	90	60

When you start DEA Solver and process the levels which were prepared in Table 3, you get the results that are given in Figure 1. On the left side of Figure 1 the results of the assessment of laptop efficiency using are present where the input-oriented model with a constant return to scale and on the right side of Figure 1 there are the results of the assessment of laptop efficiency using the input-oriented model with a variable return to scale.

Alternatives/laptop computers with shaded cells represent effective units, and efficiency rating is 1. The results show that almost half of the alternatives, in both models of efficiency assessment is unacceptable, given the set requirements. Namely, the requirements include good performance in terms of CPU, RAM, and GPU, but also that on the output side good/acceptable computer price remains, has a great battery life, and a good screen resolution. On the right side of Figure 1 there are the results of efficiency assessment, using the model that allows a variable return to scale. In column Returns-to-scale, you can see what kind of return to scale it was, in the efficiency assessment. The return to scale was constant, increasing or decreasing.

Figure 1. Results of efficiency assessment of laptop computer using the input-oriented model

INPUT ORIENTED MODELS								
CONSTANT RETURNS TO SCALE			VARIABLE RETURNS TO SCALE					
Efficiency scores		Efficiency scores			Scale efficiencies	Returns-to-scale	CCR score	NIRS score
1	1.0000	1	1.0000		1.0000	constant	1.0000	1.0000
2	1.0000	2	1.0000		1.0000	constant	1.0000	1.0000
3	0.8594	3	0.8750		0.9821	increasing	0.8594	0.8594
4	1.0000	4	1.0000		1.0000	constant	1.0000	1.0000
5	0.9750	5	1.0000		0.9750	decreasing	0.9750	1.0000
6	0.8946	6	0.9259		0.9662	decreasing	0.8946	0.9259
7	0.9750	7	1.0000		0.9750	decreasing	0.9750	1.0000
8	1.0000	8	1.0000		1.0000	constant	1.0000	1.0000
9	0.8125	9	0.8125		1.0000	constant	0.8125	0.8125
10	0.8125	10	0.8125		1.0000	constant	0.8125	0.8125
11	0.8065	11	0.8065		1.0000	constant	0.8065	0.8065

It is necessary, for good business decisions, to conduct an analysis of the efficiency assessment, using the output-oriented model with constant and variable returns to scale. Specifically, these models seek to achieve efficiency by maximizing output for given/current input state. The results of the efficiency assessment in this manner are presented in Figure 2.

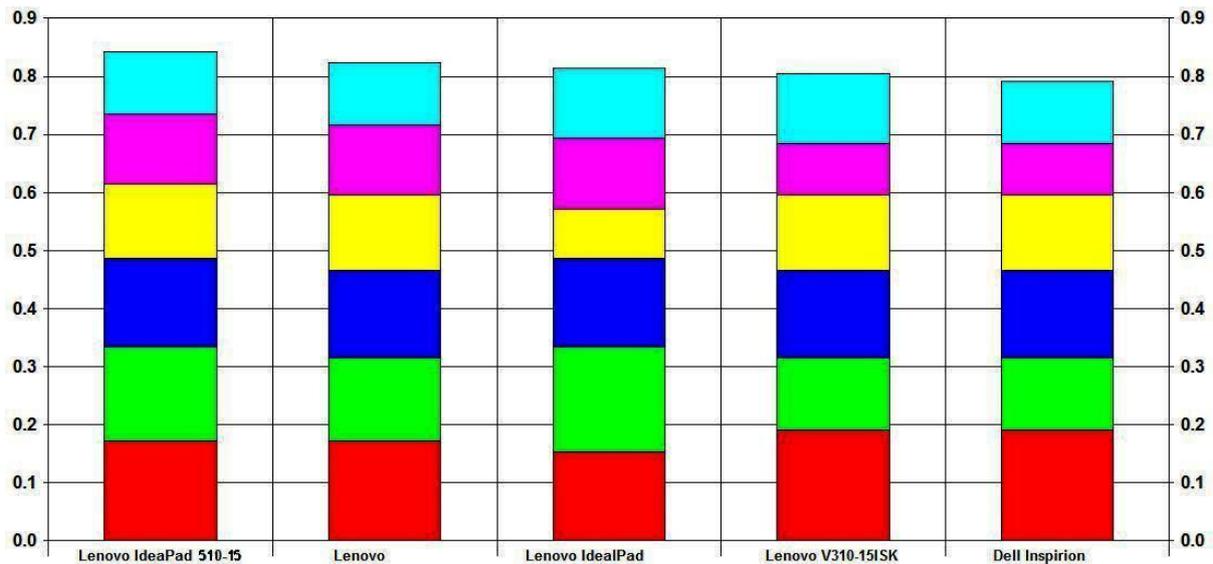
Figure 2. Results of efficiency assessment of laptop computer using the output-oriented model

OUTPUT ORIENTED MODELS								
CONSTANT RETURNS TO SCALE			VARIABLE RETURNS TO SCALE					
Efficiency scores		Efficiency scores			Scale efficiencies	Returns-to-scale	CCR score	NIRS score
1	1.0000	1	1.0000		1.0000	constant	1.0000	1.0000
2	1.0000	2	1.0000		1.0000	constant	1.0000	1.0000
3	0.8594	3	0.9032		0.9515	decreasing	0.8594	0.9032
4	1.0000	4	1.0000		1.0000	constant	1.0000	1.0000
5	0.9750	5	1.0000		0.9750	decreasing	0.9750	1.0000
6	0.8946	6	1.0000		0.8946	decreasing	0.8946	1.0000
7	0.9750	7	1.0000		0.9750	decreasing	0.9750	1.0000
8	1.0000	8	1.0000		1.0000	constant	1.0000	1.0000
9	0.8125	9	1.0000		0.8125	decreasing	0.8125	1.0000
10	0.8125	10	0.8889		0.9141	decreasing	0.8125	0.8889
11	0.8065	11	1.0000		0.8065	decreasing	0.8065	1.0000

Analysis of the results achieved by using output-oriented DEA model shows a similar situation as well as the application of the input-oriented DEA model. When it comes to output-oriented DEA model with a constant return to scale, the results are the same as in the previous case, as can be seen on the left side of Figure 2. The differences are only noted in the efficiency assessment by using the output-oriented DEA model that allows a variable return to scale, which can be seen on the right side of Figure 2. It can be noted that only two alternatives, laptop 3 and 10, are inefficient. This is explained by the fact that DMU, considering that this is the output-oriented model, is seeking to maximize the outputs and allows itself more freedom when it comes primarily to price. This analysis should be taken seriously and possibly will reject these results, especially when it comes to the procurement of a large number of laptop computers while having a limited budget.

Analysis of the efficiency assessment shows that the selection of the optimal laptop computer should be sought between models 1, 2, 4 and 8. The DEA method quickly detects the inefficient DMU. The remaining DMU were assessed as effective, but the DEA method should not be used for ranking the best alternative. For this purpose software Criterium Decision Plus was used with the version 3.04S, by Info Harvest Company.

Figure 3. Ranking of efficient DMU



Conducted ranking shows that the laptop computers made by Lenovo show better results, option 1, where the best alternative when choosing a laptop is Lenovo IdeaPad 510-15. The results can be explained by good CPU and RAM ration, as well as the affordable price. It is necessary to accentuate once again that this choice is gaining in importance only when purchasing larger quantities of computers, properly selecting it, if possible to achieve big savings.

#### 4. Conclusion

The research has shown that DEA method can be a useful tool in business decision making. This method quickly and safely identifies inefficient alternatives, which is very important at the beginning of the analysis. By combining this method with some of the models for ranking, the analysis gets full meaning and gives representative results in the selection of optimal solutions.

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